MATH0023 Algebraic Toplogy

Year:	2024–2025
Code:	MATH0023
Level:	6 (UG)
Normal student group(s):	UG: Year 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Assessment:	100% examination
Normal Pre-requisites:	MATH0014
Lecturer:	Prof FEA Johnson

Course Description and Objectives

The purpose of this course is to provide an elementary introduction to the methods of Algebraic and Geometric Topology via the homology of simplicial complexes. A good grounding in Linear Algebra is assumed, but with that exception the course is self contained.

Detailed Syllabus

Chain complexes and homological simplicial complexes and simplicial homology. Mayer-Vietoris Theorem. Computation of some low dimensional examples $H_*(\Delta^n)$, $H_*(S^n)$, $n \leq 2$. The cone construction. $H_*(S^n)$, $H_*(\mathbf{M}_1 \# M_2)$ for simplicial surfaces M_1, M_2. Geometrical realizations. Invariance of $H_*(\ \ )$ under subdivision. Homology of products. Künneth Theorem.

Classification of closed, connected piecewise linear surfaces as connected sums thus:

$$\mathbb{T}^2 \# \ldots \# \mathbb{T}^2, \quad \mathbb{RP}^2 \# \ldots \# \mathbb{RP}^2, \quad \mathbb{RP}^2 \# \mathbb{RP}^2 \# \mathbb{RP}^2 \# \cong \mathbb{T}^2 \# \mathbb{RP}^2$$

Fixed Point Theorems: in particular, Lefschetz complexes and Brouwer Fixed Point Theorems.