

# MATH0017 Measure Theory

<i>Year:</i>	2024–2025
<i>Code:</i>	MATH0017
<i>Level:</i>	6 (UG)
<i>Normal student group(s):</i>	UG: Year 3 Mathematics degrees
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	1
<i>Assessment:</i>	90% examination, 10% coursework
<i>Normal Pre-requisites:</i>	MATH0051
<i>Lecturer:</i>	Dr A Doan

## *Course Description and Objectives*

Measure theory and theory of the integral developed by Lebesgue at the beginning of the last century found numerous applications in other branches of pure and applied mathematics, for example in the theory of (partial) differential equations, functional analysis and fractal geometry; it is used to give mathematical foundation to probability theory and statistics, and on the real line it gives a natural extension of the Riemann integral which allows for better understanding of the fundamental relations between differentiation and integration. This course provides the essential foundations of this important aspect of mathematical analysis.

## *Recommended Texts*

R L Schilling Measures, Integrals and Martingales (Cambridge University Press). R G Bartle, The elements of integration and Lebesgue measure (Wiley, 1995). Texts covering some parts of the syllabus include: J F C Kingman and S J Taylor, Introduction to measure and probability (Cambridge University Press, 1966), H L Royden, Real analysis (Macmillan, 1968) Evans-Gariepy: Measure theory and fine properties of functions, CRC Press and Wheeden-Zygmund, Measure and integral (Dekker).

## *Detailed Syllabus*

- Abstract measure theory: Families of sets,  $\sigma$ -algebras, Borel sets, measurability of functions, measures, null sets, completeness, Borel measures, examples including the Lebesgue measure in  $R^n$ .
- Theory of integral: Definition and basic properties of integral with respect to a measure, Fatou's lemma, monotone convergence theorem, dominated convergence theorem.
- Construction of measures: Caratheodory construction, product measures, Fubini's theorem; definition of Lebesgue measure in  $R^n$  and its determination by translational invariance, Lebesgue integral on the line and Riemann integral, calculation of Lebesgue integral in  $R^n$  including the use of the substitution theorem but not its proof.
- One or two topics selected from the following:
  - (a) Riesz representation theorem.
  - (b) Absolute continuity of measures, decomposition of measures into absolutely continuous and singular parts, Radon-Nikodym theorem.
  - (c)  $k$ -dimensional measures in  $R^n$ , area formula, proof of the substitution theorem for Lebesgue integral, calculation of  $k$ -dimensional measures and integrals.