MATH0029 Graph Theory and Combinatorics

Year:	2024–2025
Code:	MATH0029
Level:	6 (UG)/7(PG)
Normal student group(s):	UG Year 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Assessment:	90% examination 10% coursework
Normal Pre-requisites:	MATH0057 recommended
Lecturer:	Dr J Talbot

Course Description and Objectives

The course aims to introduce students to discrete mathematics, a fundamental part of mathematics with many applications in computer science and related areas. The course provides an introduction to graph theory and combinatorics, the two cornerstones of discrete mathematics. The course will be offered to third or fourth year students taking Mathematics degrees, and might also be suitable for students from other departments. There will be an emphasis on extremal results and a variety of methods.

Recommended Texts

B Bollobás, *Modern Graph Theory* (Springer); B Bollobás, *Combinatorics* (Cambridge University Press).

Detailed Syllabus

– Binomial coefficients, convexity. Inequalities: Jensen's, AM-GM, Cauchy–Schwarz. Graphs, subgraphs, connectedness, Euler circuits, cycles, trees, bipartite graphs and other basic concepts. Vertex colourings. Graphs with large girth and large chromatic number.

– Extremal graph theory: Dirac's theorem. Ore's theorem. Mantel's theorem. Turán's theorem (several proofs including probabilistic and analytic). Kövári–Sós–Turán theorem with applications to geometry. Erdős–Stone theorem. Stability. Andrásfai–Erdős–Sós theorem.

– Set Systems: Basic definitions; set systems and the discrete cube. Chains and antichains. Sperner's lemma. The LYM inequality. Intersecting families; the Erdos-Ko-Rado theorem (probabilistic and compression proofs). Isoperimetric problems: local LYM inequality, Kruskal–Katona theorem. The linear algebra method: Fisher's inequality, Ray-Chaudhuri–Wilson theorem.

– Ramsey theory: Ramsey's theorem. Upper and lower bounds including probabilistic ideas. Schur's Theorem. Fermat's Last Theorem is false in $\mathbb{Z}_p$ for any sufficiently large prime p. Van der Waerden's theorem on arithmetic progressions.