MATH0025 Mathematics for General Relativity

Year:	2024–2025
Code:	MATH0025
Level:	6(UG)/7(PG)
Normal student group(s):	UG: Year 3 Maths degrees and Year 3 Physics degrees PG: MSc Physics
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Pre-requisite:	MATH0016 for Mathematics students MATH0043 for UG Physics students
Assessment:	90% examination, 10% coursework
Lecturer:	Prof B Hartmann

Course Description and Objectives

The course introduces Einstein's theories of special and general relativity. These theories, introduced in the early 20$^{\text{th}}$ century, along with quantum theory, provide the modern framework for the description of the fundamental physical theories of gravity and electromagnetism.

Special relativity deals with physics in the absence of gravity. It requires a rethink of many familiar concepts (such as what it means for events to be simultaneous) because of the constancy (and finiteness) of the speed light. We will be looking at the basic physical concepts of mass, momentum, energy and electromagnetism within this framework and their mathematical description. No prior familiarity of Maxwell's equations will be assumed.

General relativity is a profound generalisation of special relativity which incorporates gravity. The mathematical description of general relativity requires the mathematical language of differential geometry which uses the notions of metric, connection and curvature, which will be introduced from scratch.

The earliest tests of general relativity where the observation that light is bent by massive objects such as the sun, the precession of the perihelion of the planet Mercury, gravitational redshifts and radar echo delays. Some of these will be discussed at the end of the course.

Recommended Texts

(i) C. G. Böhmer, *"Introduction to General Relativity and Cosmology"*, World Scientific.

(ii) R. M. Wald, *"General Relativity"*, University of Chicago Press.

(iii) B. F. Schutz, *"A first course in general relativity"*, Cambridge University Press.

(iv) M. P. Hobson, G. Efstathiou and A. N. Lasenby, *"General Relativity, An Introduction for Physicists"*, Cambridge University Press.

(v) J. Foster and D. J. Nightingale, *"A Short Course in General Relativity Paperback"*, Springer.

(vi) L. P. Hughston and K. P. Tod, *"An Introduction to General Relativity"*, LMS Student Texts 5, Cambridge University Press.

(vii) N. M. J. Woodhouse, *"General Relativity"*, Springer.

Detailed Syllabus

- Vectors and gradients.
- Tangent vectors and tensors.
- Manifolds and metrics.
- Geodesics.
- Covariant differentiation.
- Curvature.
- Riemann, Ricci and Einstein tensors.
- Some physics background.
- Covariant Maxwell equations.
- Geometry and gravity.
- Einstein field equations.
- Weak field approximation.
- Gravitational coupling constant.
- Gravitational waves.
- Schwarzschild solution.
- Classical tests of GR.
- The Schwarzschild radius.
- Horizons and Black holes.