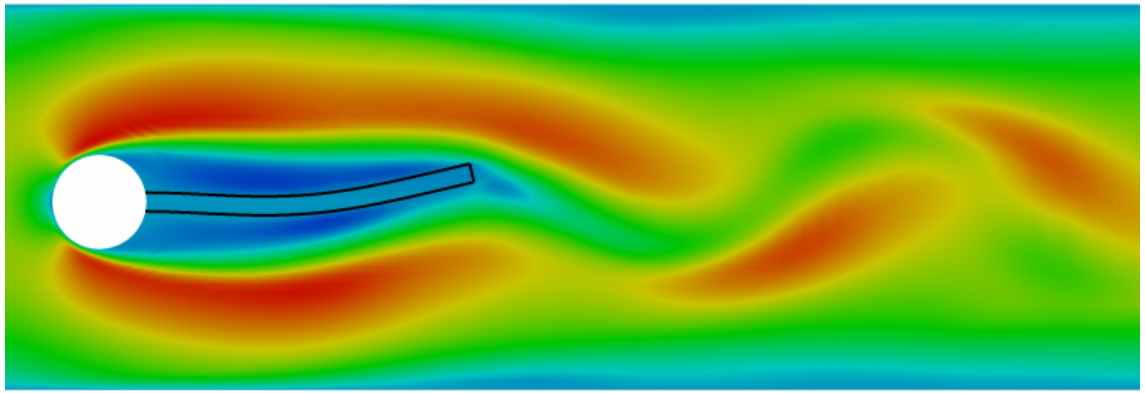

Computational Methods for Interface Problems Workshop

03 – 04 January 2019

Location:
Room 505, Department of Mathematics, UCL
25 Gordon Street, London, WC1H 0AY, UK

Computational Methods for Interface Problems Workshop
Thursday 3rd - Friday 4th January 2019

Thursday 3rd January 2019

9:00-9:30	Registration & Coffee	
9:30-10:00	Vanessa Styles	Numerical approximations of a tractable mathematical model for tumor growth
10:00-10:30	Charles Elliott	PDEs in cell biology
10:30-11:00	Ricardo Ruiz-Baier	Two mixed formulations for stress-assisted diffusion problems in biomechanics
11:00-11:30	Coffee Break	
11:30-12:00	Daniela Capatina	Local flux reconstructions for an elliptic problem and extension to a contact problem
12:00-12:40	Susanne Claus/Pierre Kerfriden	Cut finite element methods for unilateral contact problems with application in composite materials with fibrous reinforcements
13:00-14:00	Lunch	
14:00-14:30	Guillaume Delay	Hybrid high-order methods for an elliptic interface problem on unfitted meshes
14:30-15:00	Franz Chouly	Hybrid high-order methods combined with Nitsche's method for contact problems
15:00-15:30	Stefan Frei	Nitsche-type formulations for fluid-structure interactions and contact
15:30-16:00	Coffee Break	
16:00-16:30	Christoph Lehrenfeld	Geometrically higher order unfitted space-time FEM for problems involving moving domains
16:30-17:00	Maxim Olshanski	A finite element method for PDEs in time-dependent domains
17:30	Reception	

Friday 4th January 2019

9:30-10:00	Peter Hansbo	FEM/CutFEM with Dirichlet boundary value correction
10:00-10:30	Cuiyu He	A posteriori error estimates for cut finite element method with boundary correction
10:30-11:00	Silvia Bertoluzza	The virtual element method on domains with curved boundaries
11:00-11:30	Coffee Break	
11:30-12:00	Stéphane Bordas	Modelling and identifying cracks in 3D structures
12:00-12:30	Mats Larson	CutFEM for mixed dimensional problems
12:30-13:00	Arnold Reusken	Stream function formulation of surface Stokes equations
13:00-14:00	Lunch	

The Virtual Element Method on Domains with Curved Boundaries

Silvia Bertoluzza

IMATI, Italy

Abstract

Discretization methods based on polygonal/polyhedral tessellation are particularly well suited to discretizing problems with interfaces. They allow, for instance, to use directly or after some simple post-processing the grid obtained by cutting along the interface an unfitted triangular/tetrahedral or quadrilateral/hexahedral mesh. Focusing on a second order elliptic model problem we consider, among such methods, the Virtual Element Method, which relies on an underlying conforming subspace of H_1, which is, however, never explicitly constructed (whence the name virtual). As a first step towards the use of such a method for the solution of problems with an interface, we adapt it to the case of domains with a smooth curved boundary, by following the approach by Bramble, Dupont and Thomée, which we need to modify in order to deal with the virtual nature of the discretization space. We will present both a theoretical analysis and a set of numerical tests, and we will discuss several issues arising when further extending the method to the case of interior interfaces, such as, for instance, its robustness in the presence of very small edges.

Modelling and Identifying Cracks in 3D Structures

Stéphane Bordas, Konstantinos Agathos and Eleni Chatzi

Université du Luxembourg, Luxembourg

Abstract

We review some the recent developments in the field of extended/generalized finite element methods for three-dimensional (3D) fracture. We first discuss the challenges associated with modelling 3D cracks, with a focus on conditioning of the set of equations, in particular for multiple enrichment functions and large enrichment radii.

We then discuss possible applications to crack identification in complex 3D structures and conclude with some perspectives for future work.

Local Flux Reconstructions for an Elliptic Problem and Extension to a Contact Problem

Daniela Capatina

University of Pau, France

Abstract

We present in this talk a uniform framework for the computation of conservative local fluxes for an elliptic problem, for some classical finite element methods of arbitrary order on triangular meshes: conforming, nonconforming and discontinuous methods, see [1]. The computation of these $H(div)$-conforming fluxes is done by local post-processing of the finite element solution, avoiding the solution of any mixed (local or global) problem. The considered reconstructions coincide in particular cases with other approaches from the literature. Optimal error estimates for the reconstructed fluxes are proved, which are in agreement with the numerical tests presented. In addition, we establish relations between the fluxes for the different methods when the stabilization parameter of DG tends to infinity.

The approach can be generalized to more complex model problems and can be immediately used in the context of a posteriori error estimation. In what follows, we consider a contact problem, modelled for the sake of simplicity by Signorini's equation. In [2], the authors introduced a Nitsche formulation of the contact condition and a P_1-continuous finite element approximation, for which they proposed a residual-based a posteriori error estimator. The analysis is carried out under a saturation assumption.

We present next a recent extension of the previous approach to the Nitsche formulation of the Signorini problem. The local conservative flux allows us to obtain a robust a posteriori error estimator, without any additional assumption.

References

[1] R. Becker, D. Capatina, R. Luce: *Local flux reconstructions for standard finite element methods on triangular meshes*, SIAM J. Numer. Anal. 54 (4), p. 2684 - 2706, 2016

[2] F. Chouly, M. Fabre, P. Hild, J. Pousin, Y. Renard: *Residual-based a posteriori error estimation for contact problems approximated by Nitsche's method*, IMA J. of Numer. Anal. 38 (2), p. 921-954, 2018

Cut Finite Element Methods for Unilateral Contact Problems with Application in Composite Materials with Fibrous Reinforcements

Susanne Claus and Pierre Kerfriden

University of Copenhagen, Denmark and
MINES ParisTech, PSL Research University, France

Abstract

In this presentation, we will introduce several stabilised cut finite element schemes for unilateral contact problems. The contact conditions in our schemes are enforced using the LaTIn [1] domain decomposition approach, in which both interface fields and fluxes are treated as unknowns, and are determined iteratively. We stabilise our scheme using ghost-penalty stabilization [2]. The presentation will be split in two parts. In the first part, we will discuss our LaTIn-CutFEM [3] scheme for multiple elastic bodies in frictionless and frictional contact. In the second part, we will present a LaTIn-CutFEM scheme for 1D fibrous materials embedded in 3D elastic bodies including damage modelling.

References

[1] Ladevéze, P. *Nonlinear Computational Structural Mechanics - New Approaches and Non-Incremental Methods of Calculation.* Springer Verlag, (1999).

[2] Burman, E. *Ghost penalty.* C. R. Math. Acad. Sci. Paris. 348:1217–1220 (2010).

[3] Claus, S., and Kerfriden P. *A stable and optimally convergent LaTInCutFEM algorithm for multiple unilateral contact problems.* IJNME 113(6):938-966 (2018).

Hybrid High-Order Methods Combined with Nitsche's Method for Contact Problems

Franz Chouly, Karol Cascavita and Alexandre Ern

Université de Bourgogne, France

Abstract

Hybrid High-Order (HHO) methods have been introduced recently, first for linear diffusion and linear elasticity. HHO methods are formulated in terms of face unknowns, which are polynomials of arbitrary order $k \geq 0$ on each mesh face, and in terms of cell unknowns, which are polynomials of order $l \geq 0$, with $l \in \{k, k \pm 1\}$, in each mesh cell. The cell unknowns can be eliminated locally by static condensation leading to a global transmission problem posed solely in terms of the face unknowns. HHO methods offer various assets: they support polyhedral meshes, lead to local conservation principles, and their construction is independent of the space dimension.

This work aims at extending the HHO framework to problems with contact and friction. Contact and friction are formulated using Nitsche's method, which advantage is to remain a primal and consistent method. We focus on the scalar Signorini problem to tackle the main difficulties associated to the design of the method and its mathematical analysis. These difficulties are mostly i) the non-consistency of the HHO approximation, and ii) the appropriate design of Nitsches contact terms from face/cell unknowns in order to preserve optimal convergence properties.

Hybrid High-Order Methods for an Elliptic Interface Problem on Unfitted Meshes

Erik Burman, Matteo Cicuttin, Guillaume Delay and Alexandre Ern

Ecole des Ponts ParisTech, France

Abstract

Interface problems are classically meshed by fitting the mesh to the interface. Such methods can be very expensive especially when the interface has a complex geometry, which happens for instance in the field of geology. Indeed, besides the meshing process itself, fitting the mesh can induce a lot of small elements near the interface which severely increases the computational cost.

In order to ease the generation of the mesh, we propose to use an unfitted method, i.e. the method allows the use of meshes that do not fit the interface. More precisely, we will design and analyse a cut Hybrid High-Order (HHO) method for the Poisson problem with an interface. The choice of HHO [1] also enables the use of polyhedral meshes involving different types of polyhedra. This method is well suited to treat intricate geometries which can then be meshed by several types of polyhedra.

The unknowns are attached to the cells and faces of the mesh. A static condensation procedure enables to rewrite the problem in term of the face unknowns only. We then have to solve a coupled problem written on the skeleton of the mesh. Robustness of the method with respect to cut cells is achieved by a local cell-agglomeration procedure [2]. Inspired by the analysis in [3], we prove an optimal a priori error estimate for the numerical solution in the energy norm and present some numerical simulations.

References

[1] Di Pietro, Daniele A and Ern, Alexandre: *A hybrid high-order locking-free method for linear elasticity on general meshes*, Computer Methods in Applied Mechanics and Engineering, 283, p. 1 - 21, 2015

[2] Johansson, August and Larson, Mats G: *A high order discontinuous Galerkin Nitsche method for elliptic problems with fictitious boundary*, Numerische Mathematik, 123 (4), p. 607-628, 2013

[3] Burman, Erik and Ern, Alexandre: *An unfitted Hybrid High-Order method for elliptic interface problems*, Numerische Mathematik, 56 (3), p. 1525–1546, 2018

PDEs in Cell Biology

Charles Elliott

University of Warwick, UK

Abstract

I discuss surface PDE problems arising in cell biology.

Nitsche-type Formulations for Fluid-Structure Interactions and Contact

Erik Burman, Stefan Frei and Miguel A. Fernández

University College London

Abstract

We develop Nitsche-based formulations for fluid-structure interaction (FSI) problems with contact. Our approach to model contact is based on the works of Chouly and Hild [2] for contact problems in solid mechanics. Using a suitable extension of the fluid equations below the contact surface, we are able to formulate the FSI interface and the contact conditions simultaneously in equation form on a joint interface-contact surface $\Gamma(t)$. Due to the continuous switch between interface and boundary conditions, the so-called "chattering" phenomenon known in the engineering literature, is prevented.

To deal with the topology changes in the fluid domain at the time of impact, we use a fully Eulerian approach for the FSI problem. We show a stability result and present numerical examples to investigate the performance of the method.

References

[1] E. Burman, M.A. Fernández, S. Frei: *A Nitsche-based formulation for fluid-structure interactions with contact*, arXiv e-prints: `http://arxiv.org/abs/1808.08758` (2018)

[2] F. Chouly, P. Hild. *A Nitsche-based method for unilateral contact problems: Numerical analysis*. SIAM J Numer Anal 51(2):1295–1307 (2013)

[3] S. Frei. *Eulerian finite element methods for interface problems and fluid-structure interactions*, Dissertation, Heidelberg University (2016)

FEM/CutFEM with Dirichlet Boundary Value Correction

Peter Hansbo

Jönköping University, Sweden

Abstract

We propose a boundary value correction approach for cases when curved boundaries are approximated by straight lines (planes). The approach allows for optimal order convergence for higher order polynomials in the setting of Nitsches method on cut elements. We give applications to the Laplace equation as well as the Stokes equation. We also discuss an extension to the case of Lagrange multipliers, optimal up to polynomial degree 3.

A Posteriori Error Estimates for Cut Finite Element Method with Boundary Correction

Erik Burman, Cuiyu He and Mats G. Larson

University College London

Abstract

In this work we study the a posteriori error estimation for the CutFEM method on elliptic problems. We consider the problem with non-polygonal boundary and the analysis takes into account the boundary approximation by piecewise linear segments. The reliability and efficiency are proved to be robust on the location of boundary-domain intersection.

CutFEM for Mixed Dimensional Problems

Mats G. Larson

Umea Universitet, Sweden

Abstract

Geometrically Higher Order Unfitted Space-Time FEM for Problems Involving Moving Domains

C. Lehrenfeld, J. Preuss and F. Heimann

Georg-August-Universität Göttingen, Germany

Abstract

Two major issues in the design and realization of higher order unfitted finite element methods on time-dependent level set domains are time integration and accurate numerical integration. We present an approach which allows for a higher order accurate and robust time integration for domains that are prescribed by level set functions. The approach is based on parameter mappings of the background mesh. We combine this approach with a space-time discretization to obtain robust and provable higher order methods in space and time. The space-time method, its conception, implementational aspects, a priori error estimates and numerical results will be discussed.

A Finite Element Method For PDEs in Time-Dependent Domains

M. Olshanski

University of Houston, USA

Stream Function Formulation of Surface Stokes Equations

Arnold Reusken

RWTH Aachen University, Germany

Abstract

In this presentation we treat several aspects related to surface (Navier-) Stokes equations in stream function formulation. We consider a smooth connected (not necessarily simply connected) oriented hyper-surface in three-dimensional space without boundary. Appropriate surface gradient, divergence, curl and Laplace operators are defined in terms of the standard differential operators of the ambient Euclidean space. These representations are very convenient for the implementation of numerical methods for surface partial differential equations. A derivation of the surface Helmholtz decomposition

and its relation to the surface Hodge decomposition are explained. Based on this Helmholtz decomposition a well-posed stream function formulation of a class of surface Stokes problems is derived. This results in a fourth order scalar surface PDE for the stream function. A particular finite element method for this stream function PDE is proposed and results of numerical experiments with this method are presented.

Two Mixed Formulations for Stress-Assisted Diffusion Problems in Biomechanics

Ricardo Ruiz-Baier

University of Oxford, UK

Abstract

In this talk I will introduce a new mathematical model for the computational modelling of the active contraction of cardiac tissue using stress-assisted conductivity as the main mechanism for mechanoelectrical feedback. The coupling variable is the Kirchhoff stress and so the equations of hyperelasticity are written in mixed form and a suitable finite element formulation is proposed. Next I will introduce a simplified version of the coupled system, focusing on its analysis in terms of solvability and stability of continuous and discrete mixed-primal formulations. Some numerical tests will be presented, however not including interface problems.

Numerical Approximations of a Tractable Mathematical Model for Tumor Growth

Vanessa Styles

University of Sussex, UK

Abstract

We consider a free boundary problem representing one of the simplest mathematical descriptions of the growth and death of a tumor. The mathematical model takes the form of a closed interface evolving via forced mean curvature flow where the forcing depends on the solution of a PDE that holds in the domain enclosed by the interface. We derive sharp interface and diffuse interface finite element approximations of this model and present some numerical results.