MATH0035 (Algebraic Number Theory)

Year:	2018–2019
Code:	MATH0035
Old code:	MATH3704
Level:	6 (UG)
Normal student group(s):	UG Year 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Structure:	3 hour lectures and 1 hour problem class per week
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0034 (previously MATH7701), MATH0053 (previously MATH7202) and M
Lecturer:	Dr RM Hill

Course Description and Objectives

Algebraic number theory is one of the foundations of modern number theory. An algebraic number field is a finite algebraic extension of the field of rational numbers, and algebraic number theory studies the arithmetic of algebraic number fields: the ring of integers in the number field, the ideas and units in the ring of integers, the extent to which unique factorization holds, etc.

As well as being interesting objects in their own right, number fields can be used to prove results about the ordinary integers; a very advanced application is the proof of Fermat's last theorem.

Recommended Texts

(i) Ian Stewart and David Tall, *Algebraic Number Theory and Fermat's Last Theorem.*

(ii) Saban Akaca and Kenneth S. Williams, *Introductory Algebraic Number Theory.*

Detailed Syllabus

- Review of field extensions, algebraic numbers, minimal polynomials, conjugates, the Primitive element theorem.
- Algebraic integers, discriminants, integral bases, norms and traces. Cyclotomic and quadratic fields.
- Non-unique factorization of algebraic integers, unique factorization into prime ideals, fractional ideals, class group, norms of ideals.
- Lattices and Minkowski's Lemma, finiteness of class number, calculation of class numbers.
- Applications to some Diophantine equations.