MATH0021 (Commutative Algebra)

Year:	2019–2020
Code:	MATH0021
Old code:	MATH3201
Level:	6 (UG)
Normal student group(s):	UG: Year 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Structure:	4 hours lectures per week. Assessed coursework.
Assessment:	90% examination, 10% coursework.
Normal Pre-requisites:	MATH0053 (previously MATH7202)
Lecturer:	Dr J López Peña

Course Description and Objectives

The aim of this course is to study modules over commutative rings, building on the foundations established in MATH7202 and develop the framework of modules over commutative rings as a generalization of both vector spaces and abelian groups. We will study families of "nice" rings, such as Principal Ideal Domains (PIDs) and Noetherian rings, and modules over them, including the classification theorem for finitely generated modules over a PID, which generalises both the basis theorem for vector spaces and the classification theorem for finitely generated abelian groups. Knowledge of modules and their use will be further deepened by presenting the theory of module extensions and some of its applications.
This course will provide a solid foundation of commutative rings and module theory, as well as help developing foundational notions helpful in other areas such as number theory, algebraic geometry, and homological algebra.

Recommended Texts

1. M. Atiyah and I. MacDonald, *An introduction to commutative algebra*
2. C. Weibel, *Introduction to homological algebra*
3. S. Lang, *Algebra*

Detailed Syllabus

- Modules, fg modules, free modules. Submodules, quotient modules, module homomorphisms, kernel and image, isomorphism theorems, presentations of modules.
- Short exact sequences of modules, extensions of modules. Definition of Ext^1 in terms of extensions. Definition of Ext^1 from a presentation. The homomorphism lifting property for free modules. Equivalence of the two definitions of Ext^1.
- Principal Ideal Domains (PIDs), Noetherian rings. Quotients of rings. Hilbert's basis theorem. Modules over PIDs and Noetherian rings. Properties and computation of Ext^1 over Noetherian rings and PIDs. Classification of fg modules over a PID. Applications.
- Sections of extensions and derivations. Description of Ext^1 in terms of derivations. Local rings. Localizations. Derivations over local rings. Cotangent space. Geometric interpretation.