MATH0006 Algebra 2

Year:	2024–2025
Code:	MATH0006
Normal student group(s):	UG: Year 1 Mathematics degrees
	and Year 2 Mathematics and Statistics degrees
Level:	4 (UG)
Value:	15 credits (= 7.5 ECTS credits)
Term:	2
Assessment:	The final weighted mark for the module is given by: 85% examination, 15% coursework. In order to pass the module you must have at least 40% in both the examination and the final weighted mark.
Normal Pre-requisites:	MATH0005
Lecturer:	Dr ML Roberts

Course Description

This course continues from MATH0005. It has two main components. The first is an introduction to the theory of groups; the idea of a group is central to much of algebra, and pervades many areas of mathematics. The second is a continuation of the study of linear algebra started in MATH005; linear algebra underlies much advanced pure mathematics, as well as being an important tool in applications of mathematics. In particular, determinants are introduced, and the diagonalisation of matrices considered.

Recommended Texts

Recommended books for this course are (for linear algebra) A Guide to Linear Algebra by Towers (MacMillan), and (for groups and number theory), A Guide to Abstract Algebra, by Carol Whitehead (Macmillan). These are both quite helpful, elementary and not too expensive. There are of course many alternatives. Two other possible linear algebra books are Elementary Linear Algebra by Anton (Wiley, student edition) and Linear Algebra by Lang (Wiley, hardback). The first of these is a very full, helpful and elementary text; the second is more sophisticated than the others. A possible alternative for the group theory is Groups, by Jordan and Jordan (Edward Arnold), which goes a bit further with group theory and examples.

You are not required to buy books for this course, although you may find it helpful to do so. If you do buy, please look at the books first to see if you find them suitable for you.

Detailed Syllabus

1. **Number Theory.** Primes. Euclids algorithm. The h, k-lemma. Unique factorization. Euclids theorem.

2. **Groups.** Equivalence relations. Definition of groups, basic properties, order of an element. Subgroups, cyclic groups. Lagrange's Theorem and applications. Group homomorphisms, kernel and image. Group isomorphisms. Examples, including the symmetric group, symmetry groups of geometrical figures and modular arithmetic.

3. **Linear algebra.**

   (i) Brief revision of ideas of subspace, linear independence, linear span, basis and dimension.

(ii) Definition and examples of determinants. Properties, including proof that $\det(AB) = \det(A)\det(B)$. Calculations. Adjoint and inverse.

(iii) Eigenvalues and eigenvectors, characteristic equation, diagonalisation of matrices, applications. The Cayley-Hamilton Theorem.