Mathematics in Finance

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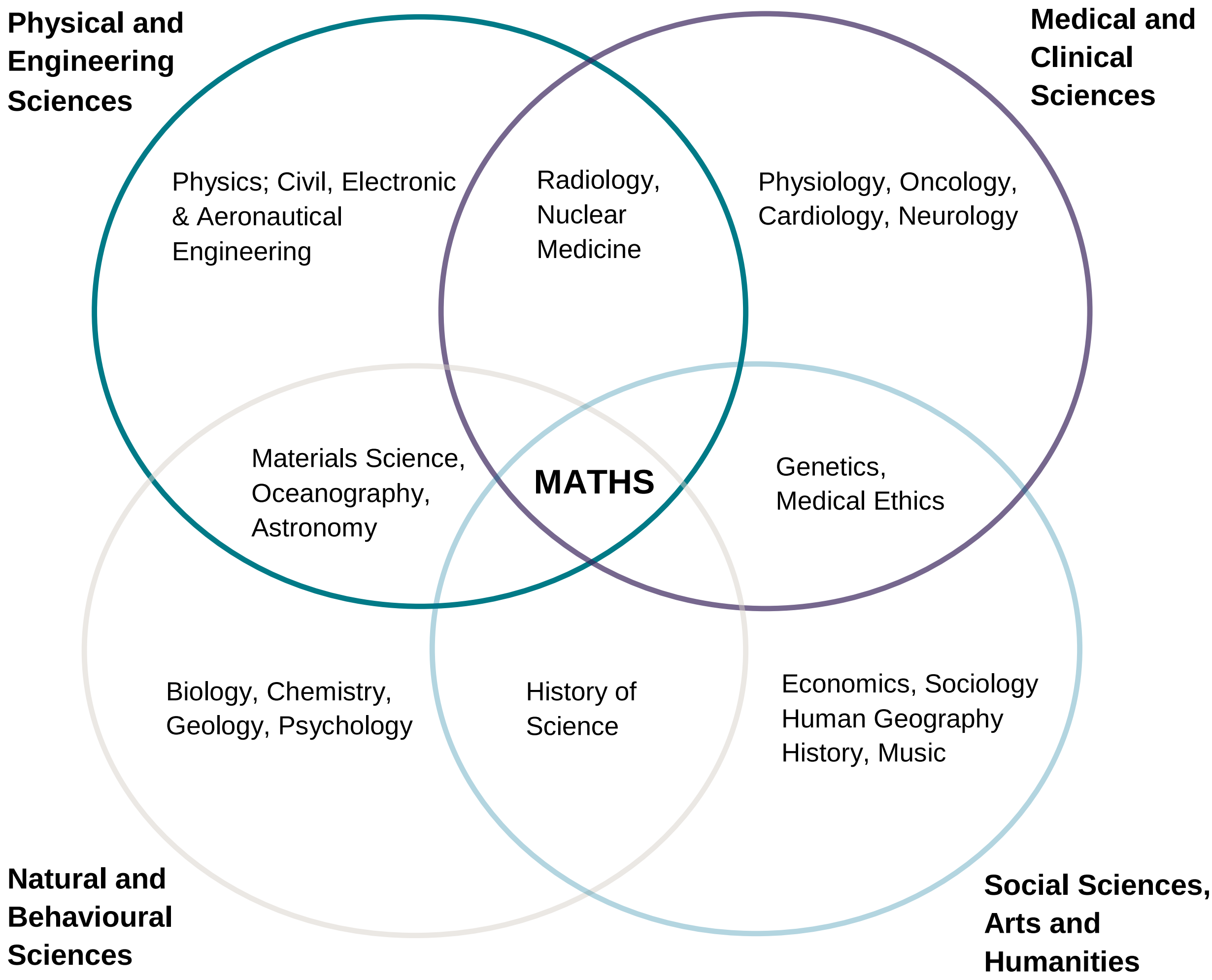
These days it is hard to escape financial news; whether we watch the news reports on TV, breeze through the newspapers, or look at our hand-held devices. Terms such as **derivatives**, **LIBOR**, **short selling**, **quantitative easing** or **FTSE 100** are constantly reminding us that finance is the most global of industries.

The financial crisis has been the chief headline across worldwide news bulletins and remains a fierce topic of discussion, with its effects still conspicuous today. There has never been a more crucial time for understanding the underlying mechanics of the complex products traded in the markets, together with a responsible approach to managing the associated risk.

Undeniably, the way finance has developed in recent years can be attributed in large part to mathematics, which has played a central role and remains the chief driving force allowing the financial markets to become increasingly sophisticated.

There is little doubt that mathematics has 'hijacked' most disciplines and its appeal and influence is noticeable in most branches of knowledge. In addition to the traditional areas of scholarship that depend on maths for its framework; less obvious academic themes are also enjoying the tangible advances being made due to the reliance on maths, such as Political Science, Medical Research and Sociology.



'Quantitative Finance' as a branch of modern finance continues to be one of the fastest-growing areas within the corporate world. The sophistication and complexity of modern financial products, has acted as the motivating factor for new mathematical models and the subsequent development of associated computational schemes.

Alternative names for this subject area are Mathematical Finance, Financial Mathematics, or Financial Engineering. Pricing, trading, management and risk control of complex financial products such as derivatives all depend on mathematics for a responsible approach.

As investment decisions for predicting risk and return are being increasingly based on principles taken from the Quantitative Finance arena, the field provides a challenge for both academics and practitioners. Although relatively young, financial mathematics has developed rapidly into a substantial body of knowledge and established part of mathematical science.

The evolution of financial assets is random and depends on time. Asset prices are examples of *stochastic processes* which are random variables indexed (parameterized) with time. If the movement of an asset is discrete it is called a *random walk*. A continuous movement is called a *diffusion process*. Hence the need for defining a robust set of properties for the randomness observed in an asset price realization, which is **Brownian Motion**. This was named after the Scottish Botanist who in 1827, while examining grains of pollen of the plant Clarkia pulchella suspended in water under a microscope, observed minute particles, ejected from the pollen grains, executing a continuous fidgety motion.

The origins of quantitative finance can be traced back to the start of the twentieth century. Louis Jean-Baptiste Alphonse Bachelier (March 11, 1870 - April 28, 1946) is credited with being the first person to derive the price of an option where the share price movement was modelled by Brownian motion, as part of his PhD, entitled *Théorie de la Spéculation* (published 1900). Thus, Bachelier may be considered a pioneer in the study of financial mathematics and one of the earliest exponents of Brownian Motion.

Five years later Einstein used Brownian motion to study diffusions; which described the microscopic transport of material and heat. In 1920 Norbert Wiener, a mathematician at MIT provided a mathematical construction of Brownian motion together with numerous results about the properties of Brownian motion - in fact he was the first to show that Brownian motion exists and is a well-defined entity. Hence Wiener process is also used as a name for this.

The field of mathematical finance has become particularly prominent due to the much-celebrated Black-Scholes-Merton equation (referred to as the Black-Scholes equation) written in 1973 by Fischer Black, Myron Scholes and Robert Merton, for which the latter two were awarded the Nobel Prize for economics in 1997.

Fischer Black passed away in 1995; the Nobel Prize is not awarded posthumously. Many have contributed ground-breaking theories and Nobel Prize winning results, to the development of financial mathematics, but undoubtedly the defining moments in this field's evolution would be Bachelier's PhD and the paper by Black-Scholes-Merton. In fact the work of the latter has successfully made them a household name and a branch of popular science. 'Black-Scholes' rolls off the tongue effortlessly by all regardless of any involvement with finance or mathematics.

Whilst many argue that collaboration between mathematicians and industry is far from optimal, Quant Finance gives an example of a fruitful partnership. Financial Engineering has the attraction of being one of only a few areas of mathematics that plays a central role in current developments in its domain of application. It has an ideal relationship with the 'real world' while it both draws from and has direct implications upon every-day financial practice in the commercial arena. The result has been the creation of new financial markets and novel financial instruments which are traded in them, which would not have been possible without the role of mathematics.

Like any area of applied mathematics, solving quant finance problems consists of three major elements.

1. **The mathematical model formulated from financial terminology**: Setting up the mathematical framework consisting of a Partial Differential Equation (PDE) or Stochastic Differential Equation (SDE) together with suitable boundary and/or initial conditions, based on the real world industrial finance principles, of the problem we wish to model.
2. **Numerical/computational implementation which includes coding**: Applying appropriate numerical methods (depending on computational requirements in terms of economy and efficiency) to reduce the mathematics to an algorithmic form. This leads on to expressing the algorithm in language-

independent format (or pseudo code), followed by writing a computer program(s).

3. **Analysis and interpretation of the result**: The most important part is studying the results obtained and understanding them. In addition being able to explain the output to both a technical and non-scientific audience, so a solid understanding of the finance based principles is equally important in the concluding stages.

This requires a solid foundation and confidence in the use of the relevant branches of mathematics. Quantitative Finance embraces the complete range of pure and applied mathematical subjects, which include probability and statistics, partial differential equations, numerical analysis, computation and operational research. Consequently an extraordinary number of quantitative-based scientists from a wide variety of backgrounds have moved into this area of research. In addition, the interdisciplinary nature of this subject matter has meant successful collaborative work being conducted by mathematicians, economists, finance professionals, theoretical physicists, and computer scientists. Even the psychologists are now playing a role through behavioural finance.

Unless a mathematical modeling problem is ideal/simple, it is unlikely that an analytical/closed form solution can be obtained. As with all industrial advances, which rely on information technology, the field of finance (in particular Financial Engineering) has benefited from the availability of computing power and programming design; enabling mathematicians to study increasingly difficult problems. There is no doubt that since the nineties Object Oriented Programming (OOP) has created much excitement. As a branch of software development OOP has been a major innovative theme, and C++ is the most widely used language, which supports the object-oriented paradigm. OOP is a design philosophy that superseded languages such as Pascal and C; defined broadly as procedural/structured programming this was the principle of reducing a code into smaller and independently functioning parts. It supported efficient programming when applied to moderately complex systems.

With the requirement of larger and further complex programs, this mode of programming was not so effective. Whilst OOP has inherited the best ideas from structured programming, what makes it vastly different is that it encourages the decomposition of the problem into related subgroups or self-sustainable 'objects'. Each object contains its own related data and instructions. This functionality promotes reusability and maintainability thus reducing the overall complexity of the problem.

In the financial markets C++ continues to retain its status as a 'sexy' language and arguable the most popular language in the financial markets. However what makes the field of Quantitative Finance so exciting and dynamic is the fast pace at which it detects and adopts new technologies. The programming language *Python* is rapidly becoming the standard in scientific computing, receiving much excitement about the application to mathematical finance; its appeal continues to grow in both academia and industry. It is simple to use; available on multiple platforms; easy to maintain; free to download and has a growing amount of add-on modules. It is particularly easy to interface with C++.

In the last two decades, there has been great interest in acquiring knowledge in financial mathematics, ranging from one-term university modules to lengthier taught course programmes. Programmes that are aimed at leading graduates towards technical careers which include quantitative analysts (quants), quant developers, and quant traders, in investment banks, hedge funds and other financial institutions. Advanced instruction that is both demanding in mathematics and related to practice, concurrently, has become a joint concern and a success factor for both educational bodies and the capital markets.

This was the motivating factor behind the decision by the mathematics department at UCL to create a MSc Financial Mathematics degree; the first cohort admitted in autumn 2012. The emphasis in the new UCL Financial Mathematics programme is to develop mathematical skills, programming proficiency and confidence in exploring financial data. The department also offers two financial maths modules at the 3rd year BSc. and 4th year MSci levels.

Those familiar with the Michael Moore documentary, *Capitalism A Love story* will recall the scene where Moore stands outside the New York Stock Exchange asking for an explanation of what a *derivative* is.

There are three broad classes of financial derivative:

1. Futures and Forwards
2. Options
3. Swaps

Undoubtedly it is derivatives that gives the field of Quant Finance its attraction, mystique and fear in equal measures – invoking many emotions. The right to buy or sell a financial asset in the future for a predefined amount is not always intuitive. So here's an example.

Imagine you'd like to sell your house in, say, a year's time. Of course you don't know what the market will be doing during that period, and you want to be sure that you'll achieve at least the current value of the house, let's say $1,000,000. Now suppose I come along and am interested in buying your house in a year's time. I want to be sure that I don't pay much more than its current value. I can enter into a contract with you, called an **option**, which stipulates that on a specified date in a year's time – in this case a year from now (called the **expiry**), I can buy the house for $1,000,000 (called the **strike price**) if I choose to. For this contract, which derives its value from the price of the house (the house is the **underlying**), I pay you a fee, called the *option price* (or premium), say $104505. I am essentially paying you for allowing me the privilege of locking in at today's price. I thus have the option of buying your house if I so choose, in which case you then have an obligation to sell me the house. If I decide not to exercise this right, that is perfectly fine, as the choice is mine - you will still keep the $104505 fee which is paid at the time we enter into the contract. Each party in this contract has a name. You are the *writer* (and have obligations) and I am the *holder* (and have choices).

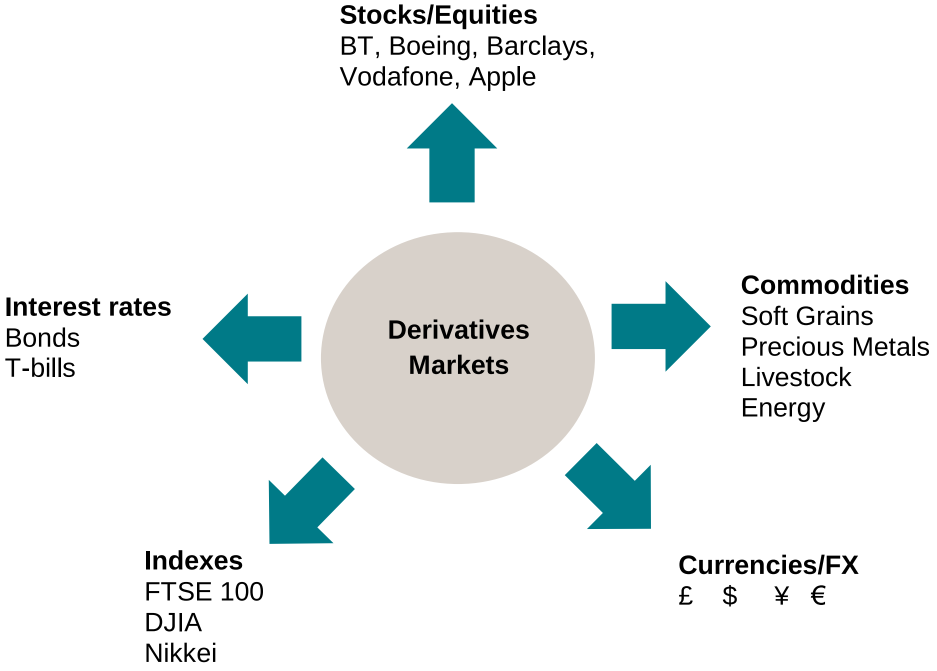
At the end of the one year period, we will be faced by one of two possible scenarios; the house value will have decreased or increased. If in a year's time the value of the house has gone down to $950,000, I certainly won't pay £1M for something which is worth £950,000 in the market, so you will have the house and the contract fee.

If the value has gone up to $1.5M, I will of course buy the house and can make an immediate profit by selling the property on the market. In this way both parties have attempted to minimise their risk – house prices falling from your point of view, and house prices going up from my perspective.

Think of the derivative as a ticket which I purchased, allowing me to acquire your house. The security in this case is the house, from which the contract 'derives' its value. Hence we can now formally write: *A derivative or derivative security is a financial contract that derives its value from an underlying cash-flow instrument*.

The earlier example is of a European call option, where nothing is potentially bought before the year is up. Another example is an American option, where I can buy at any time during the course of the 12-month period (up to and including), and then there are other variants such as the Asian option, where the option price is based upon the price of the house, averaged over the time period involved. The latter is an example of an *exotic option*. Where did the earlier figure of $104,505 come from? It certainly isn't a random number – there is a lot of maths behind arriving at this value. It is precisely what forms the basis of derivative pricing.

In 'real life' the underlying asset usually isn't a house, but other types of assets that are traded in the global financial markets. However the example described is to give in simple terms an idea of how a derivative's contract works and a few key terms. The types of assets (or cash-flow instruments) which form the underlying in a derivative are shown in the following diagram.



Quant Finance has come a long way since the pioneering work of Bachelier. However, finance continues to benefit from the effect of mathematics and gives it an unfair advantage. Not only have exciting markets and complex financial products emerged due to its powerful influence, but the unrelenting development of new applied academic subjects have resulted in the continued expansion of knowledge and learning. Nowhere has the role of mathematics in the real world been more conspicuous than in finance.