MATH0005 Algebra 1

Year:	2021–2022
Code:	MATH0005
Level:	4 (UG)
Normal student group(s):	UG: Year 1 Mathematics degrees
Value:	15 credits (= 7.5 ECTS credits)
Term:	1
Assessment:	The final weighted mark for the module is given by: 85% examination, 15% coursework. In order to pass the module you must have at least 40% for both the examination mark and the final weighted mark.
Normal Pre-requisites:	A* in A-level Mathematics and Further Mathematics
Lecturer:	Dr M Towers

Course Description and Objectives

MATH0005 is intended as an introduction to the methods of modern Algebra and Discrete Mathematics, primarily through the detailed study of a problem of particular importance and wide applicability, namely linear equations. In addition, a number of other important aspects of modern mathematics are introduced, in particular, Logic, Set Theory, Functions and Mappings, Permutations, Fields.

The main aim of the course is to lay a solid foundation for the rest of the three years.

Recommended Texts

The material on linear algebra is very well covered in a large number of excellent texts. Making a specific recommendation is largely a matter of taste. However,

(i) *Guide to Linear Algebra* by David Towers, published by MacMillan, covers the basics and is quite cheap.

(ii) *Elementary Linear Algebra* by Howard Anton, published by Wiley, is more thorough, though still elementary.

(iii) *Linear Algebra* by S. Lang, published by Wiley is more sophisticated.

The material on Logic may also be found in many places, for example:

(i) *Foundations of Mathematics* by I. Stewart and D. O. Tall, published by Oxford Scientific Publications.

(ii) *Discrete and Combinatorial Mathematics* by Ralph P. Grimaldi, published by Addison-Wesley.

Detailed Syllabus

Logic

Propositional calculus via truth tables. Minimal formulations $(\vee, \neg)$, $(\wedge, \neg)$, $(\Longrightarrow, \neg)$. Elementary Predicate Calculus.

Rules for manipulating quantifiers by reference to finite models.

Duality under $\neg$.

Methods of proof: Contradiction. Induction.

Linear Equations

Matrix formulation and elementary operations. Reduction to row echelon form.

Solution of system in canonical form.

Vector Spaces

Subspaces of $\mathbb{R}^n$, $\mathbb{Q}^n$: Linear independence. Basis and dimension.

Elementary Set Theory

Sets introduced via defining predicates.

Subsets, unions, intersections, complements.

Boole-De Morgan indentities.

Mappings

Injectivity. Surjectivity. Bijectivity. Invertibility.

Permutations. Decomposition into

(i) disjoint cycles

(ii) adjacent transpositions.

$\mathrm{Sign}(\sigma\tau) = \mathrm{Sign}(\sigma)\mathrm{Sign}(\tau)$ via Laplace's Formula.

Linear Mappings

Linear mappings and matrices

$$\dim \mathrm{Ker} + \dim \mathrm{Im} = \dim \mathrm{Domain}.$$

Fields

$\mathbb{Q}$, $\mathbb{R}$, introduction to $\mathbb{C}$. Finite fields $\mathbb{F}_p$.