MATH0013 Analysis 3: Complex Analysis

Year:	2024–2025
Code:	MATH0013
Level:	5 (UG)
Normal student group(s):	UG: Year 2 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Assessment:	The weighting of the module is 80% exam, 9% written coursework, 5% test, 4% quizzes, 2% participation. In the participation mark, 1% will be awarded for missing no more than 3 tutorials and 1% will be awarded for giving a presentation in a tutorial. (The final exam for MATH0013 will take place during the main exam period, April/May 2024)
Normal Pre-requisites:	MATH0003 (MATH0004 recommended)
Lecturer:	Prof A Sobolev

Course Description and Objectives

This is a course on complex functions. The treatment is rigorous. Starting from complex numbers, we study some of the most celebrated theorems in analysis, for example, Cauchy's theorem and Cauchy's integral formulae, the theorem of residues and Laurent's theorem. The course lends itself to various applications to real analysis, for example, evaluation of definite integrals and finding the number of zeros of a complex polynomial in a region.

Recommended Texts

Relevant books are: (i) Stein, E. and Shakarchi, R., *Complex analysis* (Princeton Lectures in Analysis); (ii) Priestley, H. A., *Introduction to complex analysis* (Oxford); (iii) Stewart, I. and Tall, D., *Complex Analysis* (CUP); (iv) Churchill, J. W. and Brown, R., *Complex Varialbles and Applications* (McGraw-Hill Higher Education, 8^th edition).

Detailed Syllabus

The complex numbers and topology in $\mathbb{C}$
Review of complex numbers. Neighbourhoods, open and closed sets. Convergence of sequences. The Riemann sphere.

Functions on the complex plane
Continuous, holomorphic functions, Cauchy–Riemann equations, and power series. Harmonic functions.

Elementary functions $e^z, \sin z, \cos z, \log z, z^\alpha$ and conformal mapping. Linear fractional transformations.

Cauchy's Theorem and its applications
Integration along curves. Goursat's theorem, Cauchy's theorem in the disc, Cauchy's integral formulas. Taylor's theorem, Laurent's theorem, Liouville's theorem, Morera's theorem. The Fundamental theorem of algebra.

Meromorphic functions
Zeros, poles, residue theorem, standard examples of integrals with consideration of the needs of methods courses, the argument principle, Rouché's theorem, Casorati–Weierstrass' theorem, the identity theorem. Maximum modulus principle.