MATH0048(Mathematical Analysis)

Year:	2019–2020
Code:	MATH0048
Old code:	MATH6404
Level:	5 (UG)
Normal student group(s):	UG: Students outside Mathematics
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Structure:	3 hour lectures per week and 1 hour problem class
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	Suitable for first year students with A in Further Maths A-level, or second year students with good results in ECON0006 (previously ECON1004)
Lecturer:	Dr J Galkowski

Course Description and Objectives

This module is an introduction to mathematical analysis, one of the most important and well-developed strands of pure mathematics with many elegant and beautiful theorems, and also with applications to many areas of mathematics, theoretical statistics, econometrics, and optimisation.

The aim is to introduce students to the ideas of formal definitions and rigorous proofs (one of the fundamental features of modern mathematics, and something that is not familiar from A-level), and to develop their powers of logical thinking.

This module is a prerequisite for Complex Analysis, MATH0013 (previously MATH2101) and provides a useful foundation for courses such as Logic, MATH0050 (previously MATH6801).

The module is intended for second or third year students in departments outside Mathematics, particularly in Economics or Statistics. Students taking this module should be mathematically able and will normally have demonstrated this by achieving a strong result in a module such as MATH0047 (previously MATH6403) or having an A* in Further Mathematics A-level.

Recommended Texts

Haggarty, Fundamentals of Mathematical Analysis (2nd edition).
Other recommended books are
(i) Binmore, Introduction to Mathematical Analysis (CUP);
(ii) M. Spivak, Calculus (Publish or Perish);
(iii) R. Bartle and D. Sherbert, Introduction to Real Analysis (Wiley);
(iv) M H Protter and C B Morrey, A first course in real analysis (Springer).

Detailed Syllabus

Sets of numbers ($\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$) and their properties (addition, multiplication, orderings). Irrationality of $\sqrt{2}$. Axioms of an ordered field, archimedean property.

Upper and lower bounds, completeness axiom (sup and inf), $\mathbb{Q}$ is not complete, $\mathbb{R}$ is complete. Definitions of max and min, comparison with sup and inf, intervals. Sequences, definition of convergence. Bernoulli's inequality, triangle inequality.

Examples of the use of the definition of convergence, statement of sum/product/quotient rules, sandwich theorem. Basic examples of null sequences.

Convergent sequences are bounded. Definition of divergence to $\pm\infty$, monotone sequences, bounded and monotone $\Rightarrow$ convergent, some recursive examples, subsequences. Bolzano-Weierstrass theorem. Subsequences of a convergent sequence. Examples of divergent sequences.

Convergence of series $\sum_n a_n$, basic properties, geometric and telescoping series, comparison test, $\sum_n |a_n|$ converges $\Rightarrow \sum_n a_n$ converges. Definition of absolute/conditional convergence, alternating series test. convergence/divergence of $\sum_n 1/n^p$. Ratio test. Power series.

Definition and convergence of $\exp(x)$, $\cos(x)$, $\sin(x)$. Definition of e, a^x for $x \notin \mathbb{Q}$. Definitions of limits and continuity for functions, examples, characterisation of limits and continuity via sequences. Algebra of limits, sandwich theorem.

Continuity of polynomials, rational functions and $\exp(x)$. Intermediate value theorem and applications.

Continuous functions on compact intervals are bounded and attain maximum/minimum. Continuity of inverse functions, logarithms. Definition of the derivative, some basic examples, proof that $(e^x)' = e^x$, characterisation of the derivative via linear approximation, differentiable $\Rightarrow$ continuous.

Sum rule and product rule for derivatives. Chain rule, derivative of x^n, derivatives of inverse functions, derivative of $\ln x$. Derivatives of $\sin x$ and $\cos x$. Local extremum theorem, Rolle's theorem, mean value theorem, $f' > 0 \Rightarrow$ strictly increasing.

Riemann integration. Upper and lower Riemann (Darboux) sums. Relation between monotonicity and convexity of a function and various types of its Riemann sums. Fundamental Theorem of Calculus.