A MODULE ON SIMILARITY

TEACHER VERSION

CORNERSTONE MATHS

About the cover

Once upon a time, *London Trending*, a digital magazine, was founded by an artist and a computer programmer, both graduates of the University of London. *London Trending* is delivered via iPads, tablets, mobiles and desktop computers. It is dedicated to keeping Londoners up to date on all the trends and happenings around the city.

The stories in this work are fictional. All characters and events appearing in this work are fictitious. Any resemblance to real persons, living or dead, is purely coincidental.

London Trending: A module on similarity
Developed jointly by SRI International, Center for Technology in Learning (United States) & London Knowledge Lab, Institute of Education, University of London (UK)

U.S. contributors: Ken Rafanan, Teresa Lara-Meloy, Jennifer Knudsen, Anna Werner,
Phil Vahey, Jeremy Roschelle, Natasha Arora, George Roy, Vivian Fueyo
UK contributors: Bola Abiloye, Eileen Coan, Celia Hoyles, Phillip Kent, Richard Noss,
Teresa Smart, Alison Clark-Wilson

These materials are provided through a grant from the Li Ka Shing Foundation and Hutchison Whampoa Limited. These materials are based upon work developed by SRI International in the SunBay Digital Mathematics project. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the the Li Ka Shing Foundation or Hutchison Whampoa Limited.

Original works: Cover photos: Ken Rafanan
Office building: Andrew Middleton (http://www.flickr.com/photos/amid/)
Admiralty Arch: J Donohoe (http://www.flickr.com/people/jboyes/)
Trafalgar Square fountain: Olivier Bruchez (http://www.flickr.com/people/bruchez/)
Trafalgar Square pool: Phil Ostroff (http://www.flickr.com/people/guvnah/)
Tower Bridge: Kim Rickwood (http://www.flickr.com/people/kim_rickwood/)
Winston Churchill: Yousuf Karsh / Library and Archives Canada / PA-165806
David Beckham: U.S. Marine Corps (http://www.flickr.com/people/marine_corps/)
Laura Robson and Andy Murray (http://commons.wikimedia.org/wiki/User:Snowmanradio)
Usain Bolt: Brunel University (http://www.flickr.com/people/bruneluniversity/)
Jessica Ennis: AdamKR (http://www.flickr.com/people/adamkr/)
Elizabeth Armitstead: Peter Borcherds (http://www.flickr.com/people/palizbay/)
All others by Ken Rafanan

Preface to the Teacher's Edition

I. Unit Overview

This unit focuses on geometric similarity. Pupils work with dynamic geometry files, manipulating shapes to deduce changeable and unchangeable properties of similar shapes. The activities start with non-geometric drawings to support informal notions of similarity to geometric shapes. This progression is based on research that shows this is a productive way to introduce similarity.

Investigation	Key Mathematical Ideas	Key Technology Experiences
Introduction Welcome to the Graphics Department (15 minutes)	→ Context of the unit: *London Trending* is a digital magazine that is available on many devices. These devices all have different display sizes, hence the need to decide whether a variety of graphics are mathematically similar to each other.	
Investigation 1 Mathematical Similarity (70 minutes)	→ Understanding mathematical similarity helps us communicate with others about key features of things in the world, such as photographs. → The informal definition of mathematical similarity is exactly the same shape, not necessarily the same size. → *Shape* characterises one type of figure (e.g., parallelogram) as opposed to parallelogram versus triangle.	Translate, rotate, and enlarge shapes, show/hide gridlines and shapes.
Investigation 2 On the Grid (65 minutes)	→ Context: The Graphics Department needs to put the same image of the London Eye on several devices (e.g., tablet, computer, phone). → The heights and the widths of mathematically similar rectangles are related by a common multiplier.	Play/pause animation, rotate and translate shapes, and use grid as measurement tool.

Investigation	Key Mathematical Ideas	Key Technology Experiences
Investigation 3 Scale Factor (100 minutes)	→ Context: The Graphics Department is using a software program that makes enlargements by using scale factor. → Scale factor is the multiplier by which the lengths in the original shape result in the enlargement. → Scale factors greater than 1 result in copies larger than the original; scale factors less than 1 (but greater than 0) result in copies smaller than the original. → Congruence is a special case of similarity, with a scale factor of 1.	Measure sides, colour sides, use scale factor slider, use measurement table, and use snapshot.
Investigation 4 Broken Scale Factor (60 minutes)	→ Context: One of our artists, Eileen, found free software without a scale factor slider but with two strange other sliders. She says that the software can still be used to create mathematically similar copies. → Scaling a shape so that it creates a mathematically similar copy requires that all lengths of the shape be scaled by the same number.	Use sliders and measure sides.
Investigation 5 More Than Lengths of Sides (50 minutes)	→ Context: *London Trending* have a new advertising client whose logo is embedded in a parallelogram. → Equal corresponding angles are necessary for similarity.	Rotate and translate shapes, measure angles, and use angle slider.

Investigation	Key Mathematical Ideas	Key Technology Experiences
Investigation 6 Ratios (65 minutes)	→ Context: The Graphics Department received directions to make pairs of images in the ratio of 3 to 1. → A ratio shows the multiplicative relationship between two numbers or quantities. → Ratios can be simplified in the same way as fractions. → Two ratios that simplify to the same unitary fraction are equivalent. → *Between* ratios show the relationship between corresponding sides in similar shapes.	Label vertices, colour sides, measure side lengths, and possibly use ratio checker.
Investigation 7 Between Ratios and Within Ratios (95 minutes)	→ Context: Investigating an animation comparable to the animation starter from Investigation 2: On the Grid. → *Within* ratios and *between* ratios are two different ways to compare sides in similar rectangles. → *Within* ratios compare sides of the same shape. → *Within* ratios are unchanging across a family of similar shapes (i.e., the height:width ratio in a family of rectangles).	Translate and rotate shapes, play/pause animation, use ratio checker, and measure side lengths.
Investigation 8 What Changes and What Stays the Same? (60 minutes)	→ For a set of similar shapes, the shape and corresponding angles are unchanging. → For two similar shapes, the ratios of corresponding sides, the scale factor and the ratios of lengths within a shape are unchanging. → For three or more similar shapes, the ratio of lengths within a shape is unchanging, and the scale factor and ratios of corresponding sides vary together.	Translate vertices, translate and rotate shapes, measure sides and angles.
Investigation 9 Build Your Own (40 minutes)	→ Context: *London Trending* needs a new front page. → Using similar shapes can help pupils make a more aesthetically pleasing cover.	Translate and enlarge shapes.

Investigation	Key Mathematical Ideas	Key Technology Experiences
Investigation 10 Supplementary Activity: So the Angles in Triangles are Important? (55 minutes)	→ Context: The graphics department are keen to know if there is a quicker way using their angle measuring software to check whether triangles are similar or not. → When the three angles in any triangles are the same, then the triangles must be similar. .	Measure angles and sides, use measurement table and snapshot.
Investigation 11 Supplementary Activity: Positioning Images Precisely (55 minutes)	→ Pupils learn about centre of enlargement. → Context: In positioning images on a page, using a coordinate system enables the exact position of the copy to be predicted. → When shapes are enlarged about the origin, there is a multiplicative relationship between the coordinates of the original and the corresponding coordinates of the enlargement. → When shapes are enlarged about a centre that is not the origin, there is a 'two stage' rule that connects the coordinates of the original with the corresponding coordinates of the enlargement.	Play/pause animation, use scale factor slider, and use show/hide.
Investigation 12 Supplementary Activity: Within Ratios in Right-Angled Triangles (61 minutes)	→ This investigation introduces pupils to the foundations of trigonometry. → Within ratios in right-angled triangles are commonly used in mathematics within problem solving. → The most common within ratios are called trigonometric or 'trig' ratios and are used to calculate lengths of missing sides or angles.	Use angle sliders, and measure angles and sides.

Mathematics Goals

Pupils can do the following:

- Identify the invariants and variants in shapes that are mathematically similar.
- Recognise that pairs of mathematically similar polygons have a one-to-one geometric correspondence of sides and vertices such that
  - The corresponding angles are equal.
  - The within-shape ratios between corresponding side lengths are equal.
  - The between-shape ratios between corresponding side lengths are equal.
  - The sides of one polygon can be transformed into the other using a multiplicative scale factor.

Pupils demonstrate this understanding in the following ways:

- Across two or more similar polygons, they are able to
  - Identify corresponding sides and corresponding angles.
  - Calculate within-shape ratios and between-shape ratios.
  - Use ratios within-shape and between-shape to find a missing side.
  - Calculate the enlargement scale factor.
  - Determine and explain their reasoning about whether or not the polygons are congruent.
- They are able to reason about similarity of two or more polygons using information about sides and angles, such that they can
  - Determine whether or not two or more polygons are similar.
  - Recognise that adding a constant amount to all the sides of a polygon will not always result in a similar polygon.
  - Explain their reasoning about whether or not two or more polygons are similar.
  - Sketch a polygon that is similar to one that is given.
  - Recognise a definition of similar polygons.

Pupils understand that a ratio relates two numbers multiplicatively and can work with ratios, including simplifying them.

II. National Curriculum Addressed

Key Processes

At Key Stage 3, through the mathematics content pupils should be taught to:

Develop fluency

- apply appropriate calculation strategies and degrees of accuracy to extend their understanding of the number system to include fractions and decimals.
- move fluently between different representations using language and properties precisely with 2D shapes.

Reason mathematically

- extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically
- establish when to use additive, multiplicative or proportional reasoning from the underlying structure of a problem when working numerically
- begin to reason deductively in geometry
- develop reasoning in different areas of mathematics (geometric similarity) and begin to express their arguments formally.

Solve problems

- develop their mathematical knowledge, in part through solving problems and evaluating the outcomes
- develop their use of formal mathematical knowledge to solve and devise problems
- begin to model situations mathematically and express the results using a range of formal mathematical representations
- apply elementary knowledge to multi-step and increasingly sophisticated problems

Subject content

Number

Pupils should be taught to:

- understand the relation between operations and their inverses and identify the inverse of a given operation where this exists
- use a calculator and other technologies to calculate results accurately and then interpret them appropriately
- round numbers and measures to an appropriate degree of accuracy

Ratio, proportion and rates of change

Pupils should be taught to:

- use ratio and scale factor notation and methods involving scaling.
- calculate missing quantities and totals using given ratios, including reduction to simplest form
- use multiplicative reasoning where two quantities have a fixed ratio including graphical representations

Geometry and measures

Pupils should be taught to:

- use concrete and digital instruments to measure line segments and angles in geometric figures, including interpreting scale drawings
- derive and illustrate properties of triangles, quadrilaterals and other plane figures (e.g. equal lengths and angles) using appropriate language and technologies
- identify and construct congruent triangles, and construct similar shapes by enlargement, including on coordinate grids
- solve problems involving spatial properties on coordinate grids
- apply angle facts, triangle congruence, similarity and properties of named quadrilaterals to conjecture and derive results about angles and sides, using transformational, axiomatic and property-based deductive reasoning
- use side ratios in similar triangles to solve problems in right-angled triangles
- interpret mathematical relationships both algebraically and geometrically.

III. Implementation Suggestions

Curriculum opportunities

The unit provides opportunities for pupils to (1) develop confidence in an increasing range of methods and techniques; (2) work on sequences of tasks that involve using the same mathematics in increasingly difficult or unfamiliar contexts or increasingly demanding mathematics in similar contexts; (3) work on open and closed tasks in a variety of real and abstract contexts that allow them to select the mathematics to use; (4) work on tasks that bring together different aspects of concepts, processes and mathematical content; and (5) work collaboratively as well as independently in a range of contexts.

Materials

- Pupil workbook
- Pencil and rough paper
- Rulers and calculators
- Computers for pupils
- Whole class display: Interactive whiteboard, computer with projector, or document camera

Computer Use

There are computer-based activities associated with each of the investigations in this unit. Pupils should use the software as often as possible.

Whole class discussions	1 whole class display visible to all
Group work	1 computer for every 3 pupils
Homework	dependent on activity

Classroom Organisation

Many activities begin with whole class discussion, then pupils use what they have learnt in a group work activity. These are our assumptions for recommending whole class, group work, and independent work.

1. Whole class discussion, teacher leads.
    - Whole class display is used.
    - Plenaries are whole class discussions.
    - Everyone can hear everything everyone says.
    - Pupils take notes in their workbooks.
2. Group work, teacher circulates.
    - Pupils work in pairs or trios, seated so each team member can see the shared computer screen and write in his/her own workbook.
    - Pupils should work together to come up with common solutions.
    - Each pupil should complete his/her own workbook.
3. Homework or independent work.
    - Pupils can work alone or in groups.
    - Focused on practice of content in activity or remembering old material relevant for the next day's activity.

General Teaching Tips

- Encourage explanations of correct and incorrect answers.
- Important instructional routine in the unit: Predict. Check. Explain.
  - Pupils predict –have a go– what will happen in a simulation or what will happen as a result of a simulation.
  - Pupils use software or calculations to check their predictions.
  - Pupils explain any differences between what they predicted and what happened.
- Let pupils do more talking than you do. During whole class mode, let them answer questions and challenge the answers others give. The think-pair-share routine is suggested, and you can use your own strategies for engaging pupils in the plenary.
- Balance whole class work with individual and group work. Pupils need each.
- Allocate additional time for pupils to become familiar with the software during the first lesson.
- Specify the time for whole class discussion so pupils know when to stop using the computers.

Overcoming Possible Pupil Misconceptions

These are all wonderful opportunities for teaching.

- Giving explanations. Pupils may need help in giving explanations. One issue is knowing the difference between describing a procedure and explaining why it works. You may want to model some explanations for pupils in the beginning.
- Additive misconception. Many pupils start out by thinking that adding the same length to each side of a rectangle will result in a similar rectangle. This is widespread and developmental. You can help by showing counterexamples, where adding clearly makes a warped shape. The animation activities and broken scale factor activity are opportunities to do this.
- Ratio. Pupils may not have applied ratio to measurements using decimals before. This concept requires coordination of two quantities instead of the single quantities or two discrete they may be accustomed to from primary school. You will need to help pupils make the transition to this higher level of conceptual understanding.
- Shape and similarity. Pupils may begin by thinking that "same shape" means that both shapes are rectangles, for example, rather than that they are similar in shape. Pupils may also think that "similar" means somewhat alike, not understanding its mathematical meaning. We use "mathematically similar" in the module to help with this, but you may still need to help pupils sort out this vocubulary.
- Angles and sides. When dealing with shapes other than rectangles, pupils must consider the lengths of sides AND size of angles to determine similarity. Because the unit begins with rectangles, they may need help transitioning to considering angles, too.

Contents

Introduction: Welcome to the Graphics Department

Key Ideas

→ Context of the unit: *London Trending* is a digital magazine that is available on many devices. These devices all have different display sizes, hence the need to decide whether a variety of graphics are mathematically similar to each other.

Starter — Whole Class | 10 minutes

- Present the slideshow (LondonTrendingIntro.pptx) to the class to provide the context for the unit.
- Discuss briefly with the class how the mathematical concepts and applications are important for many professions.

Main Activity — Whole Class | 5 minutes

- Read and discuss the context of the unit: creating mathematically similar graphics at *London Trending* magazine.

Pupil Difficulties

Pupils may have difficulty...

- describing the changes they make to the shapes for the magazine cover.

Pupils may have misconceptions...

- thinking that because two shapes are, say, rectangles, they are similar.

Introduction: Welcome to the Graphics Department

→ *Key Learning:* By the end of this activity, you should be able to explain why a graphic artist must be able to use mathematics to design a magazine cover.

Welcome to the Graphics Department

Welcome to *London Trending* digital magazine. Every day, thousands of Londoners download our pages to their mobiles, tablets, and desktop computers. The Graphics Department is responsible for making sure that the graphics and photographs look right on all the different devices.

We need staff who work well with others and can do the mathematics necessary to ensure that the graphics are perfect.

Investigation 1: Mathematical Similarity

Key Ideas

→ Understanding mathematical similarity helps us communicate with others about key features of things in the world, such as photographs.

→ The informal definition of mathematical similarity is exactly the same shape, not necessarily the same size.

→ *Shape* characterises one type of figure (e.g., parallelogram) as opposed to parallelogram versus triangle.

Starter — Group | 15 minutes

- Display Activity 1.1 on the interactive whiteboard. Give the class the context for this activity: The *London Trending* editor is planning a new front page for the magazine. The editor likes the layout for the portrait orientation. She needs a layout for a landscape orientation. She does not want a lot of white space (space on the cover layout not occupied by a picture).
- Organise pupils to work in groups to create a layout with the shapes so that they fill as much of the cover page (the black rectangle) as possible. Pupils will have to enlarge shapes and some white space may be left over, but the goal is to make something aesthetically pleasing.
- Allow groups to explore the functionality of the software. Then check that all groups are able to use the translate and enlarge controls.
- This activity is meant to be fairly difficult and provides some motivation for the learning in the following investigations.
- Ask pupils to share what they noticed in the activity. Encourage early hypotheses but do not formalise them at this point.

Main Activity — 45 minutes total

Big Ben — Group | 20 minutes

- Pupils should have a chance to use the software on their own. If necessary, demonstrate how to rotate an object. Circulate to ensure pupils are on the right track.
- As they fill out the table, some pupils may focus on the whole image (fatter, skinnier Big Ben). Some may focus on a piece of the figure (the clock face circle being more squished in the “not good” copy).
- In Activity 1.2, a line segment (actually, a very narrow rectangle) is provided. This line segment may be used to demonstrate properties of mathematically similar shapes and images.

Vocabulary — Whole Class | 5 minutes

- Read out loud with whole class. Check for understanding.

Question 3 — Group | 5 minutes

- Let pupils struggle to answer Question 3 before discussing it during Plenary.
- Highlight to pupils that a copy that is exactly the same size and shape as the original is also a good copy. This will lead to the idea of congruence later in the unit.

Check Your Understanding — Individual | 15 minutes

- For the Card Sort, pupils may cut the cards out of their workbooks. Each pair of pupils should have a shuffled set of cards. Ask the pupil pairs to determine which card has the original image and label that card as “original”. The pair should then place that card between them and sort the cards into two groups, those that look like the original and those that do not. Ask the pupils to name the qualities of each group, for example, what features make the cards that look like the original do so?
- Here, pupils apply their developing notion of similarity to photographs. Only qualitative properties need be cited.
- Some pupils/classes will be ready for more quantitative explanations, for example, heights or widths are equal or the original has a greater width than height.

Plenary — Whole Class | 10 minutes

- Draw together to discuss Questions 1 to 3 to start to generate understanding of similarity. Include discussion about the circle staying perfectly circular and how shapes are distorted or warped. More formal properties will be developed later.
- Discuss the difference between *mathematically similar* and the everyday use of the term *similar*.

Pupil Difficulties

Pupils may have difficulty...

- describing mathematically the changes they make to the shapes for the magazine cover.
- using the language of explanation.

Pupils may have misconceptions...

- thinking that because two shapes are, say, rectangles, they are similar.

Investigation 1: Mathematical Similarity

→ *Key Learning:* By the end of this activity, you should be able to decide whether a copy is a good likeness of the original.

Starter

Cover Layout

We are planning a new cover for the *London Trending* magazine. Our editor likes the draft shown below because she thinks the differently shaped pictures go together nicely. Because so many of our magazine's readers use tablet computers, she wants to make sure that we have a layout that works for landscape orientation as well.

1. Open Activity 1.1. Now, using Activity 1.1, create a new layout with the shapes so that you fill as much of the new cover page as possible. You may change each shape as much as you wish so long as it does not look stretched or warped.

2. Sketch your version of the layout below.
   In your sketch note the lengths, ratios, and angles you are using for each shape and explain how to change each part you want changed.

Multiple good solutions are possible. Use this activity as a formative assessment to inform you as to your pupils' understanding of mathematical similarity.

Main Activity

Big Ben

There is disagreement in the Graphics Department. We have many images of Big Ben. Which are good copies of the original? Which are not good? What do you think “good” means?

1. Open Activity 1.2. There is one original image of Big Ben (in blue) and several copies (in orange).
   - A good copy is exactly the same shape but can be a different size.
   - A bad copy is not exactly the same shape as the original.

2. In the table below, list the bad copies and **explain** how each is bad. You can translate and rotate the shapes if it helps. You can show and hide the gridlines as well. We have provided a model explanation for one of the bad copies.

Bad Copy	Explanation
2	It is the same height, but it is wider than the original.
3	It is the same width but shorter than the original.
4	It is twice as tall, but it is not twice as wide. Note that some pupils may begin to quantify here, which is a good strategy to build on in the next activities.
7	It is leaning over (distorted through a shear) and not exactly the same shape as the original.

Point out the grid in the Cornerstone Maths software to support pupils in beginning to quantify their observations. Emphasise the differences, including the sheared version that is “leaning”.

Vocabulary

Good copies are *mathematically similar* to the original. Mathematically similar shapes are exactly the same in shape as the original but can be different sizes. Mathematically similar copies are also called *enlargements*. An enlargement is a copy that is larger (or smaller) than the original but maintains exactly the same shape as the original.

3. How are *enlargements* of Big Ben the same as the original?

The copies all look the same as the original. They are not warped. Copies with doubled width also have doubled height and vice versa. The clock face is always in the shape of a circle. One of the copies is identical to the original in size and shape (Copy 5).

Introduce the term *congruence* when discussing Copy 5.

Check Your Understanding

4. Sorting Winston Churchill Cards
   Decide which card has the original image, label that card as “original”, and place that card on the table. Sort the cards into two sets: those that look the same as the original and those that do not. **Explain** to your partner how you sorted the cards into sets.

1

2

3

4

5

6

7

8

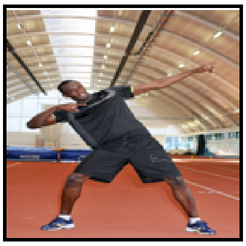
5. “Usain Bolt Visits London” is our headline in the magazine. We have these images to choose from:

Original

A

B

C



A. Which copies (if any) are *not* mathematically similar to the original, and how do you know?

Copies A and C are not mathematically similar to the original. Copy A is stretched out compared with the original. Copy C is more like a square than the original.

B. Which copies (if any) are enlargements of (mathematically similar to) the original, and how do you know?

Answers may vary. The similar shapes are the same because Usain Bolt looks the same; his body is in the right proportion. Help pupils see that everything looks the same between the original and Copy B. However, they are not the same size. Copies A and C are warped.

You may encourage pupils to use more mathematical language in their answers. For example, Copy C has a height and width that are equal, but the original has greater height than width.

6. “British Tennis Stars” is our headline in the magazine. We have these images to choose from:

Original

D

F

E

A. Which copies (if any) are *not* mathematically similar to the original, and how do you know?

Copies F and E are not mathematically similar to the original. Copy F is more like a square than the original; copy E is stretched out compared with the original.

B. Which copies (if any) are mathematically similar to the original? How do you know?

Copy D is mathematically similar to the original because it is not warped or stretched. In fact, it is the same size and the same shape as the original; it is congruent.

This is an opportunity to note the special case of congruence.

Investigation 2: On the Grid

Key Ideas

→ Context: The Graphics Department needs to put the same image of the London Eye on several devices (e.g., tablet, computer, phone).
→ The heights and the widths of mathematically similar rectangles are related by a common multiplier.

Starter — Whole Class | 15 minutes

- An animation, the starter is intended to be short and engaging. (A more complex animation challenge occurs later in the unit.)
- Ask pupils to explain their reasoning as they state which parallelogram is always mathematically similar.
- Qualitative responses are sufficient.

Main Activity — 40 minutes total

Question 1 — Whole Class | 10 minutes

- Pupils predict, on the basis of visual cues, which copies are similar to the original.
- Pupils begin to quantify mathematical similarity using a grid.

Question 2 — Group | 10 minutes

- Here, the quantification of patterns lays the foundation for ratios.
- Ensure that pupils are filling in the table appropriately.
- Some pupils may confuse height and width, which does not necessarily affect the relationship between the shapes, but this approach might be confusing in discussion with the whole class. If this happens, you can briefly mention the role of conventions in mathematics: “We all agree that height is the vertical measure so that we know where to look when someone mentions height”.

Questions 3 - 5 — Group | 20 minutes

- Have pupils work through Questions 3 to 5 in teams; you will discuss their answers during the Plenary.
- Pupils may uncover relationships within the similar shapes: that the height is always ¾ of the width in contrast to the height-to-width ratio in the non-similar shape. This is a good insight that will be used later in categorising two kinds of ratios, between and within.

Plenary

Whole Class | 10 minutes

- Review the responses to Questions 3 to 5.
- Elicit visual and numeric evidence for pupils' responses.
- Restate the key idea: One number can be used to multiply the height and width of the original to produce a mathematically similar copy (enlargement). This means that the height and width of mathematically similar shapes grow and shrink (*in the same way* or *by the same multiplier*). If the width of a rectangle grows and its height does not, then they are not growing in the same way.

Pupil Difficulties

Pupils may have difficulty …

- relating their informal understanding of similarity to the numerically based reasoning in this activity.

Pupils may have misconceptions …

- thinking that the difference or sum of the height and width are relevant for comparing similar shapes.

Minor Rounding Errors

In the software, many measurements have been rounded. Occasionally, rounding errors will occur.

Investigation 2: On the Grid

→ *Key Learning:* By the end of this activity, you should be able to use numbers to determine whether an image is an enlargement of an original.

Starter

We have another disagreement in the Graphics Department. Our staff made four animated copies of the blue rectangle, but they cannot agree on which is always an enlargement (mathematically similar copy) of the original. Use the Cornerstone Maths software to help us decide.

1. Open Activity 2.1 and watch the animation.

2. Which copies (if any) are always mathematically similar to the original blue rectangle? **Explain** your answer.
   You can pause the animation and rotate and translate the shapes to investigate.

> The orange and red rectangles are mathematically similar to the original. The pink rectangle is a non-rectangular parallelogram, and the green rectangle increases only in width.
> A more challenging animation is toward the end of the unit.
> Use this question to convey the concept that the height and width must change at the same rate, using informal language (e.g., they double).

Main Activity

The Graphics Department needs to put the same image of the London Eye on several devices (e.g., tablet, computer, mobile). The staff want to know whether the various copies look good. In addition to judging how they look by eye, we can use measurements to decide which shapes are mathematically similar.

1. Open Activity 2.2.
   Look at the four London Eye images. First, without counting or measuring, determine which of the copies are enlargements of the original. How do you know?

> Copies 1 and 2 are similar to the original because they do not look warped—the circle stays a circle. Now that the pupils have established qualitative mathematical similarity via the circles, they are ready to quantify it, as below.

2. If you have resized any of the images, refresh or reopen Activity 2.2. Now count and record the height and width of the London Eye images in the table below. Tick the rows of the copies that are enlargements.

	Height (in grid squares)	Width (in grid squares)	Tick if an enlargement
Original	6	8	
Copy 1	12	16	enlargement
Copy 2	3	4	enlargement
Copy 3	6	4	

3. Use the information in the table above to compare the relationship between the height and width of the original shape with those of each copy to uncover an important relationship for all similar shapes.

A. i. Fill in the missing multiplier that relates the original and Copy 1.

		Height	Width	
Original	× 2			× ___
Copy 1				

The height and width of the original and Copy 1 should be copied from the table in question 2.
The correct multiplier for width is 2.

ii. Write the relationship (in words) between the height of the original and the height of Copy 1.

The height of the original mulitplied by 2 gives the height of the copy.
Or a mathematically equivalent statement such as the height of the copy is 2 times that of the original.

iii. Write the relationship (in words) between the width of the original and the width of Copy 1.

The width of the original mulitplied by 2 gives the width of the copy (and other equivalent statements).

B. i. Fill in the missing multiplier that relates the original and Copy 2.

		Height	Width	
Original	× ____			× ____
Copy 2				

The height and width of the original and Copy 2 should be copied from the table in question 2. The correct multiplier for both height and width is 1/2.

ii. Write the relationship (in words) between the height of the original and the height of Copy 2.

The height of the original mulitplied by 1/2 gives the height of the copy (and other equivalent statements).

iii. Write the relationship (in words) between the width of the original and the width of Copy 2.

The width of the original mulitplied by 1/2 gives the width of the copy (and other equivalent statements).

C. i. Fill in the missing multiplier that relates the original and Copy 3.

		Height	Width	
Original	× ___			× ___
Copy 3				

The height and width of the original and Copy 3 should be copied from the table in question 2. The correct multiplier for height is 1; the correct multipler for width is 1/2.

ii. Write the relationship (in words) between the height of the original and the height of Copy 3.

The height of the original mulitplied by 1 gives the height of the copy (and other equivalent statements).

iii. Write the relationship (in words) between the width of the original and the width of Copy 3.

The width of the original mulitplied by 1/2 gives the width of the copy (and other equivalent statements).

4. On the basis of these observations, what is the relationship between an original and a mathematically similar rectangle?
HINT: Use the words height and width in your answer.

Both the height and width of an orginal are multiplied by the same multiplier to give the height and width of a mathematically similar copy. Expect pupils to use less formal language such as. "They are both half the length of… or both twice the length of…."
Height and width in the original have to be multiplied by the same number to produce the mathematically similar copy. Multiplying the height by one number and the width by a different number will produce a non-similar copy.

5. Why would the relationship hold true for all mathematically similar shapes?

Because when you increase the length and width in relation to their original size (like half again as long), it makes the shape appear not warped, which means mathematically similar. Pupils may also use the inverse logic: When the length and width are increased differently, the resulting image is warped.

Investigation 3: Scale Factor

Key Ideas

→ Context: The Graphics Department is using a software program that makes enlargements by using scale factor.

→ Scale factor is the multiplier by which the lengths in the original shape result in the enlargement.

→ Scale factors greater than 1 result in copies larger than the original; scale factors less than 1 (but greater than 0) result in copies smaller than the original.

→ Congruence is a special case of similarity, with a scale factor of 1.

Main Activity — 90 minutes total

Question 1 — Group | 10 minutes

- This question is meant to get pupils to form an intuitive sense of how scale factor works. Quantification follows.

Questions 2 and 3 — Group | 25 minutes

- Each group of pupils will create one similar pair of London Eye graphics with scales that are greater than 1, so that, as a whole class, you will have many cases to analyse and use in finding the relationship between scale factor and mathematically similar shapes.
- Demonstrate how to select, measure and colour corresponding sides. Each side should be a different colour. Let pupils choose the colours.
- Support pupils as they start quantifying the relationship between scale factor and similar shapes.
- Walk around and assess whether pupils are colouring the corresponding sides correctly.
- Help the pupils see that the multiplier between corresponding sides is the same as the scale factor.
- Lead a mini-plenary to answer Question 3. Show the software and run through several scale factors you gather from pupils. Elicit visual and numeric evidence for pupils' responses.
- As a supplementary question, you may wish to have pupils compare the diameter of the circle in the copy to the circle in the original. Pupils may need to translate and scale the original so the the circle can be better measured. They should make the connection that effect of the scale factor on the diameter of the copy's circle is the same as the effect on its height and width.

Questions 4 - 12 **Group | 15 minutes**

- Before pupils start work on these questions, demonstrate how to use the software to help in their work.
  - Demonstrate how to drag shapes on top of one another to help identify corresponding sides.
  - Demonstrate how to use the measurement table in the software by dragging side measurements into the table's cells.
  - Demonstrate how to take a snapshot of measurements in the measurement table.
- Pupils investigate what happens when the scale factor is 1. Help them see that the ratios between corresponding sides are the same as the scale factor.
- Introduce the term *congruent* when you have a discussion in the plenary.
- Pupils calculate the scale factor (of less than 1) given the lengths of the original and copies.
- A common error is for pupils to do the setup ratios correctly but not in the correct order to see the relationship between the original and the copy, thus deriving the inverse scale factor (i.e., a scale factor of 2 instead of ½).
- Help pupils notice that when the scale factor is less than 1 the copy is smaller than the original, and when it is greater than 1 the copy is larger than the original. Help them see that the ratios between corresponding sides are the same as the scale factor.

Questions 13 and 14 — Group | 20 minutes

- Support pupils as they summarise what they have learnt and make explicit the multiplicative relationship connection between scale factor and ratios of corresponding sides.
- Note that $y = kx$, where k is the scale factor, x is the original length and y is the scaled length in the enlargement. You need not introduce this notation but simply the concept that the scale factor is the multiplier by which the lengths of the original shape result in the lengths of the enlargement.

Vocabulary — Whole Class | 5 minutes

- Read the vocabulary section and discuss congruence as a special case of similarity, with its own set of properties.

Check Your Understanding — Individual | 15 minutes

- Pupils apply what they have learnt. Help pupils use the grid to find the lengths of two corresponding sides.

Plenary — Whole Class | 10 minutes

- Use the many pairs of the London Eye that pupils created in your discussion.
- Gather patterns and mathematically true observations (or raise them if pupils have not noticed them).
- AB * scale factor = A'B'
- Scale factor = ratio of corresponding sides (between two shapes).
- Congruent shapes have a scale factor of 1.
- Ask pupils to summarise what they know about the following: scale factor, ratio of lengths of corresponding sides, enlargement, mathematically similar.
- Ask pupils why congruent shapes are similar.

Pupil Difficulties

Pupils may have difficulty …

- associating between ratios and scale factor.
- feeling free to make predictions that might be wrong.

Pupils may have misconceptions …

- assuming that similar shapes cannot be congruent.
- setting up the ratio between original and copy incorrectly and deriving inverse scale factors.

Investigation 3: Scale Factor

→ *Key Learning:* By the end of this activity, you will be able to explain how to use a *scale factor* (common multiplier) in creating an enlargement.

The Graphics Department is using a software program that makes enlargements (mathematically similar copies) by using scale factor. Our staff are unclear about how the software works. You can help us by investigating how the scale factor works.

Open Activity 3.1. In this activity, the copy of the London Eye is always mathematically similar to the original and related to the original by a scale factor.

1. Move the scale factor slider.

   A. List several things that stay the same as you move the scale factor slider.

   The look of the shapes stays the same. The angles stay the same. The orientations of the shapes stay the same. The pupils might see that the heights are always ¾ the widths (or something less formal).

   B. List several things that change as you move the scale factor slider

   As the scale factor increases, the copy becomes larger. If the scale factor decreases, the copy becomes smaller. The pupils might see that the ratio from the side in the original to the copy is changing (the informal version of across ratios).

2. Use the scale factor slider to make a large copy that still fits on the screen.

   A. Write the scale factor that you used.

   Answers will vary, but the scale factor must be greater than 1.

   B. Compare the height and width of your large copy with the original. What was the effect of the scale factor on the copy's height and width (use the rectangular border).

   Answers will vary, but the pupil should describe the scale factor (which must be greater than 1) as being the same as the multiplier that is applied to the original that gives the copy's new measurements.

3. Our designer Fatima says that the software works perfectly to create enlargements. Aiden says that the software does not create good copies every time. Who is right and why?

Pupils should see that the scale factor is the multiplier used on the original to obtain to the copy. Pupils may reason (visually and numerically) from multiple cases and should be encouraged to use their intuition about good and warped copies to reason more generally.

4. Open Activity 3.2. The copy (red triangle) is always mathematically similar to the original (blue triangle). Measure and colour the matching sides of the original and the copy. For example, AC in the original matches with A'C' in the copy (both sides are the second longest side of their triangle). You may wish to rotate the triangles to put them in the same orientation.

Corresponding Sides

In mathematics, we use the term corresponding for things that match in particular mathematical ways. Matching sides in mathematically similar shapes are called corresponding sides.

A. Identify the corresponding sides for the two triangles and record them below.

__AC_ corresponds to _A'C'__
_____ corresponds to _____
_____ corresponds to _____

5. Drag the measurements for both the copy and the original into the table. Make sure that your table is organized taking corresponding sides into account. Take a snapshot of the measurements.

6. Set the scale factor slider to 3. Take a snapshot of the measurements. What is the relationship between the measurements of corresponding sides in the copy and the original?

Answers may vary. A multiplier of 3 is used to get the lengths of the copy's sides from the original's corresponding sides (or some mathematically equivalent statement).

7. Set the scale factor slider to 2. Take a snapshot of the measurements.
   What is the relationship between the measurements of corresponding sides in the copy and the original?

   Answers may vary. A multiplier of 2 is used to get the lengths of the copy's sides from the original's corresponding sides (or some mathematically equivalent statement).

8. **Predict**: If you set the scale factor to 1, what will the lengths of the corresponding sides in the copy be?

	A'B'	B'C'	A'C'
Copy	4	6	5

Answers will vary since these are predictions.

   A. **Check**: Set the scale factor to 1. Make sure that your predictions were correct.

   B. **Explain**: Explain the effect of a scale factor of 1.

   You multiply each original length by the scale factor of 1 to find the lengths in the enlargement. So, when the scale factor is 1, the lengths of the enlargement are the same as the original's.

9. **Predict**: If you set the scale factor to 0.5, what will the lengths of the corresponding sides in the copy be?

	A'B'	B'C'	A'C'
Copy	2	3	2.5

Answers will vary since these are predictions.

   A. **Check**: Set the scale factor to 0.5. Make sure that your predictions were correct.

   B. **Explain**: Explain the effect of a scale factor of 0.5.

   A scale factor of 0.5 results in corresponding lengths that are 0.5 times the length of the original.

10. Experiment with some more scale factors that are less than 1. Use the table below to record your measurements.

A.

	AB	BC	AC
Original	4	6	5

	A'B'	B'C'	A'C'	Scale Factor
Copy	2	3	2.5	0.5
Copy				
Copy				

B. When a copy is created using a scale factor of less than 1, what can you say about the relationship between the lengths of the original's and the copy's corresponding sides?

Pupils' responses should show that they understand that a scale factor of less than 1 results in a copy with corresponding side lengths that are smaller than the original's.

11. Describe what a scale factor is.

A scale factor describes the multiplicative relationship between a mathematically similar copy and an original. It is the number by which you can multiply the length of a side in the original to obtain the length of the corresponding side in the copy. When the scale factor is increased, the lengths of the sides of the enlargement increase. When the scale factor is decreased, the lengths of the sides of the enlargement decrease.

12. Describe how to use scale factor to find the lengths of sides in a mathematically similar copy when you know the lengths of the original.

You multiply each original length by the scale factor to find the lengths in the enlargement.

13. Sketch an original and three enlargements as detailed below.

A. Sketch a polygon that will be your original.	B. Sketch an enlargement that is related to your original by a scale factor greater than 1. When the scale factor is greater than 1, the lengths of the sides of the copy are greater than the original's.
C. Sketch an enlargement that is related to your original by a scale factor between 0 and 1. When the scale factor is less than 1, the lengths of the sides of the copy are less than the original's.	D. Sketch an enlargement that is related to your original by a scale factor that is exactly 1. When the scale factor is exactly 1, the lengths of the sides in the copy are equal to the original's.

14. Given an original shape and an enlargement, what can you say about the scale factor and the relationship between corresponding sides?

The scale factor has the same value as the multiplier between the length of a side of a mathematically similar copy and its corresponding side in the original.

Vocabulary

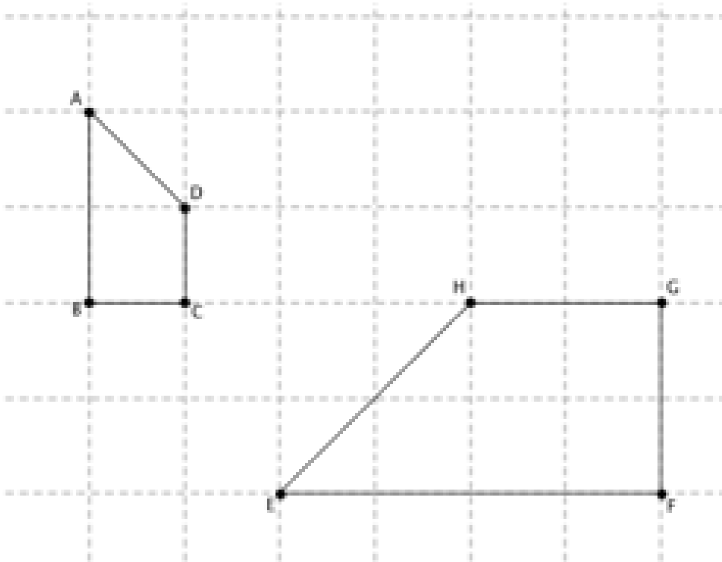
Congruent
When an original and a copy are exactly the same size and shape, they are called *congruent*. Congruent shapes are related by a scale factor of 1.

15. What is true about congruent shapes? Use what you learnt in Activity 3.2 (Question 8).

	This is true Sometimes / Always / Never
The lengths of corresponding sides in congruent shapes are equal.	Always
The scale factor between congruent shapes is 1.	Always
Congruent shapes are similar.	Always
Similar shapes are congruent.	Sometimes

Check Your Understanding

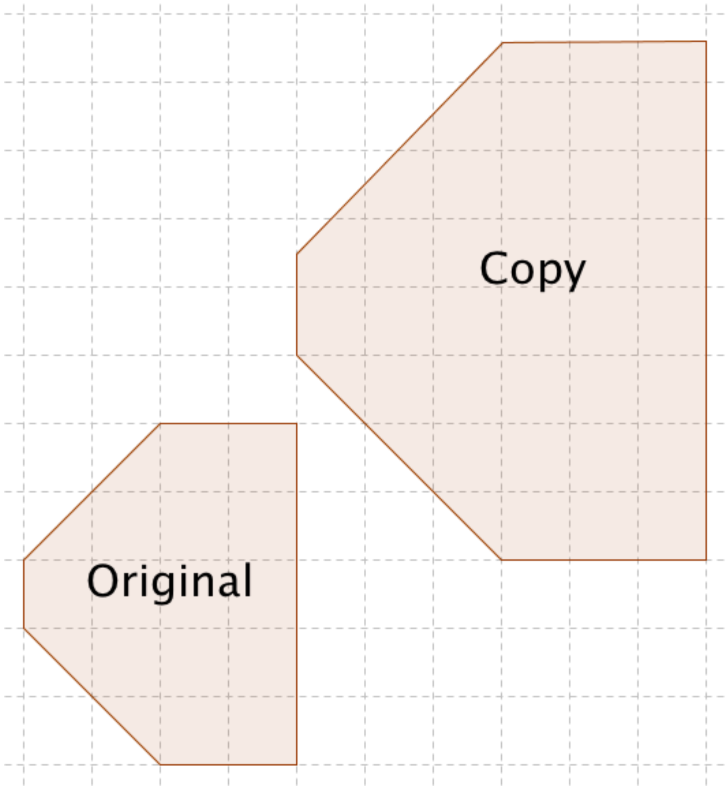
16. Figure EFGH is an enlargement of Figure ABCD and so is mathematically similar.
    Find the scale factor and explain your reasoning.



Scale factor = 2

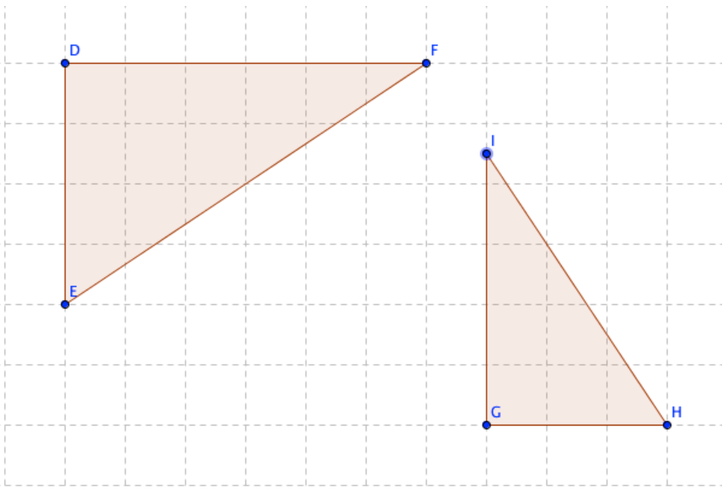
The scale factor is 2.
GH (with a length of 2) corresponds to DC (with a length of 1 unit). 2/1 = 2.
Also, FG (2) corresponds to BC (1). 2/1 = 2.

17. Find the scale factor between these two mathematically similar polygons:



Scale factor = 1.5

18. Triangle GHI (a copy) is mathematically similar to Triangle DEF (the original). Find the scale factor between these two triangles:



Scale factor = 0.75

Investigation 4: Broken Scale Factor

Key Ideas

→ Context: One of our artists, Eileen, found free software without a scale factor slider but with two strange other sliders. She says that the software can still be used to create mathematically similar copies.

→ Scaling a shape so that it creates a mathematically similar copy requires that all lengths of the shape be scaled by the same number.

Main Activity — 50 minutes total

Question 1 — Whole Class | 10 minutes

- Point out the lack of scale factor and the way the two sliders work.
- Pupils explore what happens when they can scale the length and width of a shape independently of each other.

Questions 2 - 4 — Individual | 10 minutes

- Pupils create a copy that is not an enlargement.
- Pupils are prompted to address the additive misconception by comparing the original and copy visually (the copy will be warped when pupils simply add the same amount to each length).
- You may wish to re-open Activity 4.1 and include a second example where pupils subtract the same number from each length.
- Pupils need to coordinate lengths to make a similar copy.
- Ask pupils to share their responses to Question 5 as part of a mini-plenary.

Questions 5 and 6 — Group | 20 minutes

- Make sure to connect scale factor, height and width and the common multiplier.
- You can have pupils, in pairs, challenge each other. Instead of using one of the side lengths given in the activity, one pupil makes up a length; the other must fill in the table for it. Then they can exchange roles

Question 7 — Group | 10 minutes

- In pairs, pupils write explanations of how to manipulate the height and width of a copy to make it mathematically similar to an original.

Plenary — Whole Class | 10 minutes

- Discuss how to make a mathematically similar copy using the length sliders. Make sure pupils recognise that the heights and widths of the original and copy must be related multiplicatively and bring in scale factor in relation to this idea.

Pupil Difficulties

Pupils may have misconceptions …

- thinking that adding the same amount to both sides of a rectangle will create a similar rectangle.

Investigation 4: Broken Scale Factor

→ *Key Learning:* By the end of this activity, you will understand why scale factor applies to a whole shape.

We have an interesting development in the Graphics Department. One of our artists, Eileen, found a piece of software without a scale factor slider but with two strange other sliders. She says she is able to use the software to create mathematically similar copies.

1. Open Activity 4.1. In this activity, the copy of the London Eye is *not* always mathematically similar to the original. There is no scale factor slider, but there are two other sliders. Play around with them and describe what they do.

   Slider 1 changes the width, and slider 2 changes the height.

2. Re-open Activity 4.1. Use the height slider to increase the copy's height by 12 units. Use the width slider to increase the copy's width by 12 units. **Explain** why the copy *is* or *is not* mathematically similar to the original.

   Pupils may describe visual distortions to the London Eye and numeric evidence: The ratio of the corresponding sides is not the same.

3. Use the two sliders to make a mathematically similar copy that is not the same size as the original. **Explain** how you know your copy is similar to the original. Write down the scale factor.

   Answers will vary.
   Pupils can use tables or scale factor (use of two multipliers that are the same) to explain their answers.

4. Describe what Eileen did to make a copy that was mathematically similar to but a different size than the original.

   The width and height have to be multiplied by the same scale factor.

5. Make another mathematically similar copy where the side length A'B' is 12 units.

   A. Find what the other lengths must be for the copy and the original to be mathematically similar.

   B. Fill in the information below.

Length of side AB (original)	Length of side AD (original)
4	3
Length of side A'B' (enlargement)	**Length of side A'D' (enlargement)**
12	9

   C. What is the scale factor that relates the copy to the original?

   Scale factor is 3. The lengths in the enlargement are three times as long as the lengths in the original.

6. Make an enlargement where the side length A'D' is 1.5 units.

   A. Find what the other lengths must be for the copy and the original to be mathematically similar.

   B. Fill in the information below.

Length of side AB (original)	Length of side AD (original)
4	3
Length of side A'B' (enlargement)	**Length of side A'D' (enlargement)**
2	1.5

Check that pupils are using the sliders correctly to set up common multipliers.

   C. What is the scale factor that relates the copy to the original?

The scale factor is 0.5. The lengths in the copy or enlargement are half the lengths in the original.

7. With your partner, devise a set of instructions to use Activity 4.1 so that anyone can create mathematically similar enlargements.

Multiple correct answers are possible.

- Manipulate the height and width of the copy so that the pairs of corresponding sides in the copy and the original are related by the same multiplier.
- Move the sliders until the enlargement has a real (not warped) circle in it like the one in the original.

Teacher Notes

Investigation 5: More Than Lengths of Sides

Key Ideas

→ Context: *London Trending* have a new advertising client whose logo is embedded in a parallelogram.

→ Equal corresponding angles are necessary for similarity.

Starter — Whole Class | 10 minutes

- Focus the pupils' attention on Copy 7, the copy of Big Ben that has angles different from those in the original. This step sets the stage for the lesson as pupils will begin to see that the measures of the angles in a shape are also critical in determining similarity.

Main Activity — 30 minutes total

Questions 1 - 3 — Group | 20 minutes

- Pupils focus on corresponding angles, noting that in mathematically similar shapes the corresponding angles are equal. The static parallelogram is used as a counterexample. Although it is mathematically similar at the activity's initial state, it will not be mathematically similar as soon as angle A has been changed. Encourage pupils to try many examples and ask them to generalise from what they have seen in Question 2.
- For Question 3, an answer based on visual inspection is fine.

Check Your Understanding — Individual or Homework | 10 minutes

- This activity is an opportunity for pupils to develop explanations about the role of angles in determining whether two shapes are similar.
- Encourage pupils to calculate all of the angles within the shapes.

Plenary — Whole Class | 10 minutes

- Discuss answers to "Check Your Understanding". Make sure pupils listen to each other's explanations and respond to them.

Pupil Difficulties

Pupils may have difficulty …

- cuing in to changing angles as a way to make shapes non-similar after so much work on sides.

Pupils may have misconceptions …

- finding that the ratio between sides is sufficient to show similarity for any shape.

Investigation 5: More Than Lengths of Sides

→ *Key Learning:* By the end of this activity, you will be able to explain how enlargements require more than equivalent ratios of corresponding sides.

Starter

You have explored how the lengths of sides between mathematically similar shapes are related. But mathematically similar shapes are related by more than just the lengths of their sides.

1. Open Activity 5.1. Look at Copy 7. **Explain** how you know that Copy 7 is not mathematically similar to the original.

Pupils will most likely say that it is warped. Help them see that this means the angles have changed.

Main Activity

London Trending has a new advertising client whose logo is embedded in a parallelogram. We want to try out different versions of the logo, all of which must be mathematically similar so that they look "the same".

1. Open Activity 5.2. At the start, the three parallelograms are mathematically similar.

   A. Before changing any of the parallelograms, **predict** what the relationship is between a particular set of three corresponding angles, for example DAB, D'A'B', and HEF.

   The corresponding angles are equal to each other.

   B. Now **check** by moving the angle slider and compare the three parallelograms as they change. **Explain** how they are the same and how they are different. Which two are always similar and why?

   Nothing changes except for the angles. Side lengths and thus the ratios remain the same.
   The angles of the original and Copy 1 change together. Because the angles change together, they keep the same shape, so they are mathematically similar.
   Encourage pupils to try many cases.

C. Move the scale factor slider. What happens to corresponding angles in the two mathematically similar parallelograms? What happens to the sides?

The corresponding angles are always equal to each other. Encourage pupils to try many cases or watch as they move the slider slowly.

2. Given what you know so far, what is the relationship between corresponding angles in mathematically similar shapes?

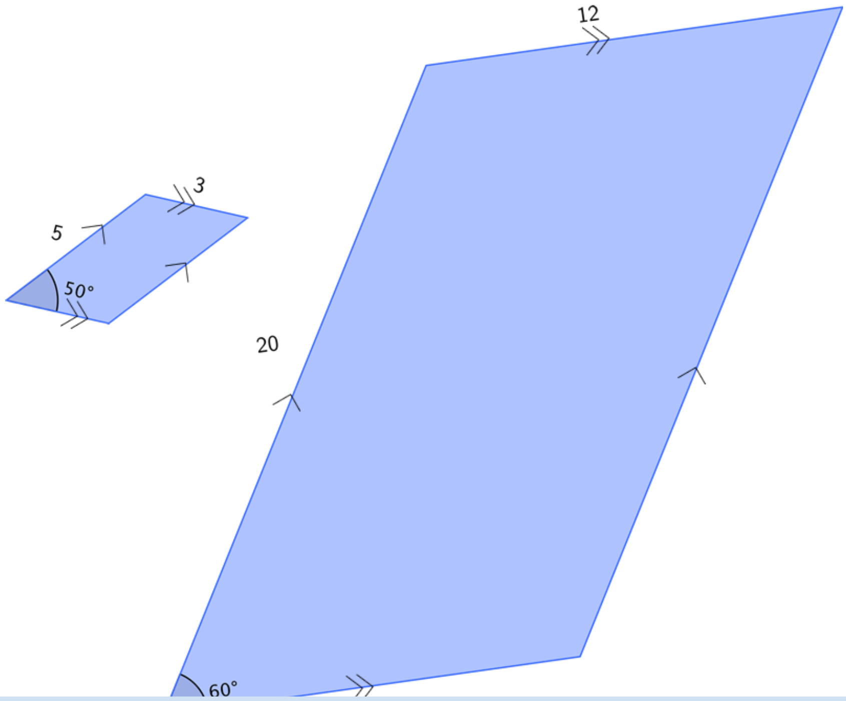
Mathematically similar shapes always have equal corresponding angles.

3. Why would that be true?

If the corresponding angles are not the same, then the shapes look warped and thus not similar.

Check Your Understanding

4. One of our artists, Sirin, says that the parallelograms below are similar, explaining that both of the multipliers are equivalent (5 x 4 = 20 and 3 x 4 = 12) and that the scale factor is 4. Is Sirin correct? **Explain.**



Sirin is not correct. Although there is a common multiplier between the pairs of sides of the two parallelograms, mathematically similar shapes must also have equal corresponding angles. The angles in the parallelograms here are not equal to each other (that is, the lesser angle from one figure is not equal to the lesser angle in the other figure, and the greater angle in one figure is not equal to the greater angle in the other figure).

Teacher Notes

Investigation 6: Ratios

Key Ideas

→ Context: The Graphics Department received directions to make pairs of images in the ratio of 3 to 1.
→ A ratio shows the multiplicative relationship between two numbers or quantities.
→ Ratios can be simplified in the same way as fractions.
→ Two ratios that simplify to the same unitary fraction are equivalent.
→ *Between* ratios show the relationship between corresponding sides in similar shapes.

Main Activity — 55 minutes total

Introduction to Ratios — Whole Class | 10 minutes

- This section is more like direct teaching than most.
- Read together with the whole class. Check for understanding.

Questions 2 - 7 — Whole Class | 10 minutes

- Support pupils as they label vertices, colour corresponding sides and measure the lengths.
- Help pupils as they create their tables, noting the importance of comparing lengths in the same way (copy length : corresponding original length).
- Help pupils see the pattern in the three ratios; this activity will lead to the discussion of equivalence.

Questions 8 - 11 — Whole Class or Group | 15 minutes

- Read together the explanation about ratio equivalence. Check for understanding.
- Walk around checking the pupils’ sketching, labelling and tables.
- Record some of the pupils’ answers to Question 7 for discussion (now and during the plenary)

Vocabulary — Whole Class | 10 minutes

Emphasise the multiplicative nature of the relationship expressed by the ratios. Even though there are special cases like congruence where addition does happen to work, the additive misconception is extremely common and must be countered with evidence. The additive misconception is addressed more experientially in Broken Scale Factor: Investigation 4.

Check Your Understanding — Individual or Homework | 10 minutes

- Pupils apply what they have learnt. Use these questions to assess pupils' understanding of equivalent ratios and unitary ratios.

Plenary

Whole Class | 10 minutes

- Ensure that pupils understand the commonalities among the scale factor (common multiplier) they used earlier in the unit and the between ratios.

Pupil Difficulties

Pupils may have difficulty …

- understanding the connection between a ratio and the scale factor (multiplier) that they learnt previously.
- understanding how a unitary ratio is formed.
- setting up the ratio between original and copy incorrectly and deriving inverse scale factors.

Pupils may have misconceptions …

- assuming that ratios are the same as fractions.

Investigation 6: Ratios

→ *Key Learning:* In this activity, you will learn to use ratios to determine whether two shapes are mathematically similar.

Introduction to Ratios

The Graphics Department received directions to make pairs of images in the ratio of 3 to 1. We need to figure out what that means and how to do it.

You are probably familiar with ratios such as 3 boys to 2 girls or 4 pencils for every 3 notebooks. Ratios relate two numbers using multiplication. For example, a 2 to 1 ratio (written as 2:1) means that the first term is two times as large as the second term in the ratio. (We call the numbers in the ratio *terms*.)

Between Ratios

When two shapes are mathematically similar, we can compare the copy with the original using a ratio. For example, one side of a copy is 3 times as long as the corresponding side on the original. The ratio between the copy and the original is 3:1. This is called a *between* ratio. It relates *a side on one shape* to its *corresponding side* in a *mathematically similar shape*.

1. Think back to On the Grid: Investigation 2. How did you describe the relationship between the side measurement of an original and its corresponding side's measurement in an enlargement? Keep this in mind as you work through this investigation.

   In On the Grid: Investigation 2, pupils used a multiplier to describe the relationship between corresponding sides of an original and an enlargement. The *between* ratio and the multiplier are just two different representations of that relationship.

2. Open Activity 6.1. Label the vertices of the two triangles.

3. Colour the corresponding sides of the triangles so that they match.

4. Identify the corresponding sides for the two triangles and record them below.

   AB corresponds to DE
   BC corresponds to EF
   CA corresponds to FD

5. Measure the sides of the triangles. Drag the side measurements into the Measurement Table so you can see the *between* ratios.

6. The table below shows a ratio between one length of the enlarged triangle and its corresponding length in the original triangle. Write the other two between ratios.

copy length: corresponding original length
9:3
15:5
12:4

7. What is the same about the three *between* ratios in the table above?

In all the ratios, the first term is 3 times the second term.

8. Ratios can be simplified in the same way as fractions. For example, 16:4 = 4:1 and 8:2 = 4:1, so these two ratios are equivalent. Show that all the ratios in the table above are equivalent.

3:1 = 9:3 = 15:5 = 12:4
You may wish to show pupils how they could use the Ratio Equivalence Checker to check their work.

9. In the space below, sketch and label two right-angled triangles that are mathematically similar. The original triangle should have sides of 3, 4, and 5. The corresponding sides in the enlarged triangle should have lengths of 4.5, 6, and 7.5.

10. Write the *between* ratio for each pair of corresponding sides.

enlargement length: corresponding original length
4.5:3
6:4
7.5:5

11. What is the same about the three *between* ratios you wrote? (Hint: Ratios can have fractions or decimals as one of their terms.)

All the ratios are equivalent to (or simplify to) a 1.5:1 ratio.
Pupils may respond with something like: "The first number is half as much again."

Vocabulary

Equivalent ratios have the same multiplicative relationship between their terms. For example, we can say that 1:3 = 4:12 because on each side of the equal sign, we can multiply the term in the first ratio by 4 to get the term in the second ratio. Notice that this works only for multiplication; adding does *not* work.

We can simplify ratios as we do with fractions. So, 6:3 = 4:2 = 2:1. We call 2:1 a *unitary* ratio because one of its terms is 1. It is easy to compare ratios when they are simplified to unitary ratios. Do not be surprised if the other term turns out to be a decimal: It is perfectly all right.

Equivalent ratios have the same unitary ratio.
12:6 and 4:2 *are* equivalent. They both simplify to 2:1.
35:10 and 7:2 *are* equivalent. They both simplify to 3.5:1.
6:3 and 6:2 are *not* equivalent. They simplify to different unitary ratios, 2:1 and 3:1.

12. Describe what is the same and what is different about the *between* ratios and the common multiplier you used in On the Grid: Investigation 2, and scale factor.

The common multiplier and scale factor are essentially the same. Scale factor describes with one value the multiplicative relationship between an enlargement and an original. The between ratio describes the same relationship with two terms and is equivalent to the scale factor. A unitary between ratio has as its first term the value of the scale factor.

Check Your Understanding

13. Explain why 5:2 is equivalent to 10:4.

Because 5:2 and 10:4 both simplify to 2.5:1, they are equivalent.

14. Write the unitary ratios for the following:

Ratio	Unitary Ratio
12:3	4:1
15:5	3:1
8:2	4:1
16:5	3.2:1

15. Tick the pairs of ratios that are equivalent.

Ratio Pair	Equivalent
3.5:2 and 11:3	not equivalent
12:3 and 16:4	equivalent
7:2 and 6:1	not equivalent

Teacher Notes

Investigation 7: Between Ratios and Within Ratios

Key Ideas

→ Context: Investigating an animation comparable to the animation starter from Investigation 2: On the Grid.

→ *Within* ratios and *between* ratios are two different ways to compare sides in similar rectangles.

→ *Within* ratios compare sides of the same shape.

→ *Within* ratios are unchanging across a family of similar shapes (i.e., the height:width ratio in a family of rectangles).

Starter — Whole Class | 15 minutes

- Ask pupils to sort the rectangles in Activity 7.1 into groups of similar rectangles.
- Lead the discussion to raise the idea that the relationship (the ratio) of height to width in a family of similar rectangles is the same for each rectangle.
- Lead pupils to talk about the ratio of height to width for each group of similar rectangles.

Main Activity — 60 minutes total

Questions 1 - 3 — Whole Class | 15 minutes

- Be sure to call on pupils who have used different ratios for finding missing sides.
- Read the text in the box together and make sure pupils understand the difference between *between* and *within* ratios.

Question 4 — Whole Class | 10 minutes

- Let the animation run through a few times for pupils to make sure they understand what is going on. Encourage pupils to focus on one shape at a time.

Questions 5 - 9 — Group | 20 minutes

- Pupils should make their own choice here and use the methods provided to establish mathematical similarity.
- Pupils should use within ratios to explain their answer for Question 9.
- If pupils need additional help in using the ratio checker, you may wish to structure the activity in the following way:
  - Pause the animation.
  - Colour and measure the height of the original and each copy using colour 1.
  - Colour and measure the width of the original and each copy using colour 2.
  - Use the ratio checker to compare the width:height ratio for the original and a copy and then run the animation. Do this for each copy in turn.
- You may wish to have a discussion about why the pink copy has a within ratio that is equivalent to the original's but is still not similar to the original.

Questions 10 - 12 — Group | 15 minutes

- Pupils need to write their own definitions for the terms.

Practice/Homework — 10 minutes total

Check Your Understanding — Individual or Homework | 10 minutes

Plenary — Whole Class | 10 minutes

- Invite pupils to describe contrasting methods for finding mathematically similar shapes. This leads naturally to a description of what *mathematically similar* means.

Pupil Difficulties

Pupils may have difficulty …

- distinguishing within and between ratios.
- seeing geometric connections among similar shapes (that are not based on measurement).

Investigation 7: Between Ratios and Within Ratios

Starter

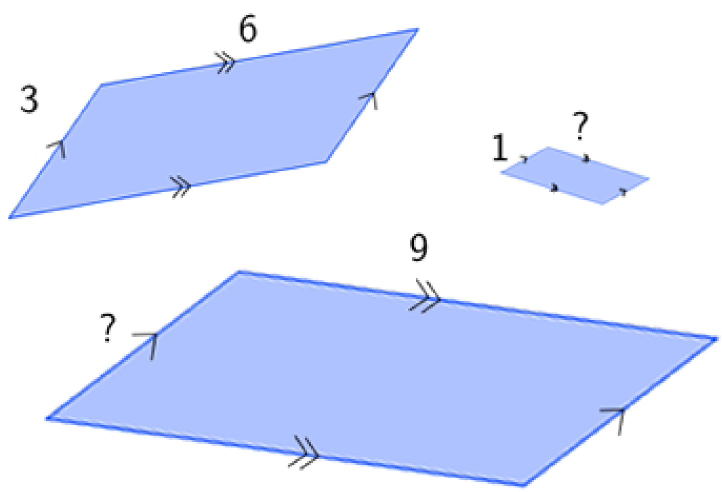
1. Open Activity 7.1. Use the Show/Hide tool to hide the gridlines.

   A. By eye, sort the rectangles into groups of mathematically similar rectangles.

   B. Within each group of rectangles, describe what looks the same and what looks different (include the words height and width).

> There are two groups of rectangles. One group (consisting of the Pink, Green and Purple rectangles) has a within ratio (pupils may call this the height to width ratio) of 3:1, and the other group (consisting of the Blue, Red and Brown rectangles) has a within ratio of 5:4. Pupils should also describe that there are various scale factors relating the rectangles within each group.

Main Activity



These three parallelograms are mathematically similar.

1. Work out the missing lengths of the sides of the parallelograms.

> The three parallelograms are 3 x 6, 4.5 x 9, and 1 x 2. Some pupils will probably use the *between* ratios (and scale factors) to find the missing sides. Others may use the easy-to-identify 1:2 within ratios.

2. **Explain** how you found the missing sides.

> Pupils should explain the use of the type of ratio(s) they used to determine the missing sides.

3. **Explain** another way to find the missing sides.

Note: Call on pupils who used *within* ratios and pupils who used *between* ratios. Ask which was easier and also distinguish them and label them as *between* and *within* ratios.

Two Different Ratios

We call the ratio of the lengths of two sides of the same figure a *within* ratio. We call the ratio of the lengths of corresponding sides of two different figures a *between* ratio. These are not formal mathematical terms, but they are useful labels to distinguish these two types of ratios.

Remember the first animation that the graphics team made? They've made another with four animated copies of the original (blue) quadrilateral. Now it is much harder to tell which quadrilateral copies are always mathematically similar copies of the original. Work with the animation file to help us figure this out.

4. Open Activity 7.2. Watch the animation.

5. Which copies (if any) are always mathematically similar to the original (blue) quadrilateral?

The orange copy is always mathematically similar to the original.

6. Which copies (if any) are never mathematically similar to the original (blue) rectangle?

The pink copy is never mathematically similar.

7. Which copies (if any) are sometimes mathematically similar to the original (blue) quadrilateral?

The brown and green quadrilaterals are mathematically similar to the original only sometimes. Green is similar only when it is congruent to blue. The brown quadrilateral is similar only twice, when it is congruent to blue and when its *within* ratios match the original's *within* ratios.

8. Try out one or more ways to show that you are right.

   A. Put the quadrilaterals on top of each other, aligned at one corner.
      - Does this help show which quadrilaterals are mathematically similar? How?

This is an open exploration. Pupils can try one or more of these methods. If pupils pile quadrilaterals, they should align the lower left vertex. This will lead to geometric comparisons. You can have pupils rotate and translate the supplied line (actually a very narrow rectangle) to use it as a ray through the aligned and opposite vertices to help them see which quadrilaterals are mathematically similar.

   B. Use the grid and measurements and the ratio checker.
      - Does this help show which quadrilaterals are mathematically similar? How?

Using the grid may support quantification of comparisons without the precision of measurements. Turning on the measurements may support pupils' constructing ratios for their investigation.

9. Which quadrilaterals (if any) are always mathematically similar to the original (blue) one and which are not? **Explain** your answer using ratios.

The orange quadrilateral is always similar; it keeps the 2:1 *within* ratio no matter the state. Explanations can include qualitative and quantitative answers.
Pink is never similar because its corresponding angles are not equal. The other two quadrilaterals are similar only sometimes. Green is similar only when it is congruent to blue. Brown is similar only twice, when it is congruent to blue and when its *within* ratios match the original's *within* ratios.

10. Describe how you tested to see whether the quadrilaterals were mathematically similar.

Answers will vary:
You can superimpose a shape to see whether it is congruent to another shape.
You can compare the within ratios of the shapes.
You can compare the ratios of corresponding sides.

11. Describe *mathematically similar* in your own words.

At this point, pupils' descriptions of mathematical similarity should be multidimensional:
1.Qualitatively – Mathematically similar shapes look the same without distortions;
2.Quantitatively – *Between* ratios for pairs of corresponding sides of mathematically similar shapes will be equivalent;
3.Quantitatively – *Within* ratios will be equivalent for mathematically similar shapes; AND
4.Quantitatively – Mathematically similar shapes must have equal corresponding angles.

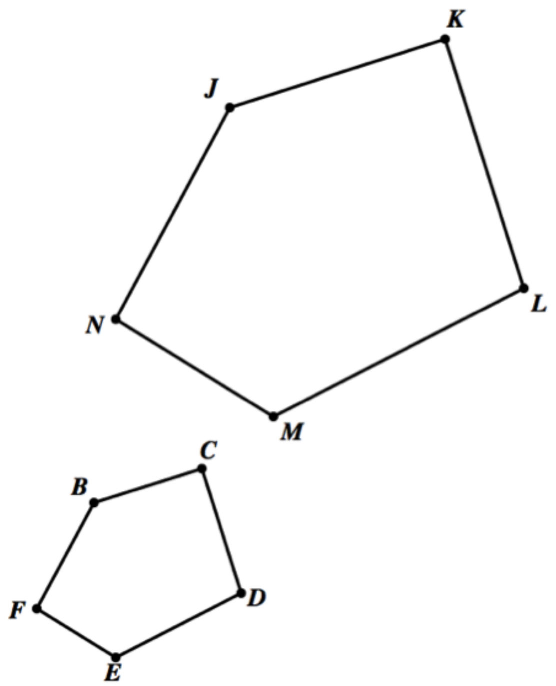
12. Compare your description of mathematically similar with the description that you wrote in Investigation 3: On the Grid, Question 4. How has your description changed?

Encourage pupils to think about how they have learned to look at shapes and pictures more mathematically.

Practice/Homework

Check Your Understanding

Pentagon BCDEF is similar to pentagon JKLMN.



1. Express as many equivalent *within* ratios for the two pentagons as you can.

EF:FB = MN:NJ
And other statements showing the equivalent *within* ratios.

2. Express as many equivalent *between* ratios for the two pentagons as you can.

DE:LM = BC:JK
And other statements showing the equivalent *between* ratios.

Investigation 8: What Changes and What Stays the Same?

Key Ideas

→ For a set of similar shapes, the shape and corresponding angles are unchanging.

→ For two similar shapes, the ratios of corresponding sides, the scale factor and the ratios of lengths within a shape are unchanging.

→ For three or more similar shapes, the ratio of lengths within a shape is unchanging, and the scale factor and ratios of corresponding sides vary together.

Main Activity — 50 minutes total

Questions 1 and 2 — Whole Class | 15 minutes

- From a paper-and-pencil exercise, pupils predict which features of a family of similar shapes are changing and unchanging.

Question 3 — Group | 10 minutes

- Pupils use Activity 8.1 to help them fill out the chart in Question 2 of the starter.

Questions 4 - 6 — Whole Class or Individual | 15 minutes

- Pupils determine whether each statement is true *always/sometimes/never*. This chart summarises the findings from all activities so far.
- Introduce this activity with a whole class discussion about why similar shapes are sometimes the same size (the first statement to be evaluated). Bring out the idea of congruency as a special case of enlargement where the scale factor = 1.
- Ask pupils to share their answers with a partner, and then choose some of the pairs to share with the whole class.

Check Your Understanding — Individual or Homework | 10 minutes

- Pupils solidify the understanding that similar shapes have sides in ratio and also congruent corresponding angles. Encourage pupils to calculate all of the angles in each shape.

Plenary — Whole Class | 10 minutes

- Discuss the chart in Question 1.
- Contrast two similar shapes with a family of similar shapes and describe the role of scale factor and how the ratios change or do not change in the different situations.

Pupil Difficulties

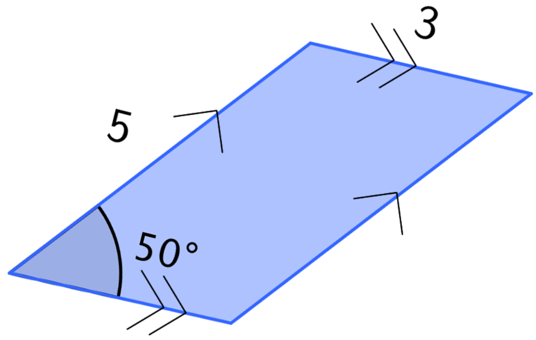
Pupils may have difficulty …

- drawing similar parallelograms where angles matter.
- distinguishing the logical reasons why between ratios change across multiple similar shapes while within ratios do not.

Extension

Have pupils draw sets of shapes that can be used to point out the importance of considering angles in determining similarity.

Investigation 8: What Changes and What Stays the Same?



1. Near the parallelogram above, sketch three mathematically similar copies with different scale factors. Remember, a similar shape can be smaller than the original. Label the sides and angles and write down the scale factor.

2. Imagine creating *many* more enlargements of the parallelogram above.
   Predict: Complete the table below with your predictions of what changes and what does not change across all your enlargements and the original—given that they are all mathematically similar. (We completed the first one for you.)

Characteristic of Shape	Changes Between Enlargement and Original Choose *Sometimes / Always / Never*
Size of shape	Sometimes *changes because when the shape is congruent the shapes are the same size.*
Ratio between one side on the original and the corresponding side on any copy *(between ratio)*	Sometimes (ratio changes as the scale factor varies, but the ratio does not change given congruence)
Lengths of sides	Sometimes (do not change when congruent)
Ratio between two of the sides on the original and the corresponding sides on copy (*within ratio*)	Never
Angles	Never *changes because corresponding angles in similar shapes are always the same*
The way a shape is oriented	Sometimes (copies can be rotated)
Scale factor	Sometimes (does not change when congruent)
Overall look of the shape	Never

You have made some predictions in the table above about mathematically similar parallelograms. Do your predictions also work for other shapes, like triangles? Use Activity 8.1 to **check**. It shows three triangles that will always be similar. Change the triangles and check your predictions by comparing the original with the two copies.

3. Would any of your predictions in Question 2 need to change if the shapes were triangles instead of parallelograms?

Pupils should recognize that their predictions should not change.

4. Now think about **all** shapes. Decide whether each statement in the table is sometimes, always, or never true.

Statement	Choose *Sometimes / Always / Never*
Similar shapes are the same size.	Sometimes
Similar shapes have equal corresponding angles.	Always
Shapes with equal corresponding angles are mathematically similar.	Sometimes (all rectangles have equal corresponding angles but only some rectangles are similar)
A mathematically similar copy is a warped version of the original.	Never
Congruent shapes are similar.	Always
Similar shapes are congruent.	Sometimes (congruent shapes / shapes related by a scale factor of 1 are similar)

Between Ratios

For similar shapes, the *between* ratios of corresponding sides do *vary*. They are equivalent to the scale factor of enlargement and so they vary as the scale factor varies.

Within Ratios

For similar shapes, the *within* ratios of corresponding sides *do not vary* as the scale factor changes.

5. Celia says, “Similar shapes always have the same *within* ratio”.
   Bola says, “They do not. A 3-4-5 triangle is similar to a 6-8-10 triangle, but the ratio of 3:4 is not the same as the ratio of 4:5”.
   Teresa says, “But 3:4 is always equivalent to 6:8”.

   Write a statement that would help the three settle their dispute and better understand within ratios.

   Pupil statements should explain that similar shapes always have the same *within* ratio between pairs of corresponding sides.

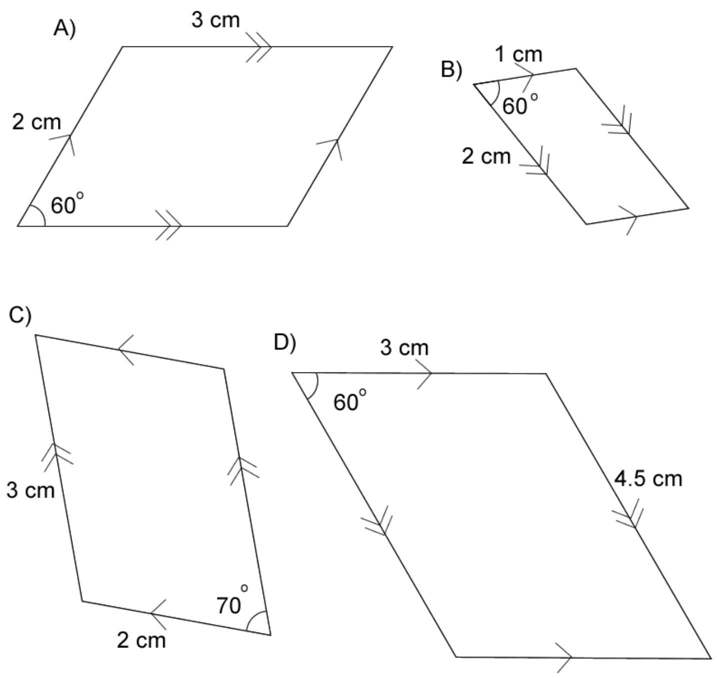
6. Maddie says, “Similar shapes always have the same *between* ratios”.
   Jaxon says, “They do not. A 3-4-5 triangle is similar to both 6-8-10 and 9-12-15 triangles, but there are two different *between* ratios”.
   Jumbo says, “But the *between* ratio between the 6-8-10 triangle and the 3-4-5 triangle is always 2:1”.

   Write a statement that would help the three settle their dispute and better understand between ratios.

   Pupil statements should explain that between any two static similar shapes there will always be one *between* ratio (equivalent to the scale factor) but among a family of similar shapes, there can be multiple *between* ratios.

Check Your Understanding

7.



Determine and show which of the parallelograms above are similar.

A and D are similar because they have equal corresponding angles, and either between or within ratios of sides are the same.

8. Sketch a parallelogram that is mathematically similar but *not* congruent to parallelogram B. Label the lengths of the sides and angles.

Investigation 9: Build Your Own

Key Ideas

→ Context: *London Trending* needs a new front page.
→ Using similar shapes can help pupils make a more aesthetically pleasing cover.

Main Activity — 30 minutes total

Discussion — Group | 10 minutes

- Pupils can use the software to check their answers.
- Check that pupils are considering congruent angles and equivalent ratios (whether *between* or *within*).

Questions 1 - 5 — Group | 20 minutes

- In this activity, pupils must develop a new layout for the magazine. This entails scaling the pictures and developing a new juxtapositioning of the pictures.
- The new cover has been rotated 90 degrees and is enlarged by a scale factor of 1.5. Because of the rotation and enlargement, there are multiple good solutions to the problem.
- This activity provides pupils with opportunities to apply what they have learnt in the module. Pupils should be able to describe their recommended transformations efficiently using scale factors. Additionally, pupils should be able to explain how their transformed pictures are mathematically similar to the originals.
- While the activity calls for a sketch, make sure that pupils are spending their time concentrating on the mathematics and not the fidelity of the sketch. Pupils don't need to sketch the pictures. Sketches of labelled rectangles are sufficient to support their transformation descriptions and explanations.

Plenary — Whole Class | 10 minutes

- Have pupils explain how they made their cover, mentioning how they used similarity.
- Being able to explain how one solves a problem is one of the best ways to demonstrate and develop understanding.

Pupil Difficulties

Pupils may have difficulty …

- explaining how they know that their transformed shapes are similar to their originals.

Investigation 9: Build Your Own

Think back to the beginning of this module on similarity when you were tasked with designing a new cover for the *London Trending* magazine. Now use what you have learnt to really impress the editor. Remember that our editor likes the draft shown below because she thinks the differently shaped pictures go together nicely. Because so many of our magazine's readers use tablet computers, however, she wants to make sure that we have a layout that works for landscape orientation as well. You will sketch a new layout and use mathematics to describe it.

1. Open Activity 9.1.

2. If our editor likes your layout, she will send it to our programming team. So, before you change anything, you must measure and record any part of any shape that you will want to change, including lengths, ratios, and angles. You can use the picture above to record your measurements.

3. Now, using Activity 9.1, create a new layout with the shapes so that you fill as much of the new cover page as possible. You may change each shape as you wish as long as it is mathematically similar to its original (remember to consider the within ratios and equal corresponding angles).

4. Sketch your version of the layout below.
   Remember, this sketch is meant to be sent to our programming team. So, in your sketch you must note the lengths, ratios, and angles you are using for each shape and explain how to change each part you want changed.

   Multiple good solutions are possible. Sketches should include notes on scale factors used and any relevant within and between ratios. Not all pictures need to have had the same scale factor applied.

5. The editor sometimes questions whether pictures have been warped or distorted in preparing them for new layouts. Use mathematics to explain how you know that each of your transformed pictures is mathematically similar to its original.

   Pupils should explain how their sketched layouts meet the requirements of their assignment. Strong explanations will most likely include explanations relying on the within ratios being equivalent in both the transformed and original shapes.

Investigation 10: Supplementary Activity: So the Angles in Triangles are Important?

Key Ideas

→ Context: The graphics department are keen to know if there is a quicker way using their angle measuring software to check whether triangles are similar or not.

→ When the three angles in any triangles are the same, then the triangles must be similar.

.

Starter — Whole Class | 10 minutes

- Use Triangles' Angles 1 activity in Investigation 10: Supplementary Activity: So the Angles in Triangles are Important? for this starter activity.
- Show the animation and ask pupils whether they think the two triangles are similar.
- Pause the animation and remind pupils how to rotate a shape so that it is easier to identify the corresponding sides.
- Remind pupils how to change the colour of the corresponding sides.

Main Activity — 35 minutes total

Questions 1 - 3 — Whole Class or Group | 20 minutes

- Use the activity Triangles' Angles 2 for this phase of the lesson.
- The triangles are labeled such that the pairs of corresponding sides are not immediately obvious to pupils.
- Show pupils how to measure the angles within the triangles.
- Show pupils how to drag the vertices of the original triangle.
- Walk around to assess whether the pupils are identifying and recording corresponding sides correctly. Show them how to overlap the shapes to identify and check.
- Remind pupils how to drag the length measurements to the Measurement Table and take and review Snapshots.

Questions 4 - 6 — Group | 5 minutes

- If possible, allow pupils to answer this question on their own. This is their first prediction based on a few cases. However, support the pupils to see that, across the class they have looked at many examples (albeit for a scale factor of 2).
- Ask pupils to discuss whether they think that this rule always applies to other shapes. (There is a counter-example if you compare non-similar rectangles).

Question 7 — Group | 5 minutes

- If possible, allow pupils to answer Question 7 on their own to revise their original conjecture (from Question 4). This is their first prediction based on a few cases. However, support the pupils to see that, across the class they have looked at many examples (albeit for a scale factor of 2).
- Ask pupils to discuss whether they think that this rule always applies to other shapes. (There is a counter-example if you compare non-similar rectangles).

Check Your Understanding — Individual or Homework | 5 minutes

- This activity assesses whether pupils understand that, if their sketched shapes are enlargements of each other, then the corresponding angles should be equal.
- Observe whether their sketches are seemingly similar—with the same number of sides, vertices and congruent angles. Assess the correct labelling of sides and angles.

Plenary — Whole Class | 10 minutes

- Ask pupils to share (think-pair-share or whole class) their responses to Question 7.
- Engage them in a discussion to generalise what they observed about corresponding sides and angles. All the pairs of triangles they tried out show the same pattern (because that is how the software file was built); the corresponding side lengths of the copy are double the side lengths in the original and the corresponding angles are equivalent.

Pupil Difficulties

Pupils may have misconceptions …

- thinking that the sides of similar shapes are always double the original, based on examples.

Investigation 10: Supplementary Activity: So the Angles in Triangles are Important?

→ *Key Learning:* By the end of this activity, you will be able to use the angles in triangles to determine if they are mathematically similar.

Open Investigation 10: Supplementary Activity: So the Angles in Triangles are Important and open the activity Triangles' Angles 2. The two triangles you see are always mathematically similar, no matter how you drag them around. You are going to find relationships among the measurements in the shapes.

1. Identify and colour the corresponding sides of the two triangles.

   AB corresponds to DE

   BC corresponds to EF

   CA corresponds to FD

   Pupils should use the methods developed in previous lessons. AB corresponds to DE, etc.

2. Play around: Drag points B and C. Try to make the length of at least one side a whole number of grid squares long. What happens to the corresponding sides of triangle DEF as you drag?

   The corresponding sides change, too, so that the triangles continue to look similar. They are still always double.

3. Now keep the triangles fixed.

   A. Use the Measure tool to find the side lengths and angles for both triangles.

   B. Drag the side length measurements to the table.

   C. Use the Snapshot tool to capture your picture and measurements.

   D. Drag points B and C to change the measurements of the triangles. Use the Snapshot tool to capture at least two more sets of measurements.

4. State the relationship between measurements of corresponding sides in the two triangles.

Corresponding sides in the copies always will be double those of the original.

5. Review your Snapshots. What do you notice about the angles in your pictures?

Answers may vary
When the corresponding angles are equal, the triangles are similar.
The corresponding angles are the same, even when you drag the triangles about.

6. Explain how you would know whether two triangles were similar by looking at the angles only.

Answers may vary.
Similar triangles have the same three angles.

7. Compare the measurements you captured for each pair of similar triangles. How do the relationships in the table show that the shapes are mathematically similar?

The sides in the copy are double the corresponding sides in the original. The patterns show that they are mathematically similar because all the lengths of the original can all be multiplied by the same number to obtain their corresponding lengths in the copy.

Check Your Understanding

8. In the space below, sketch a triangle. Label the vertices and estimate the size of the angle. Write these on your drawing. Remember that the three angles should total 180°.

   Now sketch a rotated enlargement of your triangle. Label the vertices and write the size of the angles on your drawing.

   answers will vary corresponds to answers will vary

   answers will vary corresponds to answers will vary

   answers will vary corresponds to answers will vary

Check that pupils' sketches demonstrate an understanding that the set of three angles in each triangle should be the same.

Teacher Notes

Investigation 11: Supplementary Activity: Positioning Images Precisely

Key Ideas

→ Pupils learn about centre of enlargement.

→ Context: In positioning images on a page, using a coordinate system enables the exact position of the copy to be predicted.

→ When shapes are enlarged about the origin, there is a multiplicative relationship between the coordinates of the original and the corresponding coordinates of the enlargement.

→ When shapes are enlarged about a centre that is not the origin, there is a 'two stage' rule that connects the coordinates of the original with the corresponding coordinates of the enlargement.

Main Activity — 55 minutes total

Questions 1 and 2 — Whole Class or Group | 15 minutes

- Pupils should have a chance to use the software on their own. They should become familiar with how the scale factor slider affects the Copy. Circulate and support pupils to use vocabulary such as centre of enlargement, similar, ratio, scale factor etc.
- Support pupils to notice the multiplicative relationship between the scale factor and the distance of each point from the centre of enlargement, which supports the prediction of the co-ordinates.

Question 3 — Whole Class or Group | 10 minutes

- Pupils should be supported to establish that changing the shape does not affect the multiplicative relationship that was concluded in Question 2.

Questions 4 and 5 — Whole Class | 20 minutes

- These questions will challenge the pupils' previous conclusions, which may have included a simple multiplicative relationship between the scale factor and the original coordinates. By moving the centre of enlargement, they may need support to notice and describe the two distinct steps.

Check Your Understanding — Whole Class or Individual | 10 minutes

- Use this task to assess how well pupils have understood the effects of changing the centre of enlargement.
- Encourage pupils to draw diagrams and make general statements about their learning.

Positioning Precisely: Pupil Difficulties

Pupils may have difficulty …

- Working simultaneously with x and y coordinates, in particular within questions 4 and 5. (using colour may help)

Pupils may have misconceptions …

- Noticing that it is the lengths that are multiplied by the scale factor when an image is enlarged but this is not the same as the coordinates.

Investigation 11: Supplementary Activity: Positioning Images Precisely

Julia is worried that the Graphics Department are not always sure where to position the images on the page. She has decided to find out if using coordinates might help.

Open Investigation 11 and the activity, Positioning Images 1.

The triangles ABC and AB'C' are similar and have their *centres of enlargement* at A.

1. Move the scale factor slider.
   Describe what you notice using mathematical language in as many ways as you can.

   Answers will vary.
   The shapes are rotating about A, (or (0,0) or the origin).
   The slider changes the size of the Copy.
   The slider enlarges/reduces the Copy by the scale factor.
   The shapes stay together (in the same orientation).
   When the scale factor is zero, the Copy disappears, etc.

2. Pause the animation and move the animation slider so that the bases of the triangles are on the x-axis.

   A. What are the coordinates of A?

   A = (0, 0)

   B. What are the coordinates of B and C?

   B = (4, 0)

   C = (0, 3)

   C. Use the scale factor slider to enlarge the Copy using a scale factor of 2.
   What are the coordinates of B' and C' in the enlarged copy?

   B' = (8, 0)

   C' = (0, 6)

D. Investigate what happens to the coordinates of B' and C' as you change the scale factor.
Try to notice what is staying the same and what is changing.

Coordinates of B'

Coordinates of the centre of enlargement A	Coordinates of B	Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5
(0, 0)	(4, 0)	(4, 0)	(8, 0)	(12, 0)	(2, 0)

Coordinates of C'

Coordinates of the centre of enlargement A	Coordinates of C	Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5
(0, 0)	(0, 3)	(0, 3)	(0, 6)	(0, 9)	(0, 1.5)

E. Use the Show/Hide tool and hide the Copy. Move the scale factor slider to 4. Predict the new coordinates of B' and C' in the first column of the table.

Scale factor = 4

	Predict	**Check**
Coordinates of point B'	will vary	(16, 0)
Coordinates of point C'	will vary	(0, 12)

F. Check by showing the Copy. Write the actual coordinates in the table above.

G. Explain how you used the scale factor to work out the new coordinates.

Answers will vary.
I multiplied the coordinates of B and C by the scale factor.

Does your explanation still work if you change the shape?

3. Hide the Copy.

Move slider x so that the coordinate of B is at (2, 0) and move slider y so that the coordinate of C is at (0, 6).
Move the scale factor slider to 3.

A. Predict the new coordinates of B' and C' in the first column of the table.

Scale factor = 3

	Predict	**Check**
Coordinates of point B'	will vary	(6, 0)
Coordinates of point C'	will vary	(0, 18)

B. Check by showing the Copy. Write the actual coordinates in the table above.

C. Explain your procedure for predicting the new coordinates.

(answers will vary)

D. Explain how changing the shape did or didn't change your procedure for predicting the new coordinates.

Does your procedure for predicting new coordinates still work if you move the centre of enlargement?

4. Open the activity, Positioning Images 2.
   Enlarge the copy by scale factor 2.

   A. What are the coordinates of A?

   A = (1, 3)

   B. What are the coordinates of B and C?

   B = (5, 3)

   C = (1, 6)

   C. Investigate what happens to the coordinates of B' and C' as you change the scale factor.
   Try to notice what is staying the same and what is changing.

Coordinates of the centre of enlargement A	Coordinates of B	Length of AB
(1, 3)	(5, 3)	4 units

Coordinates of B'

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5
(5, 3)	(9, 3)	(13, 3)	(3, 3)

Coordinates of the centre of enlargement A	Coordinates of C	Length of AC
(1, 3)	(1, 6)	3 units

Coordinates of C'

Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5
(1, 6)	(1, 9)	(1, 12)	(1, 4.5)

   D. Does the procedure that you used in Question 3 still work? Can you think why this is?

Answers will vary, e.g.
No, because the centre of enlargement has moved [from (0, 0)], something gets added.
No, because the numbers are bigger.

E. Use the Show/Hide tool and hide the Copy. Move the scale factor slider to 4. **Predict** the new coordinates of B' and C' in the first column of the table.

Scale factor = 4

	Predict	Check
Coordinates of point B'	will vary	(17, 3)
Coordinates of point C'	will vary	(1, 15)

F. **Check** by showing the Copy. Write the actual coordinates in the table above.

G. Write a procedure for predicting the coordinates B' and C'.

Answers will vary e.g.
"Start with a coordinate from the original, take off the centre and then times by the scale factor and then add the centre back on".

5. Now move both the Original and the Copy so that the centre of enlargement (A) is at (5,1).
Enlarge the Copy by scale factor 2.

A. What are the coordinates of A?

A = (5, 1)

B. What are the coordinates of B and C?

B = (9, 1)

C = (5, 4)

C. Investigate what happens to the coordinates of B' and C' as you change the scale factor.
Try to notice what is staying the same and what is changing.

Coordinates of B'

Coordinates of the centre of enlargement A	Coordinates of B	Length of AB	Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5
(5, 1)	(9, 1)	4 units	(9, 1)	(13, 1)	(17, 1)	(7, 1)

Coordinates of C'

Coordinates of the centre of enlargement A	Coordinates of C	Length of AC	Scale factor = 1	Scale factor = 2	Scale factor = 3	Scale factor = 0.5
(5, 1)	(5, 4)	3 units	(5, 4)	(5, 7)	(5, 10)	(5, 3.5)

D. Does the procedure that you used in Question 3 still work? Can you think why this is?

Answers will vary e.g.
'Yes it does, because you take off the centre and then add it back – so it does not matter what the centre is'

Check Your Understanding

6. Write a short note to Julia to describe what you have learnt about positioning images precisely on a coordinate grid.

Answers will vary e.g.
Pupils should use versions of their rule and diagrams.

Teacher Notes

Investigation 12: Supplementary Activity: Within Ratios in Right-Angled Triangles

Key Ideas

→ This investigation introduces pupils to the foundations of trigonometry.

→ Within ratios in right-angled triangles are commonly used in mathematics within problem solving.

→ The most common within ratios are called trigonometric or 'trig' ratios and are used to calculate lengths of missing sides or angles.

Main Activity — 36 minutes total

Questions 1 and 2 — **Group | 15 minutes**

- Pupils should have a chance to use the software on their own. They should become familiar with how each slider changes the shape. Circulate to ensure pupils are on the right track.
- As they fill out the table, pupils need to focus on naming sides and angles accurately. We are using Angle B to represent the interior angle formed at the vertex B.
- Pupils must conclude that the triangles are right-angled and similar. If they do not all do this, ask some pupils to read out their correct response to the class before moving on to Question 3.

Question 3 — **Whole Class | 10 minutes**

- Support the students to notice what happens to angle B as the sliders are moved.
- Tell the pupils that the two sliders are used to change the angles B and C.

Vocabulary — **Whole Class | 5 minutes**

- Read out loud with whole class. To assess pupils' understanding ask them to sketch another right-angled triangle in a different orientation and label the important parts. Mini whiteboards would support this assessment.

Question 4 — **Whole Class | 2 minutes**

- Once the pupils have established that all of the copies are similar right-angled triangles (which are also isosceles), this activity supports them to notice that the calculated ratios of the sides have some special properties.
- Emphasise the equivalence of the 'within ratios' when expressed as both fractions and decimals.
- Use the opportunity to discuss any rounding issues if they arise (the errors are more obvious when the scale factors are less than one because the side measurements are rounded to 1 d.p.).

Question 5 | Whole Class | 2 minutes

- Again, once the pupils have established that all of the copies are also similar right-angled triangles this activity supports them to notice that the calculated ratios of the sides have some special properties. By changing the angles (45° 45° 90° to 30° 60° 90°), the within ratios also change. The three ratios are different for the different right-angled triangles.

Question 6 | Whole Class | 2 minutes

- Pupils may struggle to adjust the sliders to achieve an angle of exactly 20°, 40° etc. Reassure pupils that working to one decimal place is sufficient.
- The pupils use the measured lengths to calculate the three within ratios for a new set of right-angled triangles using a scale factor that they have chosen for themselves. This provides an opportunity for the students to compare their calculations and to accept that the three ratios are constant for all similar right-angled triangles. During discussions, emphasise this important generalisation.

Plenary | 25 minutes total

Vocabulary | Whole Class | 5 minutes

- Read out loud with whole class. Use an on-screen calculator to check some selected pupil responses to Question 6.

Question 1 | Whole Class | 5 minutes

Pupils could exchange workbooks and compare their calculated values against the stored values in their calculators for each of the three ratios. This will provide an opportunity to discuss the measured and calculated values and the more accurate values stored within the calculator.

Check Your Understanding | Whole Class | 15 minutes

- Here the pupils apply their developing understanding by writing any statements that are true in relation to the given information.
- Encourage pupils to express their ideas in words, algebraic statements and, where possible to use their calculators to work out the missing information.
- Collate the pupils' ideas from their mind maps to build a class picture.

Positioning Precisely: Pupil Difficulties

Pupils may have difficulty …

- Focusing on the idea of a 'given' angle, which enables the opposite and adjacent to be clearly defined.
- Noticing the calculated values that should be constant due to rounding errors.
- Appreciating that it is the calculated value of the ratio that is described as either the cosine, sine or tangent of the angle.
- Knowing the size of one additional angle is enough to determine similarity when comparing right angle triangles.

Investigation 12: Supplementary Activity: Within Ratios in Right-Angled Triangles

→ *Key Learning:* By the end of this activity, you should know that the ratios of side lengths for right angled triangles are a starting point for trigonometric ratios which you will study later.

Main Activity

For this investigation, you will need your calculator.

1. Open Investigation 12 and the activity, Within Ratios in Right-Angled Triangles. Move the sliders in turn.
   What do you notice about the angle and side measurements? Record this in the table below.

Slider	Which lengths/angles stay the same?	Which lengths/angles change?
slider 1	the lengths AC and A'C' angle A and A' (students may notice that it remains 90°)	the lengths AB, BC, A'B' and B'C' the angles, B, C, B' and C'
slider 2	the lengths AB and A'B' angle A and A' (students may notice that it remains 90°)	the lengths AC, BC, A'C' and B'C' the angles, B, C, B' and C'
scale factor	AB, BC, AC all the angles stay the same	A'B', B'C', A'C'

2. What can you say about the original and copy that is true no matter how you change the sliders? (Use precise mathematical language)

Answers will vary.
The triangles are both right-angled triangles.
The copy is an enlargement of the original.
Angle A and A' are 90°.
The triangles ABC and A'B'C' are similar.

3. You are now going to focus on the angles B and B'. Begin by hiding the angle measures (by selecting the vertex and then unclicking the Measure tool button) of A and A', C and C'.
   Change the sliders so that, slider 1 = 1.5, slider 2= 2.0 and scale factor = 2.

   A. Measure and record the size of the angles B and B'.

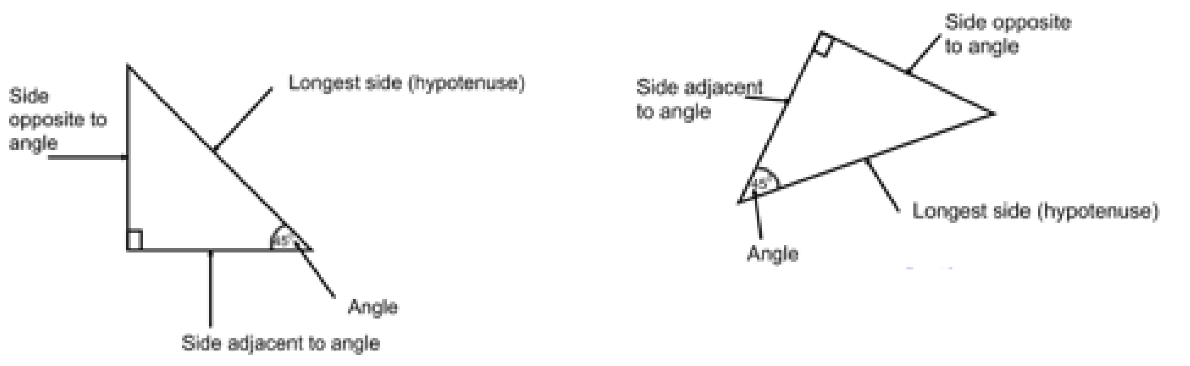
      B and B' = 45°

   B. Work out the size of the angles at C and C'.

      C and C' = 45°

Vocabulary

Vocabulary - Naming the sides within right-angled



4. Check that the sliders are: slider 1 = 1.5, slider 2= 2.0. Then in the table below
   - Record the angle (at B and B')
   - Record the lengths of the 'opposite', 'adjacent' and the 'longest side (hypotenuse)' in ratios that contain them.
   - Using a calculator, find and record the within ratios. Write the ratios to 2 decimal places.

When the scale factor is left blank, choose your own scale factor and record it in the table.

Triangle	scale factor	Angle at B and B'	Length of side opposite to B'	Length of side adjacent to B'	Length of hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
Copy 1	1.0							
Copy 2	3.0							
Copy 3								
Copy 4								

Triangle	scale factor	Angle at B and B'	Length of side opposite to B'	Length of side adjacent to B'	Length of hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
Copy 1	1.0	*45.0°*	*6 units*	*6 units*	*8.5 units*	$\frac{6}{8.5} = 0.71$	$\frac{6}{8.5} = 0.71$	$\frac{6}{6} = 1.00$
Copy 2	3.0	*45.0°*	*18 units*	*18 units*	*25.5 units*	$\frac{18}{25.5} = 0.71$	$\frac{18}{25.5} = 0.71$	$\frac{18}{18} = 1.00$
Copy 3	*answers will vary*	*45.0°*	*answers will vary*	*answers will vary*	*answers will vary*	$\frac{\text{answers will vary}}{\text{answers will vary}} = 0.71$	$\frac{\text{answers will vary}}{\text{answers will vary}} = 0.71$	$\frac{\text{answers will vary}}{\text{answers will vary}} = 1.00$
Copy 4	*answers will vary*	*45.0°*	*answers will vary*	*answers will vary*	*answers will vary*	$\frac{\text{answers will vary}}{\text{answers will vary}} = 0.71$	$\frac{\text{answers will vary}}{\text{answers will vary}} = 0.71$	$\frac{\text{answers will vary}}{\text{answers will vary}} = 1.00$

A. What stays the same when the scale factor is changed and what changes when the scale factor is changed?

Answers will vary.
The angles at B and B' stays the same.
For each copy, the calculation of Opposite/Hypotenuse stays the same.
For each copy, the calculation of Adjacent/Hypotenuse stays the same.
For each copy, the calculation of Opposite/Adjacent stays the same.
Opposite/Hypotenuse and Adjacent/Hypotenuse have the same values.
The side lengths change.

5. Change the sliders so that, slider1 = 1.6, slider 2 = 3.7 and scale factor =1.0. Fill in the table below. You will need to change the scale factor each time.

Triangle	scale factor	Angle at B and B'	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
Copy 1	1.0				
Copy 2	2.0				
Copy 3					
Copy 4					

Triangle	scale factor	Angle at B and B'	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
Copy 1	1.0	*60.0°*	$\frac{11.1}{12.8} = 0.87$	$\frac{6.4}{12.8} = 0.5$	$\frac{11.1}{6.4} = 1.73$
Copy 2	2.0	*60.0°*	$\frac{22.2}{25.6} = 0.87$	$\frac{12.8}{25.6} = 0.5$	$\frac{22.2}{12.8} = 1.73$
Copy 3	*answers will vary*	*60.0°*	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.87$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.5$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 1.73$
Copy 4	*answers will vary*	*60.0°*	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.87$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.5$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 1.73$

A. What stays the same and what changes now?

Answers will vary.
The angles at B and B' stays the same
For each copy, the calculation of Opposite/Hypotenuse stays the same
For each copy, the calculation of Adjacent/Hypotenuse stays the same
For each copy, the calculation of Opposite/Adjacent stays the same
[Pupils may also notice that Opposite/Hypotenuse and Adjacent/Hypotenuse are no longer equal]

B. Compare the two tables above. What is the same about them and what is different about them?

The angles B and B' stay the same in each table but are different across both tables.
The calculated values of each of the three ratios are the same in each table but different across both tables.

C. Suggest a reason for why you think this is happening.

Answers will vary.
If the angle B (and B') change, then the within ratios will change.
In the first triangle (45°, 45°, 90°), the opposite and adjacent sides are the same length, therefore a pair of the within ratios were the same.

6. You are now going to explore what happens to these within ratios when you change the angle B'.

   A. Move the scale factor slider to choose your Copy size. This should stay the same throughout this activity.

   B. Move the sliders so that angle B' is approximately 20° (to 1 d.p.). Complete the table, you will need to change angle B' each time.

My scale factor = ___________						
Size of angle B'	Length of Opposite	Length of Adjacent	Length of Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
20°						
40°						
50°						
70°						

My scale factor = ___________						
Size of angle B'	Length of Opposite	Length of Adjacent	Length of Hypotenuse	$\frac{\text{Opposite}}{\text{Hypotenuse}}$	$\frac{\text{Adjacent}}{\text{Hypotenuse}}$	$\frac{\text{Opposite}}{\text{Adjacent}}$
20°	*answers will vary*	*answers will vary*	*answers will vary*	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.34$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.94$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.36$
40°	*answers will vary*	*answers will vary*	*answers will vary*	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.64$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.77$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.84$
50°	*answers will vary*	*answers will vary*	*answers will vary*	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.77$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.64$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 1.19$
70°	*answers will vary*	*answers will vary*	*answers will vary*	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.94$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 0.34$	$\frac{\text{answers will vary}}{\text{answers will vary}} \approx 2.75$

   C. As the angle at B increases, what do you notice about each of the calculated ratios?

   Answers will vary. As the angle B' increases, the value of Opposite/Hypotenuse also increases. As the angle B' increases, the value of Adjacent/Hypotenuse decreases. As the angle B' increases, the value of Opposite/Adjacent also increases.

   D. There are some common values (or relationships) in the table. For example compare the ratios for 20° and 70° or 40° and 50°. Can you explain this?

   Answers will vary. Because the triangles are right-angled triangles, the 20°, 70°, 90° triangle is mathematically similar to the 70°, 20°, 90° triangle, the ratios Opposite/Hypotenuse and Adjacent/Hypotenuse swap over.

Plenary

Vocabulary

Vocabulary: Within ratios for right-angled triangles are special

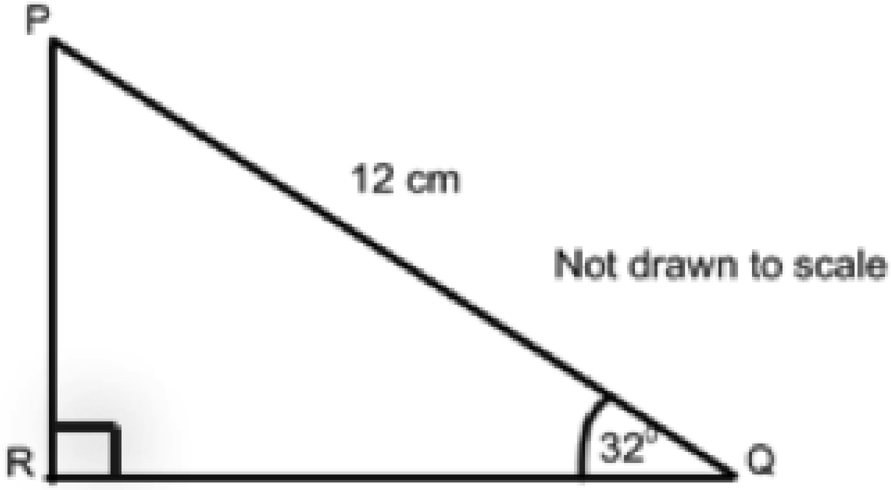
These special ratios can be used to find lengths of sides in right triangles. They are so commonly used that they are given special names.
The ratio (length of side Opposite to the angle)/(length of Hypotenuse) is called the Sine or Sin of the angle.
The ratio (length of side Adjacent to the angle)/(length of Hypotenuse) is called the Cosine or Cos of the angle.
The ratio (length of side Opposite to the angle)/(length of side Adjacent to the angle) is called the Tangent or Tan of the angle

1. The sine, cosine and tangent of any angle can be provided by your calculator. Use your calculator to check the values for the sine, cosine and tangent for angles 45°, 60° and any other value in table 3 above. What do you notice?

   Answers will vary. The calculator gives many decimal places for some calculations. The numbers are approximately the same as those in the table.

Check Your Understanding

2.



The sine of 32° is 0.55 to 2 decimal places.

What other information about triangle PQR can you infer?
Write as many facts or relationships as you can around the picture as a mind map.

Answers will vary.
PQ is the hypotenuse
Angle P = 58°
sine 32° = (Length of PR)/12
(Length of PR)/12 = 0.55 therefore Length of PR = 12 X 0.55 = 6.6 cm
cosine 32° = (Length of RQ)/12
cos 58° = 0.55
etc.

Appendix A: Software Guide

Note that not all controls and tools are available in all activities.

Selecting a Shape
To select a shape: a) check that no control buttons are selected; and b) click inside the shape. A selected shape will have circles shown on its vertices and the shape's sides will be slightly bolded.

Controls (these are along the bottom of the screen)

Translate
To translate (move) a shape: a) click on the translate button; and b) click inside the shape and drag the shape to the desired location.

Enlarge
To enlarge (and reduce) a shape: a) click on the enlarge button; b) click on the shape's enlargement grab point and drag the shape to the desired enlargement.

Rotate
To rotate a shape: a) click on the rotate button; b) click on the shape's rotation grab point and drag the point around to the rotate the shape.

Tools (these are along the top of the screen)

Measure Side Lengths
To measure a side length: a) check that no action buttons are selected; b) select the shape by clicking inside it; c) select the side you wish to measure (the pointer will be a hand when the side is selectable); and e) select MEASURE.

Measure Angles
To measure an angle: a) check that no action buttons are selected; b) select the shape by clicking inside it; c) select the vertex whose angle you wish to measure (the pointer will be a hand when the vertex is selectable); and e) select MEASURE.

Colour Sides

To colour a side: a) check that no action buttons are selected; b) select the shape by clicking inside it; c) select the side you wish to colour (the pointer will be a hand when the side is selectable); and e) select COLOUR and choose the colour you wish.

Colour Shapes

To colour a side: a) check that no action buttons are selected; b) select the shape by clicking inside it; and c) select COLOUR and choose the colour you wish.

Label a Side

To label a side: a) check that no action buttons are selected; b) select the side's shape by clicking inside it; and c) select the side you wish to label (the pointer will be a hand when the side is selectable); d) select EDIT; e) type the side's label into the dialogue box; and f) click Update Label.

Label a Vertex

To label a vertex: a) check that no action buttons are selected; b) select the shape by clicking inside it; and c) select the vertex you wish to label (the pointer will be a hand when the vertex is selectable); d) select EDIT; e) type the vertex's label into the dialogue box; and f) click Update Label.

Show/Hide a Shape or Gridlines

To show or hide a shape: a) click on SHOW/HIDE; and b) click on the name of the object you wish to show or hide. Objects that are currently showing will have a tick mark next to their name in the list.

Cornerstone Maths is a collaboration among:

SRI International®

londonknowledgelab
exploring the future of learning with digital technologies

李嘉誠基金會
LI KA SHING FOUNDATION
Education • Healthcare • Culture • Community

HWL Hutchison Whampoa