De Morgan Association Newsletter

from the Department of Mathematics UCL

Issue - 15 - 2007

Editor - Michael O'Neill

DE MORGAN ASSOCIATION DINNER

WEDNESDAY 6 JUNE 2007

Professor Brian Davies speaking at the De Morgan Association Dinner

Following pre-dinner drinks in the Old Refectory, where members had the chance to renew acquaintances, even if briefly, we adjourned to the Jeremy Bentham Room for dinner.

Our Guest of Honour was Professor Brian Davies, Fellow of the Royal Society and a distinguished Professor of Mathematics from that other place in the Strand. The title of his after dinner address "*Whither Mathematics ?*" highlighted the challenges which face modern mathematicians, particularly the complexity of rigorous proof that now has to be applied to the solution of problems. Professor Davies pointed out that mathematics developed at an astounding rate during the last century and, even in pure mathematics, proofs of important results now often depend on extensive computation. For instance, independently the classification of finite simple groups involved a collaborative enterprise by more than a hundred mathematicians over a period of thirty years. Computation often involves algorithms whose structure is difficult to verify. If these developments continue, will they alter the very nature of the subject?

Papers by Professor Davies can be viewed via http://www.mth.kcl.ac.uk/staff/eb_davies.html

This year witnessed a change of guard and the establishment of a new order within the Department following the arrival of the new Head of Department, Professor Dima Vassiliev. He breaks with a tradition in that he follows three alumni in succession, Ambrose Rogers, Edwin Power and David Larman, who held that position, spanning a period close to fifty years, and has penned his honest and perceptive thoughts on his arrival at UCL later in this Newsletter.

This year saw the departure of two members of the academic staff and the arrival in September 2007 of no less than *seven* new members. That, together with the increasing numbers of first year students despite a severe entrance requirement involving A grades in Mathematics and Further Mathematics, suggests that the UCL Mathematics Department continues to be in good shape.

It is with sadness we report the death of Professor Hugh Michael, who was a distiguished member of the academic staff over a period of 37 years.

■ **Michael E O'Neill**
(Emeritus Professor or Mathematics)

Astor Professor of Pure Mathematics

In 2007, Professor Keith Ball was elected to the Astor Chair of Pure Mathematics, one of the two named endowed Chairs of Mathematics within the Department of Mathematics. Keith becomes only the *seventh* holder of the prestigious Chair.

In 1902, two Astor Chairs were established at UCL – one in English History and one in Pure Mathematics – and endowed from a benefaction made by the famous New York immigrant William Waldorf Astor. The seven holders of the Astor Chair of Pure Mathematics have been:

1904 – 1922:	Micaiah John Muller Hill
1924 – 1944:	George Barker Jeffery
1945 – 1958:	Harold Davenport
1958 – 1986:	Claude Ambrose Rogers
1986 – 1989:	Paul Cohn
1991 – 2006:	David Preiss
2007 –	Keith Ball

■ **Michael E O'Neill**
(Emeritus Professor or Mathematics)

Birthday Party

On Wednesday 9th 2007 May the Department held a joint party for Jürgen Eckhoff and Peter McMullen to celebrate their 65th birthdays.

Peter McMullen

Peter joined the Mathematics Department in 1970, having previously held a visiting position at Michigan State University. He had then recently solved one of the great problems of discrete geometry namely the 'Upper Bound Theorem'. Put briefly, the bounded solutions of a finite number of linear inequalities form a geometric object called a polytope e.g. cubes, tetrahedral etc. The problem was to determine the largest number of vertices a polytope could have for a given number of constraints.

Peter threw himself enthusiastically into College life becoming a well known figure at the Science Society, the Professorial Dining Club and at the 'Head of Steam' Public House. The Department owes Peter a tremendous debt of gratitude for his work in editing 'Mathematika' which is one of the leading journals in Mathematics. He has written several books and his creation of the Polytope Algebra has led to considerable international research and a deeper understanding of polytopes.

In 1974, Peter had the honour of being an invited speaker at the International Congress of Mathematicians in Vancouver and in 2006 was elected an honorary member of the Austrian Academy of Sciences. In 2005 he took early retirement so as to be able to concentrate more fully on research.

Jürgen Eckhoff

When Peter McMullen took early retirement, we were delighted that Jürgen Eckhoff could join us for two years; on leave from the University of Dortmund. Jürgen is an expert on Discrete Geometry. Now he is also retiring and plans to live in Freiburg in the midst of the Black Forest. With the world renowned Mathematics Institute of Oberwolfach, on his doorstep, in particular the library, this is every mathematician's dream retirement spot.

Looking Forward

This is my first article for the De Morgan Association Newsletter: I joined UCL as Head of the Mathematics Department in July 2006 and have just finished my first year. This year saw big changes in the department, with a number of permanent academic staff leaving and many more coming. David Preiss left for Warwick, Mitch Berger left for Exeter and Josef Hofbauer returned to Vienna. David and Mitch worked in the department for a long time, since 1990 and 1989 respectively, and significantly influenced its development. Josef spent only a few years with us. All three will be sadly missed. We wish David, Mitch and Josef luck in their careers and hope they will maintain links with UCL.

Our new arrivals are Rod Halburd and Slava Kurylev from Loughborough, Minhyong Kim from Purdue, Yiannis Petridis from the City University of New York, Nadia Sidorova from Bath, Alex Sobolev from Birmingham and Jean-Marc Vanden-Broeck from East Anglia. These appointments were the result of a marathon recruitment process which generated 230 job applications from all over the world and involved many weeks of processing and many days of interviewing. I am glad to report that all the permanent staff appointed were our top choices, that is, nobody turned down a UCL offer. In the current competitive climate this is a remarkable outcome. Our new appointments will raise the research profile of the department and provide welcome relief on the teaching front.

Given the increase in staff numbers it was most fortunate that the department was given extra office space in the neighbouring Kathleen Lonsdale Building (KLB). Of course, having a split site department is not the ideal solution but I am not inclined to complain (at least for the time being) knowing how difficult it is to get any space at UCL. And the KLB surely has character: it is the place where Nobel laureate Sir William Ramsay, the chemist who discovered noble gases, used to work. Rumour has it that the sample Sir William used when discovering helium has recently been found in the basement of the KLB. I expect in another hundred year's time someone will find important theorems left behind by our staff.

As my predecessor David Larman taught me, our undergraduate intake is the bread and butter of the department. And here the department's performance is truly outstanding and is continuing to improve. Last year we increased our entry requirements by asking for an A in Mathematics and an A in Further Mathematics for most degree programmes. I initially thought this to be risky as very few UK mathematics departments have made Further Mathematics an entry requirement; say, I recall this discussion being held at Bath (one of the best UK mathematics departments) and the decision was not to proceed along this route. Amazingly, our gamble paid off! Despite the increase in entry requirements we ended up with 107 EU and 87 overseas students starting with us in 2007. I use this opportunity to thank our Admissions Tutor Robert Bowles and express the hope that he will devise new ways of stopping students coming to UCL in such huge numbers.

Looking back I have no regrets about leaving Bath. Yes, my first year at UCL has been extraordinarily difficult but it was also great fun and I feel that I am doing something useful. I was pleasantly surprised by the support I received from my colleagues within the department as well as the UCL administration. I realise that an academic is supposed to lambast their administration at every opportunity but I just can't bring myself to do this. I have previously worked in two British universities, Sussex and Bath, and it is the first time I deal with a university administration which is so dynamic. In particular, all the changes that happened in the department within the last year would have been impossible without the enthusiastic support of our new Dean, David Price. Don't know, maybe my views are simply the result of me "switching to the dark side" and becoming a full-time bureaucrat.

I just hope that one day things will calm down, the mass of urgent "to do" Post-its will disappear from my desk and I will find the time to prove a theorem. Or, at least, a lemma.

■ **Dima Vassiliev**
(Head of Department)

UCL Centre for Origins

The Faculty of Mathematical and Physical Sciences has encouraged the foundation of an interdisciplinary centre where different departments of the faculty are involved. While each department is one of UK's (and world's) leading institutions in their respective fields, there is currently very little joined research between these departments. One of the main aims of the centre is to further increase the international impact of UCL's research programme in the mathematical and physical sciences. First steps have been taken in the last year and the departments of Astronomy, Astrophysics and Atmospheric Group, Earth Sciences, High Energy Physics, Mathematics and the Mullard Space Science Laboratory have held regular small discussion meetings to identify research themes overlapping the departments.

This resulted in the following broad questions that the centre wishes to address: What is the composition of the Universe (e.g.~baryons, dark matter, dark energy)? What is the origin of mass? How did galaxies, stars and planets form? What is the role of black holes? What are the conditions to cause extreme environments and events (e.g.~gamma ray bursts, active galactic nuclei, solar flares)? What are the properties of neutrinos? What is the origin of matter/antimatter asymmetry? What is the nature of spacetime and how to incorporate extra spatial dimensions?

Whereas not all of the above research themes have a direct overlap with the Department of Mathematics, quite a few do require fresh mathematical input to push forward the frontiers of our current knowledge.

The first Origins town meeting was held on the 29 August 2007 in the Harrie Massey Lecture Theatre where the Department of Mathematics was represented by Dmitri Vassiliev and Christian Böhmer. The half-day formal meeting was followed by small discussion groups focusing on selected themes, like Mathematical foundations.

In the following, some of the ideas that were discussed in the Mathematical Foundations group are summarised:

Dark energy In 1917 Einstein introduced the cosmological constant in the field equations. He aimed to construct a static cosmological solution, known as the Einstein static Universe. When Hubble during the 1920s observed that the Universe is expanding, interest in the cosmological constant faded. However, during the 1990s observations of distant supernovae suggested that the cosmological constant is indeed needed for agreement between observation and theory. During the last decade, the terminology changed towards dark energy instead of cosmological constant. One of the main reasons for this change is due to various models of an evolving scalar field that lead to similar effects.

However, once the constant was no longer assumed to be a constant, various problematic issues have been raised. For example, from a Quantum Field Theory point of view it makes sense to interpret the cosmological constant as the vacuum energy density. This interpretation yields severe problems as the theoretically expected value and the actually observed value differ by a factor of the order 10^{123} the observed value being very small. Various models suggest that the cosmological constant may be the imprint of a higher dimensional geometry, where its smallness is related to projections.

Elementary particles To a mathematician the Standard Model of particle physics appears to be an empirical construction in which the agreement with experiment is achieved by an appropriate choice of sufficiently many physical parameters and sufficiently many underlying groups. Despite the great successes of the Standard Model confirmed by various experiments, most mathematical methods used in these calculations are not well defined and the theory contains many divergent integrals and sums. In the path integral approach to Quantum Electrodynamics for instance, from a mathematical point of view, there is no justification for how these divergences are dealt with. On the other hand, on the physical side, these `calculations' agree with observations with astonishing accuracy. The Large Hadron Collider, located at CERN, will in 2008 reach an energy regime such that the mathematical form of the Standard Model may be changed in a way unprecedented in the last two decades.

Modified gravity theories In recent years there has been great interest in modified gravity theories. There, the usual Ricci scalar in the Einstein-Hilbert action is replaced by an arbitrary function of the Ricci scalar. This leads to fourth order field equations and therefore opens various interesting question that can be addressed from a mathematical point of view. For instance, Rod Halburd has suggested that methods from integrable systems may be used to analyse these equations. This may lead to a totally new view of modified gravity theories where new mathematical input may greatly increase our present understanding of the theory.

Generalised geometries The basic equations of the theory of general relativity are the Einstein field equations. Their left-hand side is given by the Einstein tensor (Ricci tensor minus one half the Ricci scalar times the metric) while the right-hand side is determined by the matter content. In the original formulation of 1915 is was assumed that the spacetime manifold is uniquely characterised by the metric. In 1918 Weyl introduced the concept of gauge transformations in gravity (in the form of non-metricity), however his theory turned out not to agree with observations. Decades later the idea has been taken up again in the context of Quantum Field Theories and led to Gauge Field Theories on which the Standard Model of Elementary particles is based. In 1922 Cartan realised that it might be natural to consider manifolds with torsion, the skew-symmetric part of the connection. Although many of these ideas greatly benefit from beautiful geometrical insights, none to date really explains the laws of Nature. It is hoped, however, that eventually the principles that unify the theory of gravitation and the theory of elementary particles will be revealed.

■ **Christian G. Böhmer**
(One-Year Lectureship)

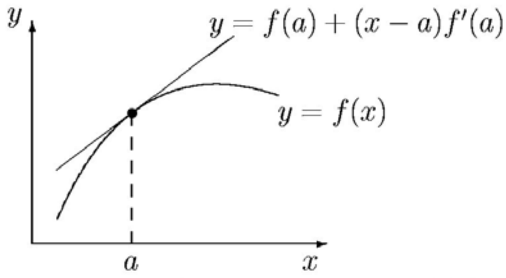
The Power of Linearisation

The structures of mathematics can be very complex, from the Feigenbaum cascade into chaos in nonlinear dynamical systems to the monster group in algebra. But it's amazing how powerful the very simplest mathematical tools can be, particularly when you're applying mathematics to real, physical systems.

The act of linearising is so simple, you probably don't really think about it any more. In the most basic form, I'm just talking about the first couple of terms of a Taylor series: for a well-behaved function $f(x)$, near $x = a$ we have

$$f(x) \approx f(a) + (x-a)f'(a)\,.$$

Plotting $y = f(a) + (x-a)f'(a)$ gives a straight line, so what we have found is the best straight-line approximation to $y = f(x)$ in the neighbourhood of $x = a$.



Best linear fit to the curve $y = f(x)$.

In two dimensions, we can find the best planar approximation to a surface $z = f(x, y)$ in the neighbourhood of $(x = a, y = b)$:

$$f(x,y) \approx f(a,b) + (x-a)\frac{\partial f}{\partial x}(a,b) + (y-b)\frac{\partial f}{\partial y}(a,b)$$

Think for a moment about the last time you went hill-walking. Forget about the little stones and irregularities along the way and just imagine the overall, smoothed shape of the landscape you walked. Most of the time, you were either walking up a slope or down a slope – which means that the best-fit plane, matching your slope, wouldn't have felt much different (locally) from the real surface you were climbing. It's only at a locally flat region - at the top of the mountain, or cresting a shoulder - that the best-fit plane fails to capture the essential properties of the landscape. At these *critical points*, linear information isn't enough to describe the variation of height with position; but the critical points are rare, and on most of the hillside the linear approximation does a good job.

We can rewrite the plane formula using vector notation as

$$f(\mathbf{x}) \approx f(\mathbf{a}) + (\mathbf{x} - \mathbf{a}) \cdot \nabla f(\mathbf{a})$$

and this gives us the beginning of the Taylor series for a scalar function in any number of dimensions.

So where do we use this tool? One excellent example comes up in the study of oceans and atmospheres. In order to make predictions - weather forecasts or, for example, calculations of where the fallout from a nuclear test is going to end up - we need to solve the problem of fluid flow on a rotating sphere. We also need to be able to study structures which are much smaller than the whole sphere (think about the size of the highs and lows on a weather map: a few hundred miles across, which is a tiny fraction of the size of the earth). To have any hope of starting this task, we need to study small regions of the earth's surface but still take account of the rotation forces.

The earth's rotation has two effects in terms of the forces we feel. The first is just centrifugal: if you're standing at the equator, you feel a little lighter than you would at the poles (and not just because you're not wearing arctic gear). That's very easy to take into account mathematically: just change g.

The other effect is truly rotational: the Coriolis effect. (You may have heard that the Coriolis effect drives which way the water swirls on its way out of the bath. This isn't true, as the force is too small over a fluid volume as small as a bathtub - but that's another story.) This has no effect at the pole, but puts a natural curve onto the velocity of anything moving horizontally on the rest of the earth's surface. Like the centrifugal force, it only exists because we insist on viewing everything from a fixed point on the earth, which means our observation frame is rotating.

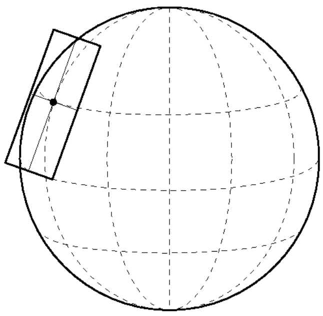
The size of the Coriolis force at latitude ϕ is proportional to $\sin\phi$ (remember $\phi = 0$ at the North pole and π at the South pole). If we're looking at a fairly small object (relative to the earth) like a hurricane, then the complexity of using the whole sine function is too much. Instead, we use the best planar approximation to the surface of the earth near some fixed latitude ϕ_0. Moving onto this plane gives a linear approximation to the Coriolis force:

$$f = K \sin\phi \approx K \sin \phi_0 + (\phi - \phi_0) K \cos \phi_0$$

Geophysicists relabel this as

$$f \approx f_0 + \beta y \text{, with } y = R\,(\phi - \phi_0)$$

where R is the radius of the earth, and so the best-fit plane for the Coriolis force is known as the beta-plane.



The β-plane.

So where else does linearisation have a part to play?

My research area is the dynamics of complex fluids, and one of my current projects is trying to predict when flows of molten polymers will become unsteady. These polymers are using in making a huge variety of plastic objects, and when the processing flows go wobbly, the result is products that have to be thrown away, wasting precious resources.

We can start to predict these problems using *linear stability theory*. The basic idea is to take a complicated system of partial differential equations (PDEs), solve for the nice flow we would like (which is usually steady in time) and then find the linear set of PDEs which govern the dynamics of the system as long as it stays in the neighbourhood of the steady state.

Because the new system is linear, the solution can only depend exponentially on time: $\exp(\sigma\, t)$ and we end up solving an eigenvalue problem (which is *linear* algebra, after all) to find σ. If σ is positive then the solution grows in time and gets further from the steady state - so we have a prediction of unsteady flow.

Right now, I'm looking at situations so complex that both the steady problem and the eigenvalue problem have to be solved on a computer: and we're just starting to predict the sort of flow problems that are observed in experiments. And all through the power of linearisation.

■ **Helen J Wilson**
(Senior Lecturer)

Professor David Hugh Michael

Hugh Michael follows in that long line of distinguished British applied mathematicians who were motivated to apply mathematical analysis to explain and understand phenomena in the real world. That lineage includes G G Stokes, Lord Rayleigh and G I Taylor, whose work inspired Michael's research. It was perhaps no coincidence that the supervisor for his PhD at Cambridge, G K Batchelor, had in turn been supervised by Taylor.

Michael's earliest research was in hydrodynamic stability and he soon became involved in investigating the stability of fluid motions in the presence of magnetic and electric fields. Electrohydrodynamic stability came to particular prominence in his studies when he participated in a landmark conference at MIT in 1969 organised by electrical engineer J R Melcher with G I Taylor. This fortuitously led to a collaboration with one of Melcher's research students which established the bifurcation mechanism of unstable modes in a composite liquid/solid insulating layer. Michael realised that analogous mathematical analysis would yield better theoretical agreement with Taylor's experiments on the bursting of a charged cylindrical film. As M J Lighthill was to say later of Michael, there were few who successfully corrected an error in Taylor's work. This led to a personally fulfilling period of collaboration with Taylor on capillary instability of fluid films, culminating with a presentation at the 1972 IUTAM Conference in Moscow. Michael's studies also included investigations in the mechanics of droplets and suspensions, with several papers on the stability of dusty gas flow and the interaction of large bodies with multi-phase flows. His contributions to the Theory of Stokes Flow, particularly the formation of eddies, followed in the steps of W R Dean, another mentor and one-time Goldsmid Professor of Mathematics at University College London.

Hugh Michael greatly admired those researchers, like Taylor, who carried out experimental work alongside the development of analytical theory. This was demonstrated in his many successful interactions with physicists and engineers, and reached an outstandingly fruitful period when he conducted excellent innovative research with the Non Destructive Testing Centre based in the Mechanical Engineering Department at University College London. Here he was responsible for groundbreaking research in NDT that produced the theoretical solution to an NDT technique known as ACFM. Michael produced a brilliant mathematical solution in alternating current field theory allowing for the first time ever the prediction of the size of semi-elliptical surface fatigue cracks in tubular steel structures. These structures are of the type used by Oil and Gas companies in the North Sea and his work helped to ensure the safe production of oil and gas for many years. The work led to the award of the Roy Sharpe Prize by the British Institute of Non-Destructive Testing in 1989 and was published by The Royal Society.

- **W D Dover**
- **M E O'Neill**

Hugh joined the department in January 1953 as an Assistant Lecturer having held the post of Assistant Lecturer at Royal Holloway College for precisely three months. (The correspondence shows that they were somewhat annoyed!) In 1963 he was appointed Reader and then to a Professorship in 1983. It should be borne in mind that promotion beyond lecturer was then very difficult, particularly with the government decree that at least 60% of staff should be lecturers. He enjoyed several periods of sabbatical leave, particularly to the University of Melbourne in 1968-9. Hugh took semi retirement in 1986 but continued teaching on a part time basis and carrying out research at the London Centre of Marine Technology until retiring fully in 1990.

I had the pleasure (I hope it was mutual) of having first year tutorials from Hugh. In those days the tutorials were one to one. He was an extremely kind and patient tutor.

■ **David Larman**

Department News

Appointments

The following have recently joined the department:

Professor Jean-Marc Vanden-Broeck as Chair in Applied Mathematics

Professor Minhyong Kim as Chair in Pure Mathematics

Professor Yarsolav Kurylev as Chair of Mathematics

Professor Alex Sobolev as Chair in Pure Mathematics

Dr Yiannis Petridis as Senior Lecturer in Pure Mathematics

Dr Nadia Sidorova as Lecturer in Pure Mathematics

Dr Rob Halburd as Reader in Mathematics

Dr Christian Böhmer has accepted a one year lectureship.

Kate Daubney joined the Department as a secretary on 19 March 2007 for one year while Mumtaz Abdul Ghafoor is away on maternity leave.

PhD Awards

Students who have recently obtained PhD's from the Department include:

Tim Edwards – *Algebraic 2-Complexes over Low Dimensional Infinite Abelian Groups*

Andrew Ellis – *Modelling Chute Delivery of Greens in Food-Salting Processing*

Natalia Garcia Colin – *Applying Tvereberj Type Theorems to Giometric Problems*

Alan Hinds – *Surf-zone Vortices near Piecewise Flat Topography*

Wajid Mannan – *Low Dimensional Algebraic Complexes over Integral Group Rings*

Owen Rump – *Non-rotating and Rotating Free Surface Flows over Topography*

Anastasia Sofoniou – *The Parametrically Excited Pendulum System and Applications to Ship Dynamics*

Prizes awarded July 2007

The following graduates were awarded prizes:

Vikesh Barchha - Filon Prize.

Jignesh Halai - Donald Davies Prize.

Lina Jalal - Mathematika Prize.

Mark Lim - Stevenson Prize.

Edwin Lutton - Hill Prize.

Nirav Parekh - Sessional Prize.

Vijal Patel - Andrew Rosen Prize.

Justyna Petke - Kesleman Prize.

Jade Savage - Mathematika Prize.

Robert Williams - Bosanquet Prize.

Tasir Virji - Andrew Rosen Prize and the Ellen Watson Memorial Scholarship in Applied Mathematics.

Chenyin Xu - Jeffrey Prize.

Xiaole Zhang - Sessional Prize.

The following were awarded Dean's List Commendations: **Jignesh Halai, Lina Jalal, Mohammad S Motala and Tasir Virji.**

Sylvester Scholarship Fund

The Sylvester Scholarship Fund was set up in 1997, on the centenary of the death of JJ Sylvester, one of the most gifted scholars of his generation. The Fund aims to award a scholarship to help support a gifted graduate mathematician.

PEOPLE

Ebru Aydemir (2006) is now studying in Oxford reading for an MSc in Mathematical Modelling and Scientific Computing and plans to continue on to a PhD in Applied Mathematics.

Chris Houghton (1960) lived in Cardiff for thirty-nine years, working mainly as a Lecturer in Pure Mathematics at Cardiff University. He has now retired and has moved to Cambridge.

Imtiaz Sandia (1985) has had a change of job and now lectures in Computer and Network Systems at the University of Westminster.

William Walker (2000) has spent three years teaching English in Japan. Upon returning to the UK he took the Diploma in Computer Science at Cambridge and is now working as a business analyst at the IT consultancy firm Detica.

STOP PRESS
For your diary

The Mathematics Department
De Morgan Association Dinner will be held on

Wednesday 4 June 2008

All those on our database will be sent an invitation to the next De Morgan Association Dinner. Make sure you send us addresses of anyone else who may want to receive an invitation and remember to keep the Department and Alumni Relations Office of UCL informed of any changes of your address.

We would welcome news and contributions for the next newsletter which should be sent to: Michael O'Neill, The De Morgan Association, Department of Mathematics, University College London, Gower Street, London WC1E 6BT. E-mail: meo@math.ucl.ac.uk.