MATH0033 Numerical Methods

Year:	2021-2022
Code:	MATH0033
Level:	6 (UG) / 7 (PG)
Value:	15 credits (= 7.5 ECTS credits)
Normal student group(s):	UG Year 3 Mathematics degrees PG MSc Scientific Computing Physics and Astronomy
Term:	1
Assessment:	80% examination, 20% homework
Pre-requisites:	MATH0004, MATH0006, MATH0011
Lecturer:	Dr I Smears

Course Description and Objectives

Many phenomena in engineering and the physical and biological sciences can be described using mathematical models. Frequently the resulting models cannot be solved analytically, in which case a common approach is to use a numerical method to find an approximate solution. The aim of this course is to introduce the basic ideas underpinning computational mathematics, study a series of numerical methods to solve different problems, and carry out a rigorous mathematical analysis of their accuracy and stability.

The overarching goal is to provide the fundamental tools for the solution of large-scale differential equations. Such problems require the solution of several subproblems and in this course we introduce numerical methods for the most important building blocks: solution methods for nonlinear equations and systems, solution methods for large linear systems, and solution methods for ordinary differential equations. For each method we typically ask two questions:

1. Under what circumstances is the numerical solution a good approximation of the true solution?

2. How much better does the approximation become if we are able to devote more computational resources to its calculation?

To answer these questions we will draw on tools from analysis including the mean value theorem, Taylor's theorem and the contraction mapping theorem.

The homework consists of theoretical exercises where different properties of the methods are explored and computational exercises that serve as a hands-on illustration of the theoretical material. The final computational exercise combines several of the elements in the course for the solution of an initial boundary value problem for a PDE. The computational exercises are designed to be solved using MATLAB and several MATLAB scripts are distributed during the course. However, in principle any programming language may be used depending on the experience and interest of the student.

Recommended Texts

- Süli, Endre; Mayers, David F. *An Introduction to Numerical Analysis.* Cambridge University Press, Cambridge, 2003. x+433 pp. ISBN: 0-521-81026-4; 0-521-00794-1

- Quarteroni, Alfio; Sacco, Riccardo; Saleri, Fausto. *Numerical Mathematics.* Second edition. Texts in Applied Mathematics, 37. Springer-Verlag, Berlin, 2007. xviii+657 pp. ISBN: 978-3-540-34658-6; 978-3-642-07101-0

Detailed Syllabus

- Numerical solution of nonlinear equations: the bisection algorithm, fixed point iteration, Newton's method and its relatives.

- Numerical solution of large linear systems: iterative methods and preconditioning, Jacobi and Gauss-Seidel methods, Richardson methods and the gradient method, the conjugate gradient method.

- Methods for ordinary differential equations: the Cauchy problem, the basic one-step methods, explicit and implicit methods, zero-stability and absolute stability, convergence, extensions to system of equations. Introduction to boundary value problems using finite differences.

- Theoretical foundations of computational mathematics: conditioning, stability, consistency, convergence, the equivalence theorem.