MATH0051 Analysis 4: Real Analysis

Year:	2021–2022
Code:	MATH0051
Level:	6 (UG)
Normal student group(s):	UG: Year 2 and 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0004
Lecturer:	Dr D Hewett

Course Description and Objectives

This course introduces students to the foundations of modern mathematical analysis, reinforcing the concepts of convergence and continuity studied in the first year in the context of functions of a single real variable, and extending them to the setting of general metric and topological spaces. We introduce some powerful new concepts such as compactness, uniform convergence and contraction mappings, which, as an illustrative application, we use to prove well-posedness of initial value problems for ODEs. On top of its intrinsic elegance, the material studied in this course also prepares students for further study in functional analysis, partial differential equations, variational methods, numerical analysis and spectral theory.

Recommended Texts

Two recommended books are Rudin, *Principles of Mathematical Analysis*, and Sutherland, *Introduction to Metric and Topological Spaces.*

Detailed Syllabus

- Metric and normed spaces. Convergence of sequences. Open and closed sets. Complete metric spaces, completion of a metric space.
- Compactness and sequential compactness, their equivalence in metric spaces. Compactness in $\mathbb{R}^n$, Heine-Borel Lemma. Compactness, ε-nets and totally bounded sets.
- Continuity. Continuous real-valued functions on compact spaces are bounded and attain their bounds.
- Pointwise and uniform convergence of sequences and series of real-valued functions on metric spaces. Weierstrass M-test. Uniform convergence and its relation to continuity and Riemann-integrability.
- Weierstrass Approximation Theorem.
- The space of continuous functions $C(X)$ and its completeness when X is compact. Arzelá-Ascoli theorem. Compactness in $C(X)$. Relative compactness. Applications.
- Contraction Mapping Theorem.
- Picard-Lindelöf Theorem. Reduction of a higher order ODE to a first order ODE system, dimension of the solution space. Continuous dependence on the initial conditions.

- Topological spaces. Continuity, homeomorphisms, and quotient spaces. Hausdorff spaces. Connectedness, path-connectedness.