

Professor Alan Sokal
Discrete Geometry and Combinatorics Seminar
08 February 2012

Title: "Some wonderful conjectures at the boundary between analysis, combinatorics and probability."

Abstract:

I discuss some analytic and combinatorial properties (most of which are at present only conjectural) of the entire function

$$F(x,y) = \sum_{n \geq 0} x^n / n! \cdot y^{n(n-1)/2}.$$

This function (or formal power series) arises in numerous problems in enumerative combinatorics, notably in the enumeration of connected graphs, and in statistical mechanics in connection with the Potts model on the complete graph ("mean-field" or Curie--Weiss Potts model). This circle of problems also touches on the theory of integrable systems in classical mechanics (Calogero--Moser system). If time permits I will discuss an analogous problem for the "partial theta function":

$$\Theta_0(x,y) = \sum_{n \geq 0} x^n y^{n(n-1)/2}$$

in this case some striking results can be proven, by using identities for q-series. For details, see <http://www.maths.qmul.ac.uk/~pjc/csgnotes/sokal/> and <http://arxiv.org/abs/1106.1003>.