MATH0018 Functional Analysis

Year:	2024–2025
Code:	MATH0018
Level:	6 (UG)
Normal student group(s):	UG: Year 3 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	90% examination, 10% coursework
Normal Pre-requisites:	MATH0051
Lecturer:	Dr M Karpukhin

Course Description and Objectives

Elementary analysis mostly studies real-valued functions on the real line $\mathbb{R}$ or on n-dimensional space $\mathbb{R}^n$. Functional analysis, by contrast, shifts the point of view: we collect all the functions of a given class (for instance, all bounded continuous functions) into a *space of functions*, and we study that space (and operations on it) as an object in its own right. Since spaces of functions are nearly always infinite-dimensional, we are led to study analysis on infinite-dimensional vector spaces, of which the most important cases are Banach spaces and Hilbert spaces. This course provides an introduction to the basic concepts of functional analysis. These concepts are crucial in the modern study of partial differential equations, Fourier analysis, quantum mechanics, probability and many other fields.

Recommended Texts

There is no single book that is perfect for all the topics in this course. You will therefore have to get into the (good) habit of reading the relevant sections in *several* textbooks (each one with its own idiosyncrasies, gems and flaws) and then synthesizing what you have learned. With each topic I will give you a detailed reading list, but here is a preliminary list of useful reference books:

(i) Kolmogorov and Fomin, *Elements of the Theory of Functions and Functional Analysis*

(ii) Conway, *A course in Functional Analysis*

(iii) Rynne and Youngson, *Linear Functional Analysis*

(iv) Kreyszig, *Introductory Functional Analysis with Applications*

(v) Giles, *Introduction to the Analysis of Metric Spaces*

(vi) Giles, *Introduction to the Analysis of Normed Linear Spaces*

(vii) Royden, Fitzpatrick *Real Analysis* (Chapter II)

Detailed Syllabus

- Metric spaces and their basic properties. Completeness and completion. Compactness and equivalent properties.

- Normed linear spaces and Banach spaces. Examples: Sequence spaces ℓ^p $(1 \leq p \leq \infty)$ and c_0; spaces $C(X)$ of bounded continuous functions. Proofs of completeness of these spaces. Special properties of finite-dimensional normed linear spaces.
- Hilbert spaces. Basic properties. Orthogonal systems and the orthogonalization process. Representation of linear functionals on Hilbert space.
- Zorn's lemma and the Hahn-Banach theorem.
- Linear functionals and duality. Dual of ℓ^p is ℓ^q. Second dual and reflexive spaces.
- Baire category theorem. Uniform boundedness theorem, open mapping theorem, closed graph theorem.
- Weak and weak-$*$ topologies. Weak-$*$ compactness of the unit ball in the dual space. Compact operators.