

MATH0013 Analysis 3: Complex Analysis

<i>Year:</i>	2021–2022
<i>Code:</i>	MATH0013
<i>Level:</i>	5 (UG)
<i>Normal student group(s):</i>	UG: Year 2 Mathematics degrees
<i>Value:</i>	15 credits (= 7.5 ECTS credits)
<i>Term:</i>	1
<i>Assessment:</i>	85% examination, 10% coursework, 5% mid-term problem sheet
<i>Normal Pre-requisites:</i>	MATH0003 (MATH0004 recommended)
<i>Lecturer:</i>	Prof A Sobolev

Course Description and Objectives

This is a course on complex functions. The treatment is rigorous. Starting from complex numbers, we study some of the most celebrated theorems in analysis, for example, Cauchy's theorem and Cauchy's integral formulae, the theorem of residues and Laurent's theorem. The course lends itself to various applications to real analysis, for example, evaluation of definite integrals and finding the number of zeros of a complex polynomial in a region.

Recommended Texts

Relevant books are: (i) Stein, E. and Shakarchi, R., *Complex analysis* (Princeton Lectures in Analysis); (ii) Priestley, H. A., *Introduction to complex analysis* (Oxford); (iii) Stewart, I. and Tall, D., *Complex Analysis* (CUP); (iv) Churchill, J. W. and Brown, R., *Complex Variables and Applications* (McGraw-Hill Higher Education, 8th edition).

Detailed Syllabus

The complex numbers and topology in \mathbb{C}

Review of complex numbers. Neighbourhoods, open and closed sets. Convergence of sequences. The Riemann sphere.

Functions on the complex plane

Continuous, holomorphic functions, Cauchy–Riemann equations, and power series. Harmonic functions.

Elementary functions e^z , $\sin z$, $\cos z$, $\log z$, z^α and conformal mapping. Linear fractional transformations.

Cauchy's Theorem and its applications

Integration along curves. Goursat's theorem, Cauchy's theorem in the disc, Cauchy's integral formulas. Taylor's theorem, Laurent's theorem, Liouville's theorem, Morera's theorem. The Fundamental theorem of algebra.

Meromorphic functions

Zeros, poles, residue theorem, standard examples of integrals with consideration of the needs of methods courses, the argument principle, Rouché's theorem, Casorati–Weierstrass' theorem, the identity theorem. Maximum modulus principle.