MATH0053 (Algebra 4: Groups and Rings)

Year:	2018–2019
Code:	MATH0053
Old Code:	MATH7202
Level:	6 (UG)
Normal student group(S):	UG: Year 2 and 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Structure:	3 hour lectures and 1 hour problem class per week. Assessed coursework.
Assessment:	90% examination, 10% coursework. In order to pass the module you must have at least 40% for both the examination mark and the final weighted mark.
Normal Pre-requisites:	MATH0006 (previously MATH1202)
Lecturer:	Prof FEA Johnson
Problem Class Teacher:	Mr J Nicholson

Course Description and Objectives

The course is divided into two parts in the approximate ratio 3:2.

The intention in Part 1 is to transmit a thorough familiarity with, and working knowledge of, groups of small order (certainly all groups of order ≤ 15); to achieve the classification of these groups, and to impart a clear understanding of the principles by which their classification is effected, particularly in regard to the construction of subgroups of given order, culminating in, rather than beginning with, the full generality of Sylow's Theorem.

The subject matter of Part 2 is Ring Theory. Beyond an elementary acquaintance with fields, no prior example knowledge is assumed. Here the intention, again by constant reference to explicit examples, is to achieve a good working knowledge of the main ideas and techniques of elementary (commutative) ring theory.

Recommended Texts

J Moody, *Groups for Undergraduates*; S Lang, *Algebra.*

Detailed Syllabus

Review of the basic results in the theory of finite groups, including Lagrange's Theorem and Cauchy's Theorem. Examples of groups of low order by barehands methods, including $C_2, C_3, C_3, C_2, D_6, Q_8, A_4$. Some infinite families of (finite) groups: $C_n; D_{2n}; A_n; S_n; GL_n(F), F$ a (finite) field. Review of homomorphisms, isomorphisms and automorphisms. Automorphism group of a group, with particular attention to Aut(cyclic group). Condition that a homomorphism $\varphi : C_n \to C_n$ be an automorphism, viz φ(generator) is a generator. Explicit consideration of Aut(G) for small G by barehands methods. Proof that $\mathrm{Aut}(C_n) = (Z/n)^*$.

Semi-direct product. Groups acting on sets. Stabiliser subgroups and the class equation. Sylow's Theorem. Application to classification of groups of 'small order' (e.g. groups of order pq^m where $q < p$.)

Review of fields (definition and basic examples, $\mathbb{Q}, \mathbb{R}, \mathbb{C}, \mathbb{F}_p$) and division rings (basic example $\mathbb{H}$). Definition of rings (associate, with unity) and examples $\mathbb{Z}, k[x]$. Sub-rings, ideals, quotient rings, integral domains. Principal ideals. Proof that Z, $k[x]$ are P.I.D.'s. *(Overhang from Part 1: Proof that a finite multiplicative subgroup of a field is cyclic. Unit group of $\mathbb{Z}/n$ and Euler's totient function)*

Proof that a finite integral domain is a field; more generally, an integral domain of finite dimension over a subfield in a field. Construction of extension fields as quotients $k[x]/(q(x))$ with $q(x)$ an irreducible element of $k[x]$. Explicit illustration with examples $F_p[x]/(q(x))$ with $p = 2, 3, 4$ and $q(x)$ an irreducible quadratic. Eisenstein's Criterion. Gauss's Lemma. Irreducibility over $\mathbb{Z}$ (and hence $\mathbb{Q}$) of

$$C_p(x) = x^{p+1} + ... + x + 1 \text{ for } p \text{ prime}$$

Algorithm for the factorisation of the cyclomotic polynomials $x^n - 1$ into Z-irreducible (and hence Q-irreducible) factors, $x^n - 1 = \Pi\{C_c(x) : d\,|n\}$. Informal proof that $C_d(x) = \Pi\{(x - w) : ord(w) = d)\}$. Finite subgroup of $\mathbb{F}^*$ is cyclic.

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