De Morgan Association NEWSLETTER

UCL Mathematics Department

Professor Robb McDonald
Head Of Mathematics Department

A glance through this year's De Morgan newsletter reveals a busy year with much success to celebrate. We were pleased to welcome four new Lecturers: Roger Casals, Ed Segal, Iain Smears and Ewelina Zatorska. We are especially grateful to the estate of Howard Davies, a former long-serving colleague, whose generous donation helped make one of these appointments.

The department's sustained recruitment of outstanding staff such as Ed, Ewelina, Iain, and Roger, along with excellent undergraduate and postgraduate students clearly demonstrates the health of our subject. Mathematics is, of course, a fundamental discipline, and one which is finding ever-increasing applications, often in surprising areas. A good example being our 'industrial sandpit' in summer 2017 themed on transport security. This highly successful workshop, led by Nick Ovenden, brought together UCL mathematicians, computer scientists, engineers, physicists and forensic scientists with industrial and government representatives, e.g., Motorola Solutions, Department for Transport and Defence Science and Technology Laboratory. Real practical solutions to pressing problems were discussed, and areas for further research were identified.

The Department's commitment to research and teaching of core mathematics, along with our willingness to embrace and collaborate with researchers in other disciplines, of which there are many at UCL, is something we are keen to build on. I am delighted that this ambition has now been recognised formally by the University. In September 2017 UCL's Provost and President, Professor Michael Arthur, announced a small number of 'ideas' key to UCL's evolution which (i) demonstrate genuine, grassroots academic enthusiasm, (ii) come from areas that have the potential to become world-leading and (iii) be deeply cross-disciplinary. Among these elite ideas is the ambition to co-locate UCL's Departments of Mathematics and Statistical Science to form an Institute of Mathematical and Statistical Sciences (IMSS). Professor Arthur writes of Mathematics, Statistical Science and IMSS:

'Research quality and student experience are already excellent in these subjects, but they lack scale and facilities compared to world-leading competitors. By combining them, we will be in a position to reach the global top 50 by 2021 and, ultimately, the top 25.'

The Department looks forward to working with the University in transforming the IMSS idea into reality. There is no better time to do this: mathematical science and its applications is flourishing globally. Now is an exciting time to be mathematician, especially so at UCL.

Letter From The Editor

Professor Ted Johnson

The annual dinner of the De Morgan Association was held on Friday 9th June 2017 in Senate House, University of London. The Guest of Honour was our own Rafael Prieto Curiel. Rafael was instrumental in setting up the Chalkdust magazine which has been a stunning international success for UCL Mathematics graduate students. Rafael entertained us with both mathematics and background on the invention and production of the magazine.

Robb McDonald is stepping down as Head of Department this year. Robb has calmly presided over a period of great change in the department with growing undergraduate enrolments and the establishment of powerful new and augmented research groups including geometry, number theory, inverse modelling, numerical analysis and financial mathematics.

Robb will be succeeded by Professor Helen Wilson. Helen's research is mainly concerned with theoretical modelling of the flow of non-Newtonian fluids such as polymeric materials and particle suspensions. In her inaugural lecture, written up inside with his characteristic panache by Adam Townsend, one of Helen's former research students, Helen described how fluids containing long polymer molecules behave very differently from ordinary (Newtonian) fluids. Under flow, long polymer molecules stretch giving rise to surprising flow behaviour, and producing undesirable instabilities in industrial processes. Helen is an editor of the Journal of Non-Newtonian Fluid Mechanics, the Journal of Engineering Mathematics, and Proceedings of the Royal Society A, a member of the Editorial Advisory Board for Physics of Fluids, chair of the Nominating Committee for the European Society of Rheology, and was, from 2015-2017, President of the British Society of Rheology.

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May 2018

Editor Ted Johnson

Associate Editor Kate Fraser

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De Morgan Association Dinner 2017

The New Challenge of Communicating Maths

Rafael Prieto Curiel - PhD Student, UCL

Perhaps the biggest difference between previous generations and millennials is that we, the millennials, were born and educated in a world saturated with different resources. Have you ever wondered how to solve a stochastic differential equation? I have, and I found more than 4,000 videos on YouTube teaching me how to solve them: I could spend more than 82 days watching every single video there is. What is a hyperbolic surface? There are more than 50,000 links on the internet that explain what they are and how to construct them. There are dozens of free apps for my phone about fluid dynamics and I am not sure which one to download. Fancy videos on the internet, ready for me to play, pause and rewind, as many times as I want, waiting for me to see and listen to a Fields medallist talking about her research, are already a common thing. Why even bother going to lectures or to a seminar if a link, a video or an app can explain a concept, perhaps just as well as a professor could, and which I could watch from the comfort of my own home.

Easy, immediate answers are in the palm of our hands (literally!), which means that contributing to the community by adding another maths communication project on the internet might have a negligible impact on the audience.

There is, however, one issue with the fancy videos or the great resources on the internet: all I can do is click on it and maybe ‘like it’ on social media.

For many maths and science communication projects, I am just a passive consumer, moving from one page to the next one in just a few seconds, looking for the most attractive graph or catchy title.

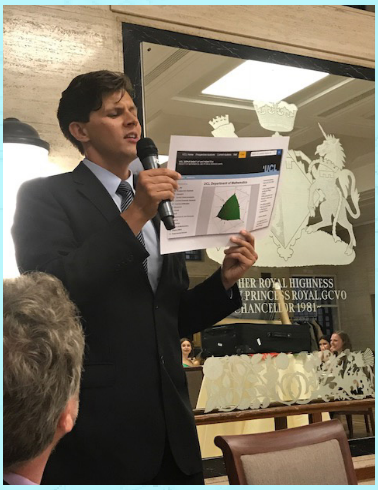
But the experience is radically different when I am an active part of the project. I remember the first time I read a printed article with my name on it or sharing on Facebook the post that I wrote or the graph that I created. It might have been a tiny contribution, but it was my contribution after all. And that, I think, is the secret with millennials. It is being able to say, to write, to express and to become an active part of a ‘something’ that makes the project not only more interesting and memorable but, more importantly, more likely to engage us Yes, there are thousands of fancy videos on YouTube, but I have never been part of any of them.

The way to engage millennials is not by providing us with yet another resource. The impact of the extra article or the new video that adds to the thousands that are already out there, lasts, at least for the viewers, perhaps a few hours. The way to engage millennials is by providing us with space. We created Chalkdust around that idea: the greatest impact that an article has will be on the author: those who took the time to do some research about a cool mathsy topic; or who thought of a nice joke or a funny flow chart. The project is all about the people who actively engage with it, they ‘own’ it pretty much like a person is the owner of their posts in social media. And since we started Chalkdust, we have published more than 300 articles from 100 authors, including a few teenagers. They share their article on social media, they show it to their parents and, it is theirs, their own contribution.

Posting interesting articles is the vehicle through which Chalkdust can contribute towards forming a better community, encourage people to write about maths and engage a generation that is already saturated with resources.

Maths has changed drastically over the past few decades - solving tedious equations is only for homework and exams. The way we conduct research has also changed - we can read today what our colleagues on the other side of the world did yesterday. Thus, the challenge is in ensuring that the way we teach and communicate maths should also adapt to the saturated number of resources, the constant flow of information, the immediate availability of resources and the desire to have a space of your own.

I say goodbye now to Chalkdust, and as new generations take the lead, I am sure that such a beautiful project will keep, for the next generations, offering us a space to communicate, share our interesting maths articles and become the owner of our own maths.



2018 De Morgan Association Dinner

Friday 8 June 2018
Senate House
Sherry at 6.45pm, Dinner at 7.30pm
Speaker - Dr Christina Pagel
“A mathematician dips her toes into politics...”

All those on the UCL Alumni database will be sent an invitation to the next De Morgan Association Dinner. Please remember to keep the Department and Alumni Relations Office of UCL informed of any changes to your address.

The Incredible Predictive Power of String Theory

Dr Ed Segal

If you pay any attention at all to popular science news, you will certainly have heard of string theory. Many physicists make grandiose claims for it – it will unify gravity and quantum theory, it's a 'theory of everything'. Others view the whole field as a gigantic scam, a mirage that has swallowed generations of graduate students and damaged theoretical physics for decades. The reason for the controversy is simple: huge numbers of theorists have been developing it, for about 30 years, and they still can't predict the result of a single experiment. They're not even close. If you want to know if string theory is correct don't hold your breath for a press release from CERN – it's not going to happen in the foreseeable future.

So is my choice of title just sarcastic? No! (Ok, maybe a bit...). String theory does make predictions, but they're of a very different kind. String theory makes predictions about pure mathematics - interesting, difficult and important predictions. And so far, every one of these predictions has been proved right! So whether it's relevant to physics or not - and personally I'm agnostic on this point – there's no question that string theory is relevant to mathematics.

To understand how string theory can make predictions about maths, let's start by going back 300 years, to Newtonian mechanics. Newton's laws describe how a particle moves around in flat 3-dimensional space, and it's trivial to generalize them to a particle moving around in flat n-dimensional space. But mathematicians are interested in lots of other spaces besides ordinary flat space. A particularly important class of spaces are manifolds, these are spaces that "close up" look like flat space, but when you zoom out they look different. A good example is the surface of a sphere – close up it looks like a flat 2d space, but globally it's very different. It turns out that you can formulate Newton's laws for any manifold (technically you first need to equip the manifold with a Riemannian metric, which is a way of measuring distances), then you get a theory which describes a particle moving around on that particular manifold.

If you study that physical theory then it's clear that you'll start to learn some things about the shape of the manifold – for example a particle moving freely on a sphere will eventually get back to its starting point, but in flat space this will never happen.

Now back to string theory. The basic physical idea in string theory is to replace your particles by tiny little loops, or "strings". These loops can move around, but crucially they can also vibrate, like the string of a musical instrument that's just been plucked. String theorists hope that all the usual particles can be described by strings vibrating in different ways, so an electron is just a string playing a particular note, and a quark is the same string playing a different note.

Like our Newtonian particles, we can think of our strings moving about in ordinary flat space if we want, but its more interesting if we let them move around in a different manifold. In fact the physics demands that we do this, because for a technical reason the theory only works properly if the strings move around in a 10-dimensional space. Since our universe appears to be only 4-dimensional, string theorists speculate that there are six additional dimensions that form a closed-up manifold, perhaps something like a 6-dimensional sphere. This is the kind of speculation that irritates more hard-headed physicists!

Strings prefer to move around on a special kind of manifold called a Kähler manifold. These are manifolds which have complex numbers built into their structure, and also the Riemannian metric is of a special form. Perhaps you remember the Riemann sphere – the complex plane curled up with an extra point at infinity – that's the simplest example of a Kähler manifold.

A good way to get more complicated examples is to take polynomials and look at the set of complex solutions, often the resulting shape will be a Kähler manifold.

So Kähler manifolds are things that lots of pure mathematicians are interested in, geometers certainly, but also algebraists (because of the polynomials) and sometimes even number theorists.

If you could understand how a string moves around in your favourite Kähler manifold, you could learn things about the shape of the manifold. Unfortunately there's a serious problem: mathematicians don't understand string theory. In fact the problem goes deeper, mathematicians don't really understand quantum field theory, and that's a piece of physics that's nearly 100 years old. And when I say we don't understand it, I don't mean that we can't solve the equations, I mean that we don't even understand what the equations mean! Physicists write down symbols, and we can't figure out what mathematical objects they're supposed to be referring to. Of course the physicists don't care about this, for them the symbols refer to physical concepts: fields, particles, and so on. Mathematicians try to interpret them as mathematical concepts: sets, functions, vector spaces etc., but we don't always succeed. We have two different mind sets, and this makes it very difficult for mathematicians to understand physicist's calculations.

The thing that bridges this cultural divide is supersymmetry. This is a rather abstract symmetry that some physical theories have; all particles in physics fall into two classes, bosons and fermions, and supersymmetry makes the two kinds swap places. Theorists love supersymmetry, and the LHC is actively looking for experimental evidence of it, but so far the results are disappointing. Of course string theorists are not dissuaded by that fact! No string theorist would dream of studying a theory that didn't have supersymmetry.

The reason supersymmetry is nice is that it makes some computations much easier. If a theory has supersymmetry then the fine details of the physics will still be hard, but certain fundamental pieces of information will be 'invariants', meaning that they do not change if we make small perturbations. Imagine that your strings are moving around in a Kähler manifold, and you deform the manifold a little bit, perhaps squeezing one part of it, and stretching another. This could make a big difference to the trajectories of individual strings.

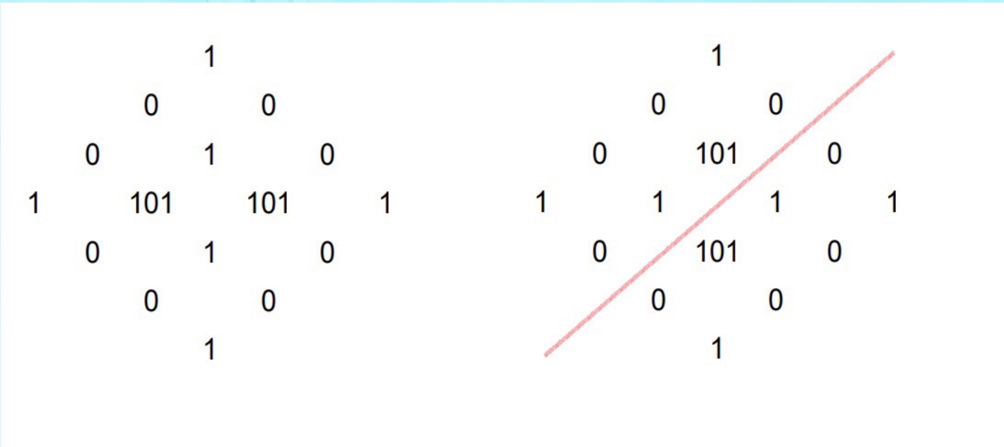


Figure 1: The Hodge diamonds of the cubic threefold (left) and its mirror

However, the invariants provided by supersymmetry are guaranteed to stay the same, and it is this robustness that makes them easy to calculate.

If you're a pure mathematician, your ears prick up at the mention of 'invariants'. We love to calculate invariants of all kinds, and we definitely love to calculate invariants of Kähler manifolds. The invariants that come out of supersymmetric string theory are exactly the kind of thing that mathematicians care about! Some of them are invariants that we already knew about, like Euler characteristics, and homology groups, and some of them are brand new

This leaves us in a rather surprising situation. Physicists' arguments – which we don't understand – can compute geometrical quantities that pure mathematicians are very interested in. Mathematicians now have to listen to the predictions made by string theorists, and their predictions are right!

Most of these predictions, though not all, centre around an astonishing phenomenon called mirror symmetry. Take your favourite Kähler manifold, and write down the physical theory for strings moving around in it. You can now do a really trivial operation on that theory – basically just swap a few plus and minus signs – and get a new physical theory of a similar kind. What does this new theory actually describe? It's definitely doesn't describe strings moving around in your original manifold, because the first theory did that, but perhaps it describes strings moving around in a different Kähler manifold. So perhaps this means that Kähler manifolds come in pairs, with the string theories for each pair being related by this trivial sign change. In this hypothesis, the pairs of manifolds are called 'mirrors' to each other.

Let's assume you believe this idea. If I hand you a Kähler manifold, then it should be possible to produce its partner, the mirror manifold. But how would you know if you'd got the correct mirror? For a start, you could compute some invariants. The most fundamental invariants of a Kähler manifold are called the Hodge numbers, this is a finite set of numbers.

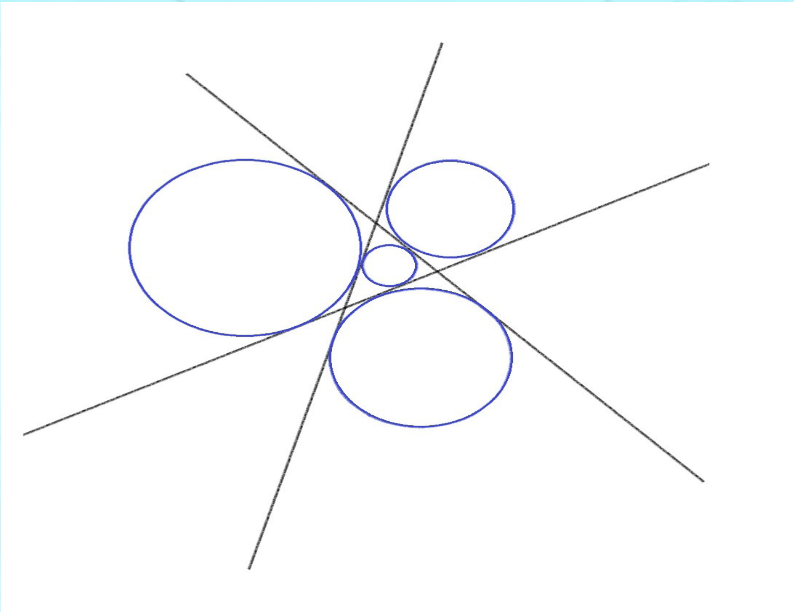


Figure 2: Circles tangent to three lines

In Figure 1 you can see the Hodge numbers of a famous Kähler manifold called the quintic threefold, this manifold is the set of solutions to a quintic polynomial in five variables. Physicists tell us that if two manifolds form a mirror pair then their Hodge diamonds are nearly the same - to get from one to the other you have to do a reflection of the array through a particular diagonal line (marked in red on Figure 1); this is where the name ‘mirror’ comes from. So now we have a fairly precise prediction: given a Kähler manifold, is there a geometric operation that will produce a new manifold, in a such a way that their Hodge diamonds are mirror images?

At first sight the answer to this question is simply ‘no’, there is no obvious geometric operation that will do this. But, for reasons I will explain in a moment, mathematicians take this idea extremely seriously. And after 20 years hard work by brilliant people, we can do it, for some examples. I find it absolutely staggering that such a trivial little operation in physics ends up requiring the most monumental effort in geometry and algebra.

So why did mathematicians believe mirror symmetry in the first place? There are now lots of good reasons, but the first really compelling reason was a result by Candelas, de la Ossa, Green and Parkes in 1991. Their result involves things called Gromov-Witten invariants, which are a deeper and more complicated invariant of Kähler manifolds, and exactly the kind of thing that both geometers and string theorists are interested in calculating. To get some kind of flavour of these invariants, imagine drawing three random lines in the plane, and then ask: how many circles can you draw that are tangent to all three lines?

This question is easy - the answer is four (see Figure 2) - but that’s because we formulated the question in flat 2-dimensional space. If you ask an analogous question in a Kähler manifold, you may get a much more complicated answer.

For example, if we ask `how many curves of degree three can we draw on the quintic threefold?’ then the answer is 317,206,375. This ridiculous number is an example of a Gromov-Witten invariant, and it was first predicted by the four physicists named above. They did it using mirror symmetry.

Mirror symmetry swaps Hodge numbers for Hodge numbers in a simple way, but it turns out that it swaps Gromov-Witten invariants to a completely different kind of invariant, which is sometimes much easier to compute. Candelas et al. had a good guess for the mirror manifold to the quintic threefold, so they simply computed the corresponding invariant for the mirror manifold. The answer was subsequently confirmed by mathematicians, without any mirror symmetry hocus-pocus.

If you can correctly predict a six-digit number then you can win the lottery. Mirror symmetry had correctly predicted a nine-digit number, and mathematicians went crazy for it. If string theory can do that, people asked, then what else might it be able to do? The answer has been ‘an awful lot’, and there is now a huge body of work on ‘pure-maths-inspired-by-string-theory’, and several Fields Medals have been won in the process.

The moral of the story is, if you want to learn something new about Kähler manifolds, read a string theory paper.

UCL Mathematical Sciences Industrial Sandpit

26th-28th June 2017

Dr Nick Ovenden

In recent years, there have been increasing demands for university research to demonstrate its impact, either in society or in contributing to economic growth. Measuring the so-called socioeconomic impact of research is an essential component of the Research Excellence Framework (REF) and the UK government is keen for university academics to engage with industry and other external partners to drive innovation and technological change. As a response to this drive, the first ever UCL Mathematical Sciences Industrial Sandpit was held over three days at the end of June where the purpose of the event was to promote academic engagement with industry, and foster new collaborative research leading to novel ideas for solving real-world problems.

Study groups, hackathons, or sandpits in various guises involving mathematics and industry already exist. The most well-known are the European Study Groups with Industry (ESGI), which stem from the Oxford Study Groups that started in the late 1960’s and were organised by Alan Tayler and Leslie Fox. ESGIs are now held annually in a number of European countries, including Denmark, The Netherlands, Portugal, Spain, and Ireland, as well as the UK.

The enthusiasm of eminent applied mathematicians from Oxford as well as other UK institutions has significantly contributed to the ESGI format being spawned into a variety of maths-in-industry events across the rest of the world.

Having organised an ESGI in the past led me to wonder why a similar event could not be hosted at UCL? After seeing the success and popularity of the more recent Integrated Think Tank events run at the Bath Centre for Doctoral Training SAMBa (Statistical Applied Mathematics in Bath) and, with the encouragement of our Head of Department and other colleagues (including director of the LSGNT, Professor Michael Singer), 2017 seemed an ideal time to organise such an event.

At the UCL Mathematical Sciences Industrial Sandpit, three external organisations, The Department for Transport, DSTL and Motorola Solutions, each offered a challenge on the theme of “security and transport”. The event was attended by academics and postgraduates from the departments of Mathematics and Statistical Science as well as some attendees from Security and Crime Science, Physics and Astronomy and Computer Science. Some visiting PhD students from the SAMBa CDT were also present.

The organisation and running of the event was greatly assisted by the help of two research facilitators from the Office of the Vice Provost Research, Dr Cat Mora and Dr Laura Fenner. The morning of the first day started with presentations from each of the external partners, detailing the challenges they are interested in tackling. This was a very active session with lots of questions from the floor and discussions continued during the ice-breaker round table session that followed. After lunch, the attendees listened to technical talks on each of the challenges, mainly given by UCL academics, reviewing the current literature and available models. A "market place" session then followed where attendees could wander between the separate tables set up for each challenge and interrogate the external partner's representatives to extract crucial details required, such as the quality of available data, in order to begin developing their own mathematical modelling ideas.

On the second day, the brainstorming continued in separate breakout rooms, one for each external partner, and, over the first coffee break, a few bright ideas emerged that started to gain traction. By the end of the day, the most convincing ideas had gradually turned into vaguely formulated research plans, drawn out on large A0 pieces of paper, that were then scrutinised by the other attendees.

The final day came down to the now fully-formed project groups each busily preparing a formal presentation in time for the final session after lunch. Professor David Price, the Vice Provost for Research, along with representatives and interested parties from the external organisations, sat in the audience to hear six interdisciplinary groups each explain their own research proposal and the resources required.

Since the sandpit took place, more detailed versions of the research proposals have been written up and discussions with the external partners are ongoing to explore the possibilities of collaborative research in the future. Hopefully the sandpit will spark further maths-in-industry events in the department, offering exciting research opportunities, in addition to raising the external profile of applied mathematics at UCL.

Inaugural Lecture - Professor Helen Wilson

Dr Adam Townsend

"I look forward to seeing what she does with those eggs", says the Dean as he makes one of the three – yes, three – introductions for Helen Wilson's inaugural lecture.

"It's not eggs!" Helen exclaims from the front. And of course, it's not. Any rheologist will tell you that eggs are shear thinning, not shear thickening, and the yellow mix in the glass bowl at the front of room 505 is actually custard.

Helen picks it up. "In the bowl, the custard is thick and gloopy, but when I slowly move the bowl around, the custard flows like a liquid. Now look what happens when I punch it" – thwack "my hand stays dry!" This is a surprising experiment. Normally when you punch a liquid, it goes everywhere: all over the floor, your clothes, the expensive new computer equipment. Not so with custard: when you shear it quickly, for example by punching it, it actually gets thicker. So thick, in fact, that it acts as a solid, keeping Helen's fist dry.

"So why is it doing this? This behaviour is pretty unusual, and is confined to custard and cornflour-and-water pastes. And we have only recently figured out why it works like this," she says.

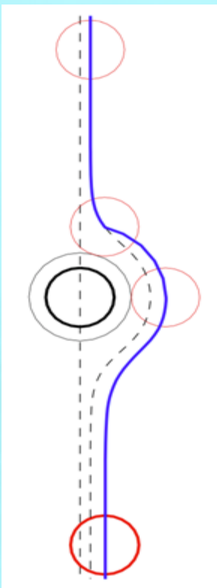


Figure 1: We can see friction acting by how one particle (red) passes another (thick black). Normally we would expect the red particle to move round the black one symmetrically, but the fact that it is offset at the end tells us that there is some normal contact force acting

Custard is an example of a non-Newtonian fluid: a fluid whose viscosity changes as you try to shear it. Many things you find around the house are also non-Newtonian – ketchup, toothpaste, blood (if you're unlucky) – but most of them get thinner as you increase the shear upon them. What makes custard become thicker then? Helen explains that it depends how exactly the fluid is non-Newtonian. Rheology, the study of non-Newtonian fluids, is constantly looking at matching what happens at the microscale with what we observe on the macroscale.

"Water, wine, honey, air: these are Newtonian fluids; fluids whose viscosities stay the same when you try to shear them," Helen demonstrates. "But for us, they're all too simple!"

There are different ways of making fluids less simple, and Helen has got stuck into quite a few of them. One way is to add particles into it: that's how you get mud or custard. Trying to understand why custard shear-thickens, and therefore why you can do exciting things like run over a pool of the stuff, means looking at how these particles interact.

Experiments have shown that we don't observe such extreme shear thickening in suspensions of attractive particles, so perhaps instead the shear thickening comes from the particles clustering together. Simulations, however, have shown that this doesn't cause enough of a viscosity jump.
We know that sand expands as is it sheared, so again perhaps the custard particles are doing this. Unfortunately, this theory leads to incorrect predictions for smooth, hard particles.

Instead, a general consensus has built up round the notion that the custard particles are coming under frictional contact, rubbing and rolling past each other due to the pressure in the system. “Recent simulations in the literature back up the idea that friction causes the change in viscosity,” says Helen. “But we came up with a really neat, really cool way of implementing the friction more exactly than others. And what did we see? No big changes in viscosity! So, you see, it is always more complicated.”

Given the history of investigations into suspensions, it is somewhat surprising that we’ve only recently got the science behind the custard experiment licked. Einstein looked at the effect on the viscosity of a fluid of adding particles back in 1906 (and then again in 1915). He was only looking at dilute suspensions – custard, of course, is very concentrated – but he found that the viscosity for a concentration ϕ scaled as $1+5/2\ \phi+O(\phi^2)$.

In the 1970s, Batchelor & Green went further and found that the ϕ^2 coefficient was about 7… so long as you were only doing certain things to your fluid. “This was good so long as the strain you were exerting on the fluid was uniaxial, biaxial or in the plane: in other words, so long as all trajectories come from infinity,” explains Helen. “The problem is, in shear flow, you don’t always get this. Particles can follow closed paths, just going round and round forever.”

This problem was tackled by Helen as a postdoc in the University of Colorado at Boulder in 1998. She was able to show that implementing contact forces severed some of these closed paths but not all, making it still impossible to calculate viscosity in dilute shear flows.

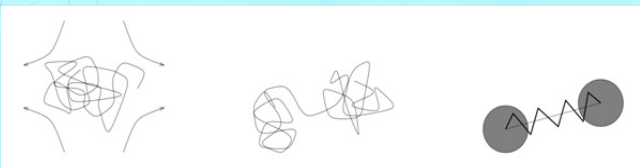


Figure 2: Long, stringy polymers can be modelled quite successfully as dumbbell-and-bead springs

But true shear thickening needs concentrated systems and that brings us to some of Helen’s recent work, where she is working with maths department alumnus Peter Kilbride, now in industry in Cambridge. While finishing his PhD in the department, he came to talk to Helen about whether shear-thickening fluids can be used for a rather innovative purpose. “A problem with cryopreservation is that when you freeze tissue, ice crystals form,” she says. “This can destroy the tissue, but what the industrial team have “found is that some approved cryoprotectants do show shear-thickening. The structural changes in the cryoprotectant as it thickens could potentially inhibit the formation of these crystals.” It’s early days, but it’s exciting work.

So we’ve seen what you can do when you put solid particles into your fluid. Another thing you can do is to put in deformable elements: this gives you blood, shampoo, or, from Helen’s latest grant, toothpaste.

“Toothpaste is really a very complex mixture,” explains Helen. “It’s a polymer gel, with rough silica particles for abrasion, but smooth silica particles for rheology. On top of that, it’s impossible to make it without small air bubbles.”

The key example for fluids with deformable microstructure is a polymer solution. Polymers are long, stringy molecules which, under Brownian motion, have a tendency to coil up if left alone in the fluid. Under the influence of some flow, however, they deform. The resulting entropic ‘desire’ to relax produces an elastic force in the fluid. “This conformation can even give the fluid a memory,” says Helen.

One way to model these polymer coils is with a two-bead dumbbell, connected by a spring. From this, we can produce a handful of analytical models for polymeric fluids. The simplest such model, the so-called upper convected Maxwell model, does a pretty good job of describing some of the interesting behaviour polymeric solutions sometimes exhibit.

Figure 3: Polymeric fluids can climb rods if you spin the rod quickly enough. (Boger & Walters, Rheological Phenomena in Focus)

If you take such a solution, place a rod vertically in it, and spin the rod in the solution, you'll find that the fluid begins to climb the rod (above). This is extremely odd: if you did the same in water, inertia would dominate and the water would move away from the rod.

This interesting effect, named after one of the founding rheologists, Karl Weissenberg, is a consequence of the elastic forces from the stretched polymers in the fluid. For a stress tensor Σ in a steady shear flow in the xy-plane, the first normal stress difference, $N_1=\Sigma_{xx}-\Sigma_{yy}$, is positive. "Rheologists talk a lot about normal stress differences," says Helen, "but you can think about them as tension in the streamlines of the flow. When the streamlines are circular, as in this experiment, the positive normal stress difference acts as a hoop stress, pulling the fluid inward. It's a lot like how surface tension acts to keep a bubble spherical."

For toothpaste, however, this model has some shortcomings. In particular, it predicts a constant viscosity under increasing shear rate. Experiments show, however, that toothpaste is shear-thinning, like ketchup or blood. Some of Helen's current work involves tweaking this model to allow it to shear-thin appropriately.

But let's jump back to 1995, a year which saw the public awaken to 'the Internet' (capital I), an era when using Fortran was socially acceptable, and the time when Helen was starting her PhD at Cambridge. Her first project, under the supervision of John Rallison, was to look at instabilities in memory fluids. "Instabilities are when flows go wrong," explains Helen. "If you're trying to extrude a material or produce something delicate like separator film in electric car batteries, this is problematic. On the other hand, if you are trying to mix some fluids, it may be desirable."

You can see these instabilities every day. "If you place a heavy fluid on top of a lighter fluid," Helen continues, "it sorts itself out pretty quickly. This is known as a Rayleigh–Taylor instability and it's driven by gravity..You can also see the effect of instability with a dripping tap .The water comes out of the tap as a slender filament, thins irregularly, and eventually breaks up into little droplets. This instability is driven by surface tension.

Helen started looking at this problem by way of linear stability theory. "You start by taking a base flow, U: an ideal, steady flow that we hope to see," she says.

Then you add a little kick to it, εu. If you then throw away any very small ε^2 terms, and Fourier-transform in the neutral direction and time, you end up with a linear system for u where the only non-trivial solutions are for selected values of the transformed time coordinate, ω. It is these values of ω which tell you about the stability of the base flow."

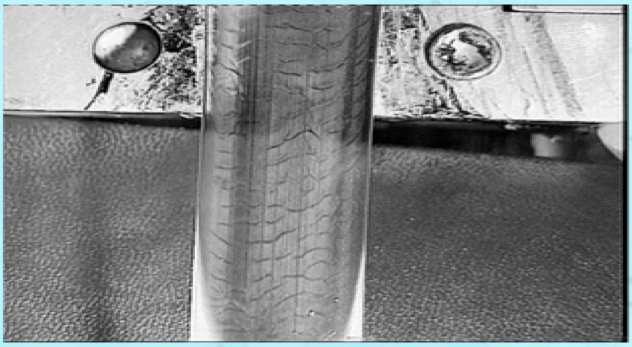


Figure 4: An instability in the extrusion of a film of polymer melt (Tim Gough, University of Bradford)

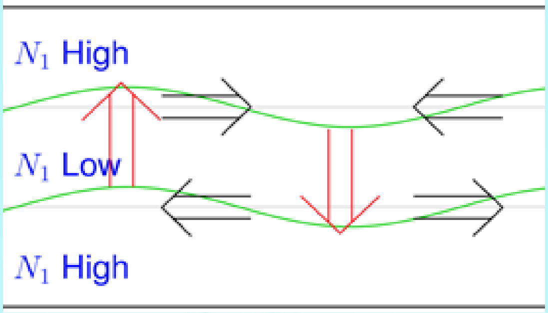


Figure 5: The three-layer interfacial instability causes a feedback loop

The problem she looked at involved three horizontal layers of fluid. The top and bottom layers had a high normal stress difference; the middle layer had a low normal stress difference. If the interfaces are perturbed (above), the difference in stresses causes a feedback loop in the fluid. The jump in horizontal tension across each interface causes the horizontal motion shown by the black arrows; then the resulting excess of fluid in the top middle of the diagram has to flow downwards, the flow marked in red. This in turn enhances the perturbation to the interface position.

Initially the layers had matched viscosities but Helen developed it to mismatched viscosities and interfaces where instead of jumps in the normal stress, it just varies steeply. One model she used for the steep variation was the empirical dumbbell described already. She had no luck finding instabilities in that case, but then serendipity intervened. "The coding was already done," she says, "so I thought, what happens if I just looked around for instabilities in different circumstances?"

Lo and behold, she happened upon a new one for a power law fluid.

So with this extra instability in the model found and thoroughly analysed, Helen was quite pleased. "I couldn't find any experimental evidence of it in the literature, but it was a good paper and a nice result, so I thought it might just sit there on the shelf," she recalls. Jump forward 14 years.

During this time she'd spent two years in Boulder, four years at Leeds, and nine years patronising the east coast mainline into UCL every day. But now an email lands in her inbox. It's from the Bordeaux group at CNRS. They'd found an instability in a straight channel while investigating the transition to elastic turbulence and were wondering if, by any chance, it might be Helen's instability. "Serendipity strikes again!"

"I'm not going to give the Chocolate Fountain Talk™," she states. What? Why not? What does she think the audience in the completely packed lecture theatre have come for? "Most of you have heard it already, and one of you has given it over 100 times." Ah, yes. Well thankfully there's a chocolate fountain in the reception next door and everyone will get to enjoy the spoils shortly. Needless to say, this work did pretty well in the media – 'Someone finally looked into the physics of chocolate fountains,' says the Washington Post headline. The article has had over 20,000 downloads and nearly 2 citations. "When an article I wrote about suspensions got over 333, Springer sent me a certificate", she muses.

In attendance tonight is the Provost, and he makes the extremely popular announcement that Helen will be taking over as head of department from next year, making her the first female head in its 192 years of existence.

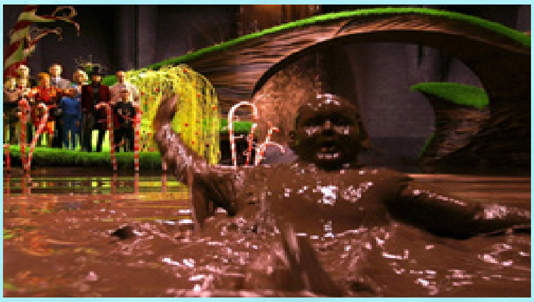


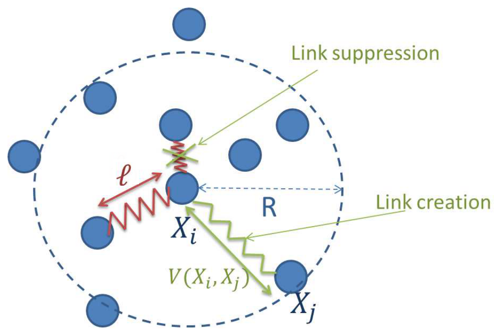
Figure 6: What Augustus Gloop falling into chocolate would look like if Hollywood did something stupid like recreate Willy Wonka and the Chocolate Factory with Johnny Depp as Willy Wonka

Dynamical Networks

Ewelina Zatorska

Complex highly dynamical networks are of significant interest in many fields of life and social sciences. Examples of such structures include neural networks, biological fiber networks such as connective tissues, vascular or neural networks, ant trails, polymers, economic interactions, etc. These systems are composed of a large number of agents – particles or cells – interacting through local interactions, and self-organizing to reach large-scale functional structures. By their continuous breaking and reforming connections, such networks are often very plastic having ability to change shape and adapt to different situations. For example, the biochemical reactions in a cell involve proteins – DNA, RNA, or gene promotors – linking or unlinking to create or break large structures [7]. Because of their paramount importance in biological functions or social organizations, understanding the properties of such complex systems is of great interest.

From the modelling point of view, the most straightforward approach would be to describe behavior of each agent and its interaction with the surrounding agents. Such description is called a *microscopic model* because it provides a precise answer where an individual agent might be at a given time and what are all the interactions around it.



Let us for example consider the 2-dimensional microscopic model that features N particles located at points $X_i \in \Omega \subset \mathbb{R}^2, i \in [1, N]$. The particles are randomly linking and unlinking to their neighbors which are located in a ball of radius R from their center.
Particles are then interacting through a network of links that we could model as springs of equilibrium length l. The detection zone for linking to close neighbours is a disk of radius R. The link creation and suppression are random in time. Let us assume, for example, that they follow Poisson processes with frequencies ν_f^N for formation of links and ν_d^N for destruction of links. If we now look at the moment in time where the number of links attached to agent X_i is equal to K, then we can compute the total energy W related to the maintenance of these links as a sum

$$W = \sum_{k=1}^{K} \tilde{V}(X_{i(k)}, X_{j(k)}).$$

In the expression above, $i(k), j(k)$ denote the indexes of particles connected by the link k, and

$$\tilde{V}(X_i, X_j) = (|X_i - X_j| - \ell)^2$$

is a pairwise potential generated by the spring with equilibrium length ℓ connecting X_i to X_j.

Particle motion between two time steps t^n and $t^n + \Delta t^n$ is then supposed to occur in the steepest descent direction to the energy W in the so-called overdamped regime:

$$X_i^{n+1} = X_i^n - \nabla_{X_i^n} W \Delta t^n + \sqrt{2D\Delta t^n}\mathcal{N}(0,1), \quad i = 1, \dots, N. \tag{1}$$

Here, $\mathcal{N}(0,1)$ is the normal distribution with mean 0 and standard deviation 1, and $D > 0$ is another system parameter called the diffusion. The last part of the equation models the fact that we might not be able to precisely determine the position of the particle already at the level of microscopic description. This equation still lacks a term modelling the creation and destruction of links between agents. In the example of cell dynamics mentioned at the beginning, the frequency of linking/unlinking depends on the size of the molecules. This is a very fast process (order of seconds). The macroscopic evolution of the cell, such as cell growth, is on the other hand much slower (order of minutes). Our description should therefore include another time unit, say ε^2, that will be much shorter than the time scale of the agent motion. It would seem that we have to solve N equations (1) in order to trace each particle, changing the energy $\frac{1}{\varepsilon^2}$ times. Therefore, the rescaled version of the microscopic model with large number of particles $N \to \infty$, large number of links $K \to \infty$, and a very fast link creation/destruction rate $\varepsilon \to 0$ makes system (1) computationally costly. We need to look for more efficient, but maybe less precise, solutions.

A way to do it would be to look for empirical distributions of the particles $f^N(x,t)$ and the links $g^K(x_1,x_2,t)$ rather to trace each particle and link separately. This means that we consider

$$f^N(x,t) = \frac{1}{N}\sum_{i=1}^{N}\delta_{X_i}(x); \quad g^K(x_1,x_2,t) = \frac{1}{2K}\sum_{k=1}^{K}\left[\delta_{X_{i(k)},X_{j(k)}}(x_1,x_2) + \delta_{X_{j(k)},X_{i(k)}}(x_1,x_2)\right],$$

where the symbol $\delta_{X_i}(x)$ is the Dirac delta centred at $X_i(t)$, with the similar definition for the two-point distribution. Postulating the existence of the following limits:

$$f(x,t) = \lim_{N\to\infty} f^N, \quad g(x_1,x_2,t) = \lim_{K\to\infty} g^K,$$

$$\nu_f = \lim_{N\to\infty}\nu_f^N(N-1), \quad \nu_d = \lim_{N\to\infty}\nu_d^N, \quad \xi = \lim_{K,N\to\infty}\frac{K}{N}$$

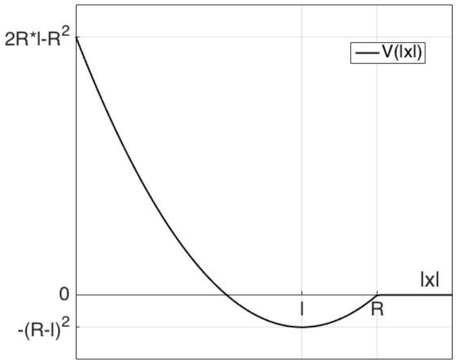
and letting $\varepsilon \to 0$ in system (1) gives rise, via the so called *mean-field limit* (see [1] and [5]), to the *macroscopic model*:

$$\partial_t f = D\Delta_x f + \nabla_x \cdot \big(f\,(\nabla_x V * f)\big) \tag{2a}$$

$$g(x,y,t) = \frac{\nu_f}{2\xi\nu_d} f(x,t)f(y,t)\chi_{|x-y|\le R}, \tag{2b}$$

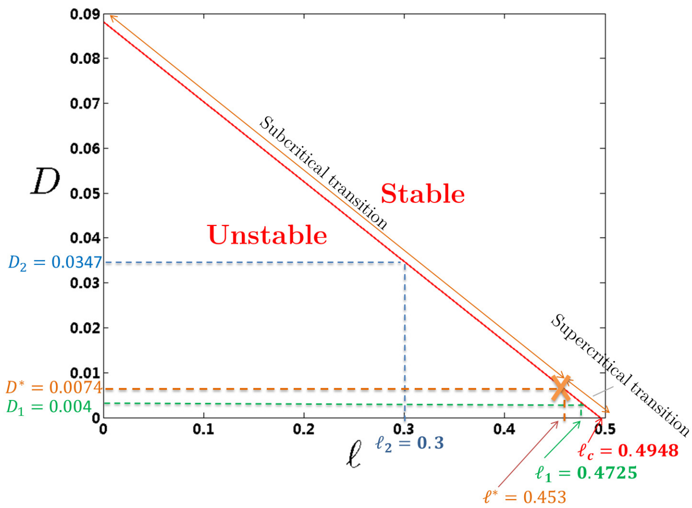
with a compactly supported potential V:

$$V(x) = \begin{cases} (|x|-\ell)^2 - (R-\ell)^2, & \text{for} \quad |x| < R, \\ 0 & \text{for} \quad |x| \ge R. \end{cases}$$



Intuitively, the bigger the ratio $\ell/R \le 1$, the stronger the tendency of the particles to repulse each others. On the other hand, if the ratio ℓ/R is small, this means that the links created between the particles should attract than repulse. This mechanism is described by the second term in the the equation for the limit distribution of particles (2a). It is sometimes called the attractive-repulsive part of equation. The first term on the right hand side is a linear diffusion term which is an effect of the random motion of the particles on the microscopic level. Another important observation is that the distribution of links in the macroscopic description (2b) is completely determined by the distribution of particles.

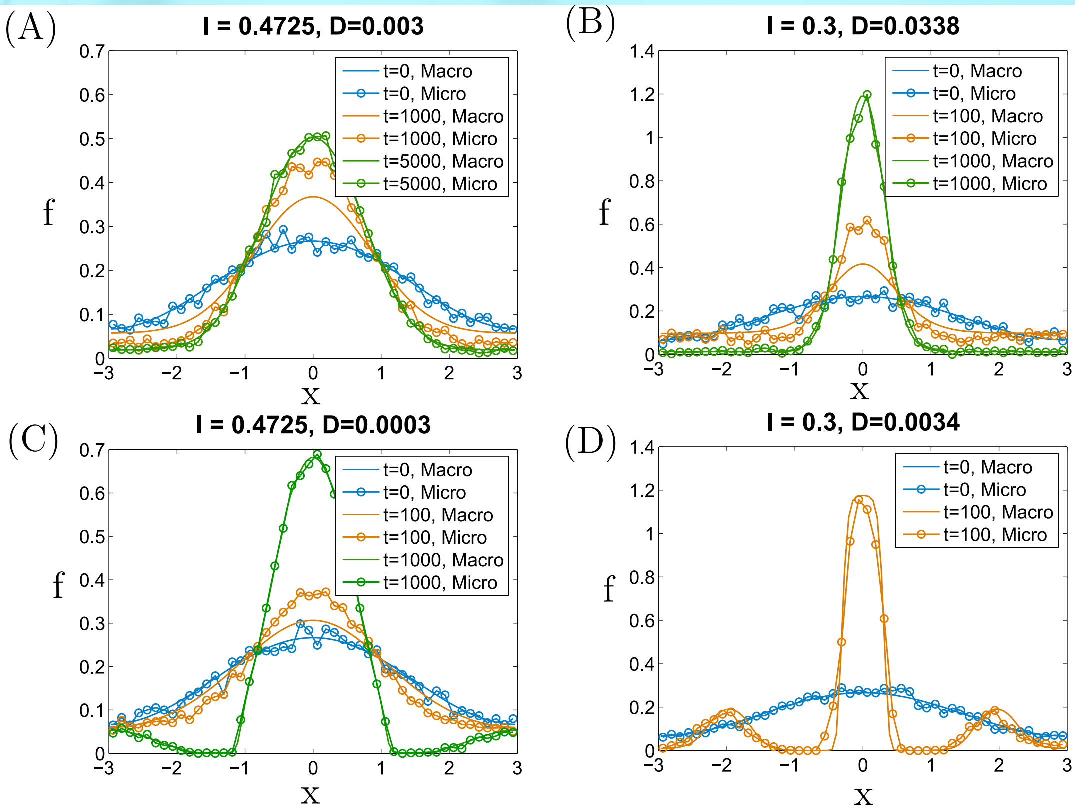
Equation (2a) is an example of nonlinear partial differential equation, and we do not know whether it has a solution in the classical sense that is different from a constant. Even if such solution exists, it would be hard to give an explicit formula for it, and so we would at least like to know some of its properties. We could, for example, check what are the values of the parameters D, ℓ, R required to balance the repulsive and attractive forces. For this reason we consider a simple one-dimensional domain $[-3, 3] \in \mathbb{R}$ with periodic boundary conditions (or a circle of circumference equal to 6) and we investigate when a small perturbation of a constant initial condition for f is amplified or damped. This corresponds to what we call instability or stability of the constant steady state. Performing Fourier analysis of equation (2a) around the constant steady state $f^{‘*} = \frac{1}{2L}$ for $R = 0.75$, we can see that the other two parameters ℓ and D should be chosen as presented in the diagram below.



Let us try to interpret this result. Parameter D measures how the perturbations of solution are smoothened by the dissipation process. If this parameter is small, any initial perturbation has a chance to expand if the potential V is more attractive than repulsive, meaning that the ratio ℓ/R should be small. This corresponds to the lower triangle in the above picture. Note also that, if the diffusion parameter is sufficiently big (bigger that 0.09), then the potential V does not play a role any more and the diffusion overcomes the local interactions, smoothing out the initial perturbations, see [2].

What is much less intuitive is that the lower triangle can be divided into two parts distinguishing the qualitative behaviour of the model beyond the linear level, [4]. Our analysis, based on central manifold reduction [6], provides a characterization of the type of bifurcation that appears at the instability onset. We can distinguish the continuous and discontinuous phase transitions (supercritical and subcritical bifurcation). In the first case, the initial perturbation first grows exponentially, but than it saturates, so that f tends to an almost homogeneous stationary state. In the subcritical bifurcation, the perturbation grows much more so that the final state may be very far from the original homogeneous state. Mathematical analysis of the macroscopic equation (2a) allows us to find a precise value of parameters ℓ and D for which the bifurcation changes type: they are denoted by ℓ^* and D^* in the diagram above, [1].

Having this information to hand, we would like to understand if and how well this captures the behaviour of actual particles at the microscopic level. Because the derivation of the macroscopic system (2) is formal, it is important to look for evidence that the limit system describes what we started with. In this particular case, we can show that the numerical solutions behave as predicted in the theory, and that we obtain – numerically – a very good agreement between the micro- and macro- formulations. In the figure below, see [2], the comparison of the density distributions f between the the macroscopic model and the microscopic one is presented for different values of parameters corresponding to the supercritical (A and C) and subcritical (B and D) bifurcations.



This numerical verification encourages us to believe that the formally derived macroscopic model is indeed a mean-field limit of the microscopic one. It also confirms that the two types of bifurcations revealed in our formal computation – subcritical and supercritical – do actually occur. More importantly, we show that the bifurcation structure is indeed relevant for the microscopic model, for which no theoretical analysis exists to date. Moreover, the same numerical method can be now used in cases where the microscopic model is too computationally intensive, for example to simulate the distribution of particles in the 2-dimensional domains, see [2, 3].

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Clifford Prize Lecture

On 10 November 2017 the department hosted the third Clifford Prize Lecture. The W.K. Clifford Prize was instituted in 2011 by the Advisory Board of the International Conference on Clifford Algebras and their Applications in Mathematical Physics (ICCA) and is awarded every three years. It is an international scientific prize for young researchers. The prize intends to encourage young researchers to compete for excellence in their research in theoretical and applied Clifford algebras, their analysis and geometry.

William Kingdon Clifford worked in our department from 1871 until his untimely death in 1879 as Professor of Applied Mathematics, holding the Goldsmid Chair. In recognition of Clifford's legacy we offer the prizewinner the opportunity to deliver a lecture at UCL.

This year we were delighted that Clifford's great grandson, Fisher Dilke, was able to attend both the lecture and our reception afterwards.

The 2017 Laureate was Dr David Kimsey from Newcastle University and his lecture was entitled "The spectral theorem for a normal operator on a quaternionic Hilbert space based on the S-spectrum".

In linear algebra the spectral theorem is a representation of an Hermitian matrix in terms of its eigenvalues and eigenvectors. This result generalises to the case of a linear self-adjoint operator acting in an infinite-dimensional complex Hilbert space. David Kimsey aims to extend this construction to quaternionic operators. The fundamental difficulty here is that, unlike complex numbers, quaternions do not form a field (quaternion multiplication is not commutative). Quaternionic spectral theory has a long and somewhat controversial history. Even the definition of the spectrum of a quaternionic linear operator is the subject of discussion amongst specialists. In recent years David Kimsey and his coauthors made substantial progress in this area.

William Kingdon Clifford

The Laureate 2017 - ICCA11 Gent

David Kimsey of Newcastle University (UK) has been selected as the recipient of the third W.K. Clifford Prize, for his outstanding mathematical research achievements in the field of quaternionic analysis with applications in quantum mechanics.

He received his Ph.D. from Drexel University with a thesis titled "Matrix-valued moment problems" (supervisor H. Woerdeman). During two postdocs his interest subsequently moved to quaternionic analysis. Spectral theory for normal operators on a quaternionic Hilbert space is a delicate and technical subject due to the noncommutativity of the quaternions. In particular, the proper notion of spectrum is not immediately obvious and turns out to be given by the recently discovered S-spectrum. Based on this notion, David Kimsey (in collaboration with Alpay and Colombo) produced a completely rigorous analogue of the spectral theorem for bounded and unbounded normal operators on a quaternionic Hilbert space. This spectral theorem is a crucial tool to formulate the axioms of quaternionic quantum mechanics and as such closed a problem formulated by Birkhoff and von Neumann in 1936.

The solution to this longstanding open problem in itself would already warrant awarding the prize to David Kimsey. However, through various collaborations, he also initiated the study of moment problems, free analysis and interpolation in the context of quaternions.

Women In Mathematics

The Women in Mathematics taster course this year was held at UCL on 3rd July 2017

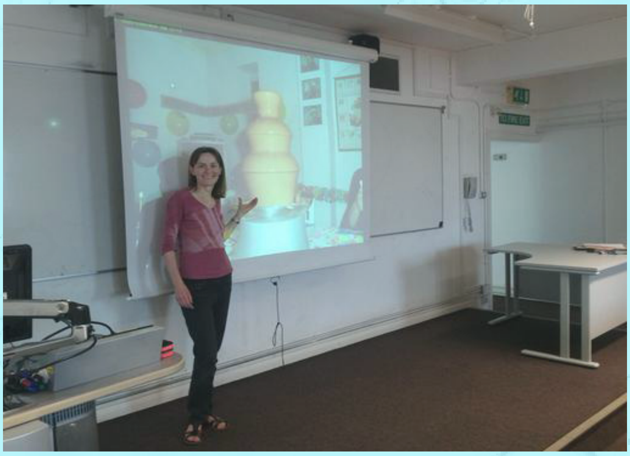
Those invited to attend the event are currently studying A level Mathematics or equivalent qualifications and are seriously considering taking a degree in Mathematics or a degree involving Mathematics.

Academic Mathematics from UCL including Dr Robert Bowles (Maths Admissions Tutor), Professor Helen Wilson and Professor Karen Page delivered formal lectures and problem solving sessions covering a variety of mathematical topics.

Advice and guidance was also provided on the admissions process and future career options.

The Women in Mathematics taster days are organised by the Careers Group, University of London and they received very positive feedback for this event

This year, the Women in Mathematics Taster Course is planned for the 2nd July 2018.



Professor Helen Wilson's chocolate fountain demonstration

Zero Tolerance Award

The Mathematics Department gained the Zero Tolerance Award for its pledge to not tolerate sexual harassment.

Students' Union UCL and UCL are committed to fighting sexual harassment. We understand that we all have a responsibility to make our University a safe space for all students and staff. Sexual harassment is any unwanted and/or persistent behaviour of a sexual nature.

We, as a department, pledge:

- To never tolerate, condone or ignore sexual harassment of any kind
- To educate students and staff about sexual harassment and why it's never ok
- To support students and staff when they talk about, report or challenge sexual harassment
- This includes making appropriate adjustments to ensure that those who have experienced sexual harassment or violence are supported in continuing their studies or work with as little disruption as possible
- To actively challenge the culture within which sexual harassment happen

Professor Robb McDonald receiving the Departmental Zero Tolerance award

Rafael Prieto Curiel

October 2017 marked the first Black Mathematician Month. Throughout the month, we aimed to celebrate diversity in mathematics, to promote the work of black mathematicians and to dismantle ideas about who can become a mathematician, hoping to create a community which is more open and inclusive.

Why do we need a Black Mathematician Month? Our experiences through the month showed us two things: firstly, after interviewing black mathematicians in the UK, the US and Nigeria we heard many tales of active discouragement from pursuing mathematics. Whether it was being told that the subject was not appropriate, being mistaken for the cleaning staff, or in one case having to quit a job because of death threats, there were stories of discrimination at every level. Being part of a silent system will not change and fix things.

One thing that every black mathematician we spoke to had in common is that they could all remember that inspiring teacher or professor who pushed them and encouraged them to become a mathematician. Their successful careers were, perhaps, highly influenced by that lecturer who shared and imprinted their passion for mathematics with them. The figure of a relevant role model was always significant, and in the majority of the cases the role model had that special place not because of the colour of their skin but because of their passion for mathematics. One way to achieve better representation might, then, be to increase the number of such role models.

Many people felt that black communities were left out of science at an early age, and that an effective but simple strategy would be the active promotion of mathematical research in schools which have large numbers of black pupils.

Black Mathematician Month was a joint effort, with support from UCL Mathematics Department, the London Mathematical Society, the Institute of Mathematics and its Applications, and others. Throughout the month of October, mathematical associations such as Chalkdust, A periodical, Plus Magazine and the Mathematical Association published relevant content such as mathematical articles, videos and interviews with black mathematicians.

To celebrate the end of the month, the UCL Mathematics Department hosted a closing ceremony. The guest speaker was Dr Nira Chamberlain, the Vice President of the Institute of Mathematics and its Applications who had recently been named the 5th most influential black person in the UK. Nira is not only highly influential, fighting for equality in mathematics but he is also a true inspiration.

Dr Nira Chamberlain

People from all over London and even further joined us at UCL to listen to Nira give a talk titled ‘The Black Heroes of Mathematics’ where he showed us that, even though the first black person to be awarded a PhD in mathematics was less than one hundred years ago (and his transcript had the word “coloured” printed across it), there have been many inspirational black mathematicians. Nira also shared with us his experiences of growing up in the UK: how he became a mathematician and how, at one point in his career, was told to become a boxer ‘since he had the right body for it’. Years later, when his son was told to become a singer rather than a mathematician, Nira decided to go back to university, completing his PhD part time with the aim of setting an example, not only for his son but for many others.

Perhaps the most valuable experience we had during Black Mathematician Month was precisely being inspired by Nira. He managed to motivate us all, continually reminding us that ‘you don’t need anyone’s permission to become a great mathematician’.

Being aware of the times when minorities are treated differently, hearing their stories and what they have to deal with, perhaps on a daily basis, does bring light to an often-ignored problem. We all need to actively work to construct a better mathematics community. We all need to destroy the intellectual stereotype. Regardless of our role, we can all contribute to a more equal society. Whether we are the empathetic classmate, the inspirational teacher or lecturer, the sympathetic colleague at a conference or the organiser who strives to have a diverse range of speakers, we all play an active role in how our community currently is and so we can all work to make it better.

We will be trying to put the lessons that we have learnt into practice throughout the year before we celebrate the next Black Mathematician Month. Whether it is trying to provide people with role models, gathering better data on university applications or just going out and talking about how beautiful mathematics can be, we believe that there is something that we can all do to help build a more representative and fairer mathematical community, to the benefit of all.

Dr Nira Chamberlain with Atheeta Ching, Rafael Prieto Curiel, Nikoleta Kalaydzhieva and Sean Jamshidi

Summer picnic at Gordon Square

Farewell drinks for Rafael from the Chalkdust team

The Chalkdust team meats Ingrid Daubechies, one of the most influential mathematicians in image processing

British Applied Mathematics Colloquium, the UCL team

British Applied Mathematics Colloquium, the UCL team

Christmas Dinner 2017

Stochastic optimal control

Iain Smears

1 Optimal control problems

Control theory is broadly concerned with the analysis of controllable dynamical systems, often with the goal of finding the best choice of control to achieve a desirable outcome. Since the pioneering works of Bellman [2] and Pontryagin in the 1950s (see [6] for a historical perspective of the latter), it has now become an essential part of modern science and engineering, and it finds diverse applications, for instance in aeronautical and aerospatial engineering, energy, robotics, as well as machine learning in computer science, to mention only a few. Concrete realizations are found in everyday items, such as thermostats at home or cruise control and ABS in cars, as well as in large scale industrial and scientific projects, such as in the planning of distribution of commercial goods or in guidance systems for atmospheric re-entry of space shuttles [9, 13].

Discrete control problems. As a result of the diversity of applications, control problems appear in many different forms, often in terms of the nature of the system being controlled. In some applications, the set of all possible configurations of the system, known as *the state space*, is naturally a finite set. For example, the state space could represent the set of all possible positions of pieces on a chessboard, with the *state* being the particular configuration at the current turn. In this example, the set of *controls*, or *actions* as they are sometimes called, is then the set of moves a player can take. It is often assumed that the evolution of the state between discrete time-steps obeys some probability distribution, for instance to represent the uncertainty of the opponent's next move. The goal is then to optimize some measure of success, typically represented by the expected value of a specified *objective function*. In such cases, we call the control problem *discrete*. It is worth noting that models of this kind are generally the starting point for *reinforcement learning* which has received considerable recent interest in Computer Science. Furthermore, control problems for specific types of such processes, such as *Markov decision processes*, are a major area of study in Operations Research [10].

Continuous control problems. In many engineering applications concerned with physical systems, the dynamics are naturally modelled by either ordinary or partial differential equations (ODEs and PDEs). In both cases, the setting is now continuous, and thus we speak of a *continuous control problems*. For ODEs and PDEs without stochastic terms, the problems are furthermore *deterministic*, in the sense that a fixed choice of controls leads to a state that follows a unique path, provided at least that the governing equations are well-posed. We refer the reader to the textbooks [13] and [14] for a presentation of the theory of deterministic control problems governed by ordinary or partial differential equations.

Here, we will focus on continuous *stochastic* optimal control problems, where the dynamics are governed by a system of stochastic differential equations [5]. Without attempting to go into too much detail, it is important to note that there are important conceptual differences between deterministic and stochastic control problems. Indeed, in the deterministic case it is possible to define the notion of an optimal path of the system, whereas in the stochastic case, this is no longer possible since the objective function is optimized in expected value only and all possible paths of the system may have some influence. For instance, a deterministic controller might consider it quite optimal to bridge a canyon using only a tightrope, yet a stochastic controller must consider a larger set of eventualities. Mathematically, these differences are significant and can be seen in the fact that certain solution methods are applicable only in the deterministic setting. However, as we shall explain in the following section, a generally applicable method for solving stochastic control problems is based on the dynamic programming principle, due to Bellman.

2 Dynamic programming

A model problem. To make ideas more precise, we now consider the following model problem of a system where the state at time $t \geq 0$ is given by the vector $X(t) \in \mathbb{R}^d$, with $d \geq 1$ denoting the dimension of the state space. The random fluctuations in the system are modelled by a k-dimensional Brownian motion $B(t)$; note that k need not equal d in general. The state vector $X(t)$ obeys the stochastic differential equation (SDE)

$$\mathrm{d}X = b(X(t), \alpha(t))\,\mathrm{d}t + \sigma(X(t), \alpha(t))\,\mathrm{d}B, \quad X(0) = x, \tag{1}$$

where $x \in \mathbb{R}^d$ is a starting position, and where $b(\cdot, \cdot)$ and $\sigma(\cdot, \cdot)$ are known functions of the current state and the *control* $\alpha(t)$ that is chosen at time t. More precisely, if Λ denotes the set of all possible controls, then

$b\colon \mathbb{R}^d \times \Lambda \to \mathbb{R}^d$, and $\sigma\colon \mathbb{R}^d \times \Lambda \to \mathbb{R}^{d\times k}$ with $\mathbb{R}^{d\times k}$ the space of d-by-k matrices. Without going into detail, note that for the purposes of analysis, it is usually necessary to make additional assumptions on the regularity of the functions b and σ, as well as on the structure of the control set Λ.

We now consider the problem of finding a control function $\alpha(\cdot)\colon [0,\infty) \to \Lambda$ that minimizes the objective functional

$$J(x, \alpha(\cdot)) = \mathbb{E} \int_0^{\tau_{\text{exit}}} \mathrm{e}^{-ct} f(X(t), \alpha(t)) \mathrm{d}t, \tag{2}$$

where $f(\cdot,\cdot)$ is a bounded real-valued function called *the running cost*, and where τ_{exit} is the exit time of the state from a specified bounded open region of $\mathbb{R}^d$, denoted by Ω. For simplicity, we include the discount factor $c > 0$ to guarantee that the integral should be finite; in economics applications, this can be justified in terms of an underlying interest rate in the system. The symbol $\mathbb{E}$ in (2) denotes the expectation as a result of starting the process at the initial point x, and evolving it under the control $\alpha(\cdot)$ via the SDE (1). To complete the specification of the problem, we must indicate the set of allowed control functions $\alpha(\cdot)\colon [0,\infty) \to \Lambda$. From a modelling point of view, it is natural to require that the controls must only depend on the information available about the system up until the current time, which can be formulated mathematically in terms of measurability conditions related to the *filtration* of the Brownian motion. Without going into technicalities, let us denote by $\mathcal{A}$ a suitably defined space of control functions that respect these requirements. We then call a minimizer in $\mathcal{A}$ an *optimal control.* We note that there exist many variants to the model problem considered here, especially with regards to the objective functional in (2); our choices here are primarily made to ensure that the following discussion remains as simple as possible.

Dynamic programming. At an intuitive level, it is clear that it would be good to know which regions of the state space are better than others, so that we may steer the stochastic process towards them. This can be represented by attributing a value $V(x)$ to each point x in the state space:

$$V(x) \coloneqq \inf_{\alpha(\cdot)\in\mathcal{A}} J(x, \alpha(\cdot)). \tag{3}$$

Note here that, although x has so far been used to denote the initial condition of the stochastic process, we can also think of it as a variable ranging over the domain Ω. Since $V(x)$ represents the value of the state being at the point x, the function V is called the *value function*. For our minimization problem above, we would naturally aim to steer the stochastic process towards regions where V is small. Using the Markov property of diffusion processes, it is possible to show under some conditions (see [5] for further details) that the value function $V(x)$ satisfies

$$V(x) = \inf_{\alpha(\cdot)\in\mathcal{A}} \mathbb{E}\left[\int_0^t f(X(s), \alpha(s)) \mathrm{d}s + \mathrm{e}^{-ct} V(X(t))\right], \tag{4}$$

for all appropriately defined stopping times $0 < t \leq \tau_{\text{exit}}$. This is the dynamic programming principle, which expresses the fact that the value of being at x is tied to the values of the points that the state $X(t)$ is most likely to reach at some later time t, as well as the running costs incurred for times between 0 and t. We note that this is a general result that is not limited to the continuous-time stochastic control problems considered here, as it is also valid in the discrete context, as well as in the deterministic setting [2]. It is therefore a central part of the theory of optimal control problems.

3 The Hamilton–Jacobi–Bellman PDE

An intuitive interpretation of (4) is that if we knew the value function, then naturally we should choose the controls minimize the right-hand side of (4), i.e. that make it most likely to move towards points where $V(X(t))$ is small, modulo the integral term due to the function f. As we shall now see, this intuitive idea is made rigorous by the associated Hamilton–Jacobi–Bellman (HJB) PDE (sometimes also just called Bellman PDE). Assuming that V is sufficiently smooth, some results from Stochastic Calculus can be applied to (4) in order to obtain the HJB equation

$$\inf_{\alpha\in\Lambda} \left[L^\alpha V + f^\alpha\right] = 0, \tag{5}$$

where L^α is a differential operator associated to each control value $\alpha \in \Lambda$, defined by

$$L^\alpha V = a^\alpha : D^2 V + b^\alpha \cdot \nabla V - cV, \tag{6}$$

where we employ the short-hand notation $a^\alpha(x) = \frac{1}{2}\sigma(x,\alpha)\sigma^\top(x,\alpha) \in \mathbb{R}^{d\times d}$, and $b^\alpha(x) = b(x,\alpha)$, $f^\alpha(x) = f(x,\alpha)$, i.e. we omit the dependence on x and indicate the dependence on the control α in a superscript. Furthermore, in (6), D^2V denotes the Hessian matrix of V, and $A : B = \sum_{i,j=1}^d A_{ij}B_{ij}$ is the Frobenius inner-product of matrices. The PDE (5) holds throughout the domain $\Omega \subset \mathbb{R}^d$, with the infimum expression being understood in a pointwise sense, at each $x \in \Omega$. The PDE is supplemented by boundary conditions on the boundary $\partial\Omega$, in particular $V = 0$ on $\partial\Omega$ for the current model problem. In fact, the assumption that V must be sufficiently smooth in the above arguments can be removed if the equation is understood in the sense of viscosity solutions [3, 4, 5], although we shall not go into further details about this here.

In going from (4) to (5), a major change of setting has occurred, namely that whereas the expectation in (4) still refers to the distribution induced by the stochastic process, (5) is a deterministic PDE that no longer directly refers to any stochastic quantities. Thus, in principle, the value function can be found using deterministic methods. Furthermore, the infimum in (4) is over the set of control *functions* $\mathcal{A}$, whereas in (5) it is over the set of control *values* Λ, which is often much simpler to work with. Furthermore, for a given point x, it turns out that minimizers α^* of (5) can be used to construct an optimal control, thereby solving the original control problem.

Challenges. In summary, the strategy for solving the optimal control problem requires finding the value function and optimal controls via (5). However, we must mention that except in rare circumstances, it is not possible to find an analytical expression for the value function. In fact, it is realistic to only expect to find approximations of the value function and the optimal controls. Ideally, we would like methods that approximate the value function and optimal controls accurately, and that work for as wide a range of problems as possible. There are many significant challenges in making this work in practice, each difficulty being related to the details of the particular application at hand. A well-known one is the *curse of dimensionality*, a phrase coined by Bellman himself: very roughly speaking, it is known that the computational resources needed to approximate a reasonable but otherwise arbitrary function of d variables to a given accuracy over the whole state space grows exponentially with d. Strictly speaking, the curse of dimensionality is not tied to the HJB PDE itself, since it also appears in discrete control problems with large state spaces. To address the curse of dimensionality, it is clear that some requirements on accuracy or generality must be relaxed; for instance, one idea used in methods such as reinforcement learning and approximate dynamic programming [9], is to find approximations (either computational or statistical) that are accurate in regions of state space that are most likely to be encountered.

Since the curse of dimensionality is a general difficulty for approximating functions of many variables, let us focus our discussion now on issues that are more specific to the HJB PDE itself. Even in low dimensions, a major challenge here is that (5) is a highly nonlinear PDE. More precisely, it is what is known as *a fully nonlinear* PDE, i.e. the solution and all derivatives appear nonlinearly inside the infimum in (5). This has far reaching consequences, the most important one for practice being that it significantly complicates the development and analysis of convergent numerical methods for approximating the value function. As we shall now explain, despite the existence of many efficient and well-understood numerical methods for linear or moderately nonlinear PDE, it is only more recently that they begun to appear for fully nonlinear PDEs.

Numerical methods. Many numerical methods for PDEs are based on discretizing the state space by a grid, or mesh, and then solving a discrete system of nonlinear equations for the unknowns that approximate the solution at the points on the grid. One family of such methods that are provably convergent for HJB equations such as (5) is the class of *monotone* schemes, which can be based on either finite difference methods [1, 8] or, as found more recently, on finite element methods [7]. They derive their name from a monotonicity property that can be viewed as a discrete version of the maximum principle enjoyed by the underlying PDE. In fact, in some cases, these methods are equivalent to solving a discrete control problem that approximates the continuous one [8]. There is still much ongoing research in developing these methods for HJB equations and for other fully nonlinear PDEs. However, these methods do present some drawbacks. For instance, monotone schemes have rather limited computational efficiency, since they are typically at most second-order accurate, with first-order accuracy being rather more common. This means that the error between V and its discrete approximation V_h is at best of order h^2, where h is the typical spacing between points on the grid. This can be written as $\|V - V_h\| = O(h^k)$ with $k \leq 2$, where $\|\cdot\|$ denotes an appropriately defined norm on a space of functions.

Due to the low-order nature of monotone methods, there have been many attempts at developing high-order methods, which have the potential to be more computationally efficient. However, the nonlinearity of the equations has been major obstacle to proving certain required stability properties of these methods and to guaranteeing their convergence to the correct solution, even for very smooth solutions. It is only recently that provably convergent high-order methods have been found, for a reasonably broad class of HJB equations [11, 12]. From a practical perspective, the key advantage of these newly developed methods is that they can attain arbitrarily high-orders of accuracy, as long as the solution is sufficiently smooth or if the mesh is appropriately adapted to the solution. Simplifying to some extent, it can be proved that the error $\|V - V_h\| = O(h^k)$, for any order $k \geq 1$, for V sufficiently regular in certain Sobolev spaces. The key step in this work was based on understanding the PDE-theoretic properties of specially constructed linearizations of the HJB equation, and combining it with certain advanced discretization methods that allow for arbitrary orders of approximation.

Concluding remarks. Overall, when they are applicable, numerical methods for solving the HJB equation are among the most accurate and efficient ways of obtaining an approximate solution, and there is now a range of methods to handle many, although not all, of the most common computational challenges encountered in applications. There has been significant recent progress in the convergence analysis of these methods, despite the strong nonlinearity of the PDE. However, outstanding challenges still remain. In particular, we have already mentioned the curse of dimensionality, and it will be interesting to see if some of the ideas that have been recently successful for discrete-time control problems can be carried over to continuous-time problems.

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Singapore Summit

James Cann - PhD Student, UCL

Quite often, messages find their way into my UCL email inbox which seem too good to be true: 'Earn 5000 Canadian Rupees per hour from your armchair', or else 'One small step - and leap into your perfect postdoc position'. And so I met 'Spend a week in Singapore and meet Nobel Prize winners' with not undue scepticism.

The culpable sender: Professor Robb McDonald. Surely our esteemed Head of Department hasn't developed a taste for spam and taken up phishing. Curious, I read that 'The Global Young Scientist's Summit is a gathering of 200 PhDs and post-docs from all over the world, together with globally recognised scientific leaders (recipients of the Nobel Prize, Fields Medal, etc.) in Singapore. UCL has been invited to nominate 5 attendees - maximum of 1 per department'.

To my lasting surprise, on the 21st January I found myself walking aboard the 7pm flight, departing a damp and dismal Heathrow. Fourteen hours, eight time zones and two surprisingly tasty foil-sealed flight meals later, I was mid-afternoon in the tropical city-state of Singapore.

Nanyang Technological University's abundantly green west-of-island campus played host to the summit - a varied programme of plenary lectures, small group discussions, and tours of the city. One degree north of the equator, Singapore is a sovereign city-state, nestled at the foot of Malaysia, its main island separated to the north by a narrow strait of water. London might be multicultural, but it doesn't boast four official languages: Malay, Mandarin, Tamil and English; neither does its outdoor temperature rarely stray from a 23 - 31°C window, its humidity seldom fall below 80%. For breakfast there were lotus steamed buns, chicken and prawn fried rice, 'century eggs', pandan leaf jam, baked beans oriented by baby corn and spice.

Just a taste of the plenary lectures: 'Solar Cells inspired by Green Leaves' was the theme of Michael Graetzel's talk. Frances Arnold described how imitating evolutionary mutations in the lab is already helping to optimise enzyme design. Stuart Parkin talked about 'Spin Orbitronics' and engineering the next generations of memory storage devices. Fields Medallist Efim Zelmanov gave his take on 'Mathematics: Science or Art?' and Sir Michael Atiyah was even more contemplative, advising that we plant more trees and read and write poetry.

Mid-week there was a poster session. I presented my research on arithmetic random waves, which was met with enthusiasm by a multidisciplinary audience of engineers, biologists and physicists. There were posters on plate tectonics; on the effect of smartphone apps on the use of public transport in major cities; on non-intrusive imaging techniques for the brain. Wandering around the 50-or-so posters, passing impassioned conversations and careful explanations, the vitality of the participants was palpable. Young researchers sharing their work; finding a common language.

There is one phrase from the summit that has stuck in my mind. Describing the development of physics over the past centuries, Gerard ‘t Hooft highlighted our gradual appreciation of a fundamental idea: ‘The uniform motion of a system cannot be detected from within that system.’ The trip to Singapore placed me far outside of my usual context - academically, culturally and geographically. Afar, my discernibly narrowed eyes, eased open once more.

Many thanks to the Department of Mathematics and especially to Professor Robb McDonald for their kind nomination and generous support in attending the Global Young Science Summit 2018.

Team UCL (author wearing shorts)

IUTAM Symposium - Wind Waves

Professor Ted Johnson

The IUTAM General Assembly gave us permission to hold an IUTAM Symposium on Wind Waves at UCL in the week of September 4-8, 2017. We welcomed about 80 participants from around the world including about 20 from US.

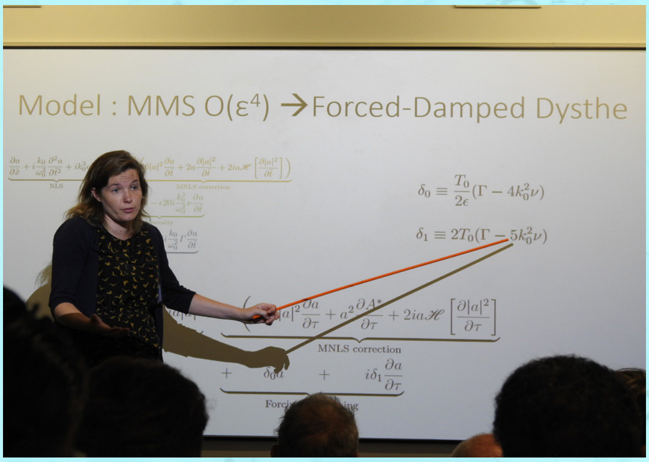
Wind-driven water waves play an essential role, both on the large scale ocean dynamics, with implications for weather and climate, and on the local scale where they affect transfer processes across the ocean-atmosphere interface, including extreme forces on marine structures, ships and submersibles. After 150 years of research, the dynamics of ideal linear and nonlinear waves, including their interaction and their evolution are broadly understood, although only recently have extremely large waves been identified through observations and in laboratory experiments.

There are however still conflicting theories about how wind generates waves, and there are only tentative theories about how wind forces affect the dynamics of extreme waves and wave groups. Current research on wind-wave dynamics by the proposers, and by other groups, is focussing on what is still a major question for water waves, namely, how in the presence of wind, do they form into characteristic groups (with or without white caps) and what are their essential properties, depending on the local atmospheric and oceanic conditions. The prediction of extreme events, such as rogue waves in the open ocean, or in shallow water, and waves driven by tropical cyclones, is becoming of increasing concern due to effects induced by climate change. Further, wind-driven waves, especially through their effect on small-scale motions and turbulence near the surface, need to be better understood in order to improve predictions of heat and mass transfer at the atmosphere-ocean interface, essential for the development of climate models.

Improved satellite observations and laboratory experiments are now becoming available to guide theoretical and modelling progress. Also, the general theme of transfer processes across a gas-liquid interface is relevant for flows in large pipes.

The Symposium took place in the Mathematics Department with the theme “Research on wind wave groups, applications to improved ocean wave modelling, and estimation of wave hazards”. This brought together theoreticians, numerical modellers, experimentalists and end-users in a forum where the latest research developments were presented, provided an environment with constructive interchanges, and with the outcome that clear directions were established for future research, and for the implementation of research advances into operational use. All sessions were held in the seminar room in the Mathematics Department at UCL, and all lunches, tea/coffee breaks and the reception were held in an adjacent room on the same floor. This gave a relaxed environment where as well as the scientific talks there was ample opportunity for informal discussions.

A special feature was the Lighthill lecture honouring the contribution of the eminent applied mathematician and fluid dynamicist Sir James Lighthill, who made profound contributions to wind wave research amongst his many accomplishments and was Provost at UCL 1979-1989.



Debbie Eeltink (Université de Genève)

There were 5 plenary talks and 41 contributed talks. The plenary talks were:

• Peter Janssen: Progress in operational wave forecasting
• Julian Hunt: (Lighthill Lecture) Mechanisms and modeling of wind-driven waves
• Guillemette Caulliez: Wind wave evolution observed in a large wind-wave tank: statistical wave properties, wave groups and wave breaking
• Ken Melville: Wind-wave breaking
• Vladimir Zakharov: Analytic Theory of Wind Driven Sea

Some highlights of the symposium were:

• Robust discussion of generation mechanisms, with some agreement about the importance of wave groups, either directly or by the necessity to consider non-monochromatic waves.
• Demonstration in laboratory experiments of the rapid development of fully two-dimensional (horizontally) wave fields, from uni-directional wind forcing.
• Role of wave-breaking in the formation and maintenance of wave groups, and through observations and numerical simulations the development of universal scaling laws for dissipation due to breaking.
• The increased capacity of DNS and LES simulations to study the air flow over wind waves in great detail, although the analogous capacity for two-phase (air and water) simulations is still in a developmental stage.
• The increased understanding and potential importance of rogue waves, not just for impact on shipping and marine structures, but for a full understanding of the wave spectrum.
• Wide range of applications for wind-wave forecasting, including surf conditions and tropical cyclone forecasts
• Importance and lack of current knowledge of the directionality of the fully-developed wind wave sea.

The proceedings of the Symposium, currently in press for Elsevier IUTAM Procedia (www.elsevier.com/locate/procedia), contain articles emanating from four of the plenary talks and a further sixteen articles emanating from the contributed talks.

Ted Johnson, Julian Hunt and Roger Grimshaw (local organisers) with Frederick Dias (University College Dublin and IUTAM Representative)

Lev Ostrovsky (University of Colorado), Efin Pelinovsky(Institute of Applied Physics, Nizhny Novgorod), Roger Grimshaw (UCL) and Tatiana Talipova (Institute of Applied Physics, Nizhny Novgorod)

Matthew Scroggs - PhD Student, UCL

My name is Peter Gustav Lejeune Dirichlet, and I am being kept against my will in a cupboard in the Kathleen Lonsdale Building.

My story starts in February 2015, when a group of PhD students in the UCL Maths Department started Chalkdust, a “magazine for the mathematically curious”. They sent me a draft copy of their first issue, and asked me if I could help out by responding to a few letters that they had received.

The quality of the rest of the magazine’s content was very high – indeed, if you haven’t read it already, you should track down a copy and read it; I believe all the issues are available at chalkdustmagazine.com. Hence, I agreed to provide responses for said letters: I helped one Chalkdust fan to ensure that her relationship was continuous while her husband at sea, and I helped another to pick the best ring to propose with (I suggested the Gaussian integers). And thus Dear Dirichlet was born.

Six months later, the students approached me again with a second collection of correspondence, and again I happily provided responses. This time, I helped Chalkdust readers sort an argument (using a branch cut), and deal with a translation problem.

My problems began a further six months later, when my entire home became infested with badgers. This left me with little to no time to reply to the letters sent to me by the Chalkdust editors, and so I missed the deadline they gave me. Two days later, I was in my local corner shop buying milk to feed my badgers, when I was grabbed by two people and thrown into the boot of a Citroën C4 Cactus.

When I awoke, I found myself in the X-Men Origins Seminar Room (XMOSR) in the Kathleen Lonsdale Building (KLB), with a set of Chalkdust editors. They told me that my letters were an absolutely integral part of their magazine, so they had procured an office for me to write my letters in. This seemed strange to me, as offices are not usually locked from the outside. And usually office aren’t called the X-Men Origins Seminar Room. And usually I don’t have to hide in a cupboard in my office, when someone else has booked my office for a seminar. But the room was well-equipped and spacious, so I happily got to work replying to a backlog of letters. A few days later, I discovered that two of my favourite badgers had also relocated to the XMOSR, where they remained, eating their way through the ever-growing stack of letters marked for my attention.

A few months later, refurbishment work in the KLB moved to our end of the building, and the area containing my office was closed. The Chalkdust editors told me that I was being upgraded to a new office, but when I arrived there, I found that my new office was actually a small cupboard in the Muirhead Room, the room used by PhD students for tutorials and other teaching duties. It was then that I realised that I was being kept against my will, while being forced to provide humorous responses to increasingly annoying letters. I vowed that I would take the first possible opportunity I had to escape.

In my year in a cupboard, I’ve overheard a great number of conversations about Chalkdust: in the summer, I heard members of the team painting a map to take to the Greenwich maths festival to demonstrate population models.

In September and October, I overheard the team putting together material for Black Mathematician Month, and doing their bit to raise awareness about minorities in mathematics. I also heard a great deal of working going into producing issue 06 of the magazine, including the goings on on Chalky Saturday, the day when the team got together to finish everything and apply the finishing touches. I finally heard the team, who had been hard at work since around 10am, finish printing everything to go home and check over just after midnight.

A few weeks after Chalky Saturday, I heard the team moving boxes containing 4000 copies of the magazine into the Muirhead room: some of them even needed to be stored in my cupboard. I hastily stole one, read it cover to cover, and highly enjoyed it; I think my favourite part was the excellent Dear Dirichlet: I must remember to congratulate whoever writes those letters...

In the weeks following the delivery, I overheard a lot of chatter about where the 4000 magazines were going: around 1500 copies were sent to 14 universities, over 150 copies were sent to schools, and many more copies were lugged to events by members of the team. Many of these were ordered online: I urge you to order a copy from chalkdustmagazine.com right now; the more you buy, the sooner I get my full cupboard space.

All the printing is paid for by the adverts placed in the inside covers: for issue 06, these were bought by G-Research and Jane Street. I did also hear someone say that the UCL Maths Department also gives a little bit towards each issue: perhaps just enough to pay for my food and my lemonade addiction.

In November, I heard members of the team making plans to go to the MathsJam Gathering, a two-day event packed full of 5 minute lightning talks, puzzles, games, cake, a competition, and so much more that I forget most of it. On the Saturday morning, when they were packing magazines to take, the team really sounded like they were struggling with the weight; but they didn't bring any copies back! Sadly, I was unable to be involved in any of these excellent activities, as I was busy replying to letters from my cupboard.

I was perhaps most disappointed at missing out on the chance to meet the Fields medallist Cédric Villani, whom three members of the team had breakfast with in February at the almost aptly named Villandry's Cafe. They wrote all about this in issue 06, so at least I got to read all about it there.

I was invited out of my cupboard once to attend the Chalkdust issue 06 launch party. I saw a large number of people at this party, all having an excellent time eating free pizza, drinking non-free drinks, and participating in the maths-themed pub quiz. It was exactly the kind of event that maths department alumni would enjoy, and I've overheard plans to hold a similar event in March for issue 07.

I hope this note reaches someone who can send help to me. Or at least send me some interesting letters to reply to.

Provost Teaching Award

The Provost's Teaching Awards were set up to celebrate the best of pedagogic prowess at UCL and to reward staff who are making outstanding contributions to the learning experience and success of our students.

They also demonstrate UCL's commitment to:

- improving teaching, learning and assessment as an ongoing process
- highlighting and rewarding achievements that support teaching and learning
- attracting and retaining world-class staff

This year, the winner of the Provost Teaching award was Matthew Scroggs, a PhD student and postgraduate teaching assistant (PGTA) in the Department of Mathematics. These awards are exceptionally competitive and this is an outstanding achievement by Matthew

Matthew has previously been awarded a MAPS Postgraduate Teaching Award and nominated for a UCLU Student Choice Teaching Award.

Matthew Scroggs receiving the award from the Provost

Choir

Emily Maw - PhD Student

The Michael Singers, our departmental choir, continue to go from strength to strength! Last term we prepared a Christmas repertoire from around the world and across the ages, which we performed at the department Christmas Party.

We also sang in the cloisters to raise money for Shelter, and at St Pancras Hospital. Our most exciting performance yet was a wassail at Bentham's Farm, the UCL allotment, where we sang to a newly-planted pear tree to bring it good luck for the spring!

This term our overarching theme is 'birds', which includes songs ranging from the Beatles' "Blackbird" to Wordsworth's "The Linnet" (set to a tune by Beethoven). We have a performance at UCLH lined up, as well as the now annual performance at the De Morgan Dinner!

Departmental Colloquium
Tuesday 14 November 2017

The Cost of the Sphere Eversion and the 16\π Conjecture

Professor Tristan Rivière (ETH Zurich)

How much does it cost...to knot a closed simple curve ? To cover the sphere twice ? to realise such or such homotopy class ? ...etc.

All these questions consisting of assigning a “canonical” number and possibly an optimal “shape” to a given topological operation are known to be mathematically very rich and to bring together notions and techniques from topology, geometry and analysis.

In this talk we will concentrate on the operation consisting of everting the 2 sphere in the 3 dimensional space. Since Smale’s proof in 1959 of the existence of such an operation the search for effective realisations of such eversions has triggered a lot of fascination and works in the math community. The absence in nature of matter that can interpenetrate and the quasi impossibility, up to the advent of virtual imaging, to experience this deformation is maybe the reason for the difficulty to develop an intuitive approach on the problem.

We will present the optimization of Sophie Germain conformally invariant elastic energy for the eversion. Our efforts will finally bring us to consider more closely an integer number together with a mysterious minimal surface.

Professor Tristan Rivière speaking at the Departmental Coloquium

Professor Barry Simon speaking at Departmental Colloquium

Departmental Colloquium
Tuesday 12 December 2017

“More Tales of our Forefathers”

Professor Barry Simon (Caltech)

Professor Barry Simon delivered a fascinating lecture on the history of mathematics at the Departmental Colloquium. The title of the lecture was “More tales of our forefathers”. The very next day he gave the same lecture at the Newton Institute in Cambridge and that lecture was recorded. Those who missed Simon’s talk can watch it here:

https://www.newton.ac.uk/news/20171114/1290378

Prizes Awarded to Undergraduate Students June 2017

First Year Prizes

Stevenson Prize
Myles Workman

Kestelman Prize
Alexandros Konstantinou

Bosanquet Prize
Yohance Osborne

Departmental Prizes in Mathematics:
Vivienne Leech
Andela Markovic
Paul Dubois
Matthew Rees
Hexin Cheng
Ulf Persson
Zhefan Zhou

Second Year Prizes

Kestelman Prize
Laura Wakelin

Andrew Rosen
Daniel Bussell

Departmental Prizes in Mathematic
Zhe Hong Lim

Third Year Prize

The Nazir Ahmad Prize
Natalie Evans

Wynne-Roberts Prize
Lingbo Ji

Finalists Prizes

Andrew Rosen Prize
Mohammed Abdallah

Ellen Watson Memorial Scholarship
Edwina Yeo

Mathematika Prize
Mihai Barbu

Bartlett Prize
Mai Buil

Castillejo Prize
Lingbo Ji

Fourth Year Prizes MSci

David G Larman Prize - Pure Mathematics
Christopher Evans

Susan N Brown Prize - Applied Mathematics
Leo Middleton

Project Prize
Felipe Jacob

Sessional Prize
Georgina Kennedy

The Institute of Mathematics and its Applications (IMA) Prizes

1 year membership of the IMA
Lingbo Ji
Mai Bui

Students who have recently obtained PhDs from the Department include:

Khadija Alamoudi (supervised by Rod Halburd) ‘The use of singularity structure to find special solutions of differential equations: an approach from Nevanlinna theory’

Samire Balta (supervised by Frank Smith) ‘On fluid-body and fluid-network interactions’

Gregorio Benincasa (supervised by Rod Halburd) ‘The anti-self-dual Yang-Mills equations and discrete integrable systems’

Bjorn Berntson (supervised by Rod Halburd) ‘Integrable delay-differential equations’

Alex Cioba (supervised by Chris Wendl) ‘Nicely embedded curves in symplectic cobordisms’

John Evans (supervised by Frank Johnson) ‘Group algebras of metacyclic type’

Adam Sanitt (supervised by John Talbot) ‘Turán problems in graphs and hypergraphs’

Pietro Servini (supervised by Frank Smith) ‘Roughing up wings: boundary layer separation over static and dynamic roughness elements’

Toby Sodoge (supervised by Jonny Evans) ‘The geometry and topology of stable coisotropic submanifolds’

Luke Swift (supervised by Erik Burman) ‘Geometrically unfitted finite element methods for the Helmholtz equation’

Adam Townsend (supervised by Helen Wilson) ‘The mechanics of suspensions’

Bin Bin Xue (supervised by Ted Johnson and Robb McDonald) ‘Dynamics of beach vortices and multipolar vortex patches’

Wenting Wang (supervised by Robert Bowles) ‘Opinion dynamics on typical complex networks and applications’

Department Teaching Award 2016-17

Dr Isidoros Strouthos

MAPS Teaching Award Nominees

Postgraduate Teaching Assistant **Belgin Seymenoglu**

MAPS Teaching Award Nominee (Staff): **Dr Christian Boehmer**

Promotions

Professor Yiannis Petridis
Professor of Mathematics

Professor Petridis works on analytic aspects of automorphic forms and their relation to the spectral theory of the Laplace operator on hyperbolic Riemann surfaces. He has applied perturbation methods to understand the location of scattering poles, which in arithmetic examples relate to the nontrivial zeros of the Riemann zeta function and other L-series. Currently he works on the distribution of modular symbols and distributional questions of closed geodesics and groups elements with restrictions of (co) homological type. Problems in quantum chaos for hyperbolic manifolds are investigated in relation to the behaviour of (more complicated) L-series.

Another line of investigation is the distribution of the eigenvalues of the Laplace operator, in particular Weyl's law. He has applied techniques from analytic number theory (exponential sums) to this problem for specific manifolds e.g. Heisenberg manifolds.

Dr Lauri Oksanen
Reader in Mathematics

Dr Oksanen's research interests include inverse problems for partial differential equations and related geometric problems such as the boundary rigidity problem and inversion of the geodesic ray transform.

Dr Andrea Macrina
Reader in Mathematics

Dr Macrina's research in Financial and Insurance Mathematics includes asset pricing and risk management, interest-rate modelling, algorithmic and high frequency trading, and the development of a probabilistic approach to asymmetric information. His research interests extend to risk measures and the modelling of market information, insurance claims reserving and hedging, and to optimal transport theory.

New Staff

Dr Roger Casals
Lecturer in Pure Mathematics

Dr Casals is a contact and symplectic topologist. Research interests: contact and symplectic topology, flexible-rigid dichotomy, h-principles and groups of contactomorphisms.He is also interested in the relations with algebraic geometry, including mirror symmetry and singularity theory.

Dr Eleni Katirtzoglou
Teaching Fellow.

Dr Katirtzolou is a Teaching Fellow at UCL. She has developed the Round Table model for teaching and learning mathematics and specializes in teaching abstract mathematics to economists.

Dr Ed Segal
Lecturer in Algebraic Geometry

Dr Segal is interested in the interactions between geometry, algebra and theoretical physics. More specifically, he works on derived categories of coherent sheaves and their various generalisations.

Dr Iain Smears
Lecturer in Applied Mathematics

Dr Smear's interests include numerical analysis of partial differential equations, Stochastic control problems & Hamilton-Jacobi-Bellman equations and a posteriori error analysis & adaptivity.

Dr Matthew Towers
Teaching Fellow

Dr Tower's research interests lie in representation theory and homological algebra, especially for restricted enveloping algebras and related algebras.

Dr Ewelina Zatorska
Lecturer in Applied Mathematics

Dr Zatorska works in the field of Mathematical Fluid Mechanics, in particular, analysis of Partial Differential Equations describing the flow of compressible and incompressible complex fluids or collective behaviour of agents. She is also interested in aggregation-diffusion equations modelling dynamical networks (of animals, polymers, etc.).

In Memoriam: Susan Brown

The Department of Mathematics is sorry to announce that Susan Brown, Emeritus Professor of Mathematics, died on Thursday 10 August 2017 at the age of 79.

Professor Susan Brown's career at UCL spanned over 40 years during which time she established herself as one of the UK's leading contributors to theoretical fluid mechanics. She made important and long-lasting contributions to viscous flow theory, separation, hypersonic boundary layers, vortex breakdown and hydrodynamic instability. It was her collaboration with Professor Keith Stewartson FRS which must surely rank as one of the 20th century's most productive partnerships in fluid mechanics. Together they published 29 papers and pioneered early developments of 'triple-deck' theory, which, in turn, enabled resolution of long-standing questions in steady and unsteady trailing-edge flows, and addressed associated important aerodynamic applications. Another area for which Professor Brown was especially renowned was a series of discussions of critical layers, especially effects of viscosity and nonlinearity and applications to geophysical flows such as atmospheric jets. Professor Brown was a superb analyst and her work was characterised by clever and determined use of asymptotic methods and allied computations.

Susan North Brown was born 22 December 1937 in Southampton. She was an undergraduate at St Hilda's College, Oxford, where she obtained a first class degree in mathematics and a junior mathematical prize, in 1959. She then studied for 2 years for a DPhil in Oxford under the supervision of Professor G. Temple before moving to the University of Durham to complete her DPhil. It was here she started her remarkable and long-lasting collaboration with Stewartson, first through a temporary Lectureship in Durham, a Lectureship in Newcastle and then, in 1964, her Lectureship at UCL, Readership in 1971 and eventually as Professor from 1986. We believe that Professor Brown was the first female in the UK to be appointed to a Professorship in Mathematics, a source of pride for the department, but a date which seems all too recent from a gender equality point of view.

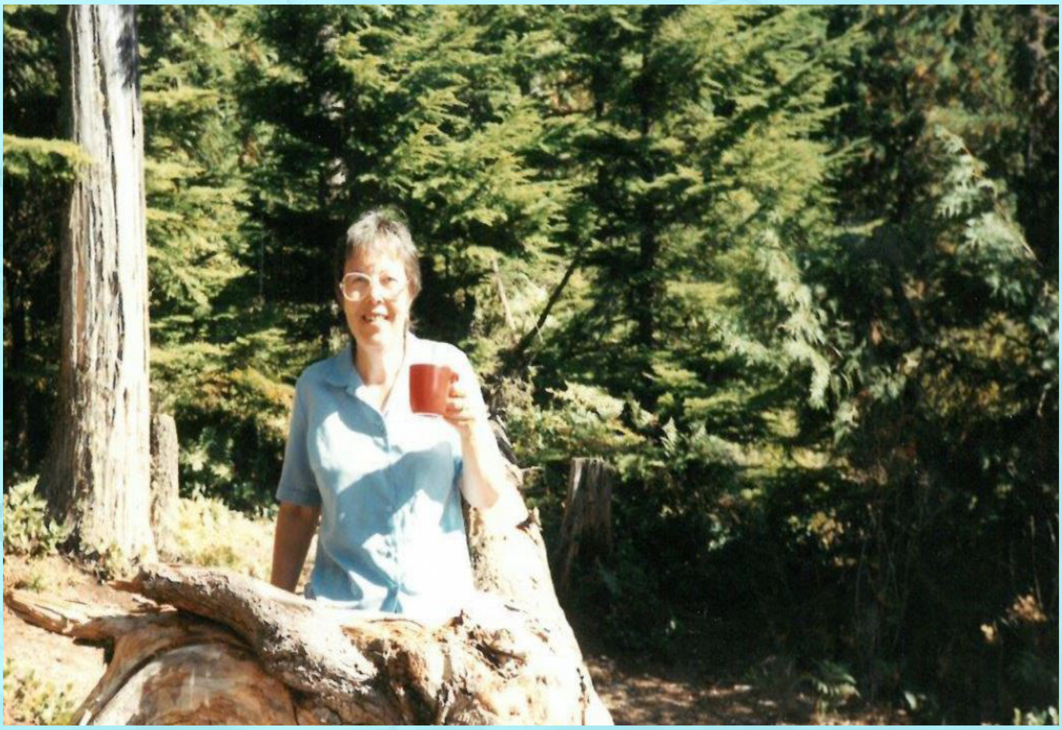
Professor Brown earned an international reputation for her research. In addition to Stewartson, Professor Brown's collaborators included HK Cheng, Norman Riley, Frank Smith FRS and Sidney Leibovich. During her UCL career, she was a visiting researcher at several overseas institutions including Cornell, University of Southern California and she also spent time as a visiting consultant at the Royal Aircraft Establishment in Farnborough. She successfully supervised 6 PhD students.

In 2003 a meeting was held at UCL to honour her 65th birthday (along with Professor Michael O'Neill) with guest speakers including her former PhD student Professor Peter Daniels, and Norman Riley and Sidney Leibovich.

Professor Brown was also an outstanding teacher, inspiring to both students and staff, many of the latter being mentored by Susan. She was often assigned large first year classes in mathematics and engineering. She also contributed to the smooth running of the department, notably her efficient Chairing of the Mathematics Examination Board. Professor Brown had a fine sense of humour and was ever-present and ever-reliable; her loyalty will be treasured in the department.

Frank Smith, Professor of Mathematics remembers -

Memories of Susan are many. As a doctoral student I first met her in 1970 on a visit to the department. She recalled her first visit about five years earlier, when the back of the college on Gordon Street reminded her of a bomb site. It took many years to improve. She and Keith understood my research immediately, clearly knew the directions it should take, and helped enormously. She was already a star in a thriving department and she became extremely well known internationally. We met often in London as well as during research visits to the States. Once, in 1976 in Columbus Ohio, we played possibly the worst tennis match ever against Keith and Jean Stewartson in which the longest rally lasted only two shots, whereas the laughing and embarrassment lasted long. She loved crosswords. On a train journey back to London from a British Applied Mathematics Colloquium in the early 1980s she virtually completed the Times crossword, with me still trying to understand even the first few answers. When I joined the staff in 1984 she was most welcoming. We soon organised in the college an international symposium on interactive boundary layers, which was hard work, and an accompanying book, which was harder, and our collaboration began. This carried on until 2003 and was less intense than hers with Keith had been but she collaborated well with many others on some of the hardest problems of the times. In later years she lived with Derek Moore and enjoyed their companionship until serious illness took him away. Susan was a pleasure to work with and to be with. Her modesty would make her embarrassed by all these words about her. Susan, thank you.



J J Sylvester Scholarship Fund

The J J Sylvester Scholarship Fund was set up in 1997, on the centenary of the death of J J Sylvester, one of the most gifted scholars of his generation. The Fund aims to award a scholarship to help support a gifted graduate mathematician.

You can make your gift to UCL online, by telephone or by post. Donations may be made by cheque, charity voucher or GiftAid. Any donation, large or small will be gratefully acknowledged by the College. If you are interested in knowing more about the Fund or other tax-efficient ways of supporting the Fund please do not hesitate to contact makeyourmark@ucl.ac.uk or on +44 (0)20 3108 3834.

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Sylvester was one of the greatest mathematicians to be associated with UCL and it is hoped that, through contributions made to the Scholarship Fund, we shall be able to assist in progressing the education of other mathematicians so as to realise their full potential for the benefit of us all.

Alumni Careers Advice

The department is keen to welcome alumni to its careers events and fairs for our present students. This includes alumni who have gone on to do further study.

If you are interested in this possibility, please contact Robb McDonald n.r.mcdonald@ucl.ac.uk

We would welcome news and contributions for the next newsletter which should be sent to:

Professor Ted Johnson, The De Morgan Association, Department of Mathematics, University College London, Gower Street, London WC1E 6BT.

Email: editor_newsletter@math.ucl.ac.uk.