MATH0108 Commutative Rings and Algebras

Year:	2023–2024
Code:	MATH0108
Level:	6 (UG)
Normal student groups:	UG: Year 3 Mathematics degrees
Value:	Half unit (= 7.5 ECTS credits)
Term:	2
Structure:	3 hours per week. Assessed coursework
Assessment:	90% final exam, 10% coursework
Normal Pre-requisites:	MATH0053: Algebra 4
	The material is related to MATH0076: Algebraic geometry
Lecturer:	Prof A. Yafaev

Course Description and Objectives

Modern commutative algebra was developed in the first half of the 20th century as a technical tool to study both Number Theory and Algebraic Geometry.
It turned out that algebraic objects: rings, modules etc.. can be thought of as geometric objects and this geometric intuition is often very insightful and useful.
Towards the end of the course, if time permits, we will make these ideas rigorous by defining the Zariski topology on the Spectrum of a ring.
We will provide rigorous algebraic proofs of all statements. We will also put an emphasis on examples (and counter-examples).
We will start with an example-based review of the basic notions: rings, ideals, modules, algebras, prime and maximal ideals,... Then we will introduce a fundamental tool in commutative algebra - localisation that we will study in some detail. We will emphasize the geometric meaning of this operation. We will prove the Nakayama's lemma - a fundamental and a very useful result in commutative algebra.
Then we will study in detail the notion of integral extensions of rings which is essential in both number theory and algebraic geometry. We will prove the fundamental result - Noether normalisation theorem and its consequences.

Recommended Texts

Introduction to Commutative Algebra, M.F Atiyah, I.G. MacDonald, Addison-Wesley, 1969.

Undergraduate Commutative Algebra, M. Ried, CUP, 1995

Detailed Syllabus

0. Introduction, review of the basics: rings, ideals, modules, algebras. Basic properties (eg Noetherian rings, finitely generated modules and algebras,...).
1. Prime and maximal ideals, radical ideals, local rings.
2. Localisation of rings and modules, Nakayama's lemma.
3. Integral extensions.

4. Noether normalisation theorem, Zariski's lemma, weak Nullstellensatz theorem, normal domains, Discrete valuation rings
5. Krull dimension of a ring
6. *If time permits:* Spectrum of a ring, Zariski topology.