



UCL

De Morgan
Association
Newsletter

2018/19

Letter from the Editor

Professor Ted Johnson

This year the annual dinner of the De Morgan Association was held on Friday 8 June 2018 at Senate House, University of London. The Guest of Honour was our own Professor Christina Pagel. Professor Pagel gave a fascinating talk based on the year she spent in the USA researching Republican and Democrat politicians priorities for health policy, showing how careful mathematics allows us to make connections in highly controversial areas. In her article on page 5, Professor Pagel turns her attention to examining core beliefs of voters in relation to the equally controversial question of Brexit.

Associate Editor Kate Fraser has been promoted to Senior Staffing Officer where she will manage Human Resources for the Mathematics Department. Unfortunately this means that we will lose her as the Associate Editor of the Newsletter. We will miss her cheerful enthusiasm and calm expertise.

Our new Associate Editor is Sam Hopkins, who joined the Department over the summer. Sam has rapidly mastered the brief and smoothly guided this edition. We hope you enjoy it, and encourage you to send us articles and photographs, like those of the 1979 graduates' reunion on page 4, for future editions.

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@MathematicsUCL

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Letter from the Head of Department

Professor Helen Wilson



I took over as Head of Department in September 2018, with some excitement and not a little trepidation. Just before I started, we had an astonishing summer of student recruitment: examination results were better than ever before and our undergraduate intake this year was 270 students (up from 210 the year before). That caused some consternation - where would we find rooms for all the tutorials? What rooms could hold everyone at once? But it is a very nice problem to have!

The department has been overstretched for space for many years. We moved into 25 Gordon Street as a temporary measure in 1958 and 60 years later we're still here, but much larger than we were. We now have over 50 full-time academic staff working in the department, 28 of whom are sharing offices. Worst of all, for the first time ever we have had to commandeer the undergraduate reading room as an office for PhD students. But things are changing! Building work is under way in our building, and from September 2019 the fourth floor of 25 Gordon Street will be part of the Mathematics Department. This new space will provide 14 new offices, a tutorial

room, and an extra room for PhD students (releasing the fifth floor reading room back to the undergraduate students). This excellent result is due in large part to the efforts of my predecessor, Professor Robb McDonald, who campaigned tirelessly for more space throughout his seven years as head.

In January we lost a valued colleague, Professor Slava Kurylev, to cancer. He had worked at UCL since 2007 and he brought a new perspective - the research field of inverse problems - to the department. He was a very active researcher until ill health forced him to retire, late in 2018. He will be greatly missed.

In happier news, one major highlight of the year has been new appointments. We hired two new permanent Teaching Fellows, Dr Cecilia Busuioc and Dr Matthew Towers, who joined us in January of this year. Then through the rest of the year we are hiring no less than seven new academics - Dr Shane Cooper, Dr Lorenzo Foscolo, Dr Jeff Galkowski, Dr Luis Garcia, Dr Mahir Hadzic, Dr Angelika Manhart, and Dr Ian Petrow - most of whom will join us in September 2019. Some of these were replacement posts, part of the usual turnover as staff move on, but others were part of a strategic expansion of staff numbers. Our long-term plan is to co-locate with the Department of Statistical Science, expanding both departments and creating a real UCL hub of mathematical science activity. The Institute for Mathematical and Statistical Science (IMSS) as it will be called, will have a central London location still to be determined, and should have everything our two departments need: ample office and discussion spaces, large lecture theatres, computer labs, and student common areas. This is one of UCL's top four potential future major projects - and aims to move us into the global top 50 places to study Mathematics and Statistics. This is an exciting time to be involved with Mathematics at UCL: real change is on the horizon.

UCL Mathematics 1979 Graduates' Reunion

Nick Coates



Photographs courtesy of Nick Coates

For my 60th birthday I have been organising a series of re-unions of groups of people from different periods of my life. One of these was obviously the time spent reading Mathematics at UCL from 1976 to 1979. Fortunately six of us have been meeting up at least once a year for some time and some of us knew a few more, and so on. We started with a pass list of 28 names (which in today's money seems unbelievably intimate). Gradually we were able to contact 18 of the people on that list of whom 13 were able to make the date. Sadly one of our number has already died.

Just before we met one of our number discovered a sheet with the pictures of all the starters - there were 38 of us, including quite a few that none of us remember. Of these 38 first years, 12 students switched courses or left and two others switched to mathematics.

We (together with partners) met in the Haldane Room (which a smaller group of us had last been in 10 years before for pre-dinner sherry when attending a De Morgan Society dinner) for drinks - definitely not sherry - and then an excellent meal. We shared then and now pictures, found out that others had

similar difficulties in understanding parts of the course (back then we didn't admit to this failing - no such inhibitions now after all this time) over the three years and enjoyed fond reminiscences. We were taken on a tour of the main building (we were appalled by the moved and improved refectory) and were disappointed that Jeremy Bentham was locked up: we were all hoping for the obligatory selfie with the great man. I did enjoy the Fake News exhibition, particularly the frame endeavouring to debunk the myths about UCL and Strand polytechnic, and about JB attending Council Meetings. Frankly we did not believe a word of this vain attempt to ruin our memories.

We were delighted that Professor Robb McDonald, then Head of Department, gave us a considerable part of his Saturday afternoon and took around us the oh so familiar Maths Department. We sat in our usual places in the 5th floor lecture theatre and Robb answered our questions about 'now' while we told him about 'then', and we exchanged memories of lecturers who spanned the period. He was most unlike any of the lecturers that I remembered - he even looked at us. Everyone really appreciated it.



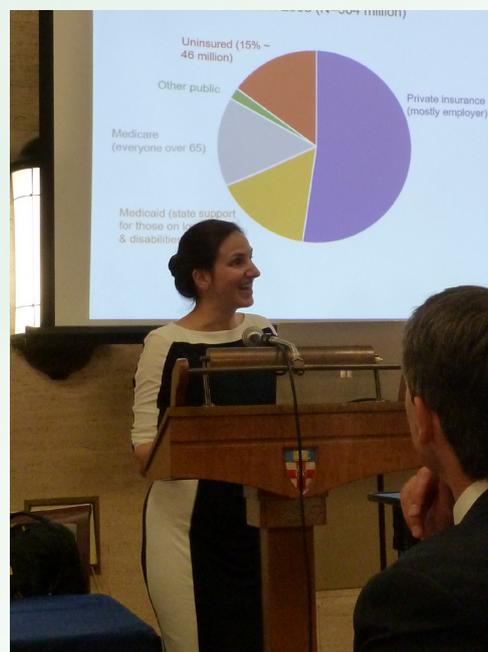
We also want to extend a really big thank you to the UCL alumni department, particularly Katie Raymond who was patient, helpful and efficient.

De Morgan Association Dinner 2018

Bringing Some Mathematics into Brexit

Professor Christina Pagel

I spent 2016/17 in America on a one year fellowship in Health Policy. My US year coincided with Donald Trump's election and the 2017 Republican attempt to repeal Obamacare. So I decided to ditch my planned project and try to understand better what Republicans actually wanted national health policy to do. I asked state politicians across the US about their priorities for the goals of health policy. The mathematically aggregated results from Republican and Democrat politicians were illuminating, highlighting enormous differences on the role of government and tackling disparities^{1,2}. I spoke about that project at the De Morgan annual dinner in June 2018 but instead of going over it again here (which requires a crash course in US health policy), I thought I would instead write about how I've extended that work into the politics of Brexit over the last year.



Christina presenting at the De Morgan Dinner (Photo Soheni Francis)

Leave and Remain have become tribal identities in the UK today and the country is bitterly divided. I was frustrated that polls asked people about their preferences regarding say the Single Market or Customs Union vs controls on immigration when to me the real question was what people wanted the UK to achieve (with e.g. the Customs Union being a potential policy mechanism to achieve a goal). Such binary questions also explicitly assume the pollster knows what voters' top priorities are. But what do UK voters actually want Britain to achieve over the next 5 years? I felt that if we could get a handle on that, then maybe we could find common ground after all – or at least use stated priorities to understand and shape what sort of Brexit (if any) parties should pursue.

My plan of action was thus to:

1. Develop a list of challenges for the UK that I could ask voters to rank in order of priority to them
2. Use Condorcet ranking methods to generate group priorities, where groups could be defined by referendum vote, party vote, age or other demographic factors
3. Use multi-dimensional scaling to create visual “importance maps” of priorities for groups and see where there were common priorities between groups
4. Understand how aligned voters within party or referendum vote were with each other

1. Develop a list of challenges

No matter how big the survey sample or how rigorous the mathematical methods, results would be meaningless without careful development of the survey questions. I had to acknowledge that I was biased, with strong opinions of my own on Brexit. That meant that it was crucial that I talked with people holding different opinions, that I tested the survey language thoroughly for relevance, potential for misinterpretation and that the priorities were meaningful.

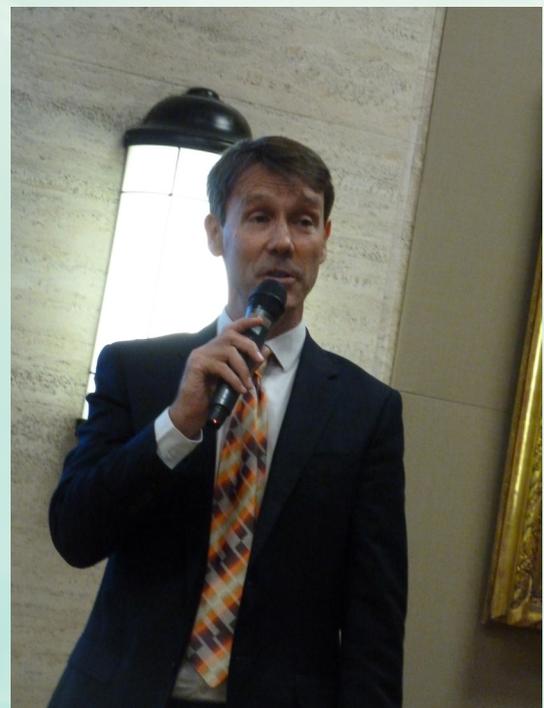
I used the website prolific.ac to run mini-surveys on groups of Leave and Remain voters in the UK, testing out different versions of a list of challenges. I wanted a list that was reasonably comprehensive, not tied too closely with policy and to comprise challenges that might be affected by Brexit. As well as asking respondents to rank the prototype list, I asked them what was missing or what was unclear. I also ran separate surveys asking respondents to describe what they thought key challenges meant to tighten the language. Overall, I engaged with almost 200 people in developing the survey, weighted towards Leave voters (70 Remain Voters, 120 Leave Voters) not least because, as an academic in London, I hardly knew any Leavers.

This engagement definitely changed the priorities chosen. I ended up including two immigration questions – one around reducing overall number of immigrants and the other restricting immigration to just high-skilled immigrants – because Leave voters described and ranked these differently in the test surveys. I also added “UK control over laws and regulations” because many Leave voters told me this was their number one issue. And indeed, in our surveys of over 15,000 people, control over laws was much more important to Leavers (and a significant number of Remainers) than immigration.

We ended with a list of 13 challenges and received funding from the King’s College think tank *UK in a Changing Europe* to run the survey through YouGov on over 7000 representative voters in July 2018.

2. Use Condorcet ranking methods to generate group priorities

I then used a Condorcet ranking method first developed by CORU’s Professor Utley in 2007³ for use in healthcare to calculate overall priorities for pre-defined groups of voters. This is an efficient graph theory method to generate rankings based on pairwise preferences between all possible pairs of challenges. The overall list of priorities for the entire sample of 7,208 voters is shown in Table 1. Ensuring strong economic growth came top overall (preferred by a majority of respondents to all other challenges), but the table hid significant differences between groups of voters. The easiest way to show these is using visual importance maps.



Professor Robb McDonald at the De Morgan Dinner (Photo Soheni Francis)

Overall Rank (N=7208)	Challenge
1	Ensuring strong economic growth
2	Ensuring that Britain has control over its own laws and regulations
3	Allowing Britain to make its own trade deals
4	Making sure there are enough jobs for everyone and that workers are paid fairly
5	Reducing the pressure on public services (e.g. by reducing demand, increasing the workforce, or with more money)
6	Ensuring everyone can afford decent housing
7	Limiting immigration to only high-skilled workers or sectors where there are significant shortages
8	Reducing the gap between richest and poorest people in the UK
9	Reducing the gap between richest and poorest regions of the UK
10	Ensuring that state benefits are distributed fairly
11	Maintaining the union of Northern Ireland, Scotland, Wales and England
12	Reducing the total number of people immigrating to the UK
13	Preserving traditional British culture

Table 1 - Ranked challenges by the whole survey population in July 2018. First published by UK in a Changing Europe⁴

3. Use multi-dimensional scaling to create visual “importance maps” of priorities for groups

If there are N respondents to the survey then each of the M options can be plotted in N -dimensional space, where option C_k has coordinates $(x_{1k}, x_{2k}, \dots, x_{Nk})$. The relative positions and Euclidean distance between the M goals in this N -dimensional space provide insight into how choices tend to be ranked by respondents. I then used ordinal Multi-Dimensional Scaling (MDS) to map points in N -dimensional space to points in 2D space so that they can be easily visualized. Clusters of options can be identified using hierarchical cluster analysis methods on the full dimensional distance matrix to generate insight – these clusters represent options that tend to get ranked similarly by respondents and so might represent a broader concept.

On the 2D map, the horizontal axis is the axis that most separates the options in N -dimensional space. Given that data points are ranks, this axis must represent importance. The vertical axis represents the next most important axis of variation in the rankings. The interpretation of the vertical axis and the interpretation of the clusters is for the observer.

The importance maps for Leave and Remain voters is shown in Figure 1 overleaf.

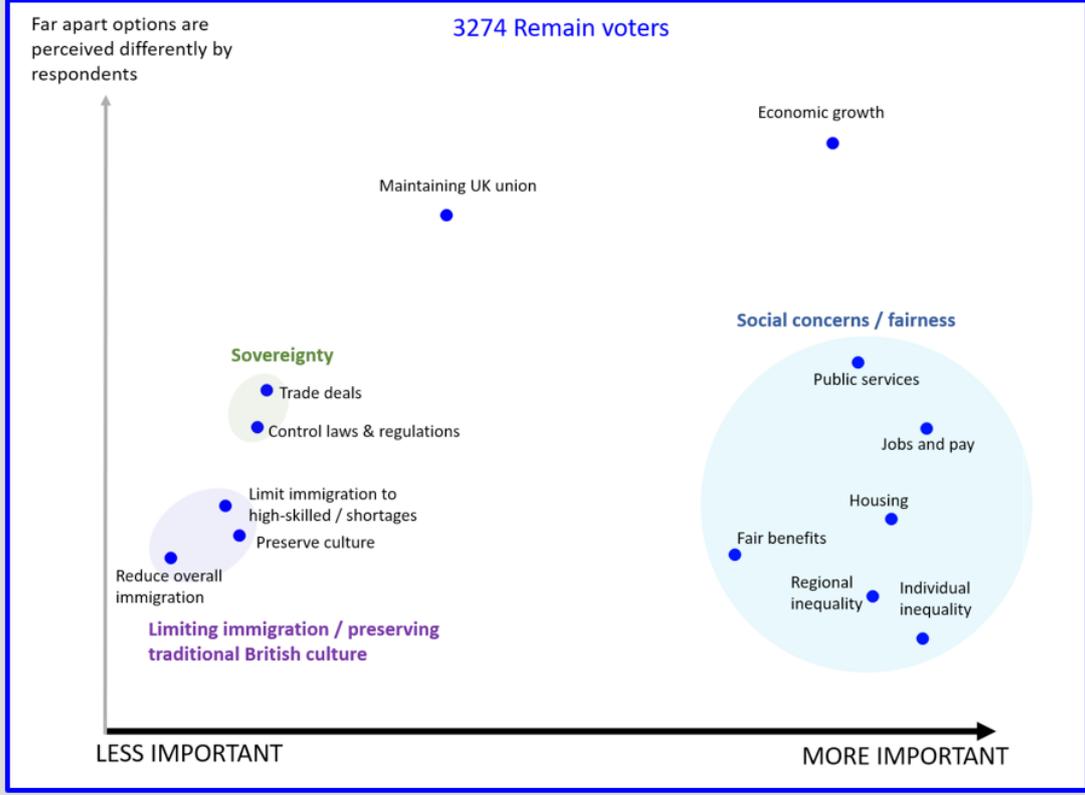
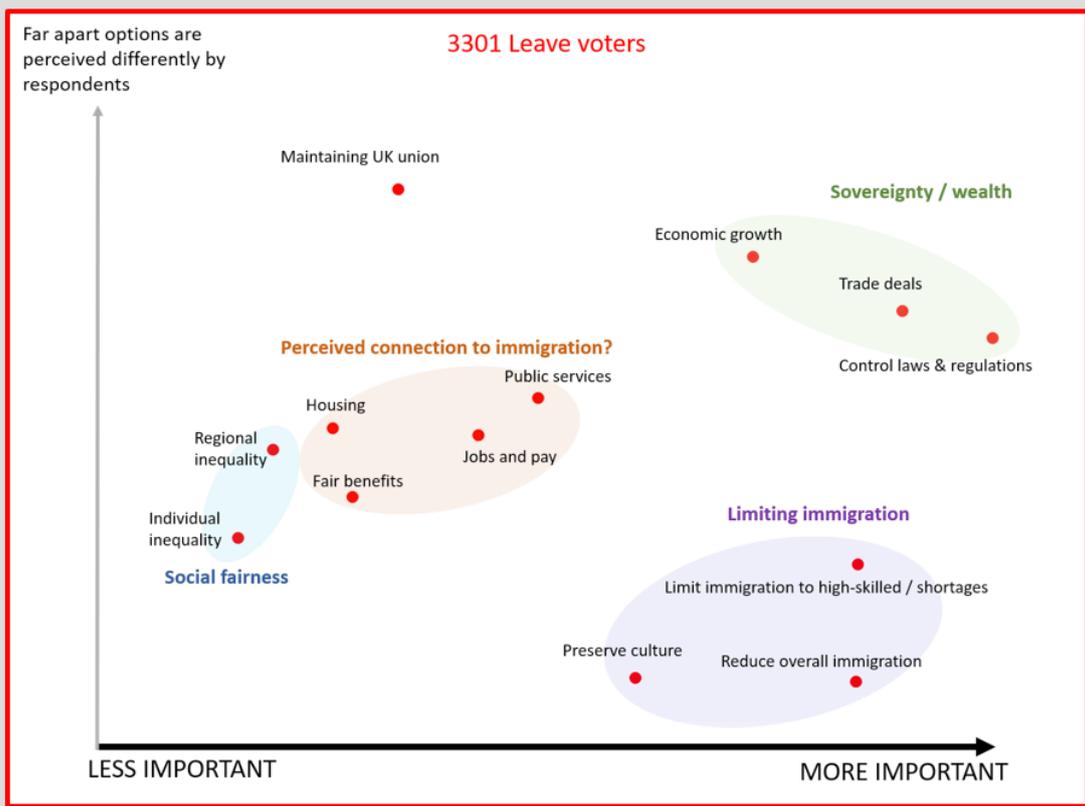


Figure 1 - Importance maps for Leave and Remain voters from July 2018

Remain voters strongly prioritised social concerns and economic growth over everything else, while Leave voters prioritized sovereignty and limiting immigration. Interestingly, economic growth for Leavers was within the sovereignty cluster of challenges – suggesting that Leave voters considered control over laws and independent trade deals as a way to ensure economic growth. Remain voters however, did not consistently link economic growth with any of the other challenges.

One really interesting thing that we found is that while there are expected large differences between Leave and Remain voters, there were also significant difference within Leave and Remain vote by party – particularly for Remain voters (Figure 2).

Conservative voters who backed Remain ranked a strong economy as their top priority, followed by the UK having control of its own laws and trade deals (even though they ultimately voted Remain). “Maintaining the union of Northern Ireland, Scotland, Wales and England” was also important to this group. On the other hand, top priorities for Labour Remainers were reducing inequality, ensuring there were enough jobs, and reducing the pressure on public services; not one of these issues made the Conservative Remainers’ top four list. The only areas of commonality were economic growth and having little concern about immigration. There has been much speculation about setting up an anti-Brexit party bringing together Labour and Conservative Remain voters, but beyond prioritising economic growth over any of the Brexit options, our survey suggests that these two groups of voters have little in common⁵.

To my disappointment, our July survey showed that the country was as divided as it seemed – while economic growth was important to most groups, voters were very divided on the relative importance of almost all the other challenges. To many, nothing was more important than the UK having control over its own laws and controlling immigration while to many others, nothing could matter less. However, the survey was one of the first to demonstrate that sovereignty issues (control over laws and independent trade) were more important to Leave voters than immigration⁶.

4. Identify different types of voter

The Condorcet rankings and the MDS importance maps gave real insight into the UK electorate – but the data I had suggested that there were other groups within the obvious ones defined by Referendum or General Election vote. The key was how to identify these groups. I explore K-means and K-medians clustering methods to identify groups of voters who tended to rank challenges similarly (this time I wanted to cluster **voters** and not **challenges**) but these methods tended to try for groups of the same size and were extremely sensitive to random starting positions (they tend to get stuck in local minima). Instead, my husband and I developed a new algorithm based on simulated annealing to find clusters of similar voters and applied this to the broader groups of Leave and Remain voters. We sense checked the algorithm using histograms of the raw ranked data and over multiple runs to ensure stability from different random starting points and it worked extremely well. By this stage we had rerun the survey with YouGov in September 2018 on over 8,000 people, this time funded by the People’s Vote campaign, so that we could identify not only different types of voters but how strongly each type adhered to their 2016 referendum vote.

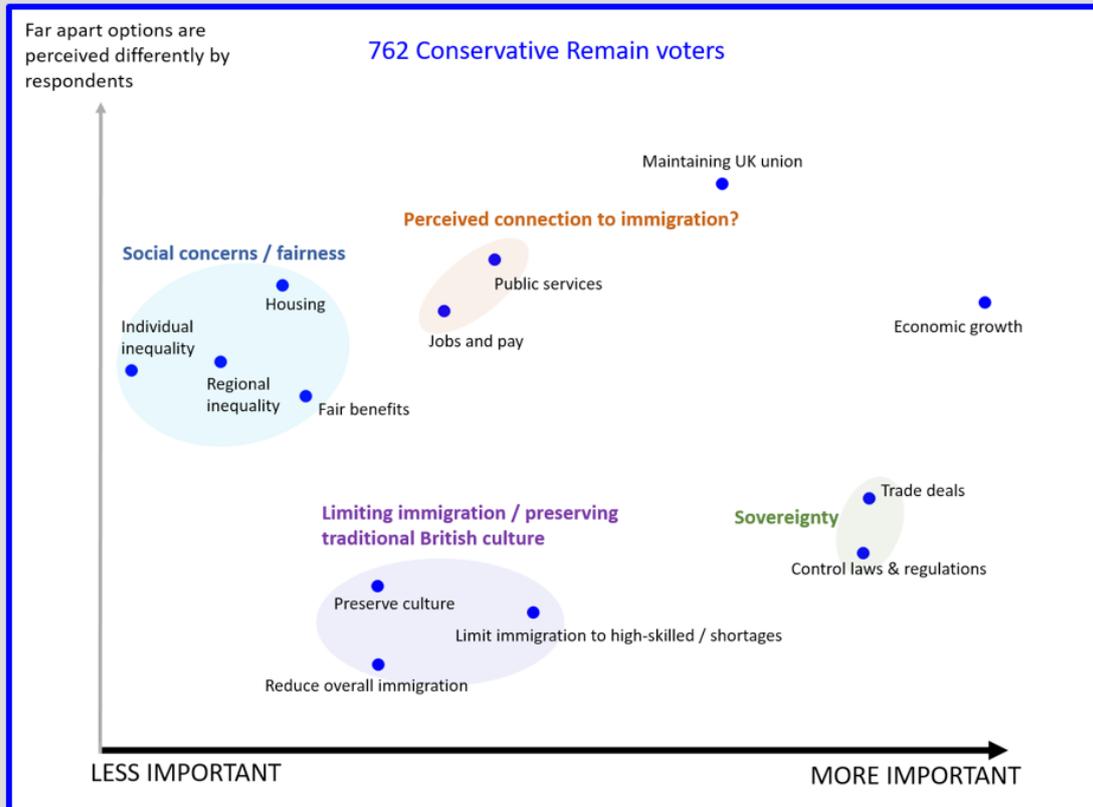
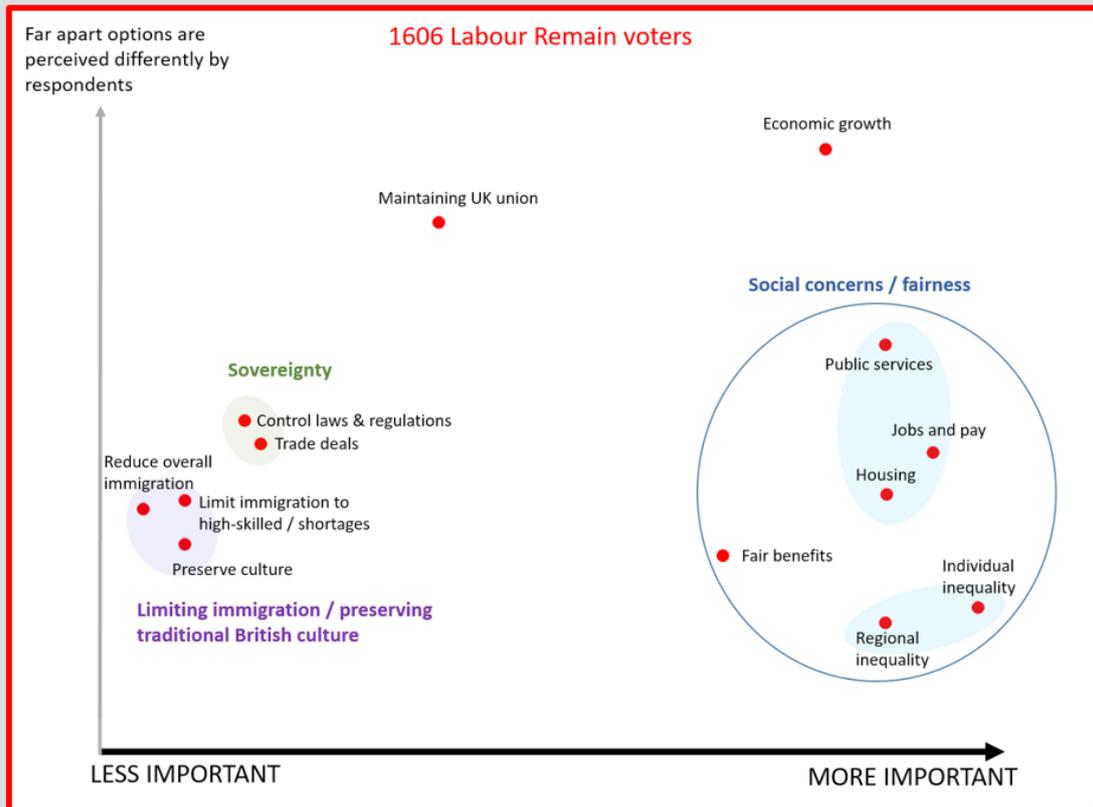


Figure 2 - Importance maps for Remain voters by party, first published by UK in a Changing Europe⁴

We identified four types of Remain voter and four types of Leave voter with the population reasonably evenly divided between types (Figure 3).

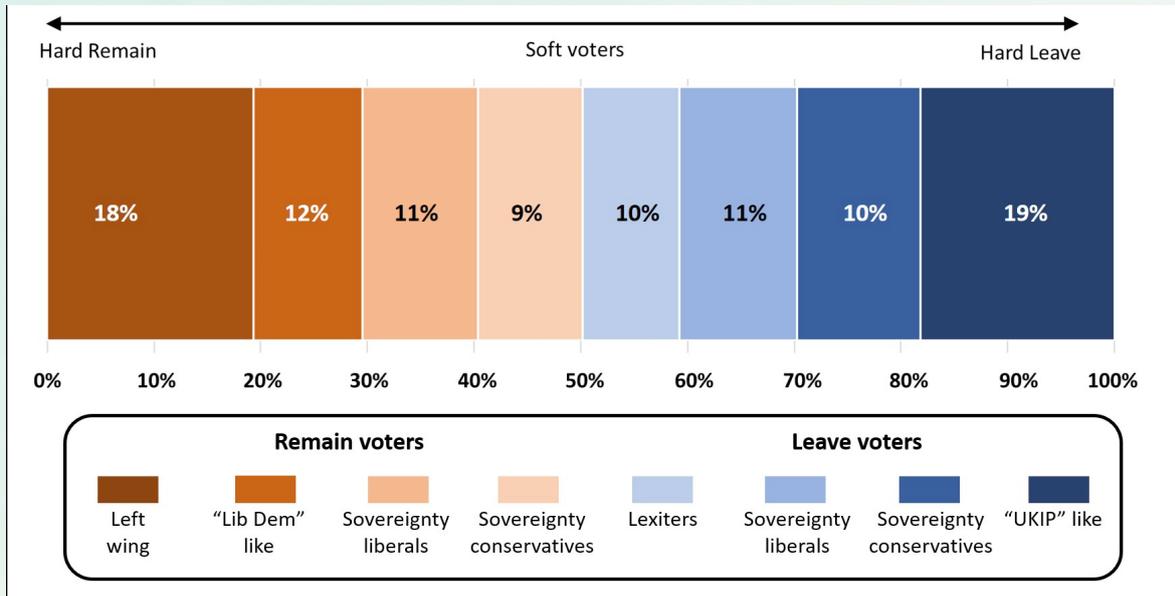


Figure 3 - Distribution of identified voter types across the population. First published by politics.co.uk⁷ in October 2018 using People's Vote data from September 2018.

An indication of the different priorities of each of the types is shown in Figure 4.

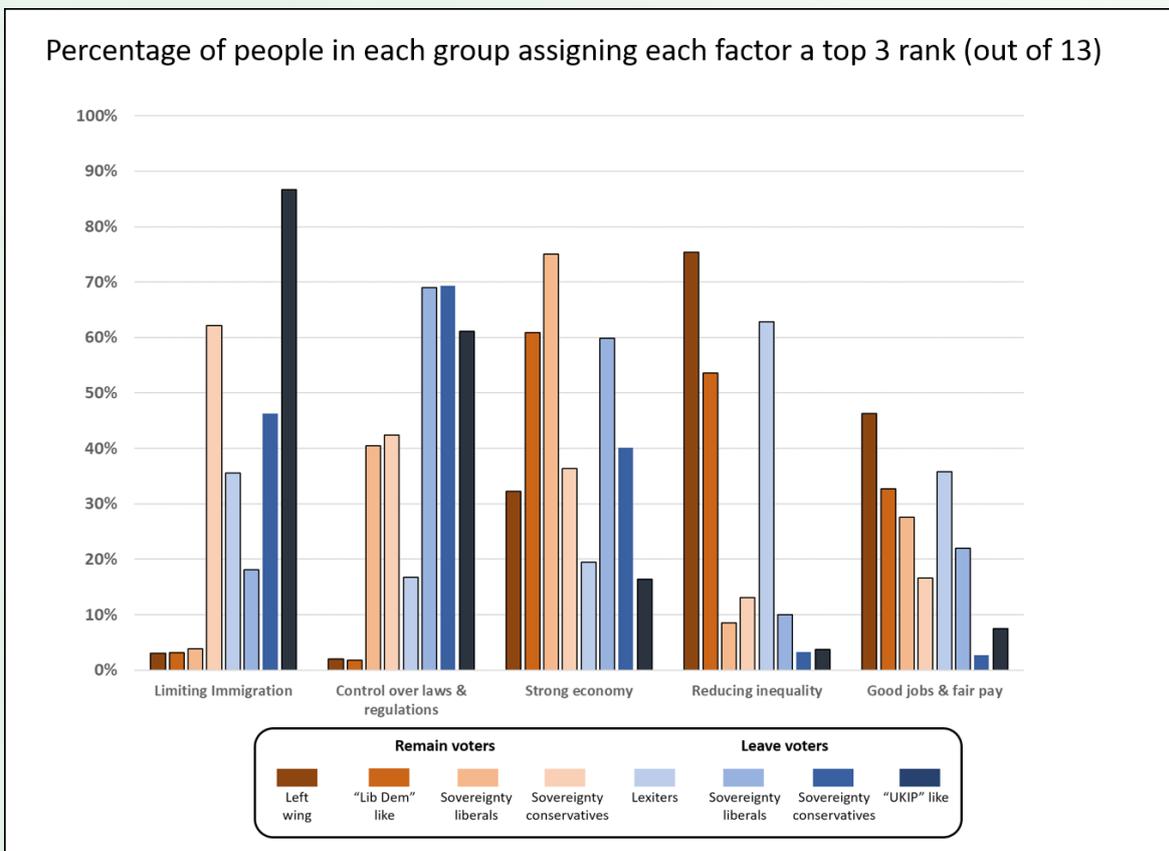


Figure 4 - Differing priorities of different types of voter

Essentially, a significant proportion of voters care a lot about sovereignty but not at all about immigration (Sovereignty liberals) – and these voters are within both the Remain and Leave camps. The media tends to report on Brexit as if there are only two tribes: “UKIP” like Leavers who care only about immigration and sovereignty and then “Left wing” Remainers who are the liberal cosmopolitan elite. In truth, these are the 20% extreme at each end of the spectrum. From the People’s Vote data we can show that these two groups are also the least likely to change their minds on Brexit (only a couple of percent). It’s the other groups that might change their minds – people who tend to care most about sovereignty but also about the economy. Crucially there are significant numbers of voters switching from Remain to Leave as well as from Leave to Remain (see Pagel and Cooper⁷ for details).

These voter types also provide significant insight into the policy priorities of the government and other parties (Figure 5). Although the country is split relatively equally across the types, the parties are not. Conservative voters in the 2017 General Election are heavily weighted not just towards Leave, but towards sovereignty (even Conservative Remain voters). Thus the government’s priority to leave the Single Market and Customs Union makes sense when considering just its voter base.

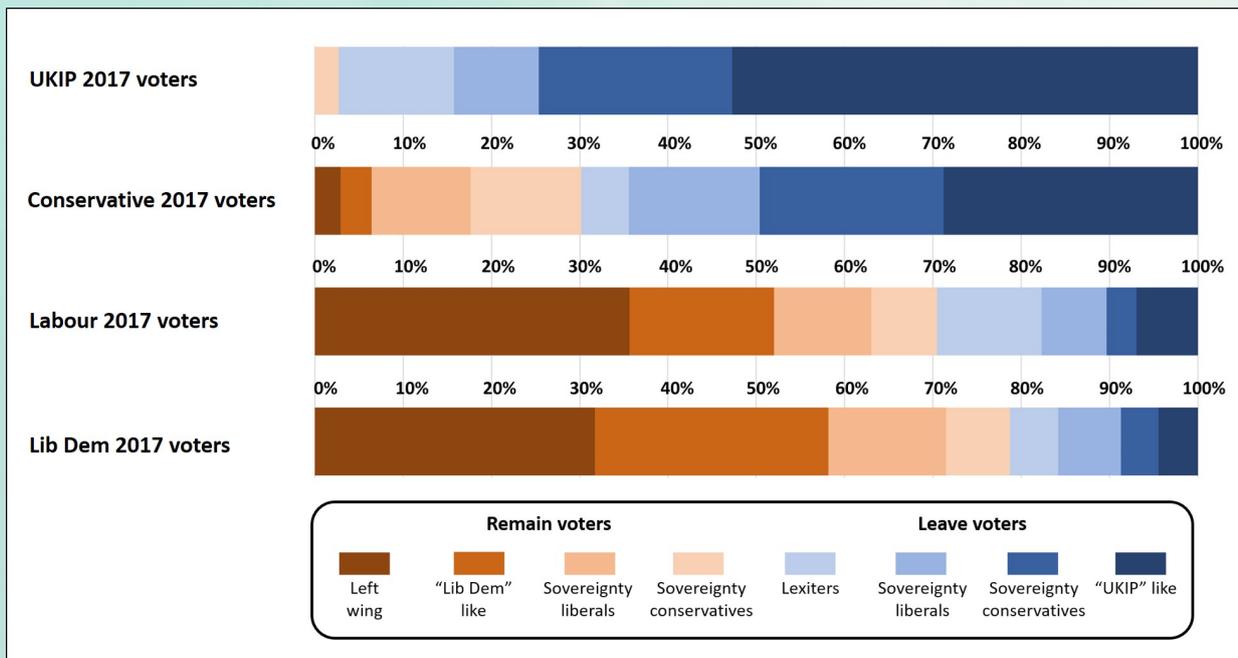


Figure 5 - How the voter types split across parties. First published by politics.co.uk⁷ in October 2018 using People’s Vote data from September 2018.

Conclusions

These different techniques are not that common in standard national polling methods although they are more common in various academic research fields. I have found my foray into this area fascinating from an academic point of view, if not that hopeful from a personal political point of view. I hope that I have at least demonstrated that some relatively straightforward mathematics can provide useful insights into politics. And finally, the fact that the results made sense and were consistent across two large surveys, shows that the initial care taken in choosing the

questions was worth it. It's so tempting to skimp on this step, to rush straight into surveying and answers. But misleading, or leading, questions lead to unreliable answers and no amount of mathematics can compensate for that.

Acknowledgements

This whole strand of work was undertaken with my friend Councillor Christabel Cooper, a data analyst and Labour Councillor for Hammersmith and Fulham. Thank you also to UK in a Changing Europe and the People's Vote Campaign for funding the YouGov surveys. My time spent on this analysis was given for free in my spare time.

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The Michael Singers performing at the 2018 De Morgan Dinner (Photo Soheni Francis)

2019 De Morgan Association Dinner

Friday 7 June at Senate House
Speaker: Rob Eastaway
'Pick a card, any card'

All those on the UCL Alumni database will be sent an invitation to the next De Morgan Dinner. Please remember to keep the Department and Alumni Relations Office of UCL (alumni@ucl.ac.uk) informed of any changes to your contact details.

Inaugural lecture – Professor Sarah Zerbes

Dr Chris Cummins (University of Edinburgh)

Professor Sarah Zerbes' inaugural lecture, on elliptic curves and the conjecture of Birch and Swinnerton-Dyer, took place on 9 May 2018. Sarah was raised in Wuppertal, and studied at the University of Cambridge, where her PhD was supervised by Professor John Coates and entitled "Selmer groups over non-commutative p -adic Lie extensions". She subsequently held a postdoctoral position at Imperial, before spending four years as a lecturer at the University of Exeter, prior to beginning her current appointment at UCL in 2012. She received the Philip Leverhulme Prize in 2014 and the Whitehead Prize in 2015, both jointly with her frequent collaborator, climbing partner and husband Professor David Loeffler.

On this occasion, Sarah's colleagues were joined in the audience by her friends and family, and one particularly special guest, Alfred Rodenbücher of the Wilhelm-Dörpfeld-Gymnasium in Wuppertal, the teacher who first awakened Sarah's enthusiasm for mathematics in general. This presented the interesting challenge of how to provide experts with some flavour of her current research while giving enough context to make it somewhat comprehensible to a less specialised audience.

Fortunately, Sarah's research addresses questions that, if not exactly graspable to laypeople, are at least related to some widely recognisable ideas. She began her lecture by situating her work within algebraic number theory, which has its classical roots in the study of Diophantine equations – that is, equations that have integer solutions. And these turn up again and again, at many

different levels of difficulty, from the kind of problems solved by trial and error in primary school to the great breakout story of Fermat's Last Theorem. Algebraic number theory generalises this object of study to rational solutions of polynomial equations (solutions that can be expressed as fractions with integer coefficients), rather than just integers; but it remains true in this setting that some questions that are relatively easy to state turn out to be profoundly difficult to answer.

For this lecture, Sarah took as a jumping-off point the idea of Pythagorean triples: rational numbers (a, b, c) such that $a^2 + b^2 = c^2$. These have been studied since antiquity – a Babylonian clay tablet dating from 1200 years before Pythagoras' lifetime, catalogued as Plimpton 322, appears to present a set of integer solutions to this equation – but named for Pythagoras because of the geometrical interpretation of (a, b, c) as the lengths of the sides of a right-angled triangle. Such a triangle would have area $\frac{1}{2}ab$. A natural and deceptively simple question is, then: which integers can be the area of such a triangle? Based on the triple $(3, 4, 5)$, we know that there exists a right-angled triangle of area 6 with rational sides. But can we, for example, produce a right-angled triangle of area 1 with rational sides?

We'll label an integer as *congruent* if it is the area of such a triangle. It turns out that not all integers are congruent: indeed, Fermat was able to prove (in 1640) that 1 is in fact not congruent. But it also turns out that various integers are congruent with non-trivial solutions: 5 is congruent, but the simplest solution for the side lengths of the corresponding triangle gives us $a = 20/3$, $b = 3/2$, $c = 41/6$; not one that leaps to the eye. Sarah noted that 2012 is also congruent, but the description of the relevant triangle spills off the page: there was only room on the slide for the first 76 digits of the numerator of c .



On closer inspection, the question of whether an integer is congruent can be restated as a question about the non-trivial rational solutions of a particular equation, $Y^2 = X^3 - n^2X$. If this equation has a solution with Y not equal to zero, n is congruent. The bad news is that there is no straightforward way of finding rational solutions of this equation. The better news is that this is an example of an elliptic curve, which is a well-studied class of mathematical objects of great importance to number theory in general.

Consequently, it is possible to say things about those rational solutions, which we can think of as the *rational points* on the curve. For instance, it is known that the set of rational points has the structure of an Abelian group, and (in a result due to Mordell, 1922) this set of points has a finite basis. That is to say, given a particular finite subset of the rational points on a given elliptic curve, we could generate all the others. The cardinality of this basis, termed the *rank* of the elliptic curve, is zero for any curve with a finite number of rational points: thus, we can recast Fermat's (1640) result as stating that the elliptic curve $Y^2 = X^3 - X$ has rank zero. However, there exist elliptic

curves with rank 1, and rank 2 – indeed, there is known to exist an elliptic curve with rank 28. The problem remains open of finding an algorithm known to finish that outputs the rank of any elliptic curve.

To get from there to the locus of Sarah's own research contribution requires a greater understanding than I am able to bluff. Her work predominantly addresses the conjecture of Birch and Swinnerton-Dyer (BSD), which concerns the behaviour of L-functions of elliptic curves. An L-function can be defined for an elliptic curve by considering the points on the curve modulo each prime number, and using these to construct an Euler product. The L-function in question converges on the complex plane for values with real parts greater than $3/2$, but also has an analytic continuation to the entire complex plane. The general intuition is that the value of the L-function can tell us something about the rank of the corresponding elliptic curve (it should be small for non-zero rank), and that this represents a promising approach to those open problems about the behaviour of elliptic curves in general.

Sarah's work involves constructing Euler systems, which she describes as the "arithmetic avatar" of L-functions. These represent a particularly promising tool for the elucidation of several important conjectures arising in connection with BSD. Her recent collaborative work has brought forth a collection of new Euler systems, each constituting a potentially transformative step in the understanding of the corresponding algebraic structure. However, cashing this out is not straightforward. Sarah herself concludes by commenting that the demonstration of the explicit reciprocity law, a crucial step in the application of one of her Euler systems to a specific open case of the BSD, is "hard". Given the standards that apply to her work, I'm inclined to take her word for this.

Tomorrow's Mathematicians Today

Yohance Osborne

3rd Year BSc Mathematics

On the 9th of February 2019, the University of Greenwich hosted a conference for undergraduate students called 'Tomorrow's Mathematicians Today' (TMT), with support from the Institute of Mathematics and its Applications. The event was conceptualised at Greenwich with the goal of granting undergraduate students the chance to experience what it's like to attend a conference on mathematics. But TMT would have the distinction that (almost) all presentations would be done by undergraduates themselves! Since its debut in 2010, the conference has been an overwhelmingly positive experience for many undergraduates across the United Kingdom, with 2019's event being its 7th occurrence, not falling short of its spirit of being a splendid day for mathematics.

A few months prior to this year's conference, students were invited to submit abstracts for talks through which they'd share their passion for mathematics in their own way with like-minded pupils. As an incentive for submitting abstracts, students were also invited to detail

outlines of their talks in what formed the initial round before the determination of the 'Best Presentation' on the conference day, with the winner to receive a £300 prize. This prize was sponsored by the Government Communications Headquarters (GCHQ), and applications for the competition underwent an assessment against the follow criteria:

1. Evidence of an excellent understanding of the mathematics involved in the problem proposed
2. Evidence of collaboration and team work either with your peers, with your tutors or with outside contacts
3. An understanding of current research in the area examined and an indication of further research that could be undertaken
4. An innovative approach to a standard problem

I managed to submit my application within 6 minutes of the deadline (yes, 6 minutes), and a month later I received remarkable news that my talk was shortlisted for the GCHQ prize! In fact, out of the 30 proposals that were accepted for the conference, my submission was among only 4 that were nominated for the prize. Without a doubt, the advice I received from Dr Iain Smears and Dr Dave Hewett in putting together my application was indispensable to such an outcome. What remained thereafter was the task of putting together a 20-minute presentation on a theme from Nonlinear Functional Analysis.

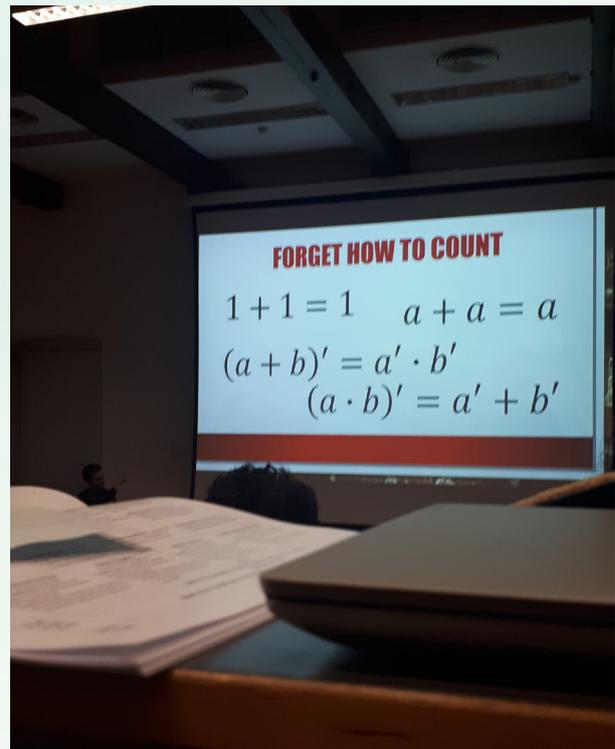
My presentation was entitled 'The Browder-Minty Theorem for Nonlinear Monotone Operators'. The Browder-Minty theorem is a fundamental result in the theory of monotone operators which provides sufficient conditions under which such operators turn out to be surjective over real reflexive Banach spaces. Within the five-week period between the beginning of term and the conference day, I was under the supervision of Dr Smears in



Yohance Osborne at the TMT conference

developing a detailed understanding of the result, as well as the essence of its proof, with the quasilinear p-Laplace problem as a featured example. Honestly, I found the experience was quite rewarding as I enjoyed learning new concepts, seeing how they come together to form an interesting proof, while also encountering many research areas in mathematics where the p-Laplace problem is looked at extensively in a variety of ways. The topic area also came to me as an extension of a module that I took in the first term on Variational Methods for linear partial differential equations. Once more, I thank Dr Smears for the enormous support he gave me on this small-scale project.

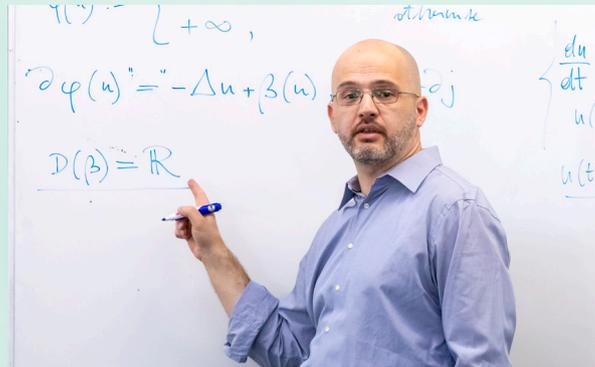
Certainly, I'd call the TMT conference a Saturday well-spent! By virtue of the typical structure of a mathematics conference, I was only able to attend nine talks on the day. Nonetheless, I truly felt inspired by the passion of the speakers, and others who attended would agree. There were many interesting topic areas, which included a Vortex Visualiser in Fluid Mechanics, the Physics of Space Plasma, Quantum Mechanics, Fractional Analysis, Boolean Logic (featuring puppies), Cryptography, variations of the Travelling Salesman Problem, and much more! And of course, one cannot forget to mention the phenomenal final segment by Noel-Ann Bradshaw which was in the form of a keynote address by the very person from which the idea of the conference was born.



Although I did not win the prize in the end, something remarkable happened after the conference. The winner for Best Presentation asked the event organisers that the prize money be split equally amongst the four finalists! I do not know the winner's reasons behind the request, but on the day the judges themselves had great difficulty in choosing the winner amongst the four talks, all of which they found were excellent.

In all, the preparations in the weeks before the conference and the conference day itself constitute one of my most memorable experiences at UCL. The Tomorrow's Mathematicians Today conference has certainly achieved its goals since its inception, and many congratulations are in order for the organisers of the conference who I sense truly put their hearts into making the 2019 event a reality.

Monotonicity and Stochastic PDEs Inaugural Lecture of Professor Carlo Marinelli



Two years have passed since Carlo Marinelli joined the professor rank at UCL. That his inaugural lecture takes place only in 2018, as pointed out by the dean Professor Ivan Parkin, only illustrates the healthy amount of appointments our department has enjoyed recently.

For this lecture, Carlo chose a subject on well-posedness of stochastic partial differential equations (SPDEs), a domain that has tingled Carlo's interest at least since 2010. He has also worked in financial mathematics and statistical topics, a wide scope supported by a mathematical background that includes a Laurea (MSc) in Electrical Engineering at the Università di Padova (1999) and a PhD in Statistics at Columbia University (2004). The list of his academic appointments is long and contains several postdoctoral positions until 2008 and a tenured assistant professor position (Ricercatore confermato) at Libera Università di Bolzano (2008-2012). He joined UCL as a reader in 2012.

Specifically, for this lecture Carlo considered the question of well-posedness for a class of SPDEs of monotone type. Armed only with a marker, the white board, and clear ideas, he guided us on a journey through the theory of existence of solutions that begins with some relevant results on classical equations in the real line, paused on the case of stationary and evolutionary deterministic PDEs, and culminated in an appreciation of the case of stochastic PDEs.

Indeed, Carlo started his presentation by reminding the audience how in the case of univariate real functions $f : \mathbb{R} \rightarrow \mathbb{R}$, one can define a solution x to the problem $f(x) = b$ whenever the function is monotonic and suitably extended so that its graph is maximal (in the sense that the graph of the extended f , now considered as a relation in $\mathbb{R} \times \mathbb{R}$, should not be properly included in any other monotone graph).

The same intuition, Carlo explained, can be used to work with operators from a Hilbert space to itself, or, more generally, from a Banach space to its dual, and from a Banach space to itself. The key point in each case is to properly extend the concepts of monotonicity and maximality. For instance, in the case of mappings from a Banach space E to itself, a mapping A is said to be *accretive* if

$$\|x_1 - x_2 + \lambda(Ax_1 - Ax_2)\| \geq \|x_1 - x_2\|$$

for all $x_1, x_2 \in D(A) \subset E, \lambda > 0$. This property can be easily shown to be equivalent to monotonicity for real functions, so it is a good way to extend the definition to more general spaces. If in addition to be accretive, A satisfies the range condition $R(I + A) = E$, the operator is said to be *m-accretive*. These would be, roughly speaking, the analogous of maximal and monotone functions in the real line. One of the central results in the deterministic theory of monotone PDEs states that the abstract ODE

$$\frac{du}{dt} + Au \ni f, \quad u(0) = u_0$$

in a general Banach space E , where A is a an *m-accretive* graph in $E \times E$, $f \in L^1(0, T; E)$, and $u_0 \in E$, admits a unique *mild* solution (a concept introduced by Crandall and Liggett), that depends continuously on u_0 and f in suitable topologies.

This result can be considered as the culmination of an intense research activity that took place in the '60s and early '70s, involving, among others, Minty, Browder, Brezis, Crandall, Pazy, and Kato.

The story, Carlo admits, is quite different when we consider stochastic PDEs. The well-posedness of equations of the type

$$du + Au dt = B(u) dW, \quad u(0) = u_0,$$

where W is a Brownian motion, cannot be shown using the deterministic PDE approach mentioned above, as it breaks down due to the lack of regularity of the Brownian term. In fact, Carlo reports that no general well-posedness theory is available for this type of equations, although several particular cases have been studied, using different tools and techniques. He explained that this situation is akin to that of the deterministic theory in the early '60s, giving a sense of limitation of the field, but perhaps also expressing some hope on possible progress towards a comprehensive general theory.

Carlo then proceeded to discuss some ideas used to solve equations where A is a semilinear operator defined on the Hilbert space $L^2(D)$, with D a bounded domain of \mathbb{R}^n . In particular, A is the sum of a linear maximal monotone operator on H and of the superposition operator associated to a maximal monotone graph in $\mathbb{R} \times \mathbb{R}$. His approach is to construct a sequence of solutions to regularised versions of the problem (for which the solution exists thanks to classical theory for equations with bounded coefficients), and to extract a convergent subsequence, the limit of which can be shown to solve the original equation. Such compactness results are obtained combining the so-called variational approach to stochastic PDEs, and *ad hoc* techniques based on convex analysis and weak compactness in L^1 spaces.

At the end of the journey, an implicit invitation to work on obtaining a general theory in the stochastic case lingers. I am confident that Carlo will play an important role in tackling this problem.

Write-up provided by Dr Camilo Garcia Trillos, UCL

Black Mathematician Month

Amin Sabir

1st Year MSci Mathematics

To celebrate the second Black Mathematician Month this year, Chalkdust magazine and UCL Mathematics Department organised a mathematics workshop day in Tottenham to sixty Year 9 and 10 students from six local schools. Black Mathematician Month aims to highlight the minimal progress of diversity in our field, and to improve it in the best way possible. With the success of last year, we were proud to showcase our passion in Tottenham. We hope that it will encourage young black students to engage more with mathematics, enjoy it and view it as an important skill for employment.



Dr Nira Chamberlain and David Lammy MP with Professor Helen Wilson and representatives of UCL Mathematics

To kick start the day, the MP for Tottenham, David Lammy gave a speech about the importance of believing that you can achieve anything, regardless of what your upbringing may have been like. For example David spoke of WhatsApp founder Jan Koum, who had to migrate to USA as a child with his parents, who both passed away soon after. David also spoke about Brexit and how his team of researchers were needed to check and

discuss how it would impact the public's finances. Each of these anecdotes served the purpose of highlighting the significance of mathematics in life's difficult situations. Ultimately, the technological and economic climate is changing quickly, and David hopes that mathematics students - such as the ones who were sat before him - will be well-placed to lead the way.



David Lammy MP addressing students

Keen to remind us that mathematics isn't all just formulas, Dr Nira Chamberlain then arrived with an enthusiastic, interactive and humorous presentation about how mathematics can be applied to the real world. All the students thoroughly enjoyed these talks and could not wait for the workshops.



Dr Nira Chamberlain presenting

These workshops included fun and intriguing topics ranging from number theory and

modelling alien civilisations via the Drake equation, to frieze patterns and topology. They were run by UCL PhD students and Dr Naz Miheisi from King's College. In the topology class, we made several types of Möbius loops, combined them and understood their properties. A discussion was also held about the Klein bottle and projective plane. The students were all trying to predict what the different shapes would turn out as, but were completely astonished by the results. Some did say they would make great Christmas decorations!

In the afternoon, UCL undergraduate volunteers formed a Q&A panel to answer some interesting and funny questions from the pupils about what it's like to study mathematics, including whether it's necessary to turn up to lectures! On a serious note, the students did find this very helpful and

informative, further establishing whether maths is for them.

To round off the day, there was a question relay between schools where the winning team got their very own Chalkdust T-shirts. The pupils did get competitive against each other, albeit for the love of mathematics!

For me, I loved this whole day and the idea of inspiring the next generation felt very rewarding and I will be hoping to do more outreach events like this in the future.

To conclude, thank you to London Academy of Excellence for hosting the event. The feedback from the schools shows that they thoroughly enjoyed the day and that it will really influence their pupils' future decisions. After all, that's what Black Mathematician Month is all about!



The Q&A panel of UCL students

Photographs courtesy Sean Jamshidi

Green Team goes for Platinum

Dr Belgin Seymenoglu Green Team

Our story starts in November when Robb McDonald attended a few Green Impact meetings, which is where he got the idea of setting up a Green Team. Early in, Robb recruited a few of us: Emily Maw, Harry Donnelly, Sukh Thiara and Gavin Prentice to the Team.



Bentham's Farm

One January morning the whole department received an email saying that a battery recycling bin had been installed in Room 610. Before then I had always struggled to find places near my office to recycle my batteries, so this was a godsend. But something else in that fateful email sparked my curiosity: the mention of a "Green Team". The same team who would proceed to remove the water cooler after a democratic debate plus vote during a Staff Meeting. Now we get all our drinking water from the filtered taps. I guess the cooler was on its last legs anyway!

We also got rid of the plastic cups, while Dr Luciano Rila received plenty of mugs from the Further Maths Support Programme. Later that month, Robb and Emily kept their eyes peeled for birds from the comfort of their homes as part of the RSPB Birdwatch.

We've also installed colourful triple bins on all floors, so you can separate your recyclables and food waste from the normal rubbish. We've also opted to print double-sided, which means less strain on the recycling bins! Our coffee machines are now fed a healthy diet of Fairtrade coffee beans. They still spit out plenty of grounds, but that's no problem. We pass them on to Gordon's Café, who send the beans out to turn them into biofuel!

Emily dedicates plenty of her time to running and promoting Bentham's Farm. Moreover, she joined the rest of the Michael Singers in singing a wassail on the day a pear tree was being planted in the farm.

I became the second PhD student to join the Green Team just when they were holding the Fairtrade cake sale. Maybe it was the matcha tea cake that lured me in! Before long, we went



Professor Robb McDonald with the new staff room greenery

on an enjoyable expedition to the nearby plant centre to turn all the money we raised into a coconut tree and another tall-ish plant; both of which are now standing proudly in the staff room.

A number of us already regularly cycle to UCL, and when the weather improved in time for April, it was the perfect time to promote cycling to work. We also advertised using the stairs in the department... and a number of you pledged to do it during the Do Nation campaign. Thanks to you, our department came first in pledges. Perhaps the record for most pledges by a single person goes to Johnny Nicholson: seventeen!

When I first joined, we were modestly aiming for a silver award, but before long we found gold within our sights. And if that wasn't enough, we had some tantalising news shortly after the audit... we had reached the dizzy heights of platinum!

We received the award during UCL's Sustainability Awards 2018 ceremony in early September. Sadly, I missed it because I was racing against time to submit my thesis, but Robb, Sukh and Emily attended, and collected our platinum award. It had a crack in it, and



Emily Maw with her Outstanding Commitment to Sustainability award

had been glued back together... which is arguably truly sustainable! There was more good news to come... Emily received the student award for Outstanding Commitment to Sustainability - her award being still in one piece!

The Green Team is now one year old, but we've already come so far. I have only been part of the team for eight months, and will likely leave UCL soon. But we did so many fun activities during my brief stay, and the team is always discussing ideas on how to be sustainable. And I did not expect the photo of me and the rest of Chalkdust meeting David Attenborough to appear on our green noticeboard!



Staff and students meeting Sir David Attenborough

If you have any ideas on how to make us even more sustainable, please tell us. Or, better still, join us!

Is that a pattern?

Rafael Prieto Curiel and Rudolf Kohulák

During the World Cup, fans watched the England vs Colombia match being defined by the most nerve-racking of penalty shootouts. One shot after the next, fans were shouting at the screen, a rollercoaster of excitement and emotions. What are the chances that Pickford, the English goalkeeper, will stop a shot from Bacca, from the Colombian team? Are penalties predictable? Where will the next player kick the ball? If we look at thousands of penalties, they actually follow a pattern! Penalty shots are more frequently aimed at specific locations of the goal, and so the goalkeeper might increase their chances of stopping a shot if they focus on these regions. Players have a *penalty profile*, meaning that they tend to kick the ball to some locations more frequently than others. Although the pattern might not be quite distinct, Pickford was able to stop Bacca's penalty, sending England through to the next round! And, perhaps, detecting a player's penalty profile increases the chance a penalty will be stopped. That is a pattern!

Patterns are one of the most interesting things to observe and to analyse, but identifying what is a pattern, and what is not, might be quite complicated. Take, for instance, the birthdays of a group of people. If a person is born on a random day, each day with equal probability, then in a group of 57 people, there is a 99% chance of celebrating two birthdays on the same day. But that is just random! The number of birthdays on any particular day might suggest an apparent pattern, coordination or synchronisation, but that wouldn't be correct. When things are merely the result of randomness, we say that there is 'no pattern' or that the pattern is random: for example, the lottery finishing with the same number in two

consecutive weeks or, in a group of 57 people, there being two birthdays on the same day.

Random things also form apparent patterns and celebrating two or three birthdays on the same day in a small group of people is just random. However, when we look at the birthdays of a lot of people, we notice that individuals are born more frequently on certain days of the year than others. For instance, few people celebrate their birthdays exactly on Christmas day. Thus, there is a pattern in people's birthdays, but it is just a slight difference between the day with the lowest number of birthdays and the day with the most.

Patterns are everywhere but are often confused with uniformity or randomness. One night you prepare your favourite dinner: fusilli with olives, enough for twenty bites, and so you use twenty olives in your recipe hoping to get one olive in each bite. You mix the ingredients very well, making sure to combine the olives with the pasta. As you eat your creation, you notice that around seven out of the twenty bites of your pasta have no olives, while one of the bites has... four olives! "But I mixed them", you repeat to yourself, infuriated. Mixing olives and fusilli, carefully combining the ingredients and giving them a **random** location on your plate does not mean that all bites have an olive, but the opposite. The random position of the olives actually concentrates them into fewer bites, while an actual pattern would be an olive for each bite. Randomness will result in nearly 40% of your bites being without olives.

What about shapes that we find in nature? Take, for instance, the spots on the back of a cheetah. Most of a cheetah's fur is covered in a similar regular pattern, with the distance between the spots being *too uniform* to be considered random. It is similar to taking a group of 365 and all of them celebrating

their birthdays on different days! *Random* would imply observing some felines which are orange from head to waist and black from waist to tail, but, in the spots of the cheetah, there is uniformity, which is not random. Uniformity is quite a remarkable pattern since it is highly unlikely to appear as a result of pure randomness. The spots observed on the cheetah are not random and so they are a pattern! A similar thing happens with the freckles on the face or the back of a person. Rather than being random, they form a uniform pattern, and the uniformity or the regularity is a pattern.



Spots on the back of a cheetah

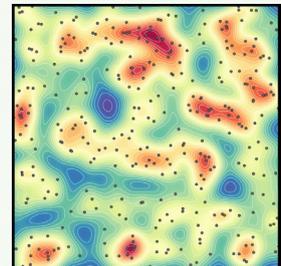
What if, instead of spots we consider stripes, like the ones on a tiger or a zebra, or even the ones in our fingerprints? They too are very uniform and too 'parallel' to be considered random. Like the orientation of a street network in a city or the ripples on a sandy beach, the stripes we observe in nature, like the stems on a leaf or the stripes on a tiger, are a pattern. Patterns in nature are extremely relevant. Rivers around the world have a similar shape, and there is a reason for this pattern. But so is the shape of beaches or the size of the craters on the moon, which also follow a pattern.

A pattern is something that either repeats itself, oscillates, alternates, concentrates or has some key aspect which makes describing it and reproducing it slightly easier. But rolling three fair dice and obtaining at least two with the same number is not a pattern: it is simply

chance – in fact, a 45% chance. Playing the roulette and getting one black, one red, one black and so on is just a coincidence. Your football team winning when you wear those old trousers is just good fortune and your pet cannot predict the results of a match - they are just being lucky.

Patterns occur in space, in time, or both. Think of the number of people at Euston station. It might be quite empty on a Tuesday at 6am, but in a matter of one hour, it will become one of the busiest stations in London: people walking in and out, waiting for their train or arriving from all over the country. And the same happens day after day, week after week. Passengers are synchronised, and so there is quite a clear pattern in the number of people at the station.

But, how do we distinguish whether what we observe is the result of randomness or an actual pattern? Compare it against something that you know is random. For example, drop a box full of pins, one by one, onto a table such that every pin is likely to land anywhere on its surface. Due to pure randomness, there will be regions on the table with more pins than others (just as there are days of the year in which more of your friends celebrate their birthdays) but it does not mean that there is a pattern, since you dropped the pins on random locations. The locations of pins are not a pattern, much as though they might look like one. Comparing against something that you know has no patterns is often the trick, while remembering that randomness also generates what would seem to be patterns!



Random points

Now, what if instead of pins on a table (like the ones in the figure), we consider something serious, like the location

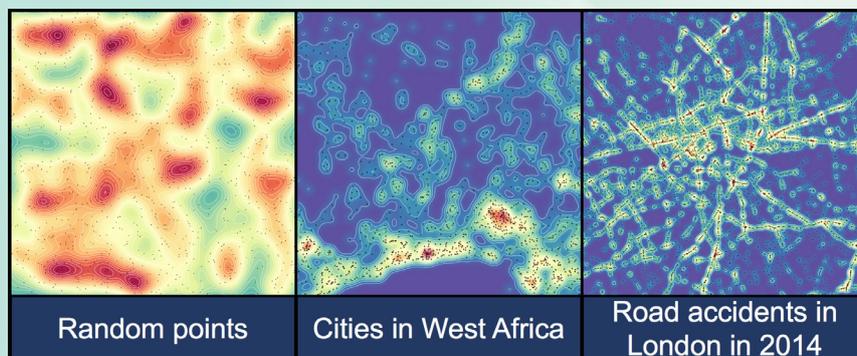
IS THAT A PATTERN?

of robberies in a city? Detecting any pattern would be a powerful tool for the police: if crime happens regularly in just a few places, then the police strategy can also be focused on those specific regions. On the other hand, if crime happens everywhere and there is no pattern, then no matter what the police strategy, their results will also be random. Like the passengers on a train station, however, humans tend to synchronise, and form patterns. In the case of crime, ongoing research shows there are clear patterns in where crime is committed. Indeed, the majority of crimes happen in just a few locations, and so the police actually use this information to plan their strategy. Gang fights follow patterns and even the places in which a serial killer commits their crimes are crucial in detecting where the criminal lives and where they will attack next.

Although comparing might be difficult, mathematics, statistics, computing and simulation are the tools for distinguishing randomness from patterns! The location of

cities and towns show natural features (like the ones in the figure below), such as rivers and mountains, and the places where people take more pictures indicates the positions of major tourist attractions of a city. Looking at the location of road accidents, it is easy to spot avenues and congested junctions and so they also form a clear pattern that is not random. Identifying patterns helps us to understand, predict and improve our cities.

Patterns go beyond aspects of nature: they are also part of our life. The number of friends that you have follows a pattern, and so are the locations that you usually go to in your city. If we analyse where the same banknote is used in two consecutive transactions, we will spot a pattern. The population of cities around the world varies according to very specific rules, and so does the shape of cities, patterns of human migration and even the size of companies. Patterns in nature, physics, our social life and even sports and gambling dictate our life.



The location of cities (using OECD data) and the location of road accidents (using TFL data) against a random pattern. All images courtesy of Rafael and Rudolf.

Equilibrium modelling of financial markets

Daniel C. Schwarz

How are prices in financial markets determined? What factors influence the demand for securities? How is income redistributed across time and across different future evolutions of the world? All these are questions economics attempts to address with the help of equilibrium theory. This theory tries to explain the formation of prices and the behaviour of demand and supply in financial markets with many interacting agents. Its origins date back to the 1870s and in particular the work of the French economist Léon Walras, who in his book *Elements of Pure Economics* (1874) suggested the first models for prices observed in economies. Today's modern approach to equilibrium theory is footed on the influential work of three economists, Kenneth Arrow, Gérard Debreu, and Lionel McKenzie, as presented by Debreu in *Theory of Value* (1959).

Due to the interaction of many agents financial markets are inherently complex and hence difficult to model. Therefore, intuitive arguments and qualitative propositions are a good starting point to elucidate the theory.

Equilibrium theory follows a bottom-up approach to modelling financial markets. It takes as its starting point the attributes of individual agents. Their activities are co-ordinated with the help of a market structure. The behaviour of agents and the structure of the market are then complemented with a set of conditions which makes precise the sense in which the decisions and expectations of agents are considered to be mutually compatible. This leads us to three main ingredients for the construction of an equilibrium model:

- the real side of the economy
- a market structure
- a concept of equilibrium

The *real side of the economy* consists of commodities and agents. Each agent is equipped with a preference ordering and an exogenously given income stream in the form of an endowment of goods. The preference ordering represents an agent's attitude towards the consumption of goods today versus consumption in the future as well as his attitude towards the variability of the unpredictable stream of consumption in the future.

One can easily imagine that, in light of their personal preferences for future consumption, some agents will not consider their initial exogenous endowment of goods optimal. The resulting desire to redistribute income over time and across

the different possible realisations of the future justifies the introduction of a financial *market structure*. The financial contracts that can be bought and sold in the market constitute claims to future consumption streams. Hence, the strategic exchange of such contracts via a market platform can smooth out an agent's future income stream.

The model is now completed by the specification of an appropriate *concept of equilibrium*. According to popular folklore the notion of equilibrium is associated with a balance of demand and supply, also known as the condition that markets have to clear. In the context of an evolving world and an unpredictable future this requires that markets clear in the present and also at all future times and in all future states of the world.

1 A first stylised economy

For the first mathematical treatment of equilibrium theory we drastically simplify the world. We assume that there are merely two dates, today and tomorrow and that the world may end up in any of S different states tomorrow:

$$t \in \{0, 1\} \quad \text{and} \quad s \in \{1, \dots, S\}.$$

Counting the state of the world today as an additional outcome there are a total of $(S + 1)$ states in this economy.

The real economy features a single good, whose value today serves as unit of account, and a total of I agents. An agent is characterised by his income stream, represented by a vector $e^i \in \mathbb{R}_+^{S+1}$, and his preference ordering over all possible future consumption streams, captured by a utility function

$$u^i : \mathbb{R}_+^{S+1} \mapsto \mathbb{R}, \quad i \in \{1, \dots, I\}.$$

Agents participate in the financial market in order to shape their consumption stream $c^i \in \mathbb{R}_+^{S+1}$. Thereby they have to make a choice between consumption today (c_0^i) and consumption tomorrow (c_1^i, \dots, c_S^i), as well as a choice regarding the amounts of consumption in each possible state of the world tomorrow.

The income transfers which agents desire are facilitated by financial contracts which can be bought or sold in the market. We assume there are J contracts, each promising to deliver $V_s^j \in \mathbb{R}$ units of the good in state s tomorrow and each available for purchase today at a price $p^j \in \mathbb{R}$. The defining properties of all contracts in this economy can thus be summarised by a $S \times J$ matrix

$$V = \begin{bmatrix} V_1^1 & \dots & V_1^J \\ \vdots & \ddots & \vdots \\ V_S^1 & \dots & V_S^J \end{bmatrix}.$$

If we wish to include today's price of the contracts in our summary we obtain the $(S + 1) \times J$ matrix

$$W(p) = \begin{bmatrix} -p \\ V \end{bmatrix}.$$

Each agent in the economy now has to make two decisions: what consumption stream does he aspire to and how does he achieve the necessary income transfers? The latter (s)he may influence by purchasing a portfolio $\theta^i \in \mathbb{R}^J$ of contracts today, thereby forgoing or earning income today in return for securing or giving up income in one of the states of the world tomorrow. (If $\theta_j^i \geq 0$ the agent buys an amount of contract j , otherwise (s)he sells an amount.)

In order to ensure the decisions of agents are mutually consistent the concept of equilibrium requires certain compatibility conditions. Specifically, a pair of consumption-portfolio variables $(\bar{c}, \bar{\theta}) \in \mathbb{R}_+^{(S+1)I} \times \mathbb{R}^{JI}$ together with a price vector $\bar{p} \in \mathbb{R}^J$ for the single good is a *financial market equilibrium*, if the equilibrium price vector \bar{p} induces agents, in their pursuit to maximise their utility functions, to choose the pair $(\bar{c}, \bar{\theta})$ and, if this choice of portfolio clears the market in each contract. Mathematically speaking, the variables $((\bar{c}, \bar{\theta}), \bar{p})$ satisfy:

$$\bar{c}^i \in \arg \max \{u^i(c^i) \mid c^i \in \mathbb{R}_+^{S+1}, c^i - e^i = W(\bar{p})\bar{\theta}^i\}, \quad i = 1, \dots, I,$$

$$\sum_{i=1}^I \bar{\theta}^i = 0.$$

In a discrete time economy, of which our setting is the simplest possible example, the mathematical analysis of financial market equilibria is relatively well understood. Under reasonable conditions on the model primitives existence, (local) uniqueness and the optimality properties of equilibria have all been studied.

The existence of equilibria, for example, can be shown to be equivalent to the existence of a fixed-point, which in many cases is guaranteed by Kakutani's fixed-point theorem. The nature of the problem is also heavily influenced by the structure of the financial market. In a nutshell, the richer the opportunities for the exchange of income streams facilitated by the financial market are, the 'juster' the resulting equilibrium allocations turn out to be and the easier the problem often is to solve. If, for example, the dimension of the matrix W equals S , any desired redistribution of income can be achieved and the resulting consumption streams are Pareto optimal—a state of the economy in which no agent can be made better off without some other individual becoming worse off as a result.

2 The study of equilibria in continuous time

Whereas the analysis of equilibria in discrete time is quite advanced, the open problems in continuous time are still abundant and challenging in nature while offering interesting links to different areas of mathematics, in particular stochastic analysis and the theory of partial differential equations.

In a continuous time setting the unpredictability of the future is modelled by an uncountable probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and the presence of time by a filtration $(\mathcal{F}_t)_{t \in [0,1]}$. The variables in the model now become stochastic processes which are progressively measurable with respect to this filtration.

Since the theoretical development in continuous time is far from complete we will illustrate the nature of the problem at the hand of a specific example. As before, there are I agents in the economy who are only permitted to consume income at the terminal time, $t = 1$, when they also receive their endowment $E^i \in \mathcal{F}_1$. Agent's utility functions are assumed to be of the specific form

$$u^i(c) = -e^{-c/\delta^i},$$

for some positive constant δ^i .

The financial market offers the opportunity to trade in one stock, whose price is represented by a stochastic process $P = (P_t)_{t \in [0,1]}$ and which pays a dividend of $V \in \mathcal{F}_1$ units of the good at the end of the time horizon. There also exists a risk-free bond B , which is assumed to have constant value $B \equiv 1$.

In our two-contract, continuous time economy a financial market equilibrium corresponds to a pair of stochastic processes $(\bar{\theta}, \bar{P}) \in \mathbb{R}^I \times \mathbb{R}$ which satisfy the following compatibility criteria: at the end of the time horizon the price of the stock matches the dividend payment V ; the trading strategy $\bar{\theta}^i$ maximises the utility agent i derives from the combined consumption of his profits from trading and his terminal endowment; the market in the stock clears at each time $t \in [0, 1]$. To wit,

$$\begin{aligned} \bar{P}_1 &= V, \\ \bar{\theta}^i &\in \arg \max_{\theta} \mathbb{E} u^i \left(\int_0^1 \theta_t d\bar{P}_t + E^i \right), \quad \forall i = 1, \dots, I, \\ \sum_{i=1}^I \bar{\theta}_t^i &= 1, \quad \forall t \in (0, 1). \end{aligned}$$

Even in this simple setting it can be highly non-trivial to merely prove existence of an equilibrium. Suppose that the probability space supports a d -dimensional Brownian motion W and that the filtration is the one generated by this Brownian motion. In this case Hao Xing and I have shown in a current working paper that there exists an equilibrium $(\bar{\theta}, \bar{P})$ if and only if there exist stochastic processes $((\bar{P}, R), (\sigma, \eta))$ which solve the system of so-called *backward stochastic differential equations*¹,

$$\begin{aligned} \bar{P}_t &= V - \int_t^1 (\sigma_u + \alpha_k \eta_u^k) \cdot \sigma_u du - \int_t^1 \sigma_u dW_u \\ R_t^i &= \frac{1}{2} \int_t^1 \frac{((\sigma_u + \alpha_k \eta_u^k - \eta_u^i) \cdot \sigma_u)^2}{|\sigma_u|^2} - |\eta_u^i|^2 du - \int_t^1 \eta_u^i dW_u, \end{aligned}$$

where α_k are positive constants and $\sum \alpha_k = 1$.

The trading strategies in equilibrium are defined in terms of the solution to the above system. Specifically, they are of the form

$$\bar{\theta}_t^i = \alpha_i + \frac{\alpha_i (\alpha_k \eta_t^k - \eta_t^i) \cdot \sigma_t}{|\sigma_t|^2}.$$

¹Here and below we rely on the Einstein summation convention of summing over repeated indexes.

One easily verifies that they indeed satisfy the market clearing condition imposed by the definition of the equilibrium.

Therefore, in order to prove that an equilibrium exists one has to prove the existence of a solution to a system of equations commonly studied in stochastic analysis. Once again the difficulty of this problem is partly determined by the relationship between the payment structure of the financial contracts in the economy and the model for the random evolution of the world (the probability space).

If, loosely speaking, the stock and the bond suffice to potentially achieve any income redistribution in the economy the above system of equations simplifies to a system of conditional expectations. In this case the well-posedness of equilibrium prices and the associated trading strategies is shown by exploiting the duality between conditional expectations and the theory of linear parabolic partial differential equations (PDEs). Interestingly enough, it does not suffice to prove the existence of a solution to this system of linear PDEs. In order to conclude the well-posedness of the equilibrium the solution needs to be nondegenerate in a specific way. Said nondegeneracy property has been proved—relatively recently in 2012—by Julien Hugonnier, Semyon Malamud, Eugene Trubowitz (2012) and relies on the construction of a solution which exhibits analytic regularity in the time variable.

In the more general case, when only a subset of all possible income redistributions can be achieved in the economy, the above system of backward stochastic differential equations does not simplify. Nevertheless one can still construct a solution to the system by considering a corresponding system of semilinear PDEs with quadratically nonlinear terms. Interestingly, the resulting system bears some resemblance to systems of PDEs which have been studied by Alain Bensoussan and Jens Frehse in—what has by now become—a long sequence of papers concerned with the construction of certain stochastic differential games. However, their results do not apply to our setting directly; once again one needs to prove the nondegeneracy of the solution to the system under study in a specific sense in order to construct a solution to the equilibrium problem.

The work undertaken so far only resembles first steps to a better understanding of financial market equilibria in continuous time. Even small deviations from the economy considered here result in systems of equations which are not well understood so far. For example, replacing the exponential utility function above with a simple power utility function of the form $u(c) = (c^\gamma - 1)/\gamma$, leads one to consider systems of quasilinear PDEs with quadratic nonlinearities about which very little is known (other than counterexamples that demonstrate non-existence in many cases!). Questions such as uniqueness or the stability of solutions are all unanswered to date and offer the opportunity to work on interesting and rich problems which are rooted in classical economic theory and will hopefully be answered one day by new, beautiful mathematical theorems.

En Fase Experimental Podcast

Dr Ruben Perez-Carrasco Clifford Fellow, UCL Mathematics

Every fortnight, between the British Library and St Pancras station, inside a wooden room in the ground floor of the Francis Crick Institute, three scientists discuss the present breakthroughs in science. Unlike many other discussions that happen in this room, this one will reach many other Spanish-speaking people around the world. We are talking about “En Fase Experimental”, the science podcast of the Society of Spanish Researchers in the United Kingdom (SRUK), where the team of hosts Teresa Rayón (Francis Crick Institute), Berta Verd (University of Cambridge) and Ruben Perez-Carrasco (UCL), together with the sound technician Carlos Bricio, have the mission to discuss not only current advances in science but also the role and challenges that STEM research faces in the current world.

In each program we interview a researcher actively contributing to the advance of science

that provides their personal perspective. This season the program featured researchers such as Sergi Castellano (UCL), expert in the extraction and computational analysis of ancient DNA, and Javier Carmona (Springer Nature), discussing his new role as an editor in Nature Medicine. These interviews motivate discussions on current challenges of science ranging from topics such as the impact and repercussion of winning a Nobel prize, to the relevance of open access science in developing countries.

In addition, inspired by the work of Jess Wade (Imperial College), in each program we dedicate a section to narrate the rousing career of female STEM researchers such as the neurobiologist Rita Levi-Montalcini or the computational meteorologist Joanne Simpson.

So if you want to participate in our discussions, or just want a excuse to learn Spanish, this is the podcast for you. You can find us on Spotify, YouTube, iVoox, and on social media #EnFaseExperimental @ComunidadCeru.



Ruben, pictured recording with fellow hosts Teresa Rayón and Berta Verd

Chalkdust Issue 08 Launch Party

Chalkdust, the magazine created by students in the department for the mathematically curious, celebrated the launch of their eighth issue in October 2018. The evening featured food, drinks and a mathematical-themed pub quiz.

The team behind the magazine has since followed this up with the introduction of Talkdust - the podcast for the mathematically curious. You can listen to this online at chalkdustmagazine.com.

Contributor Matthew Scroggs has also created a choose-your-own-adventure-style game for us about creating an issue of Chalkdust, which begins overleaf.



Photographs courtesy Matthew Scroggs

0

It's five months before the next issue of Chalkdust Magazine is due to be released and you are the magazine's new lead editor. If you want to begin by looking for articles to publish, **go to 13**. If you'd rather start by visiting the Chalkdust website, **go to 25**.

1

Two weeks later than you'd originally planned, the magazine is finally ready for printing, but the printers cannot get it ready in time. You will have to throw a launch party for an imaginary magazine. **Game over**.

2

You tweet a request for articles from @chalkdustmag. A few people reply to say they'd like to write something and will have drafts with you in around 2 months' time. **Go to 14**.

3

You share the articles among the team and start uploading them to chalkdustmagazine.com, so that people all over the world can read them. By the time you've done this, the printers have emailed you to say that the magazines are arriving today. **Go to 31**.

4

You get in contact with some people you follow on Twitter who tweet excellent puzzles (including @Cshearer41 and @puzzlecritic), and you email the presenters of the Odds and Evenings podcast to see if they've got any puzzles they'd like to print.

Together with a few puzzles you've written yourself, you end up with a small pile of top quality puzzles to fill gaps in the magazine. **Go to 7**.

5

You decide to write an entertaining article for the De Morgan Newsletter to encourage alumni to write for Chalkdust. Hopefully, they'll read it and send you an email (contact@chalkdustmagazine.com) telling you they'd like to write something. **Go to 14**.

6

A week later, you get together with the rest of the Chalkdust team and discuss the articles. You pick six that you think are among the best and would combine to make a well-rounded magazine. You assign two people to each article to work through them and check that the content of their article is correct and clearly explained. Around a week later, you receive second drafts back from the authors. If you want to print the magazine, **go to 27**. If you want to read through the articles again, **go to 21**.

7

What do you want to do next? If you want to write a crossnumber, **go to 19**. If you want to write some terrible jokes, **go to 30**. If you want to pour tea on your laptop, **go to 28**.

8

The article deadline has arrived, and a medium-sized pile of articles have been submitted. If you want to read all the articles and pick your favourites, **go to 29**. If you want to get your team to read the articles and help you pick the best, **go to 6**.

9

It rains and the magazines are turned into a soggy mess. **Game over**.

10

It's now three days until the magazine launches. If you want to bake cakes for the launch party, **go to 32**. If you want to write the quiz for the launch party, **go to 17**. If you want to tweet about how excited you are, **go to 16**.

11

You get in touch with companies that are interested in recruiting PhD students and ask them if they'd like to sponsor the next issue. G-Research, a company that researches statistical methods and their applications in finance, and TPP, a healthcare IT company, decide to sponsor you. The Department of Mathematics at UCL has also offered to give you some money towards the next issue.

What do you want to do next? If you want to look for puzzles to put in the next issue, **go to 4**. If you want to look for someone to interview, **go to 36**. If you want to look for images to use on the cover, **go to 20**.

12

Your design team make a cover that looks like this:



Once you've finished enjoying it, **go to 7**.

13

The majority of the articles that appear in Chalkdust are written by people outside the team. This means that encouraging people to write articles is one of the first jobs to be done for each issue.

Where do you want to start your search for articles? If you want to ask alumni of UCL mathematics, **go to 5**. If you want to ask on Twitter, **go to 2**. If you want to take articles from other magazines, **go to 22**.

14

What do you want to do while waiting for the articles to arrive? If you want to look for sponsorship, **go to 11**. If you want to look for images to use on the cover, **go to 20**. If you want to write an article yourself, **go to 26**.

15

You and the team move all the magazines upstairs in the lift, and put them in the Chalkdust space behind the lockers. **Go to 10**.

16

You tweet about how excited you are about the launch party. It gets retweeted by @MathematicsUCL, the Department's new Twitter account. It gets a few more retweets and likes and you realise that you're not the only one excited about launching the new issue. You next need to write the quiz for the launch party, **go to 17**.

17

You write some questions for the launch quiz. Your favourite creation is for the wipeout round: Five of the following are real Wikipedia articles on the English language Wikipedia, the other five are made up. You get three points for each correct one you identify, but if you write down any incorrect answers, you lose all your points for this question.

Elementary symmetric polynomial

Timeline of geometry

Arithmetic group theory

Inequality of arithmetic and geometric means

Inequality of addition and multiplication

Galois knot theory

Topological polynomial

Topological combinatorics

Equation solving

Topological number theory

The answers are given on the back cover of the newsletter. Once you've checked your answers, **go to 39**.

18

You send the magazine to the printers. It's a shame you didn't try to catch any typos and mistake before printing, but it'll do. **Go to 3.**

19

You design a crossnumber to be included in the magazine. Your favourite clues include:

7A. This number is equal to the sum of the factorials of its digits. **(3)**

11D. The number of squares (of any size) on a 13,178-by-13,178 chessboard. **(12)**

39A. Why is 6 afraid of 7? **(3)**

Maths Gear, a website that sells nerdy things worldwide, agree to offer a large goody bag as a prize for the crossnumber. **Go to 8.**

20

You get the team together and discuss ideas for the cover. You come up with a shortlist of two images. If you want to put pictures of hydrogen orbitals (created by Tom Rivlin) on the cover, **go to 33.** If you want to put Chladni figures (created by James Cann) on the cover, **go to 12.**

21

You and the team read through the articles again and catch errors in grammar and spelling, as well as a few typos in some equations. If you want to print the magazines, **go to 27.** If you want to typeset the articles, **go to 23.**

22

You are taken to court for copyright infringement. The Chalkdust budget cannot cover a lawyer so you are sent to prison forever. **Game over.**

23

You share the articles between the team and use LaTeX to make them look like Chalkdust articles. This takes a while and it looks like you might run out of time. If you want to get together on a Saturday to finish everything,

go to 34. If you want to let everything overrun and assume it'll be ok, **go to 1.**

24

You send the magazine around the team for proofreading. The team catch a few spacing errors, typos and other small mistakes. You correct them and now the magazine is perfect. To print the magazines, **go to 37.**

25

You open your web browser and visit chalkdustmagazine.com where you find a very informative and entertaining article about the best fonts for maths. After you finish this, you find more excellent articles: you read about mathematical crochet, then different voting systems, then you find some puzzles to solve, then you read about snowflakes and wormholes and...

By the time you have finished reading everything that looked interesting, you realise that five months has passed and you've not started work on the next issue. **Game over.**

26

You write an article about your favourite bit of maths and add it directly to the pile of submissions. (There's obviously no need for you to email to contact@chalkdustmagazine.com as anyone else would do because you're the one who reads emails sent there.)

What do you want to do next? If you want to look for puzzles to put in the next issue, **go to 4.** If you want to look for someone to interview, **go to 36.** If you want to look for images to use on the cover, **go to 20.**

27

You send a collection of unfinished drafts in different formats to the printers. They do their best, but the magazines they send you look awful. **Game over.**

28

Your laptop is ruined, and you've lost all the work you've done on the next issue. **Game over.**

29

You have terrible taste so the next issue of Chalkdust is dreadful. **Game over.**

30

You write some truly dreadful jokes. The best of them are for Dear Dirichlet, the agony uncle column, including an excellent joke about badgers. **Go to 8.**

31

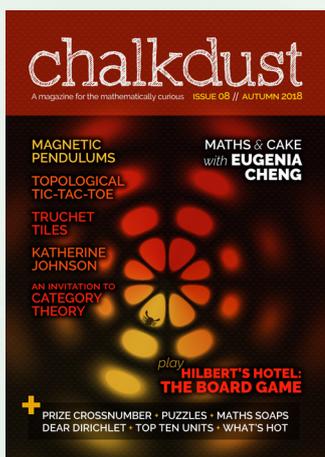
The magazines arrive. If you want to leave the magazines where they are for someone else to sort out, **go to 9**. If you want to carry them up the stairs, **go to 38**. If you want to take them upstairs in the lift, **go to 15**.

32

You bake some butterfly cakes for the launch party. You try one and they are delicious. You now need to write the quiz for the launch party. **Go to 17.**

33

Your design team make a cover that looks like this:



Once you've finished enjoying it, **go to 7**.

34

You organise Chalky Saturday, the day on which the team comes together and finishes everything ready for printing. After a long but fun day full of hard work, tea, biscuits and pizza, you finish the day by gathering everyone together to write the contents page and editorial. And it's done! You've finished the next issue. To proofread the issue, **go to 24**. To print the magazines, **go to 18**.

35

You are locked in the stationary cupboard on the 6th floor. How on Earth did you get here?! **Go to 35.**

36

You call a team meeting to discuss people who you could interview. You decide to interview *[insert name of your favourite mathematician here]* and pick two members of the team to carry out the interview. **Go to 7.**

37

You send the magazine to the printers. They will arrive in a week's time. **Go to 3.**

38

You carry all the magazine up the stairs. You're exhausted. Why didn't you use the lift? Or at least ask some people to help?! **Go to 10.**

39

You go to the Chalkdust launch quiz. It's great fun and everyone loves you and your magazine.

Congratulations, you win! Wait one month then **go to 0** to start on the next issue.

Epilogue

Written by Matthew Scroggs, UCL PhD Student and frequent Chalkdust contributor.

Elementary modular arithmetic and Archimedes' *dimensio circuli*

1 Most people who know anything about Archimedes' bounds for what we call π , that is $3 \frac{10}{71}$ and $3 \frac{1}{7}$, suppose that it has been well or even completely understood for a long time now, even centuries or millennia. However, considering the six features listed below, this is not the case; there is something new to be said on all or nearly all of them. In fact, as Knorr⁽¹⁾ says in his massive *Textual Studies*, “a close reading reveals unexpected subtleties.” One of these, not mentioned by him, is the subject of this article. The features mentioned are these:

- (a) Egyptian (“unit fraction”) arithmetic, seen in many places. Such notations survived into early modern times, although the associated techniques had already been superseded much earlier;
- (b) The inequality $\sqrt{a^2 + b} < a + b/2a$, used to obtain upper bounds;
- (c) Difference of two squares, for comparisons, avoiding the use of large numbers;
- (d) The *h.c.f. process* to reduce awkward fractions. This is what the Greeks called *anthyphairesis*, or something similar, that is reciprocal subtraction, which is described by Fowler⁽²⁾ in his work on the mathematics of Plato's Academy.
- (e) *Elementary modular arithmetic*, also for reductions;
- (f) A form of *binary degradation* to obtain lower bounds from upper bounds. *Unitary degradation* is also a possibility.

Armed with these tools, with which it is possible to make the rather sparse surviving remnants of the original text come to life again, albeit in a way not entirely identical to current reconstructions, a way can be found conformable to what is known of Greek number systems and mathematics. It's not possible, in the space available, to show a complete reconstruction, so I will concentrate on (e), one of the new ideas.

2 Little is said here about either the *geometry* of the problem, or about the proof methods due to the fourth century (B.C.) polymath, Eudoxos of Cnidos, that are applied to this geometry. This article deals mainly with the *arithmetic* of the problem, and then only with a small part of that. I will, however, mention two extraneous matters. One: the geometry used is Pythagoras' theorem (Euclid I.47) and the angle bisector theorem (Euclid VI.3), which are applied to the 30-60-90 triangles inscribed and escribed in the circle, with one side of each forming its diameter. The 30 degree angles are then bisected four times to give regular 12, 24, 48 and 96-sided polygons whose sides approximate the circles. And two: one will need good upper and lower bounds to the ratios for $\sqrt{3}:1$, and Archimedes used the ratios 265:153 and 1351:780. These are the best ratios for the given denominators, which curiously are both triangular numbers. (The number 153 also appears mysteriously towards the end of St John's Gospel!) There are numerous conjectures about how these were obtained, whether by Archimedes or his predecessors, *e.g.* by using continued fractions, or an extension of the side-and-diagonal numbers of later Antiquity, or some more recent proposals, but it would take us too far afield to describe them here. The inequalities can, of course, be easily verified.

3 First, only the bare bones of Archimedes' text have survived from Antiquity (he died in 212 B.C.). In the arithmetic stages this means intermediate results. Much effort, not all of it very plausible, has been extended to filling in the gaps. On what I call the modular stage, nothing is to be found. The two surviving source-texts are the received Greek version and the medieval Arabic version, which are very similar. I quote from the Greek version of the sequence leading to Archimedes' lower bound, in translation:

“therefore ... AT has to TG a lesser ratio than $5924 \bar{2}\bar{4}$ to 780, or 1823 to 240. For each is $4/13$ of the other.”

$\bar{2}$ is Neugebauer's notation for $1/2$; $\bar{4}$ for $1/4$. Thus $\bar{2}\bar{4} = 1/2 + 1/4 = 3/4$. These are examples of Egyptian unit fractions, as we call them, which are explained in some detail both in the Rhind Papyrus, which is in the British Museum, and in various modern texts.

Confirmation of the equality, in various ways, is easy enough, both for Ancient Greeks (this means “Greek-speaking” – they were scattered over the Eastern Mediterranean and Black Sea coastlines) and for us. The former might prefer to use *anthyphairesis* (reciprocal subtraction) as per the h.c.f. process, which leads to identical sequences of integer quotients.

One question here is “where does the $4/13$ come from?”; another (less obviously) is “how have the original ratios become so exact when they weren't so originally?” In a second and similar example, the text tells us that $3661 \frac{9}{11}$ to 240 is in the ratio of 1007 to 66, each being $11/40$ of the other. It turns out that not only the $4/13$ and $11/40$ need to be explained, as to their origin, but so also do the $3/4$ and the $9/11$, which have arisen to fit the procedure we describe. It is important to realise that the bare-bones text proceeded directly (and correctly) to $5924 \frac{3}{4}:780$ in the first example quoted above, and to $3661 \frac{9}{11}:240$ in the other one, also correctly. And further that the two ratios are very conveniently equal to $1823:240$ and $1007:66$, respectively, in which the smaller numbers involved ease the rather cumbersome arithmetical procedures available at the time, which although cipherised were not place-value. (The much earlier Babylonians already had a place-value system, with base 60, but it had not spread to the Greek-speaking world until after Archimedes' time, when it was mainly used in astronomical work.) But it does seem remarkable that the additional fractions, $3/4$ and $9/11$, fit so well, and we now explain how they might well have been obtained as parts of Archimedes' accurate chains of inequalities.

4 Having set out the two examples to be explained, here is the explanation of the first. The 5924 starts life as $3013 + 2911$, and in turn this starts as $\sqrt{(2911^2 + 780^2)}$, which is less than 3015 and a bit. This is got from 1(b), known much earlier, and which in turn can be found from the square-root formula $\frac{1}{2}(a + N/a)$ for a better approximation (actually an upper bound, as $A > G$), which is mentioned by a first century A.D. writer, admittedly later than Archimedes, but our knowledge of Greek arithmetic is very fragmentary compared with (say) that of Greek geometry. In fact,

the square root is not only less than 3015, but also than 3014. Then 3013 is an upper bound, as $3013^2 - 2911^2 = 5924 \times 102 < 780^2$, by 1(c) above. The other results are confirmed similarly. The quoted square root also = $\sqrt{(3013^2 + 4152)}$ which is $< 3013 \frac{9}{13}$ by 1(b) and the h.c.f. process, preserving the inequality. The $\frac{9}{13}$ reappears a few lines below.

Now we turn to the modular work. What we are looking for is $5924 + a/b$, such that $5924b + a$ is a multiple of a suitable factor of 780. There are many possible choices for this factor, as $780 = 2.2.3.5.13$, not all of which are suitable in giving a worthwhile multiplicative reduction. But 13 works in this respect. To find suitable fractions a/b , we observe that 5928 is the next integer above 5924 to be divisible by 13. As $5928 - 5924 = 4$, we then need to consider the sequence $\{4b \pmod{13}/b\}$ for $b = 1, 2, 3, \dots, 13$, viz. $4/1, 8/2, 12/3, 3/4^*, 7/5, 11/6, 2/7, 6/8, 10/9, 1/10, 5/11, 9/12, 0/13$. The first in this sequence in $(9/13, 1)$, and therefore suitable, is $3/4$, indicated by the *. This is not a *necessary* condition as far as the $9/13$ is concerned, but it is *sufficient*; it so happens that Archimedes' $3/4$ (as we may call it) satisfies it. Hence the $5924 \frac{3}{4}$. The necessary multiplier is then $4/13$, where the 4 is the “new” value of b and the 13 the chosen factor of 780. The 4 makes this fraction give a reduction factor of $4/13$, which is worth having, and $5924 \frac{3}{4} : 780$ becomes $1823 : 240$. We come to the choice of a suitable factor (here 13) further on, in §6.

5 For the other example, observe that $\sqrt{(1823^2 + 240^2)} < 1838 \frac{4}{5}$, also by 1(b) and the h.c.f. process. (This depends on how you do it. An alternative would have the square root equal to $\sqrt{(1838^2 + 2685)}$, and this leads to $1838 \frac{3}{4}$ by the same process.)

Now consider $1838 : 240$ (and *not yet* $1838 \frac{9}{11} : 240$; the fraction $9/11$ derives from the *subsequent* working), and observe that $1823 + 1838 = 3661$. We then note that the next integer above 3661 to be divisible by 40 (the preferred factor of 240; again see §6 for how the preferred factors are obtained) is 3680, involving $a = 3680 - 3661 = 19$ in this case. The sequence (in part) is $19/1, 38/2, 17/3, 36/4, 15/5, 34/6, 13/7, 32/8, 11/9, 30/10, 9/11^*, 28/12, 7/13, \dots, 0/40$. The first suitable fraction, which gives the best reduction factor, *i.e.* in $(4/5, 1)$ or $(3/4, 1)$, is the eleventh term, $9/11$, the above sequence being $\{19b \pmod{40}/b\}$, $b = 1, 2, 3, \dots, 40$. The multiplier is then $11/40$, and all (or almost all) is revealed. We must now explain the choice of divisors.

6 The choice of divisors needs some organisation to ensure that all possibilities are considered. The square roots involved are usually irrational. The inequality process will give a fractional upper bound, and the h.c.f. process an upper bound to that, using an odd number of steps; three is adequate. We then need a further upper bound, which I call the *modular fraction (m.f.)*, with fractional part less than 1. This *m.f.* should be big enough to maintain the chain of inequalities, but small enough not to render the eventual reduction factor worthless. This corresponds to a consecutive use of features (b), (d) and (e) of §1 above.

The two cases here, 780 and 240 ($= 2.2.2.2.3.5$), lead to 24 and 20 distinct divisors

respectively, not all of which are useful. In the case of 780, the small ones are useless, but all those divisible by 13 would do, giving $\frac{3}{4}$ as the *m.f.*, and mostly leading to $\frac{4}{13}$ as the reduction factor. The simplest case is of course 13, so Archimedes (or our version of him) is vindicated, as one would expect. Similarly, in the 240-case, the divisor 20 offers $\frac{9}{11}$, and a reduction factor of $\frac{11}{20}$, which isn't marvellous; 30 offers $\frac{13}{17}$ and a reduction factor of $\frac{17}{30}$, which seems similar, but if followed through leads to $\frac{10}{71}$, and hence $3 \frac{10}{71}$, one of Archimedes' end-results, the same as is obtained from the divisor 40, whose reduction factor is a useful $\frac{11}{40}$. The divisors 60, 80 and 120 all give $\frac{9}{11}$ for the modular fraction and either $\frac{11}{20}$ or $\frac{11}{40}$ for the reduction factor. Hence the choice of divisor has in each case been a dominant and effective one, not improvable as to its end-result.

We can't know if Archimedes did all the analysis just outlined, but it is hard to believe that both reduction factors were just lucky strikes; the arithmetic is perfectly possible in the available notations and the elementary modular arithmetic is almost trivial. Was the use of this modular arithmetic *necessary*? No, but it's certainly a clever *jeu d'esprit*, and possibly a useful exercise for any assistants he might have had, a subject on which nothing is known. It's also worth noting that the modular method described above provides a *general* procedure for reducing or simplifying fractions, irrespective of the application shown here. Although it is certainly possible to obtain the stated results by *ad hoc* methods, rather than by modular arithmetic, we shall probably never know for certain what Archimedes' actual method was.

7 Finally, modular arithmetic is usually said to have been introduced by Gauss before 1801; but it is more than likely that Archimedes got there first – it almost fits too well to be otherwise. There is nothing strange about the recovery of such a long-lost device. The early twentieth century saw the recovery of Archimedes' important *Method of Mechanical Theorems*, thought since Antiquity to be lost, and other texts including his *neusis* construction of the regular heptagon found in a medieval Arabic manuscript. Other works, now known only by name, might still be discovered.

- (1) Wilbur R. Knorr, *Textual Studies in Ancient and Medieval Geometry* (Birkhäuser, 1989). The relevant parts of this long work are at pp. 484-489.
- (2) David H. Fowler *The Mathematics of Plato's Academy* (Oxford, 1987.) Important for describing *anthyphairesis* and many of its applications.

The author, Dr Jon V. Pepper, was an Honorary Lecturer in the Department from 1988 to 2005, and was elected a member of the International Academy of the History of Science in 1971. He worked mainly on the unpublished scientific and mathematical manuscripts of the Elizabethan polymath Thomas Harriot, which included the first known rectifications of plane and twisted curves and the related construction of conformal mappings. Currently he is working to create a Website on topics in the history of mathematics.

Women in Mathematics Day

Dr Kim Moore

Established in 1826, UCL prides itself on being one of the first universities in England to admit women on equal terms with men in 1878. It is fitting, then, that the Department of Mathematics appointed one of the first female professors of mathematics, Professor Susan Brown, at a UK mathematics department. Today, the UCL Mathematics Department is led for the first time by a woman, Professor Helen Wilson, one of four female professors at the department, which has been presented with a Silver Athena Swan Award in recognition of its commitment to the recruitment and progression of female students and staff. Whilst great progress has been made, there is still work to be done, and one area where we really want to make a difference is to encourage the many talented school age women across the country to consider studying for a degree in mathematics, whether at UCL or at another institution.

The Women in Mathematics taster day at UCL was pioneered in the early 1990s by Dr Bill Stephenson, aiming at encouraging aspiring female mathematicians to study maths at UCL. This event has been held almost every year since, with this year's meeting being held on the 2nd July 2018.

There were over 20 promising young women in attendance at this year's event from London and beyond. The students were all studying for A Level Mathematics and were seriously considering studying for a degree in mathematics or a closely related subject. The event was a fantastic opportunity for the participants to learn more about studying maths at university, meet UCL staff and current students as well as meeting other like-minded potential maths undergraduates.

The day began with a talk by the Department's Professor Christina Pagel, on "Using maths in real life to help people make better decisions in the NHS and beyond". Christina is the first female Director of the Clinical Operational Research Unit at UCL, and she is passionate about improving the delivery of healthcare using mathematics. Her talk gave the aspiring mathematicians a real insight into one of the many important real world applications of mathematics.

After a short break, the students were put to work in a puzzle and problem solving session. Their challenge: work in a group to calculate the approximate number of Smarties in a sealed box. There were a wide range of interesting methods applied to the problem, with a large variation in number of Smarties guessed! Whilst some methods worked better than others, the young women learnt that there isn't always a right way to solve a problem, and working together in a group is a great way to share ideas that you might not have thought of yourself.



Dr Kim Moore, Research Fellow

In the afternoon, I gave a pure mathematics talk on "Complex numbers: Our imaginary friends". I am a postdoc at UCL, specialising in geometric analysis, with some of my work inspired by higher dimensional analogues of complex numbers: quaternions and octonions. The students learnt about what the addition and multiplication of complex and quaternionic numbers means geometrically,

and that the quaternions have a surprising real world application to three-dimensional computer graphics in computer games, which just goes to show - however abstract a piece of mathematics may look, it might just turn out to be useful one day!

The day closed with a session led by Dr Robert Bowles, Senior Lecturer and Admissions Tutor in the Department, and Kate Gault from the UCL Careers Service, on admissions and careers, to give the students an insight into the application process for a mathematics course and what potential careers might be open to them if they did complete a mathematics degree.

The day offered a great chance for some promising young women to experience what might await them in the future if they choose to study maths, and we hope to see them again in the future. We are already looking forward to next year's Women in Mathematics taster day, to be held on the 1st July 2019.

Women in STEM Society

Sophia Dryden 2nd Year BSc Mathematics

UCL Women in STEM Society was established this year with the aim to promote diversity and inclusion within STEM (Science, Technology, Engineering and Mathematics) disciplines. The society was established by four Mathematics students who noticed there was not a student society like this, despite there being a need.

The society is aiming to empower all students by offering inspiring talks from prominent women in STEM careers, networking sessions, workshops and social events.

We kicked off the year with a free pizza event to spread the word about the society, letting people know who we are and what we plan to do.

We welcome and encourage everyone to join regardless of gender or course of study.

Facebook page: @uclwomeninstem
Instagram: @uclwomeninstem



Women in STEM Society founders: Sophia Dryden, Nirojana Shanthakumar, Rhea Alexander and Zoe Steele

Financial Mathematics Team Challenge

Dr Andrea Macrina

The fifth Financial Mathematics Team Challenge (FMTC) took place from 26 June to 6 July 2018 at the University of Cape Town (UCT). Four teams participated, and the team members were MSc and PhD students from UCL, UCT, University of Oxford and ETH Zürich. From UCL, Financial Mathematics MSc students William Black, Aurore de Savigny, Yiyang Wang, Yiran Wang, and Mathematics PhD student Holly Brannelly (who led one of the teams) participated with much enthusiasm and great team spirit.

Launched for the first time in 2014, the FMTC is jointly organised by UCT's African Institute for Financial Markets and Risk Management and UCL Mathematics. Encouraged by the huge success of the first team challenge, the FMTC has meanwhile become an annual ten-day event. Each year international and South African MSc and PhD students gather to form four or five mixed teams led by a PhD student and mentored by an academic or an industry practitioner. The goal of the FMTC is to create an opportunity for postgraduate students studying in a developing country to interact with international students and academics. Students work together in teams on a topical and industry-relevant research project without any distractions. At the same time, the FMTC produces a platform for developing lasting links with overseas postgraduate students and experienced scientists.

The 2018 FMTC research projects were on *JIBAR Dynamics and Short-Dated Caplets*, *Portfolio Optimisation under Uncertainty*, *An Assessment on the Appropriateness of the use of the LFMM in South Africa*, and on *Commitment Scheduling for Private Equity Investments*.



FMTC 2018 UCL delegation. From left to right: Yiran Wang, Aurore de Savigny, Andrea Macrina, Holly Brannelly, William Black, Yiyang Wang.

The FMTC teams are assembled so that students learn to work in diverse teams while they are exposed to a beneficial dose of competition. The hosting institution benefits from the presence of the international researchers who are often invited to give talks at regular seminars and colloquium series. This creates additional opportunities for other researchers at the hosting university to exchange knowledge with the visiting academics and industry practitioners who stay for the whole duration of the FMTC. The visiting scientists have a chance to share research ideas and interests with local researchers - getting a glimpse of the dynamical and innovative environments at institutions of less-known addresses. There can definitely be exciting finds!

One month before the FMTC students arrive in Cape Town, each mentor provides their team with preliminary reading material. The participating students are thus encouraged to immerse themselves in the topic, which is proposed by their mentor, without having any knowledge of what the actual research problem is. The mentor only discloses the problem to their team on the morning of the day that the team challenge begins! The teams

scramble and the first management challenge for the team leader, the PhD student on the team, is to recognise the strengths of their team members and efficiently allocate tasks. In fact, the research problems are such that it is materially impossible for any team member to solve the problem on their own, including the team leader themselves.

Each FMTC team works seven days with one (compulsory) day off in-between, and all teams are required to present their findings on the final two days by delivering an hour and a half long presentation to all participants and a select group of assessors comprising academics and industry practitioners. In addition, each team is required to submit a written report containing their results. The reports are then made available via a repository in the form of a volume with the aim to spur future youth to embrace scientific research and contribute to the advancement of knowledge and science. On the final day, the winning team is announced and a floating trophy is awarded: the names of the winning team members are engraved on the trophy.



FMTC 2018 winning team with mentors and FMTC organisers. L-R: Mbele Bandile (UCT), Prof. Martin Larsson (ETH Zürich, mentor), Dr Andrea Macrina (UCL, organiser), Cole van Jaarsveldt (UCT), Prof. David Taylor (UCT, organiser), William Black (UCL), Dr Lisa Powers (StepStone, mentor), Johannes Wiesel (Oxford).

Students on the FMTC face a very intensive and draining period of hard work, but the learning curve is steep and often they emerge from this event transformed. They gain in confidence and self-esteem, knowing that they now have the experience to tackle challenging research questions. Some of the participants discover a deep interest in research and embark on PhD studies they thought unattainable. All of them develop a consciousness of being able to contribute to a team effort and discover the power of collaborating in teams with a common research goal. Furthermore, new friendships are born, which keep developing and open up new avenues. It is a truly touching experience to witness each year the energy and the enthusiasm generated by the participating students. Not to mention the high quality of the scientific work and results the teams produce in just seven days, which sometimes is accepted in peer-reviewed journals for publication.

Propelled by the consistent positive feedback by past FMTC participants and the industry, which over the years has grown keen to hire students with an FMTC experience, the first FMTC Brazil (FMTC-BR) was hosted by the Fundação Getulio Vargas (FGV) in Rio de Janeiro from 8 to 18 August 2018. Students from Australia, Brazil, Canada and the USA focused on research problems arising in *machine learning to solve nonlinear and high-dimensional PDEs, applied to the computation of option sensitivities, and in connection with stochastic control in algorithmic trading*. The FMTC-BR was a resounding success with students' feedback highlighting how much they had learned within a very short period of time in a friendly and motivating work environment. In addition, visiting academics delivered seminar talks and they were invited to reflect together with the Head of School of Applied Mathematics on future research directions in Financial and Insurance

FINANCIAL MATHEMATICS TEAM CHALLENGE

Mathematics and its importance and potential in Brazil.



Participants of the first FMTC Brazil hosted by FGV EMAp. ©biancagensfotografia 2018

From the start in 2014, the FMTC has endeavoured to involve the industry, chiefly by inviting them to propose cutting-edge industry research problems and by encouraging co-mentorship of an FMTC team. The goal has been to facilitate skills and knowledge transfer and to produce an opportunity for the participating students to prepare for the industry job market should they aspire to secure an industry career. An important example was the collaboration with StepStone, a global private markets firm, and their Vice President of Research, Dr Lisa Powers, who co-mentored the project on commitment scheduling for private equity investments at the 2018 FMTC in Cape Town. Reflecting on the students' experience on the FMTC, Dr Powers noted: "Working with people, whom you may not know, to solve a complex problem and then communicating your findings in writing is similar to a business development cycle that I see daily in my work."

Looking ahead it is a pleasure to see the 2019 FMTC editions in Cape Town and Rio de Janeiro already confirmed and the prospect of infecting more young and keen students with the love for scientific research. The FMTC has benefitted from government and industry

funding, and the hope is to find more firms in the private sector which recognise the importance of this initiative, especially for developing economies.



FMTC Brazil 2018. Prof Yuri Saporito (FGV, mentor) at his desk and Prof Rodrigo Targino (FGV, local organiser). ©biancagensfotografia 2018

Students described their experience on the FMTC as follows: "[The] FMTC as an experience was great. I thought it was very well thought through, especially with the mentor/team leader structure. [The FMTC] helped me personally think about my strengths (and weaknesses) going forwards in a career mindset, [...] I'd highly recommend. This was my first time using what I had learned in class to solve real world financial problem. I found this experience really interesting. I worked a lot but I liked it and this made me appreciate this subject even more than I used to. I definitely enjoyed the team work; after a couple of days I already felt my team like a family. It was very interesting, educative and [the FMTC] exposed me to material that would have required me to sit down and read lots of different books. It was a very intense experience--the learning curve was very steep, the problems were really interesting, and everyone had a very friendly, but laser-focused and serious vibe. It was awesome being able to interact with other Masters/PhD students."

Students and young researchers in developing countries sometimes can feel isolated from the big international centres of research and innovation. The enduring scarcity of Science, Technology, Engineering and Mathematics (STEM)-educated young talent produces much competition in the industry to secure the graduates with such a background. Often STEM-graduates, already after completing a first university degree, are allured to join private firms by big salaries. In areas where personal and family situations are dominated by constant financial hardship and other challenges less prevalent or altogether absent in developed economies, relatively fat remuneration, especially offered by financial and tech companies, are hard to forego in favour of an academic path promising little in terms of financial security. Such dynamics can produce vicious circles and exacerbate brain-drain from higher education institutions, which struggle to grow and foster future generations of domestic researchers and academics. Attracting established or promising scientists from cutting-edge international research hubs is even harder.



Winning team of the first FMTC Brazil, 2018. From left to right: Gabriel Jardim (FGV), Ali Al-Aradi (University of Toronto), Adolfo Correia (IMPA, Rio de Janeiro), Danilo Naiff (Universidade Federal do Rio de Janeiro), Prof. Yuri Saporito (FGV, mentor). ©biancagensfotografia 2018

The FMTC, even if just a drop in an ocean, endeavours to create opportunities in particular for keen, less privileged students at their institutions, at home, in places with much drive and aspiration, which try to break what sometimes seems to be a circle with little hope of escaping from. In 2018, three ambitious and talented South Africans completed their PhDs in Quantitative Finance at UCT, who before beginning their PhD research were poised to join the financial industry. We like to think their experience on the first FMTCs may have helped bring these bright minds closer to the fascinating world of scientific research. It is all the more rewarding to see that two of them have stayed on at UCT as Research Associates and now contribute to the training of the next generation of research students and assist the local industry with research and innovation. May they become the professors and research directors of tomorrow and pull their developing country forwards.



2018 FMTC, Cape Town. Some of the participants

For queries about the FMTC, please contact Dr Andrea Macrina (UCL Mathematics): a.macrina@ucl.ac.uk

Observing the eigenfunctions

Matteo Capoferri

January 6, 2019

An eigenfunction of the Laplace operator Δ on a bounded domain $\Omega \subset \mathbb{R}^n$ is a function $\varphi : \Omega \rightarrow \mathbb{R}$ satisfying

$$\Delta \varphi = \lambda \varphi$$

for some nonpositive real number λ , the corresponding eigenvalue. Depending on the problem at hand, different boundary conditions may be specified. More generally, Ω and Δ can be replaced by a compact Riemannian manifold with or without boundary and the Laplace–Beltrami operator, respectively.

Eigenfunctions and eigenvalues of the Laplacian are intriguing, yet elusive, animals. In spite of being fundamental objects in almost any branch of pure and applied mathematics, it is in general not possible to compute them explicitly.

Far from pure speculation, the knowledge of eigenvalues and eigenfunctions is often essential in real life applications. In fact, they can be associated with the *natural frequencies* of certain physical systems, i.e. the frequencies at which systems tend to oscillate in the absence of a driving force. The excitation of the system at a frequency that matches one of the natural frequencies results in greater and greater oscillations – a *resonance* – which may eventually damage the system and break it apart. Assume you are to build an airplane. If you want to prevent it from disintegrating, you have to make sure (among other things!) that the vibrations generated by the engines do not resonate with the natural frequencies of the fuselage of the airplane in an uncontrolled way. To do this, you effectively need to solve an eigenvalue problem and find the first few eigenvalues.

Even though we cannot compute eigenfunctions exactly, we can get a rough idea of what an eigenfunction looks like by studying its *nodal lines*, i.e. the collection of points of Ω where the eigenfunction is zero. Nodal lines divide Ω into connected regions – the *nodal domains* – where the eigenfunction is either strictly positive or strictly negative.

Quite amazingly, nodal lines and nodal domains can be observed experimentally. This realisation goes back to the eclectic German-Hungarian physicist Ernst Chladni (1756–1827) who, building upon previous studies conducted by Robert Hook (1635–1703), investigated the natural modes of vibration of rigid plates. Chladni ran a violin bow on the edge of a metal plate dusted with sand and observed the emergence of distinguished patterns – the *Chladni figures*. Normal modes of a rigid plate are solutions of the eigenvalue problem

$$\Delta^2 \varphi = \nu \varphi$$

subject to free boundary conditions, modulated sinusoidally in time. When the bow induces vibrations at normal frequencies, nodal domains oscillate up and down, so that the sand concentrates along nodal lines, drawing a pattern on the surface of the plate.

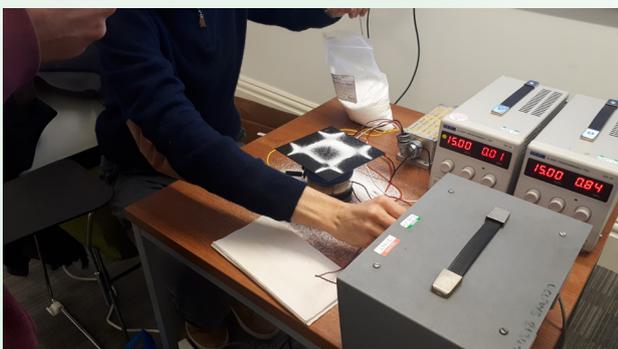
MC: PhD student & TA, Department of Mathematics, University College London, Gower Street, London WC1E 6BT, UK; matteo.capoferri.16@ucl.ac.uk, <http://www.ucl.ac.uk/~ucahmca/>

Even though Chladni did not understand the underlying maths, he appreciated the potential of his discovery and started advertising his techniques all around Europe in public demonstrations. When Napoleon attended one of such performances in Paris, he was so excited by the beauty of Chladni patterns that he authorised an extraordinary prize, sponsored by the Paris Academy of Sciences, to whoever would be able “to give the mathematical theory of the vibration of an elastic surface and to compare the theory to experimental evidence”. The only entry in the competition was by Sophie Germain (1776 – 1831), a French mathematician known for her seminal contributions to elasticity theory. Unfortunately, her submission contained mathematical mistakes and the contest was re-opened. She did not give up and started consulting with Poisson. On her third attempt, she was eventually awarded the *prix extraordinaire*, thus becoming the first woman to win a prize from the Paris Academy of Sciences.

The Chladni experiment is a rather entertaining didactic tool, which gives a first-hand experience of the eigenfunctions of the squared Laplacian. All you need is a mechanical wave driver, a signal generator & amplifier, a metallic plate and some silica sand. By means of the signal generator one can control the oscillations of the mechanical wave driver, which is connected to the plate. Varying the frequency of the signal within the audio range, approximately from 150 Hz to 20 kHz, one observes the sudden appearance of sand patterns at the natural frequencies of the plate. This can be used to verify experimentally theoretical predictions such as, for instance, the symmetries of the eigenfunctions or the behaviour of the gap between two subsequent eigenvalues as we progress in the spectrum. A similar experiment with the plate replaced by a clamped elastic membrane allows one to visualise the eigenfunctions of the Laplacian, as opposed to the Laplacian squared.

Chladni’s technique survived the centuries and is still used nowadays in industrial processes – for example, the quality control of musical instruments. As the reader may by now appreciate, it does something more than just creating beautiful patterns: it brings eigenfunctions to life.

Acknowledgements: *I would like to thank Sean Jamshidi for providing the plate and the UCL Physics Department for lending me part of the electronic equipment. I am grateful to Derek Thomas for helping me set up the demonstration.*



Pictures were kindly provided by Josh Daniels-Holgate and Andrey Ten.

Christopher Zeeman Medal

The 2018 Christopher Zeeman Medal was awarded to Dr Hannah Fry for her contributions to the public understanding of the mathematical sciences.

Hannah is an Associate Professor in the Mathematics of Cities at the Centre for Advanced Spatial Analysis at UCL, and Honorary Lecturer in the Department of Mathematics. She works alongside a unique mix of physicists, mathematicians, computer scientists, architects and geographers to study the patterns in human behaviour - particularly in an urban setting. Her research applies to a wide range of social problems and questions, from shopping and transport to urban crime, riots and terrorism.

Besides her mathematical research, she has a sustained and distinguished record of communicating mathematics to the public, with a huge portfolio of public engagement activities including books, videos, radio, TV, and public talks, which between them reach vast audiences.

The medal is awarded by the Councils of the IMA and LMS, who had this to say of her award: "*Hannah Fry's dedication in promoting mathematics to the widest possible public has not only done untold good for the subject, but has provided a powerful role model for mathematicians, most especially female ones, making mathematics feel more relevant, more humorous and most of all more human. Her spectacular success in an otherwise notoriously difficult endeavour may be ascribed to a unique and enviable set of characteristics. First, she has*

an uncanny instinct for spotting mathematics that will readily engage the public, and then of constructing the perfect context and using it to convey profound mathematical ideas that might otherwise appear dry. Second, she is effortlessly able to transcend audience boundaries and make mathematics both accessible and "cool" to an enormous range of hard-to-convince onlookers. Finally, Hannah has a tremendous capacity for her sheer hard work: the breadth and range of activities that she undertakes, the number of separate media appearances to which she commits, and the widely disparate audiences and age groups that she is able to reach, are all eloquent testaments to her ability and commitment. Perhaps her most significant achievement is to have inspired a generation of girls in a way that has not been done before."



Hannah Fry is a truly outstanding ambassador for mathematics and it is fitting that she is awarded this prize in acknowledgment of her remarkable impact.

Photo: Hannah Fry at the Data of Tomorrow Conference, by Sebastiaan ter Burg, licensed under CC BY 2.0

Postdoc Life

When not wrapped in up their research, the postdocs in the department have been venturing out into the wider world on a few hikes, organised by Kim Moore and Ben Lambert. These photos (courtesy of Dr Moore) were taken at Box Hill in Surrey, and Goring in the Chilterns. They were also able to plan an elaborate outdoor birthday surprise for Mathias Pétréolle, complete with cake.



Undergraduate Students - The Augustus De Morgan Society

Our undergraduate students have had another busy year, from the annual Fresher's BBQ, the Christmas Quiz, Spring Ball, and a number of maths-themed pub quiz nights helping to keep them occupied outside of their studies.





UNDERGRADUATE PRIZE AWARDS

Prizes Awarded to Undergraduate Students Summer 2018

First Year Prizes

Kestelman Prize

Andrew McNeill

Stevenson Prize

Huw Williams

Bosanquet Prize

Enea Sharxhi

Departmental Prizes in Mathematics

Renjie Cui

Axel Kerbec

Muhammed Uddin

Krzysztof Kacprzyk

Sadanand Ugale

Shahil Sheth

Xiaowen Huang

Second Year Prizes

Kestelman Prize

Myles Workman

Andrew Rosen Prize

Vivienne Leech

Departmental Prize in Mathematics

Yohance Osborne

Third Year Prizes

The Nazir Ahmad Prize

Ivan Zlatnik

Wynne-Roberts Prize

Zhe Hong Lim

Finalists Prizes

Andrew Rosen Prize

Oliver Street

Mathematika Prize

Laura Wakelin

Ellen Watson Memorial Scholarship

Hanhong Wei

Bartlett Prize

Zhe Hong Lim

Casillejo Prize

Jai Lathia

Fourth Year MSci Prizes

David G Larman Prize

Natalie Evans

Susan N Brown Prize

Lewis Marsden

Project Prize

Charlie Egan

Sessional Prize

Riaz Fazal

The Institute of Mathematics and its Applications (IMA) Prizes

One year membership of the IMA

Natalie Evans

Zhe Hong Lim

Panos Tofarides Prize

Georgios Nathaniel

**Students who have recently obtained PhDs
from the Department include:**

Sebastian Bahamonde (supervised by Christian Boehmer) 'Modified teleparallel theories of gravity'

Antonio Cauchi (supervised by Sarah Zerbes) 'On classes in the motivic cohomology of certain Shimura varieties'

Celso Dos Santos Viana (supervised by Jason Lotay and André Neves) 'Isoperimetric and index one minimal surfaces in spherical space forms'

Hui Gong (supervised by Alvaro Cartea) 'Automatic trading using stochastic methods'

Ryan Palmer (supervised by Martin Utley) 'Use of network analysis, and fluid and diffusion approximations for stochastic queueing networks to understand flows of referrals and outcomes in community health care'

Rafael Prieto Curiel (supervised by Steven Bishop) 'Mathematical modelling of social systems'

Luca Scarpa (supervised by Carlo Marinelli) 'A variational approach to some classes of singular stochastic PDEs'

Chunxin Yuan (supervised by Ted Johnson and Roger Grimshaw) 'The evolution of oceanic nonlinear internal waves over variable topography'

Congratulations to all!



Promotions

Associate Professor Sonya Crowe

Sonya's work focuses on operational research applied to health care. Current research interests include linking and analysing national datasets to support quality improvement of services for congenital heart disease, and using analytical and heuristic models to explore workforce innovation in home health care.



Associate Professor Ed Segal

Ed is interested in the interactions between geometry, algebra and theoretical physics. More specifically, he works on derived categories of coherent sheaves and various generalisations.

Professor Timo Betcke

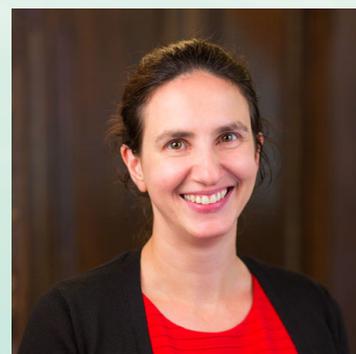
Timo is a computational mathematician working at the interface of numerical mathematics, software development and exciting applications across the natural and engineering sciences. He studied Computational Engineering as an undergraduate in Germany and then pursued his PhD in Numerical Analysis at the University of Oxford. After postdoc positions in Braunschweig and Manchester he was awarded in 2009 a Career Acceleration Fellowship at the University of Reading, from which he then moved to UCL as a lecturer in 2011. He was promoted



to Reader in 2013 and to Professor of Computational Mathematics in 2018. His main research area is the fast solution and applications of boundary integral equations in acoustics and electromagnetics. Recent projects include fast electromagnetic simulations of atmospheric particles, simulation of ultrasound treatment planning in the lower abdomen, and the development of novel software for the simulation of electromagnetic properties of meta-materials. He is the principal developer of the Bempp software package, a popular tool for the numerical solution of boundary integral equation problems.

Professor Christina Pagel Director of CORU

Christina's main interest is in using information to help people within the health service make better decisions. Often this involves mathematical modelling or other operational research techniques, and/or statistical model and data analysis but sometimes it can be just presenting the data in a more intuitive way. She wants her work to be relevant to, and used by, those working within the NHS and so is also very interested in the process of how to get theoretical knowledge into practical application. Since 2013, she has spent 1-2 days a week at Great Ormond Street Hospital as an embedded academic researcher within their critical care units.



Since 2012, Christina has been working with national bodies and specialist hospitals in the UK on methods for measuring and reporting survival after children's heart surgery which have been adopted across the UK. Most

recently this involved building a website with families to share and explain hospital results and leading a new project to link five national datasets to map for the first time the journey of people born with Congenital Heart Disease through the English health care system.

Professor Jason Lotay

Jason's interests are in differential geometry and geometric analysis, particularly geometry related with special holonomy and calibrated submanifolds, geometric flows including Lagrangian mean curvature flow and the G2-Laplacian flow, as well as instantons. Jason now works at the University of Oxford.



Department Teaching Award

The Department Teaching Award for 2017-18 was awarded to **Dr Mark Roberts**.

New Staff

Dr Matthew Towers

Teaching Fellow

Matthew is an algebraist with interests including representation theory and homological algebra, working mainly on Lie algebras in positive characteristic, especially restricted enveloping algebras and their cohomology.

Dr Cecilia Busuioc

Teaching Fellow

Cecilia's research lies primarily within the fields of Algebraic Number Theory and K-theory, with particular emphasis on arithmetical aspects of automorphic cohomology and its applications to special values of L-functions associated to motives over number fields.



The UCL Student Choice Awards

The UCL Student's Union run the annual Student Choice awards, which in 2018 received over 1,000 nominations from students across UCL to recognise the excellence of their teaching staff.

Professor Yiannis Petridis from the Department of Mathematics was shortlisted for 'Inspiring Teaching Delivery', in recognition of his clear passion for the subject, engaging teaching, and dedication to each individual student.

Departmental Colloquium 16 October 2018

The structure of stable sets: from additive number theory to model theory

Dr Julia Wolf
(University of Cambridge)

A long-standing open problem in additive number theory is the following: how dense does a set of integers have to be before it is guaranteed to contain a non-trivial arithmetic progression of length 3?

In the first half of this talk, we surveyed recent progress on this problem and the techniques used to solve it, and related questions about additive structures in finite abelian groups. In particular, we explained the idea behind the so-called "arithmetic regularity lemma" pioneered by Green, which is a group-theoretic analogue of Szemerédi's celebrated regularity lemma for graphs.

In the second half of the talk we described recent joint work with Caroline Terry (University of Chicago), which shows that under the natural model-theoretic assumption of stability the conclusions of the arithmetic regularity lemma can be significantly strengthened, leading to a characterisation of stable subsets of finite abelian groups.



Departmental Colloquium 12 March 2019

Euclid's *Elements*: This is the answer to *what* question?

Professor Piers Bursill-Hall
(University of Cambridge)

Well, we all know the answer to that: because axiomatising geometry puts it on a sound and certain footing; that's a good thing, and allows us to build the rest of mathematics on a sure and certain basis. And once you have read the *Elements* (not that you have), you might almost believe this. But until you know that axiomatics does this, you wouldn't know that there is a sure and certain foundation to geometry out there. How on earth did Euclid come up with the idea of axiomatising geometry in the first place? It is not as if axiomatising things is something human beings do naturally ... so there must be some story that leads up to Euclid's axiomatisation (and it wasn't a system to teach geometry, either). So... why? How did ancient Greek thinking about proof develop in the couple of centuries before Euclid?

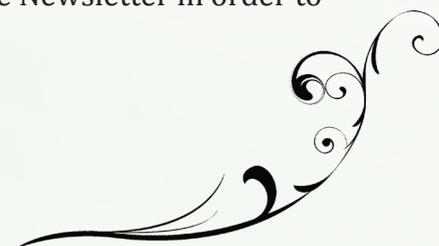


In Memoriam: Professor Yaroslav Kurylev



The Department of Mathematics is sorry to announce the death of Professor Yaroslav Kurylev on Saturday 19 January 2019.

We plan to publish a full obituary in next year's edition of the Newsletter in order to pay full tribute to Slava's career and time spent here at UCL.



J J Sylvester Scholarship Fund

The J J Sylvester Scholarship Fund was set up in 1997, on the centenary of the death of J J Sylvester, one of the most gifted scholars of his generation. The Fund aims to award a scholarship to help support gifted graduate mathematicians.

You can make your gift to UCL online, by telephone or by post. Donations may be made by cheque, charity voucher or GiftAid. Any donation, large or small, will be gratefully acknowledged by the College. If you are interested in knowing more about the Fund or other tax-efficient ways of supporting the Fund please do not hesitate to contact campaign@ucl.ac.uk or +44 (0)20 3108 3833.

For more information, including ways to give, please see ucl.ac.uk/campaign/get-involved/donate. To make a gift over the phone, using a credit or debit card, please contact the Gift Services team on +44 (0)20 3108 9167.

Sylvester was one of the greatest mathematicians to be associated with UCL and it is hoped that, through contributions made to the Scholarship Fund, we shall be able to assist in progressing the education of other mathematicians so as to realise their full potential for the benefit of us all.

Alumni Careers Advice

The Department is keen to welcome alumni to its careers events and fairs for our current students. This includes alumni who have gone on to complete further study.

If you are interested in this possibility, please contact Professor Helen Wilson at helen.wilson@ucl.ac.uk

We also encourage you to join the UCL Alumni Online Community (ucl.ac.uk/alumni) to stay up to date with the latest news, views, events, special offers and exclusive opportunities from UCL, and stay connected with your fellow alumni.

Contributing to the De Morgan Association Newsletter

We would welcome news and contributions for the next newsletter, to be sent to:

Professor Ted Johnson, The De Morgan Association, Department of Mathematics, University College London, Gower Street, London WC1E 6BT

Email: editor_newsletter@math.ucl.ac.uk

Answers to part 17 of the Chalkdust Game (page 35):

The Wikipedia articles that exist are: Elementary symmetric polynomial, Timeline of geometry, Inequality of arithmetic and geometric means, Topological combinatorics, and Equation solving.