

MATH0052 Geometry and Groups

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| <i>Year:</i> | 2021–2022 |
| <i>Code:</i> | MATH0052 |
| <i>Level:</i> | 5 (UG) |
| <i>Normal student group(s):</i> | UG: Year 2 and 3 Mathematics degrees |
| <i>Value:</i> | 15 credits (= 7.5 ECTS credits) |
| <i>Term:</i> | 2 |
| <i>Assessment:</i> | 90% examination, 10% coursework |
| <i>Normal Pre-requisites:</i> | MATH0006, MATH0013, MATH0014 |
| <i>Lecturer:</i> | Prof M Singer |

Course Description and Objectives

Geometry attempts to describe and understand the space around us. It is a central activity and main driving force in many branches of mathematics and physics. In this course we will meet some of the basic examples in geometry, build up fundamental understanding of curvature, and enhance familiarity with groups and group actions outside of pure algebra.

Recommended Texts

1. Geometry and Topology, by Miles Reid and Balazs Szendroi, Chapters 1, 2, 3, 8 and 9.
2. Curved Spaces: From Classical Geometries to Elementary Differential Geometry, by P.M.H. Wilson, Chapters 1, 2 and 5
3. Notes on Geometry, by Elmer G. Rees, Parts I and III.

Detailed Syllabus

- Symmetry groups of Platonic solids: using the orbit-stabiliser theorem to count symmetries and identifying symmetry groups by their actions. Classification of Platonic solids.
- Isometries of Euclidean space: Galilean group and orthogonal group. Every rotation is a composition of reflections. Rotations in 3D: every rotation has an axis, quaternionic picture of rotations. Rotations in 4D: quaternion action.
- Spherical geometry: geodesics, spherical triangles, spherical trigonometry. Area controls angle surplus.
- Möbius transformations; the Riemann sphere. Stereographic projection. Conformality and preservation of straight lines and circles, 3-transitivity.
- Hyperbolic geometry: hyperboloid, disc and upper-half plane models. Geodesics, distances and hyperbolic triangles. Failure of parallel postulate. Area controls angle deficit.
- Isometries of hyperbolic space as $\mathrm{PSL}(2, \mathbb{R})$. Parabolic, elliptic, hyperbolic elements and their fixed points.