MATH0014 (Algebra 3: Further Linear Algebra)

Year:	2018–2019
Code:	MATH0014
Old code:	MATH2201
Level:	5 (UG)
Normal student group(s):	UG: Year 2 Mathematics degrees and Year 3 Mathematics and Statistical Science degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Structure:	3 hour lectures and 1 hour problem class per week. Assessed weekly coursework.
Assessment:	90% examination, 10% coursework. In order to pass the module you must have at least 40% for both the examination mark and the final weighted mark.
Normal Pre-requisites:	MATH0006 (previously MATH1202)
Lecturer:	Dr I Strouthos
Problem class teacher:	Dr I Strouthos

Course Description and Objectives

The aim of this course is to complete the study of the basic concepts of linear algebra started in the first year. The topics covered have applicability in many areas of mathematics. The ring theory of polynomials over a field is studied. The theory of the diagonalization of matrices is completed and Jordan normal form is introduced. The theory of quadratic forms and orthogonal diagonalization are introduced.

Recommended Texts

Relevant books are: (i) S Lang, *Linear Algebra* (Springer); (ii) P M Cohn, *Elements of Linear Algebra* (Chapman and Hall); (iii) P M Cohn, *Algebra* (Vol 1) (Wiley); (iv) S Lipschutz and M Lipson, *Linear Algebra: Schaum's Outlines* (McGraw-Hill); (v) Morton Curtis, *Abstract Linear Algebra.*

Detailed Syllabus

Polynomial Rings Over a Field
Definitions of ring, field, unit, reducible and irreducible elements. Highest common factor. Degree function. Division algorithm, Euclidean algorithm. Unique factorisation theorem. Remainder theorem.

Diagonalization of Matrices Re-visited and Jordan normal form
Review of bases, coordinates, change of bases, representations of linear mappings by matrices and Cayley-Hamilton theorem. Direct sums. Invariant subspaces. Minimum polynomial of a linear map. Primary decomposition theorem. Characterization of diagonalizability. Jordan normal form.

Linear and quadratic forms
Linear forms and dual spaces. Symmetric bilinear forms. Quadratic forms. Congruence of matrices. Normal forms for real and complex symmetric matrices. Polarization. Sylvester's Law of Inertia. Definite and semi-definite forms.

Orthogonal Diagonalization
Real and complex inner product spaces. Review: length, angle, Cauchy-Schwarz inequality, orthogonal basis and Gram-Schmidt process. Orthogonal complements. Adjoint mapping. Orthogonal and unitary maps and matrices. Spectral theorem.