A MODULE ON SIMILARITY

CORNERSTONE MATHS

About the cover

Once upon a time, *London Trending*, a digital magazine, was founded by an artist and a computer programmer, both graduates of the University of London. *London Trending* is delivered via iPads, tablets, mobiles and desktop computers. It is dedicated to keeping Londoners up to date on all the trends and happenings around the city.

London Trending: A module on similarity
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Introduction: Welcome to the Graphics Department

→ *Key Learning:* By the end of this activity, you should be able to explain why a graphic artist must be able to use mathematics to design a magazine cover.

Welcome to the Graphics Department

Welcome to *London Trending* digital magazine. Every day, thousands of Londoners download our pages to their mobiles, tablets, and desktop computers. The Graphics Department is responsible for making sure that the graphics and photographs look right on all the different devices.

We need staff who work well with others and can do the mathematics necessary to ensure that the graphics are perfect.

Investigation 1: Mathematical Similarity

→ *Key Learning:* By the end of this activity, you should be able to decide whether a copy is a good likeness of the original.

Starter

Cover Layout

We are planning a new cover for the *London Trending* magazine. Our editor likes the draft shown below because she thinks the differently shaped pictures go together nicely. Because so many of our magazine's readers use tablet computers, she wants to make sure that we have a layout that works for landscape orientation as well.

1. Open Activity 1.1. Now, using Activity 1.1, create a new layout with the shapes so that you fill as much of the new cover page as possible. You may change each shape as much as you wish so long as it does not look stretched or warped.

2. Sketch your version of the layout below.
   In your sketch note the lengths, ratios, and angles you are using for each shape and explain how to change each part you want changed.

Main Activity

Big Ben

There is disagreement in the Graphics Department. We have many images of Big Ben. Which are good copies of the original? Which are not good? What do you think “good” means?

1. Open Activity 1.2. There is one original image of Big Ben (in blue) and several copies (in orange).
   - A good copy is exactly the same shape but can be a different size.
   - A bad copy is not exactly the same shape as the original.

2. In the table below, list the bad copies and **explain** how each is bad. You can translate and rotate the shapes if it helps. You can show and hide the gridlines as well. We have provided a model explanation for one of the bad copies.

Bad Copy	Explanation
2	It is the same height, but it is wider than the original.

Vocabulary

Good copies are *mathematically similar* to the original. Mathematically similar shapes are exactly the same in shape as the original but can be different sizes. Mathematically similar copies are also called *enlargements*. An enlargement is a copy that is larger (or smaller) than the original but maintains exactly the same shape as the original.

3. How are *enlargements* of Big Ben the same as the original?

Check Your Understanding

4. Sorting Winston Churchill Cards

   Decide which card has the original image, label that card as “original”, and place that card on the table. Sort the cards into two sets: those that look the same as the original and those that do not. **Explain** to your partner how you sorted the cards into sets.

1

2

3

4

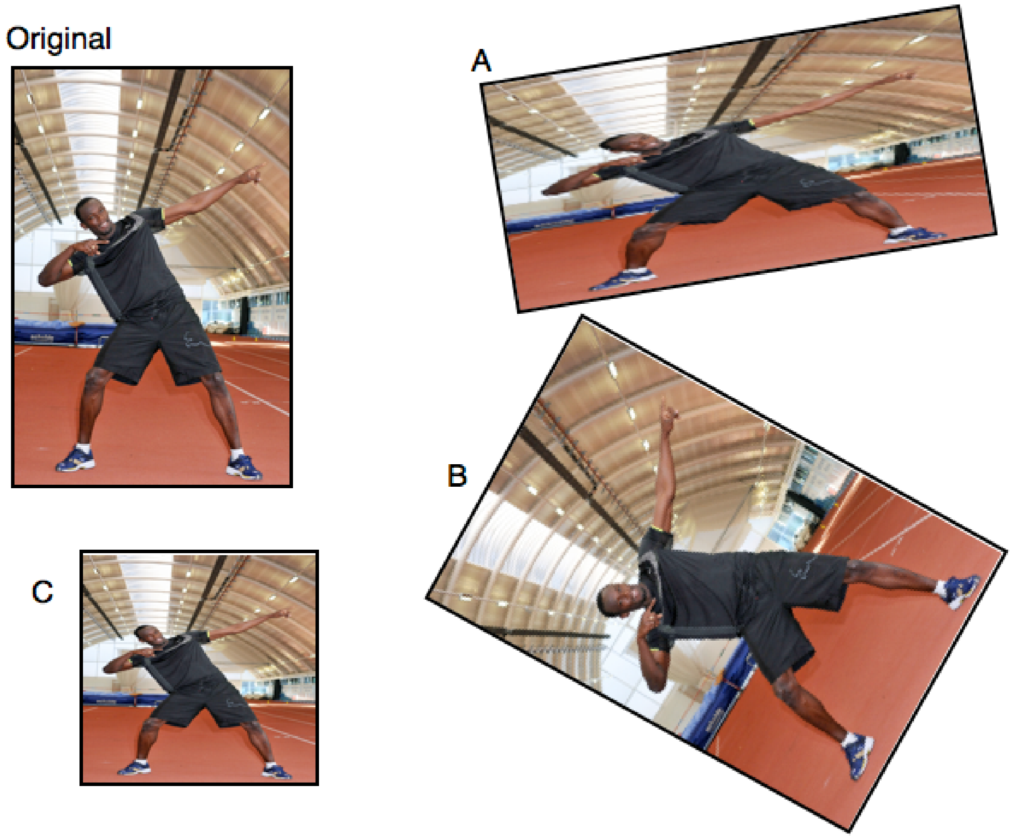
5

6

7

8

5. “Usain Bolt Visits London” is our headline in the magazine. We have these images to choose from:



A. Which copies (if any) are *not* mathematically similar to the original, and how do you know?

B. Which copies (if any) are enlargements of (mathematically similar to) the original, and how do you know?

6. “British Tennis Stars” is our headline in the magazine. We have these images to choose from:

Original

D

F

E

A. Which copies (if any) are *not* mathematically similar to the original, and how do you know?

B. Which copies (if any) are mathematically similar to the original? How do you know?

Investigation 2: On the Grid

→ *Key Learning:* By the end of this activity, you should be able to use numbers to determine whether an image is an enlargement of an original.

Starter

We have another disagreement in the Graphics Department. Our staff made four animated copies of the blue rectangle, but they cannot agree on which is always an enlargement (mathematically similar copy) of the original. Use the Cornerstone Maths software to help us decide.

1. Open Activity 2.1 and watch the animation.
2. Which copies (if any) are always mathematically similar to the original blue rectangle? **Explain** your answer.
   You can pause the animation and rotate and translate the shapes to investigate.

Main Activity

The Graphics Department needs to put the same image of the London Eye on several devices (e.g., tablet, computer, mobile). The staff want to know whether the various copies look good. In addition to judging how they look by eye, we can use measurements to decide which shapes are mathematically similar.

1. Open Activity 2.2.
   Look at the four London Eye images. First, without counting or measuring, determine which of the copies are enlargements of the original. How do you know?

2. If you have resized any of the images, refresh or reopen Activity 2.2. Now count and record the height and width of the London Eye images in the table below. Tick the rows of the copies that are enlargements.

	Height (in grid squares)	Width (in grid squares)	Tick if an enlargement
Original			
Copy 1			
Copy 2			
Copy 3			

3. Use the information in the table above to compare the relationship between the height and width of the original shape with those of each copy to uncover an important relationship for all similar shapes.

A. i. Fill in the missing multiplier that relates the original and Copy 1.

<table>
<tr><td></td><td></td><td>Height</td><td>Width</td><td></td></tr>
<tr><td>Original</td><td rowspan="2">× 2</td><td></td><td></td><td rowspan="2">× ____</td></tr>
<tr><td>Copy 1</td><td></td><td></td></tr>
</table>

ii. Write the relationship (in words) between the height of the original and the height of Copy 1.

iii. Write the relationship (in words) between the width of the original and the width of Copy 1.

B. i. Fill in the missing multiplier that relates the original and Copy 2.

		Height	Width	
Original	× ____			× ____
Copy 2				

ii. Write the relationship (in words) between the height of the original and the height of Copy 2.

iii. Write the relationship (in words) between the width of the original and the width of Copy 2.

C. i. Fill in the missing multiplier that relates the original and Copy 3.

		Height	Width	
Original	× ___			× ___
Copy 3				

ii. Write the relationship (in words) between the height of the original and the height of Copy 3.

iii. Write the relationship (in words) between the width of the original and the width of Copy 3.

4. On the basis of these observations, what is the relationship between an original and a mathematically similar rectangle?
HINT: Use the words height and width in your answer.

5. Why would the relationship hold true for all mathematically similar shapes?

Investigation 3: Scale Factor

→ *Key Learning:* By the end of this activity, you will be able to explain how to use a *scale factor* (common multiplier) in creating an enlargement.

The Graphics Department is using a software program that makes enlargements (mathematically similar copies) by using scale factor. Our staff are unclear about how the software works. You can help us by investigating how the scale factor works.

Open Activity 3.1. In this activity, the copy of the London Eye is always mathematically similar to the original and related to the original by a scale factor.

1. Move the scale factor slider.

   A. List several things that stay the same as you move the scale factor slider.

   B. List several things that change as you move the scale factor slider

2. Use the scale factor slider to make a large copy that still fits on the screen.

   A. Write the scale factor that you used.

   B. Compare the height and width of your large copy with the original. What was the effect of the scale factor on the copy's height and width (use the rectangular border).

3. Our designer Fatima says that the software works perfectly to create enlargements. Aiden says that the software does not create good copies every time. Who is right and why?

4. Open Activity 3.2. The copy (red triangle) is always mathematically similar to the original (blue triangle). Measure and colour the matching sides of the original and the copy. For example, AC in the original matches with A’C’ in the copy (both sides are the second longest side of their triangle). You may wish to rotate the triangles to put them in the same orientation.

Corresponding Sides
In mathematics, we use the term corresponding for things that match in particular mathematical ways. Matching sides in mathematically similar shapes are called corresponding sides.

A. Identify the corresponding sides for the two triangles and record them below.

__AC_ corresponds to _A'C'__
_____ corresponds to _____
_____ corresponds to _____

5. Drag the measurements for both the copy and the original into the table. Make sure that your table is organized taking corresponding sides into account. Take a snapshot of the measurements.

6. Set the scale factor slider to 3. Take a snapshot of the measurements.
What is the relationship between the measurements of corresponding sides in the copy and the original?

7. Set the scale factor slider to 2. Take a snapshot of the measurements.
   What is the relationship between the measurements of corresponding sides in the copy and the original?

8. **Predict**: If you set the scale factor to 1, what will the lengths of the corresponding sides in the copy be?

	A'B'	B'C'	A'C'
Copy			

   A. **Check**: Set the scale factor to 1. Make sure that your predictions were correct.

   B. **Explain**: Explain the effect of a scale factor of 1.

9. **Predict**: If you set the scale factor to 0.5, what will the lengths of the corresponding sides in the copy be?

	A'B'	B'C'	A'C'
Copy			

   A. **Check**: Set the scale factor to 0.5. Make sure that your predictions were correct.

   B. **Explain**: Explain the effect of a scale factor of 0.5.

10. Experiment with some more scale factors that are less than 1. Use the table below to record your measurements.

A.

	AB	BC	AC
Original			

	A'B'	B'C'	A'C'	Scale Factor
Copy				0.5
Copy				
Copy				

B. When a copy is created using a scale factor of less than 1, what can you say about the relationship between the lengths of the original's and the copy's corresponding sides?

11. Describe what a scale factor is.

12. Describe how to use scale factor to find the lengths of sides in a mathematically similar copy when you know the lengths of the original.

13. Sketch an original and three enlargements as detailed below.

A. Sketch a polygon that will be your original.	B. Sketch an enlargement that is related to your original by a scale factor greater than 1.
C. Sketch an enlargement that is related to your original by a scale factor between 0 and 1.	D. Sketch an enlargement that is related to your original by a scale factor that is exactly 1.

14. Given an original shape and an enlargement, what can you say about the scale factor and the relationship between corresponding sides?

Vocabulary

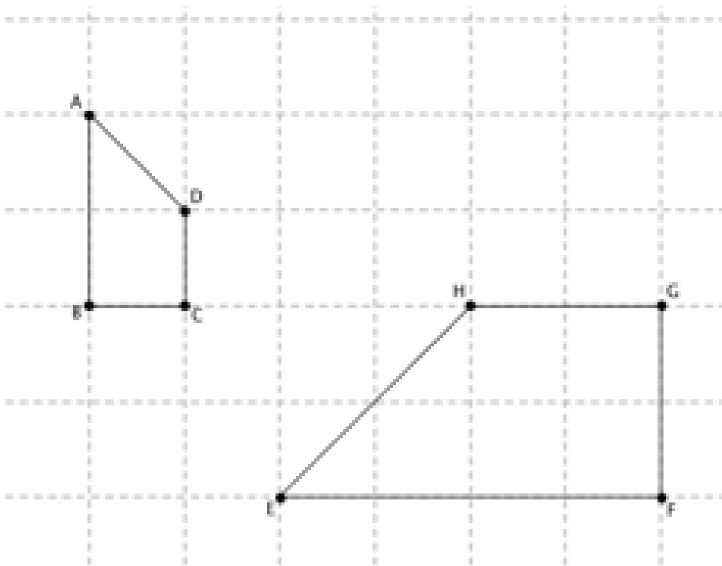
Congruent
When an original and a copy are exactly the same size and shape, they are called *congruent*. Congruent shapes are related by a scale factor of 1.

15. What is true about congruent shapes? Use what you learnt in Activity 3.2 (Question 8).

	This is true Sometimes / Always / Never
The lengths of corresponding sides in congruent shapes are equal.	
The scale factor between congruent shapes is 1.	
Congruent shapes are similar.	
Similar shapes are congruent.	

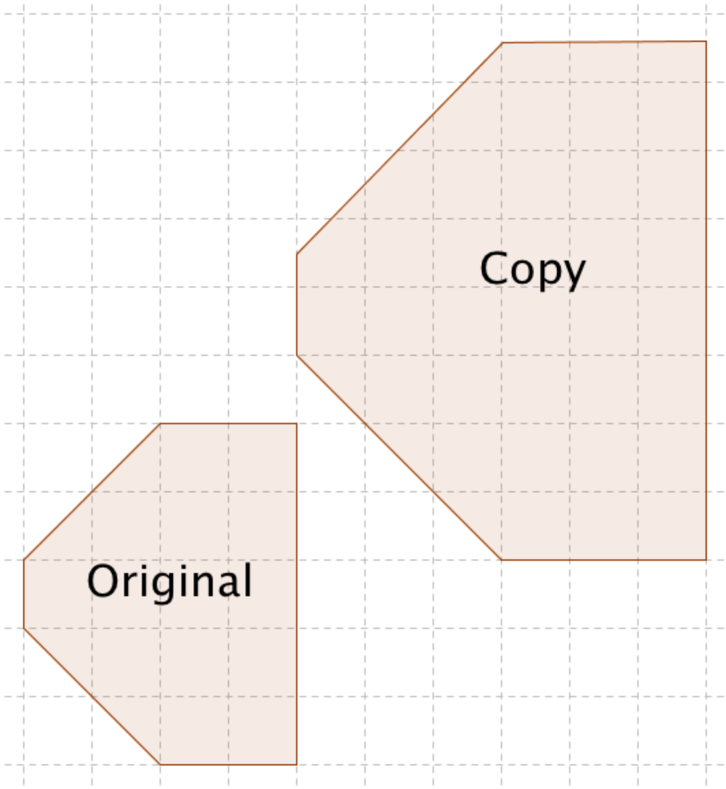
Check Your Understanding

16. Figure EFGH is an enlargement of Figure ABCD and so is mathematically similar. Find the scale factor and explain your reasoning.



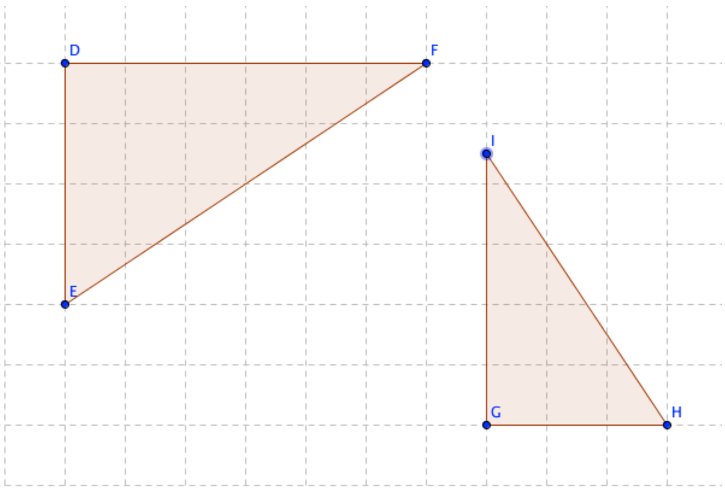
Scale factor = ____

17. Find the scale factor between these two mathematically similar polygons:



Scale factor = _____

18. Triangle GHI (a copy) is mathematically similar to Triangle DEF (the original). Find the scale factor between these two triangles:



Scale factor = ______

Investigation 4: Broken Scale Factor

→ *Key Learning:* By the end of this activity, you will understand why scale factor applies to a whole shape.

We have an interesting development in the Graphics Department. One of our artists, Eileen, found a piece of software without a scale factor slider but with two strange other sliders. She says she is able to use the software to create mathematically similar copies.

1. Open Activity 4.1. In this activity, the copy of the London Eye is *not* always mathematically similar to the original. There is no scale factor slider, but there are two other sliders. Play around with them and describe what they do.

2. Re-open Activity 4.1. Use the height slider to increase the copy's height by 12 units. Use the width slider to increase the copy's width by 12 units. **Explain** why the copy *is* or *is not* mathematically similar to the original.

3. Use the two sliders to make a mathematically similar copy that is not the same size as the original. **Explain** how you know your copy is similar to the original. Write down the scale factor.

4. Describe what Eileen did to make a copy that was mathematically similar to but a different size than the original.

5. Make another mathematically similar copy where the side length A'B' is 12 units.

   A. Find what the other lengths must be for the copy and the original to be mathematically similar.

   B. Fill in the information below.

Length of side AB (original)	Length of side AD (original)
Length of side A'B' (enlargement)	Length of side A'D' (enlargement)

   C. What is the scale factor that relates the copy to the original?

6. Make an enlargement where the side length A'D' is 1.5 units.

   A. Find what the other lengths must be for the copy and the original to be mathematically similar.

   B. Fill in the information below.

Length of side AB (original)	Length of side AD (original)
Length of side A'B' (enlargement)	Length of side A'D' (enlargement)

   C. What is the scale factor that relates the copy to the original?

7. With your partner, devise a set of instructions to use Activity 4.1 so that anyone can create mathematically similar enlargements.

Investigation 5: More Than Lengths of Sides

→ *Key Learning:* By the end of this activity, you will be able to explain how enlargements require more than equivalent ratios of corresponding sides.

Starter

You have explored how the lengths of sides between mathematically similar shapes are related. But mathematically similar shapes are related by more than just the lengths of their sides.

1. Open Activity 5.1. Look at Copy 7. **Explain** how you know that Copy 7 is not mathematically similar to the original.

Main Activity

London Trending has a new advertising client whose logo is embedded in a parallelogram. We want to try out different versions of the logo, all of which must be mathematically similar so that they look “the same”.

1. Open Activity 5.2. At the start, the three parallelograms are mathematically similar.

   A. Before changing any of the parallelograms, **predict** what the relationship is between a particular set of three corresponding angles, for example DAB, D'A'B', and HEF.

   B. Now **check** by moving the angle slider and compare the three parallelograms as they change. **Explain** how they are the same and how they are different. Which two are always similar and why?

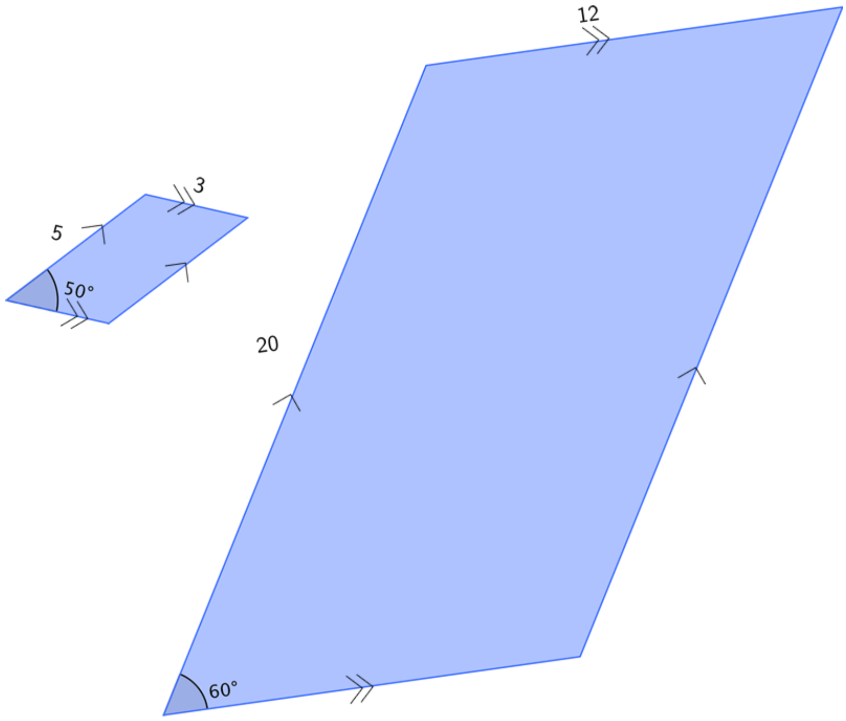
C. Move the scale factor slider. What happens to corresponding angles in the two mathematically similar parallelograms? What happens to the sides?

2. Given what you know so far, what is the relationship between corresponding angles in mathematically similar shapes?

3. Why would that be true?

Check Your Understanding

4. One of our artists, Sirin, says that the parallelograms below are similar, explaining that both of the multipliers are equivalent (5 x 4 = 20 and 3 x 4 = 12) and that the scale factor is 4. Is Sirin correct? **Explain.**



Investigation 6: Ratios

→ *Key Learning:* In this activity, you will learn to use ratios to determine whether two shapes are mathematically similar.

Introduction to Ratios

The Graphics Department received directions to make pairs of images in the ratio of 3 to 1. We need to figure out what that means and how to do it.

You are probably familiar with ratios such as 3 boys to 2 girls or 4 pencils for every 3 notebooks. Ratios relate two numbers using multiplication. For example, a 2 to 1 ratio (written as 2:1) means that the first term is two times as large as the second term in the ratio. (We call the numbers in the ratio *terms*.)

Between Ratios

When two shapes are mathematically similar, we can compare the copy with the original using a ratio. For example, one side of a copy is 3 times as long as the corresponding side on the original. The ratio between the copy and the original is 3:1. This is called a *between* ratio. It relates *a side on one shape* to its *corresponding side* in a *mathematically similar shape*.

1. Think back to On the Grid: Investigation 2. How did you describe the relationship between the side measurement of an original and its corresponding side's measurement in an enlargement? Keep this in mind as you work through this investigation.

2. Open Activity 6.1. Label the vertices of the two triangles.

3. Colour the corresponding sides of the triangles so that they match.

4. Identify the corresponding sides for the two triangles and record them below.

   _____ corresponds to _____
   _____ corresponds to _____
   _____ corresponds to _____

5. Measure the sides of the triangles. Drag the side measurements into the Measurement Table so you can see the *between* ratios.

6. The table below shows a ratio between one length of the enlarged triangle and its corresponding length in the original triangle. Write the other two between ratios.

copy length: corresponding original length
9:3

7. What is the same about the three *between* ratios in the table above?

8. Ratios can be simplified in the same way as fractions. For example, 16:4 = 4:1 and 8:2 = 4:1, so these two ratios are equivalent. Show that all the ratios in the table above are equivalent.

9. In the space below, sketch and label two right-angled triangles that are mathematically similar. The original triangle should have sides of 3, 4, and 5. The corresponding sides in the enlarged triangle should have lengths of 4.5, 6, and 7.5.

10. Write the *between* ratio for each pair of corresponding sides.

enlargement length: corresponding original length

11. What is the same about the three *between* ratios you wrote? (Hint: Ratios can have fractions or decimals as one of their terms.)

Vocabulary

Equivalent ratios have the same multiplicative relationship between their terms. For example, we can say that 1:3 = 4:12 because on each side of the equal sign, we can multiply the term in the first ratio by 4 to get the term in the second ratio. Notice that this works only for multiplication; adding does *not* work.

We can simplify ratios as we do with fractions. So, 6:3 = 4:2 = 2:1. We call 2:1 a *unitary* ratio because one of its terms is 1. It is easy to compare ratios when they are simplified to unitary ratios. Do not be surprised if the other term turns out to be a decimal: It is perfectly all right.

Equivalent ratios have the same unitary ratio.
12:6 and 4:2 *are* equivalent. They both simplify to 2:1.
35:10 and 7:2 *are* equivalent. They both simplify to 3.5:1.
6:3 and 6:2 are *not* equivalent. They simplify to different unitary ratios, 2:1 and 3:1.

12. Describe what is the same and what is different about the *between* ratios and the common multiplier you used in On the Grid: Investigation 2, and scale factor.

Check Your Understanding

13. Explain why 5:2 is equivalent to 10:4.

✎

14. Write the unitary ratios for the following:

Ratio	Unitary Ratio
12:3	
15:5	
8:2	
16:5	

15. Tick the pairs of ratios that are equivalent.

Ratio Pair	Equivalent
3.5:2 and 11:3	
12:3 and 16:4	
7:2 and 6:1	

Investigation 7: Between Ratios and Within Ratios

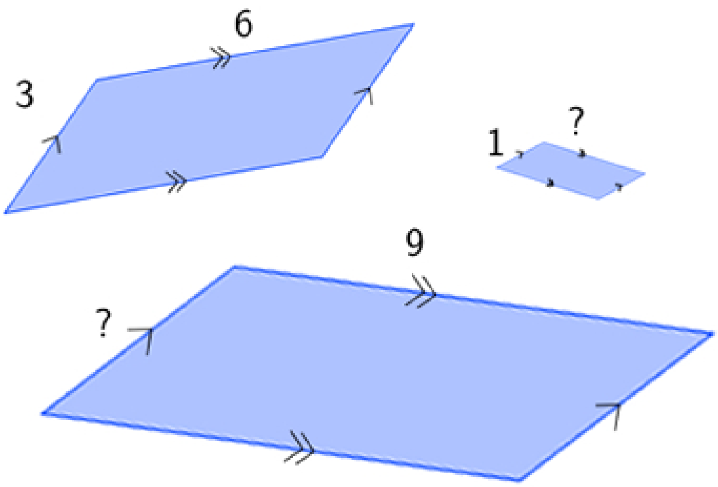
Starter

1. Open Activity 7.1. Use the Show/Hide tool to hide the gridlines.

   A. By eye, sort the rectangles into groups of mathematically similar rectangles.

   B. Within each group of rectangles, describe what looks the same and what looks different (include the words height and width).

Main Activity



These three parallelograms are mathematically similar.

1. Work out the missing lengths of the sides of the parallelograms.

2. **Explain** how you found the missing sides.

3. **Explain** another way to find the missing sides.

✎

Two Different Ratios

We call the ratio of the lengths of two sides of the same figure a *within* ratio. We call the ratio of the lengths of corresponding sides of two different figures a *between* ratio. These are not formal mathematical terms, but they are useful labels to distinguish these two types of ratios.

Remember the first animation that the graphics team made? They've made another with four animated copies of the original (blue) quadrilateral. Now it is much harder to tell which quadrilateral copies are always mathematically similar copies of the original. Work with the animation file to help us figure this out.

4. Open Activity 7.2. Watch the animation.

5. Which copies (if any) are always mathematically similar to the original (blue) quadrilateral?

   ✎

6. Which copies (if any) are never mathematically similar to the original (blue) rectangle?

   ✎

7. Which copies (if any) are sometimes mathematically similar to the original (blue) quadrilateral?

   ✎

8. Try out one or more ways to show that you are right.

   A. Put the quadrilaterals on top of each other, aligned at one corner.
      - Does this help show which quadrilaterals are mathematically similar? How?

   B. Use the grid and measurements and the ratio checker.
      - Does this help show which quadrilaterals are mathematically similar? How?

9. Which quadrilaterals (if any) are always mathematically similar to the original (blue) one and which are not? **Explain** your answer using ratios.

10. Describe how you tested to see whether the quadrilaterals were mathematically similar.

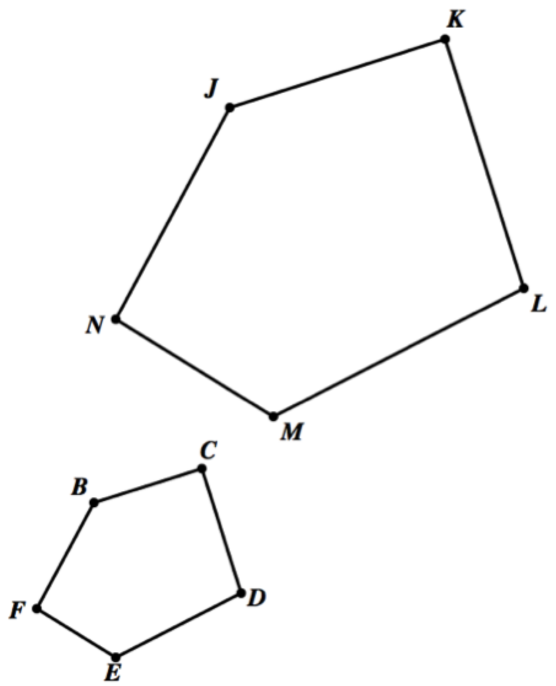
11. Describe *mathematically similar* in your own words.

12. Compare your description of mathematically similar with the description that you wrote in Investigation 3: On the Grid, Question 4. How has your description changed?

Practice/Homework

Check Your Understanding

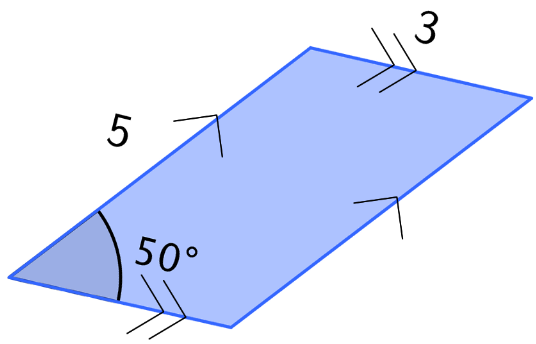
Pentagon BCDEF is similar to pentagon JKLMN.



1. Express as many equivalent *within* ratios for the two pentagons as you can.

2. Express as many equivalent *between* ratios for the two pentagons as you can.

Investigation 8: What Changes and What Stays the Same?



1. Near the parallelogram above, sketch three mathematically similar copies with different scale factors. Remember, a similar shape can be smaller than the original. Label the sides and angles and write down the scale factor.

2. Imagine creating *many* more enlargements of the parallelogram above.
   Predict: Complete the table below with your predictions of what changes and what does not change across all your enlargements and the original—given that they are all mathematically similar. (We completed the first one for you.)

Characteristic of Shape	**Changes between Enlargement and Original** **Choose *Sometimes / Always / Never***
Size of shape	*Sometimes changes because when the shape is congruent the shapes are the same size.*
Ratio between one side on the original and the corresponding side on any copy *(between ratio)*	
Lengths of sides	
Ratio between two of the sides on the original and the corresponding sides on copy (*within ratio*)	
Angles	*Never changes because corresponding angles in similar shapes are always the same*
The way a shape is oriented	
Scale factor	
Overall look of the shape	

You have made some predictions in the table above about mathematically similar parallelograms. Do your predictions also work for other shapes, like triangles?
Use Activity 8.1 to **check**. It shows three triangles that will always be similar. Change the triangles and check your predictions by comparing the original with the two copies.

3. Would any of your predictions in Question 2 need to change if the shapes were triangles instead of parallelograms?

4. Now think about **all** shapes. Decide whether each statement in the table is sometimes, always, or never true.

Statement	Choose *Sometimes / Always / Never*
Similar shapes are the same size.	Sometimes
Similar shapes have equal corresponding angles.	
Shapes with equal corresponding angles are mathematically similar.	
A mathematically similar copy is a warped version of the original.	
Congruent shapes are similar.	
Similar shapes are congruent.	

Between Ratios

For similar shapes, the *between* ratios of corresponding sides do *vary*. They are equivalent to the scale factor of enlargement and so they vary as the scale factor varies.

Within Ratios

For similar shapes, the *within* ratios of corresponding sides *do not vary* as the scale factor changes.

5. Celia says, “Similar shapes always have the same *within* ratio”.
   Bola says, “They do not. A 3-4-5 triangle is similar to a 6-8-10 triangle, but the ratio of 3:4 is not the same as the ratio of 4:5”.
   Teresa says, “But 3:4 is always equivalent to 6:8”.

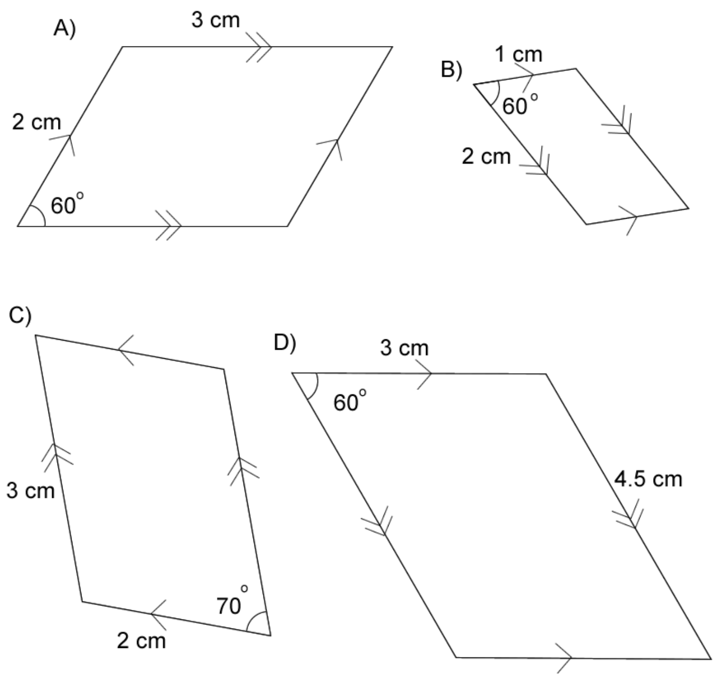
   Write a statement that would help the three settle their dispute and better understand within ratios.

6. Maddie says, “Similar shapes always have the same *between* ratios”.
   Jaxon says, “They do not. A 3-4-5 triangle is similar to both 6-8-10 and 9-12-15 triangles, but there are two different *between* ratios”.
   Jumbo says, “But the *between* ratio between the 6-8-10 triangle and the 3-4-5 triangle is always 2:1”.

   Write a statement that would help the three settle their dispute and better understand between ratios.

Check Your Understanding

7.



Determine and show which of the parallelograms above are similar.

8. Sketch a parallelogram that is mathematically similar but *not* congruent to parallelogram B. Label the lengths of the sides and angles.

Investigation 9: Build Your Own

Think back to the beginning of this module on similarity when you were tasked with designing a new cover for the *London Trending* magazine. Now use what you have learnt to really impress the editor. Remember that our editor likes the draft shown below because she thinks the differently shaped pictures go together nicely. Because so many of our magazine's readers use tablet computers, however, she wants to make sure that we have a layout that works for landscape orientation as well. You will sketch a new layout and use mathematics to describe it.

1. Open Activity 9.1.

2. If our editor likes your layout, she will send it to our programming team. So, before you change anything, you must measure and record any part of any shape that you will want to change, including lengths, ratios, and angles. You can use the picture above to record your measurements.

3. Now, using Activity 9.1, create a new layout with the shapes so that you fill as much of the new cover page as possible. You may change each shape as you wish as long as it is mathematically similar to its original (remember to consider the within ratios and equal corresponding angles).

4. Sketch your version of the layout below.
   Remember, this sketch is meant to be sent to our programming team. So, in your sketch you must note the lengths, ratios, and angles you are using for each shape and explain how to change each part you want changed.

5. The editor sometimes questions whether pictures have been warped or distorted in preparing them for new layouts. Use mathematics to explain how you know that each of your transformed pictures is mathematically similar to its original.

Investigation 10: Supplementary Activity: So the Angles in Triangles are Important?

→ *Key Learning:* By the end of this activity, you will be able to use the angles in triangles to determine if they are mathematically similar.

Open Investigation 10: Supplementary Activity: So the Angles in Triangles are Important and open the activity Triangles' Angles 2. The two triangles you see are always mathematically similar, no matter how you drag them around. You are going to find relationships among the measurements in the shapes.

1. Identify and colour the corresponding sides of the two triangles.

   _____ corresponds to _____

   _____ corresponds to _____

   _____ corresponds to _____

2. Play around: Drag points B and C. Try to make the length of at least one side a whole number of grid squares long. What happens to the corresponding sides of triangle DEF as you drag?

3. Now keep the triangles fixed.

   A. Use the Measure tool to find the side lengths and angles for both triangles.

   B. Drag the side length measurements to the table.

   C. Use the Snapshot tool to capture your picture and measurements.

   D. Drag points B and C to change the measurements of the triangles. Use the Snapshot tool to capture at least two more sets of measurements.

4. State the relationship between measurements of corresponding sides in the two triangles.

5. Review your Snapshots. What do you notice about the angles in your pictures?

6. Explain how you would know whether two triangles were similar by looking at the angles only.

7. Compare the measurements you captured for each pair of similar triangles. How do the relationships in the table show that the shapes are mathematically similar?

Check Your Understanding

8. In the space below, sketch a triangle. Label the vertices and estimate the size of the angle. Write these on your drawing. Remember that the three angles should total 180°.

   Now sketch a rotated enlargement of your triangle. Label the vertices and write the size of the angles on your drawing.

   ________________ corresponds to ________________

   ________________ corresponds to ________________

   ________________ corresponds to ________________

Investigation 11: Supplementary Activity: Positioning Images Precisely

Julia is worried that the Graphics Department are not always sure where to position the images on the page. She has decided to find out if using coordinates might help.

Open Investigation 11 and the activity, Positioning Images 1.

The triangles ABC and AB'C' are similar and have their *centres of enlargement* at A.

1. Move the scale factor slider.
   Describe what you notice using mathematical language in as many ways as you can.

2. Pause the animation and move the animation slider so that the bases of the triangles are on the x-axis.

   A. What are the coordinates of A?

      A = _______

   B. What are the coordinates of B and C?

      B = _______

      C = _______

   C. Use the scale factor slider to enlarge the Copy using a scale factor of 2.
      What are the coordinates of B' and C' in the enlarged copy?

      B' = _______

      C' = _______

D. Investigate what happens to the coordinates of B' and C' as you change the scale factor.
Try to notice what is staying the same and what is changing.

Coordinates of B'

Coordinates of the centre of enlargement A	Coordinates of B

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5

Coordinates of C'

Coordinates of the centre of enlargement A	Coordinates of C

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5

E. Use the Show/Hide tool and hide the Copy. Move the scale factor slider to 4. Predict the new coordinates of B' and C' in the first column of the table.

Scale factor = 4

	Predict	**Check**
Coordinates of point B'		
Coordinates of point C'		

F. Check by showing the Copy. Write the actual coordinates in the table above.

G. Explain how you used the scale factor to work out the new coordinates.

Does your explanation still work if you change the shape?

3. Hide the Copy.

   Move slider x so that the coordinate of B is at (2, 0) and move slider y so that the coordinate of C is at (0, 6).
   Move the scale factor slider to 3.

   A. Predict the new coordinates of B' and C' in the first column of the table.

   Scale factor = 3

	Predict	**Check**
Coordinates of point B'		
Coordinates of point C'		

   B. Check by showing the Copy. Write the actual coordinates in the table above.

   C. Explain your procedure for predicting the new coordinates.

   D. Explain how changing the shape did or didn't change your procedure for predicting the new coordinates.

Does your procedure for predicting new coordinates still work if you move the centre of enlargement?

4. Open the activity, Positioning Images 2.
   Enlarge the copy by scale factor 2.

   A. What are the coordinates of A?

   A = ________

   B. What are the coordinates of B and C?

   B = ________

   C = ________

   C. Investigate what happens to the coordinates of B' and C' as you change the scale factor.
   Try to notice what is staying the same and what is changing.

Coordinates of the centre of enlargement A	Coordinates of B	Length of AB

Coordinates of B'

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5

Coordinates of the centre of enlargement A	Coordinates of C	Length of AC

Coordinates of C'

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5

   D. Does the procedure that you used in Question 3 still work? Can you think why this is?

E. Use the Show/Hide tool and hide the Copy. Move the scale factor slider to 4. **Predict** the new coordinates of B' and C' in the first column of the table.

Scale factor = 4

	Predict	**Check**
Coordinates of point B'		
Coordinates of point C'		

F. **Check** by showing the Copy. Write the actual coordinates in the table above.

G. Write a procedure for predicting the coordinates B' and C'.

5. Now move both the Original and the Copy so that the centre of enlargement (A) is at (5,1).
   Enlarge the Copy by scale factor 2.

   A. What are the coordinates of A?

      A = ________

   B. What are the coordinates of B and C?

      B = ________

      C = ________

   C. Investigate what happens to the coordinates of B' and C' as you change the scale factor.
      Try to notice what is staying the same and what is changing.

Coordinates of the centre of enlargement A	Coordinates of B	Length of AB

Coordinates of B'

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5

Coordinates of the centre of enlargement A	Coordinates of C	Length of AC

Coordinates of C'

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5

   D. Does the procedure that you used in Question 3 still work? Can you think why this is?

Check Your Understanding

6. Write a short note to Julia to describe what you have learnt about positioning images precisely on a coordinate grid.

Investigation 12: Supplementary Activity: Within Ratios in Right-Angled Triangles

→ *Key Learning:* By the end of this activity, you should know that the ratios of side lengths for right angled triangles are a starting point for trigonometric ratios which you will study later.

Main Activity

For this investigation, you will need your calculator.

1. Open Investigation 12 and the activity, Within Ratios in Right-Angled Triangles.
   Move the sliders in turn.
   What do you notice about the angle and side measurements? Record this in the table below.

Slider	Which lengths/angles stay the same?	Which lengths/angles change?
slider 1		
slider 2		
scale factor		

2. What can you say about the original and copy that is true no matter how you change the sliders? (Use precise mathematical language)

3. You are now going to focus on the angles B and B’. Begin by hiding the angle measures (by selecting the vertex and then unclicking the Measure tool button) of A and A’, C and C’.
   Change the sliders so that, slider 1 = 1.5, slider 2= 2.0 and scale factor = 2.

   A. Measure and record the size of the angles B and B’.

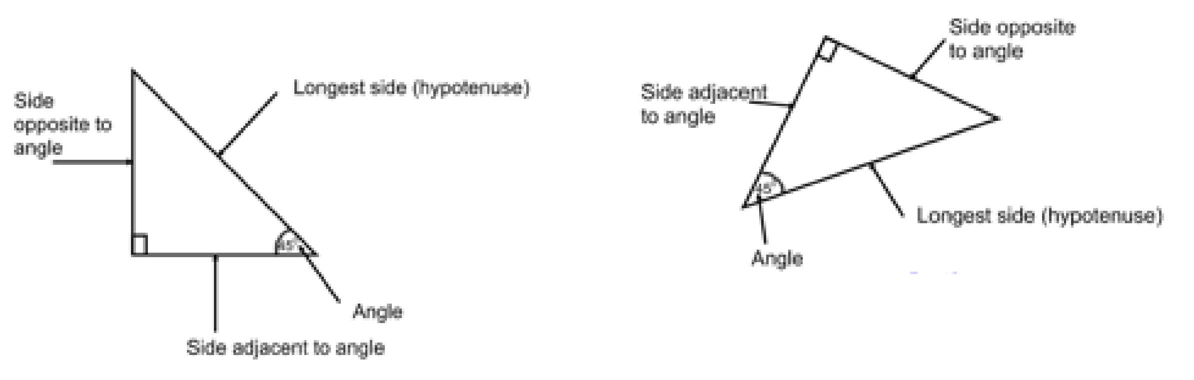
      B and B' = ______

   B. Work out the size of the angles at C and C’.

      C and C' = ______

Vocabulary

Vocabulary - Naming the sides within right-angled



4. Check that the sliders are: slider 1 = 1.5, slider 2= 2.0. Then in the table below
   - Record the angle (at B and B')
   - Record the lengths of the 'opposite', 'adjacent' and the 'longest side (hypotenuse)' in ratios that contain them.
   - Using a calculator, find and record the within ratios. Write the ratios to 2 decimal places.

   When the scale factor is left blank, choose your own scale factor and record it in the table.

Triangle	scale factor	Angle at B and B'	Length of side opposite to B'	Length of side adjacent to B'	Length of hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
Copy 1	1.0							
Copy 2	3.0							
Copy 3								
Copy 4								

A. What stays the same when the scale factor is changed and what changes when the scale factor is changed?

5. Change the sliders so that, slider1 = 1.6, slider 2 = 3.7 and scale factor =1.0. Fill in the table below. You will need to change the scale factor each time.

Triangle	scale factor	Angle at B and B'	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
Copy 1	1.0				
Copy 2	2.0				
Copy 3					
Copy 4					

A. What stays the same and what changes now?

B. Compare the two tables above. What is the same about them and what is different about them?

C. Suggest a reason for why you think this is happening.

6. You are now going to explore what happens to these within ratios when you change the angle B'.

   A. Move the scale factor slider to choose your Copy size. This should stay the same throughout this activity.

   B. Move the sliders so that angle B' is approximately 20° (to 1 d.p.). Complete the table, you will need to change angle B' each time.

My scale factor = ___________						
Size of angle B'	Length of Opposite	Length of Adjacent	Length of Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
20°						
40°						
50°						
70°						

   C. As the angle at B increases, what do you notice about each of the calculated ratios?

   D. There are some common values (or relationships) in the table. For example compare the ratios for 20° and 70° or 40° and 50°. Can you explain this?

Plenary

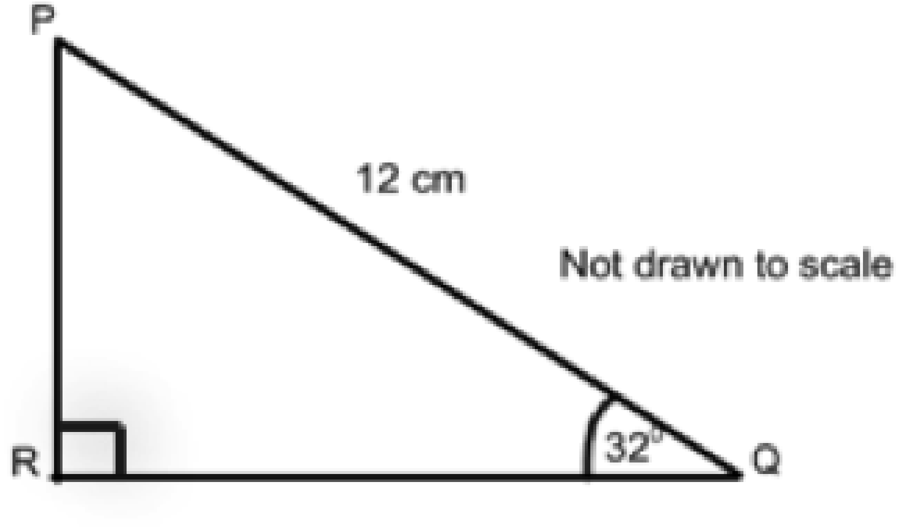
Vocabulary

Vocabulary: Within ratios for right-angled triangles are special
These special ratios can be used to find lengths of sides in right triangles. They are so commonly used that they are given special names. The ratio (length of side Opposite to the angle)/(length of Hypotenuse) is called the Sine or Sin of the angle. The ratio (length of side Adjacent to the angle)/(length of Hypotenuse) is called the Cosine or Cos of the angle. The ratio (length of side Opposite to the angle)/(length of side Adjacent to the angle) is called the Tangent or Tan of the angle

1. The sine, cosine and tangent of any angle can be provided by your calculator. Use your calculator to check the values for the sine, cosine and tangent for angles 45°, 60° and any other value in table 3 above. What do you notice?

Check Your Understanding

2.



The sine of 32° is 0.55 to 2 decimal places.

What other information about triangle PQR can you infer?
Write as many facts or relationships as you can around the picture as a mind map.

Appendix A: Software Guide

Note that not all controls and tools are available in all activities.

Selecting a Shape
To select a shape: a) check that no control buttons are selected; and b) click inside the shape. A selected shape will have circles shown on its vertices and the shape's sides will be slightly bolded.

Controls (these are along the bottom of the screen)

Translate
To translate (move) a shape: a) click on the translate button; and b) click inside the shape and drag the shape to the desired location.

Enlarge
To enlarge (and reduce) a shape: a) click on the enlarge button; b) click on the shape's enlargement grab point and drag the shape to the desired enlargement.

Rotate
To rotate a shape: a) click on the rotate button; b) click on the shape's rotation grab point and drag the point around to the rotate the shape.

Tools (these are along the top of the screen)

Measure Side Lengths
To measure a side length: a) check that no action buttons are selected; b) select the shape by clicking inside it; c) select the side you wish to measure (the pointer will be a hand when the side is selectable); and e) select MEASURE.

Measure Angles
To measure an angle: a) check that no action buttons are selected; b) select the shape by clicking inside it; c) select the vertex whose angle you wish to measure (the pointer will be a hand when the vertex is selectable); and e) select MEASURE.

Colour Sides

To colour a side: a) check that no action buttons are selected; b) select the shape by clicking inside it; c) select the side you wish to colour (the pointer will be a hand when the side is selectable); and e) select COLOUR and choose the colour you wish.

Colour Shapes

To colour a side: a) check that no action buttons are selected; b) select the shape by clicking inside it; and c) select COLOUR and choose the colour you wish.

Label a Side

To label a side: a) check that no action buttons are selected; b) select the side's shape by clicking inside it; and c) select the side you wish to label (the pointer will be a hand when the side is selectable); d) select EDIT; e) type the side's label into the dialogue box; and f) click Update Label.

Label a Vertex

To label a vertex: a) check that no action buttons are selected; b) select the shape by clicking inside it; and c) select the vertex you wish to label (the pointer will be a hand when the vertex is selectable); d) select EDIT; e) type the vertex's label into the dialogue box; and f) click Update Label.

Show/Hide a Shape or Gridlines

To show or hide a shape: a) click on SHOW/HIDE; and b) click on the name of the object you wish to show or hide. Objects that are currently showing will have a tick mark next to their name in the list.

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