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These proceedings consist of short research reports which were written for the BSRLM day conference on 11 March 2011. The aim of the proceedings is to communicate to the research community the collective research represented at BSRLM conferences, as quickly as possible.

We hope that members will use the proceedings to give feedback to the authors and that through discussion and debate we will develop an energetic and critical research community. We particularly welcome presentations and papers from new researchers.

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Extending Valsiner's zone theory to theorise student-teacher development

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This paper sketches an extension of Valsiner's 'zone theory' to theorise student-teacher development in inquiry classrooms. The paper is structured as follows. We begin with the empirical classroom study that grounds the theoretical extension we propose. We then provide the basic ideas of Valsiner's zone theory, the Zone of Free Movement (ZFM) and the Zone of Promoted Actions (ZPA) followed by illustrative results. The final section presents the substance of the paper, that over the course of the study there were transformations of students' ZFM/ZPA, of teachers' ZFM/ZPA and these transformations were interrelated.

The empirical classroom study

Abdul Hussain (2010) investigated the effect of a school year intervention to introduce inquiry methods into mathematics teaching and learning in four Bahraini state primary schools (only one school reported on here). The aim was to understand the intervention at three levels: 1) the mathematics classroom level, the extent to which mathematics classroom activity was transformed; 2) the teaching level, the views, actions and decisions of mathematics teachers, senior mathematics teacher and the teacher supervisor; 3) the whole school level, the obstacles and challenges facing senior managers, the senior mathematics teacher and the teacher supervisor. In levels 2 and 3, changes in participants' views and beliefs were investigated.

Inquiry is viewed as a social construct with various forms arising from Western 'reform' movements in the 1980s. Inquiry is viewed an active quest for knowledge that arises in activity, often investigative activity in mathematics classrooms. The study was greatly influenced by Jaworski's many papers on inquiry, e.g. Jaworski (2006) which describes three inquiry practices: inquiry in mathematics; inquiry in mathematics teaching; and inquiry in research. Our three level approach takes its cue from this division.

The first author, a Bahraini primary school supervisor under post graduate study secondment, provided initial training on inquiry classroom approaches for participating teachers at the beginning of the school year and provided support/advice, following classroom observations, during four 3-week periods over the course of the 2007-08 school year. The overriding theoretical framework is socio-cultural in as much as: teaching and learning are viewed as person, tool and sign mediated activity; the ZFM and the ZPA arose from investigations into Vygotsky's Zone of Proximal Development (ZPD); methodological tools were appropriated from explicitly socio-cultural researchers, e.g. Mortimer & Scott (2003).

With regard to methodology we only report on aspects of the study relevant to this paper. Four interrelated research questions focused on: the extent to which the three level approach changed adults' beliefs about primary mathematics teaching and learning; the extent to which the implementation changed classroom discourse; the key characteristics of instructional actions in the process of transforming classrooms into inquiry communities; the main obstacles in implementing inquiry communities. The study used a design experiment approach in a developmental research paradigm.

Four 4-week teacher preparation/support coupled with data collection periods were distributed over the school year. Data collection at the three levels included: school level – interviews with teacher supervisors and senior management, field notes and documents; teaching level – interviews with teachers and senior teachers, field notes and documents; classroom level –observations, interviews, field notes and documents.

Data analysis to address the research questions focused on beliefs, instructional actions and obstacles included data reduction (documents) and open coding of interview transcripts followed by iterative comparison of emergent themes. Data analysis to address the research question on classroom discourse included analytic induction (Flick 2006) on an initial framework, on transcripts of classroom observations related to the first and last stages of the intervention. The initial framework used Halliday's (1989) discourse constructs to extend Mortimer & Scott’s (2003) account of science classroom discourse. This framework (which was tested and developed during the study) employed Halliday’s constructs of field, mode, tenor and register to extend Mortimer & Scott’s categorisation of teacher-student discourse along two dimensions (interactive-non-interactive and dialogic-authoritative) and teacher interventions **(**shaping ideas, selecting ideas, marking key ideas, sharing ideas, checking student understanding). Classification of classroom social and sociomathematical norms (Yackel & Cobb 1996) as well as analysis of classroom discourse, i.e. initiation-response-evaluation (I-R-E), were a part of this framework.

The basic ideas of Valsiner’s zone theory

Valsiner (1987) introduces the ZFM and ZPA in relation to Vygotsky’s ZPD. The ZFM characterises the child-environment relationship, at a particular time and in a certain environment, “the child’s freedom of choice of action (and thinking) is limited by a set of constraints” (ibid, 97). The ZFM is a social construct that is created through mutual cultural interactions between the child and the adult. We note that the ZFM shapes specific cultural norms and values about permissible future actions that might occur. The ZFM plays a key role in structuring current and future actions of the child in a given environment and is dynamic, not fixed, and can be reconstructed according to the situation. The ZPA refers to the “set of activities, objects, or areas in the environment, in respect of which the child’s actions are promoted” (ibid, 99-100). The ZPA is typically a sub-zone of the ZFM which has a non-binding nature, i.e. the child has the option to comply with or to reject what the adult promotes. However, the ZPA can restructure the ZFM: through encouraging the child to go beyond existing boundaries of the ZFM; through becoming a ‘zone of required actions’ where the child has no options, i.e. the adult turns the ZPA into the ZFM. These two zones interact and “work jointly as the mechanisms by which canalization of children’s development are organized” (ibid., 101).

Valsiner’s concern is child development but his zone theory can be used for adult development and his zone theory has been used in accounts of teacher practice, e.g. Blanton, Westbrook & Carter (2005). Our approach is, to our knowledge, unique in applying it to both child (student) and adult (teacher) development.

Illustrative results

We present extracts from classroom work and discourse at the start and towards the end the intervention. The teacher is Moneer, the class has been working on long division. The start of intervention lesson we provide extracts from involved 4 tasks and we consider the third task:

A group of 732 tourists arrived at Bahrain International Airport. How many buses are required to transfer these tourists if the capacity of each bus is 48 passengers?

Moneer wants to involve the students in collaborative learning and organises them into seven groups of four students each. He asks the students to open their textbooks to page 43, writes “exercises” as the title and encourages students to employ a “read and understand” strategy. The extract starts with Moneer’s opening remarks and then skips to later in the lesson.

2 Moneer: Right, everyone knows which problem … I told you now, read and see the ideas, then determine the main ideas. Describe the task … and start.

53 Moneer: What are your ideas?

54 Student: 732 tourists arrived at Bahrain airport and the capacity of the bus is 48 passengers.

55 Moneer: Right, these are the givens of the question. Do you have anything to add more on the ideas of what was given? What did you do here, boys?

56 Student: A group of 732 tourists arrived at Bahrain International Airport.

57 Moneer: [Interrupting] At the outset I have to know the number of tourists in the group that arrived at the airport. How many? The number of the tourists.

58 Students: 732

We summarise the analysis of the lesson from which this extract is taken.

Approach Interactive/authoritative.

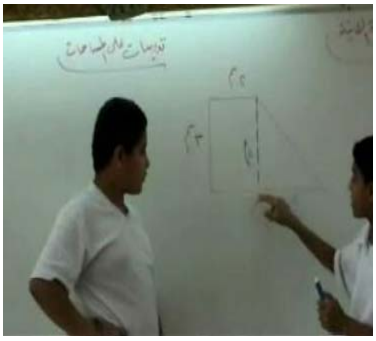
Patterns of interaction I_T-R_S-E_T and I_T-R_S-F_T-R_S-F_T

Teacher interventions Shaping, selecting, sharing, reviewing students’ ideas.

Cultural norms Teacher determines what counts as an acceptable answer. Teacher is not obliged to accept students’ ideas and mistakes. Students are not obliged to express their non-understanding or to negotiate their solutions with each other.

We interpret the approach, patterns of interactions, teacher interventions and cultural norms as similar to Valsiner’s interpretation of a lesson “the children’s ZFMs in the situation equals the ZPA – they can act only in the ways that are allowed by the teacher” (ibid., 103).

We contrast this lesson with a lesson with the same class and teacher towards the end of the intervention. Moneer designed an open task with Ahmed (senior teacher) to introduce a new topic, finding the area of a trapezium. The extract below focuses on two students, Hussain and Sayed, volunteers from one group who are at the board explaining their solution but we begin the extract with extracts that show how Moneer framed the collaborative task to the students.



11 Moneer: Now try to find the area by yourself.

12 Ali: Without a rule?

13 Moneer: Who knows, it might be that there is a rule … Don’t undervalue whatever you know. Record it.

22 Hussain: At first, we constructed a line from the beginning of the angle [pointing to the dotted line]. After that we measure it and we got umm 2.5 cm.

23 Moneer: OK boys [addressing the class], pose your questions.

24 Ali: Why, why is it 2.5?

25 Hussain: Because we computed it

26 Mahmood: Why not 3 so they become equal?

27 Ali: You computed it! Why not 3?

28 Mahmood: The length of this must be equal to the length of this, why are they different?

29 Hussain: Ha? [meaning 'say it again']

30 Ali: Why is this one 3 [the left vertical side of length 3]?

31 Mahmood: Why didn't you make them equal?

32 Hussain and Sayed: [Discussing on the board]

33 Ali: I'm saying why is it 3?

We summarise the analysis of the lesson from which this extract is taken.
Approach Interactive/dialogic
Patterns of interaction Student-student but also I_T-R_S-F_T-R_S-F_T
Teacher interventions Marking, shaping, sharing, reviewing students' ideas
Cultural norms Teacher obliged to listen to students' ideas and mistakes.

The ZFM/ZPA complex system at work here is different to that in the lesson at the start of the intervention. Students are obliged to express their ideas and negotiate solutions with each other. Students collectively determine warrants for an acceptable solution and are obliged to express their non-understanding. Mistakes are acceptable.

Interrelated transformations

We now come to the heart of this paper. We see the above illustrative extracts and analysis of lessons as evidence of a transformation of 'ZFM/ZPA complex systems' (ZFM/ZPA complex hereafter) over the course of the year but of who's complex systems are we referring – the teacher's or the students' or both? We argue that it is both and that these transformations are interrelated. Our argument is in four parts: (i) the teacher's ZFM/ZPA complex promotes the students' ZFM/ZPA complex; (ii) the actions of senior staff afford and constrain the development of teachers' ZFM/ZPA complexes (in this paper we focus, for reasons of space, on a senior teacher and workshops provided by the first author but the actions of other senior staff were also important); (iii) incremental changes in students' ZFM/ZPA complex provided positive feedback to teachers with regard to their beliefs about what students were capable of doing and about the nature of school mathematics; (iv) the cumulative effect of (ii) and (iii) over the intervention period resulted in a significant transformation of both teachers' and students' ZFM/ZPA complexes. We now consider (ii) to (iv) in turn (we omit the case for (i) as this seems uncontroversial) and provide illustrations of actions and events over the course of the intervention.

With regard to the influence of the actions of senior staff on the development of teachers' ZFM/ZPA complexes we briefly consider the start of intervention workshops and the joint work of Moneer and Ahmed. At the start of the intervention the first author conducted four 2 hour school-based workshops for all mathematics and senior teachers and one teacher supervisor. These were received enthusiastically. These workshops provided opportunities for all to discuss: inquiry-oriented learning tasks and how they might engender collaborative learning and facilitate students' interactions within groups; how to create inquiry norms; metacognitive strategies and skills; and how to pose questions that promote inquiry processes. These workshops had an immediate effect on teachers' and students' ZFM/ZPA complexes, what teachers were free to do and promote their students to do. As evidence for this statement we point to Moneer's start of intervention lesson (extract above); Moneer continued many of his prior to intervention classroom practices but he did attempt to involve students in collaborative learning and organised them into groups for this purpose. We will record this change symbolically to emphasise (by arrows) promoted actions (subscripts indicate base and incremented states):

(prior to workshops) $\text{ZFM/ZPA}_{\text{teacher1}} \rightarrow \text{ZFM/ZPA}_{\text{students1}}$

workshops $\downarrow$

(after workshops) $\text{ZFM/ZPA}_{\text{teacher2}} \rightarrow \text{ZFM/ZPA}_{\text{students2}}$

With regard to the joint work of Moneer and Ahmed, they enacted an 'inquiry cycle' which they maintained over the intervention. This cycle involved:

1. planning together
2. implementing and monitoring the lessons
3. reflecting on what was going on
4. enacting modifications and/or consolidating successful pedagogical practices.

Although we have a penchant for this cycle we do not reify it as a cycle that should be used in inquiry practice. It was a practice recommended in the workshops that was appropriated by the teachers and senior teachers in the intervention. We present below extracts from interviews, conducted towards the end of the intervention, which illustrate teacher activities and outcomes in this cycle: teacher collaboration; collaborative task design; restructured roles.

Prior to the intervention, teacher planned alone, senior teacher observed and fed back. As the year progressed there was increased collaboration:

> Moneer: I went to his house about what we are going to do tomorrow… We sit and make a complete plan, the types of activities, the stages, designing the questions… directed to the student … Then, on the next day we saw and measured through an evaluation process …

> Ahmed: When we design a task together we want the students to be speakers so … the student will come and talk and present… Sometimes during my classroom observation which covers the whole lesson … If I don't intervene then there will be great loss… of an idea.

With regard to collaborative design of learning tasks:

> Moneer: The school curriculum … presents the rule …What we do now, always … present a problem to the student… so he will feel the problem

With regard to restructured roles:

> Ahmed: Indeed, the teacher abandoned some roles … to give sufficient opportunities for the student to become a speaker.

> Moneer: My roles at the beginning were too much toward intervening. At the end of the experiment gradually I …withdrew some of my authorities … I try to keep the student to be in the centre of everything and I merely intervene …

We record this change symbolically to emphasise promoted change:

(state at a certain stage) $\text{ZFM/ZPA}_{\text{teacher(n)}} \rightarrow \text{ZFM/ZPA}_{\text{students(n)}}$

collaboration/cycle $\downarrow$

(state after collaboration) $\text{ZFM/ZPA}_{\text{teacher(n+1)}} \rightarrow \text{ZFM/ZPA}_{\text{students(n+1)}}$

We now consider incremental changes in students' ZFM/ZPA complex that provided positive feedback to teachers with regard to their beliefs about what students were capable of doing and about the nature of school mathematics and promoted change in teacher ZFM/ZPA complex. We illustrate this with one (of many) unexpected student solutions (a rarity prior to the intervention). Figure 1 presents a task (a) a student (group) solution (b). This is followed by teacher comment.

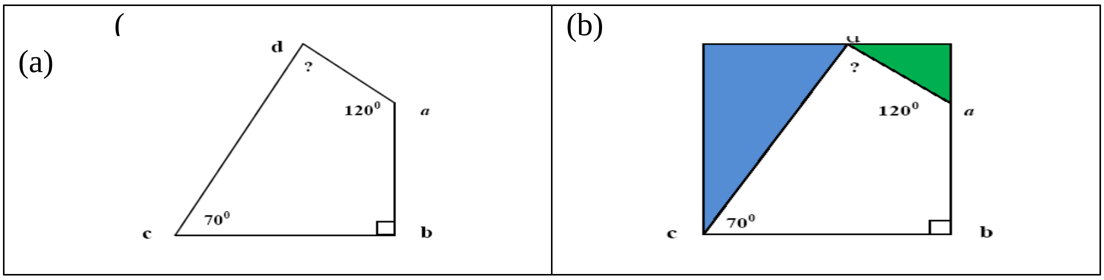


Figure 1 A task and a student (group) solution (students were expected to calculate 360-(70+90+120))

> Moneer: I extended the time for something, because …I could not overlook students' ideas. This is why it was quite long, I liked to give all of them the opportunity. Lot of thing came out from the students… New non-expected solution and also the critiques, views, articulations. Now to be honest, I can say a half or 3 quarters of the class can express and talk about what is inside, describe the shapes, bring out mathematical inferences and that by himself only.
>
> Ahmed: the articulation of ideas became a routine practice for the students.

Students' actions influenced Moneer's transformation to include: greater confident about students' capabilities (a belief); a review of students' mistakes; increased commitment to inquiry norms; a review of time (from an obstacle to an opportunity). We record this change symbolically to emphasise promoted change:

$$\mathrm{ZFM/ZPA}_{\text{teacher}(n)} \rightarrow \mathrm{ZFM/ZPA}_{\text{students}(n)}$$

$$\text{students' action} \downarrow$$

$$\mathrm{ZFM/ZPA}_{\text{teacher}(n+1)} \rightarrow \mathrm{ZFM/ZPA}_{\text{students}(n+1)}$$

The cumulative effect of student and senior teacher promoted changes resulted in a significant transformation of both teachers' and students' ZFM/ZPA complexes.

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Family mathematics/numeracy: identifying the impact of supporting parents in developing their children's mathematical skills

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For a number of years, parents have been encouraged to become involved in their children's learning. This has led to 'family learning' provision of various types being developed and funded. There have been a number of studies looking at parental involvement in their children's learning, though less with a focus on the perspective of the parents (although see Abreu and Cline 2005). The researchers have started a small scale, pilot investigation in the impact of the provision on parents in supporting their children. Previous authors (McMullen and Abreu 2010) have noted that such parental support means that parents are engaging in some aspect of teaching. The study involves interviewing parents about their motivations for learning, their views on their ability to support their child's learning, and the extent to which the courses involved have assisted this process. The data collected so far indicates heterogeneity in motivations although some possible categories are emerging which may assist planning for such programmes.

Keywords: family, adults, mathematics, numeracy, parents, impact

Introduction

Parental involvement in school education has been the object of various studies, for example the Impact project (Mertens and Vass 1990), in which it was argued that such involvement assisted the development of children's learning. Family learning programmes have been provided to help support such parental involvement by developing the skills of the adults. A study of more general adult numeracy provision (Swain et al 2005) has demonstrated that helping their children is one of the motivations for adults to attend classes. LLU+ was asked to run a number of these programmes by two London boroughs and the authors decided to use the opportunity to undertake a small scale, pilot study into the impact of such programmes.

The study

This study is an investigation into the impact that various Family Mathematics programmes have had on parents in two types of provision. The researchers have supported programmes in two London boroughs. In one borough two 60 hour programmes have run in early years centres and in the second borough seven 30 hour programmes have run in primary schools.

The learners were asked to complete short questionnaires on joining the classes and volunteers took part in semi structured interviews. The questionnaires were intended to find out why the parents joined the course and the type of support that they already provided for their children. The purpose of the interviews was to find out from the parents examples of how they support their children and the elements of the course that may have assisted this, or aspects that may need further development.

It was decided that the class teacher (the authors) would collect the data including conducting the interviews as there was a wish to minimise disruption and make the learners feel comfortable during data collection. The authors are aware that the participants are likely to be positive about the programmes (and that some of the questions chosen may be seen as ‘leading’ e.g. ‘in what ways has the course changed your own mathematics / numeracy knowledge?’) and so it was important to us that participants discussed specific examples of interventions (e.g. ‘can you describe a recent time when you have successfully helped your child with mathematics?’). In other words, the authors were more concerned with *how* the provision may have helped rather than whether it did.

It is perhaps unsurprising, given the multicultural nature of the London boroughs, that these groups contain a variety of backgrounds although we note that the groups (so far) are made up almost exclusively of women and contain very few participants identifying themselves as ‘white British’. This may say something about the wider relevance of the research and suggests an investigation into those that attend, and those that do not, would be worthwhile.

Other literature

There is a small but growing body of (international) literature on Family Mathematics provision. McMullen and Abreu (2010) found that many parents were unclear about current teaching methods and were reliant on their children’s explanations, but the children often had difficulty explaining. Their research concludes that participating in different mathematical approaches allows parents to be more positive and understand their value.

Abreu and Cline (2005) looked at the impact of children’s home culture on their maths learning in school. They looked at school maths in school as a different social practice to school maths in the home. They found few differences between groups of parents apart from some language issues. However they report that their research shows that parents do not find it easy to teach their children at home and argue that parents need support with both how maths is taught in school and strategies for bridging the home-school gap.

Ginsburg and Farina (2008) explored the roles women take when attempting to solve mathematical problems with their children. They conclude by advising that ‘adult educators should be sensitive to helping parents consciously prepare for this work.’

In order to discuss the data that is collected the team has found it useful to describe two types of provision. One type (Type I) of provision focuses on children’s learning and the content of the school curriculum, adult skills are discussed and developed as a secondary feature. Another type of provision (Type II) focuses on the development of adult skills with an awareness of children and the school curriculum. We claim that Family Mathematics programmes lie on a continuum between these two types with the longer programmes at the children centres closer to Type II and the shorter programmes closer to Type I. It is possible to see that Type I provision focuses on subject pedagogic knowledge while Type II is more of a mix between subject and subject pedagogic knowledge in the sense of Shulman (1985).

The data

The research is ongoing and at, the time of the report, 8 learners had been interviewed, 3 from each of the longer courses and 2 from one of the shorter. The following quotes are selections from the interviews which illustrate the type of responses obtained in the research.

On changes

> There's a lot of maths English I did not know. .. I never heard of it and I learn a lot now. Which changes my feelings, I become a lot more confident. Because I lost a job before because of maths… I come for my kids, to help in the future as well. (Parent B)

On how the course impacts

> Anything we are doing is helping. When I look at what we are learning back home, and what we are learning here, its different ways… When I come to class and learn myself how to do the maths, I understand my children. (Parent E)

> The things we did with the games and that with the rabbits, it makes it more exciting. (My children) really liked that one. My kids are 8 and 6. They wanted to win – there were arguments over the game. 'Can we play it again!?' (Parent G)

Table 1 displays some key characteristics of the parents and their children as described in the interviews.

Main motivation	Own maths	Child's mathematics	First Language	Home activities	Prog. type
A - Improve own maths & help children	Always loved maths. Assessed at L1	Confident at the moment but child is very young	Spanish	Counting	II
B - Improve own maths	Poor – caused job loss. Assessed at E3	Very young – too young to say	Tigrinya / Amharic. Has done ESOL classes	None noted although discusses counting	
C – To help child	Grade C at GCSE but feels she is not good at maths.	Worried that child will not enjoy maths.	English	Counting	
D – To help children	Not confident. Did not like maths. Grade G GCSE. Assessed at E3.	Quite confident and really enjoys maths.	English	Has done activities with younger child but not confident helping 9 yr old.	
E – To help children & improve own maths	Likes maths but not confident. Used to be confident in own country. L1	Not confident. Thinks maths is ok.	Somalian - difficulty with maths terminology	Had tried to help children but didn't always understand the questions.	
F - Help children	Lacked confidence. No qualifications in maths. E3 assessed.	Lacks some confidence but enjoys maths.	Chinese	Tried to help but didn't understand school methods.	
G – To help children	Fairly confident in own maths .	Confident.	French	Cooks with her children.	I
H - Help children	Enjoys maths but not confident. No quals. Assessed at E2.	Very confident and really enjoy maths	English	Counting using pennies and practical activities. Helped with homework.	

Table 1 Key characteristics of parent participants and children identified from interviews

Some categories of parents

The following have emerged as categories of the parent participants. (Note that the descriptions should be understood as reported rather than statements of fact.)

1. Confident in own maths / joined to help child:

> A – has gained ideas about helping her child and knowledge about school methods (but has also been able to make sense of what she learned at school).
>
> G – confident in own maths but not in helping child due to only knowing one method and not knowing how to explain or teach her child; gained knowledge of different methods, ideas for helping and confidence; worried about confusing her children.

These individuals expressed very positive attitudes towards mathematics, confidence in their own abilities and joined mainly to help their children (although A mentioned improving her mathematics as well). They were already helping their children through activities but were not completely confident about how to help them – either now or for the future. It is interesting to note that their children are also confident in mathematics. One of them had joined a type I course and one had joined a type II course (this may not be important to the participant and may just be a case of opportunity and location). It is not surprising therefore that they reported gaining knowledge about school methods, finding out what their children are learning and getting ideas for how to help them at home i.e. the pedagogy. One of them was worried about confusing her child but has now gained confidence in their ability to help. This learner mentioned particular strategies and methods such as using *compensation* for subtraction and the *lattice method* for multiplication, as being particularly helpful.

2. Low confidence in own maths but enjoys it / joined mainly to help child:

> E – Low confidence is linked to not being able to help due to language issues and not having studied mathematics for a long time. Used different methods in her own country; child not confident with mathematics. Improved understanding of mathematics language through doing written questions. Being able to help her child has improved her own confidence and reports her child has improved a lot.
>
> F – Low confidence is also linked to language; children previously didn’t understand her methods; through the course she gained knowledge of UK school methods and improved her language. This learner connected with her child through a course resource using a school method that he understood. Now she thinks maths is ‘magic’.
>
> H – Enjoyed mathematics but had forgotten some of it and is not very confident. She already helped with homework but has gained knowledge of methods, strategies and resources for helping her child. She has found using games fun and has reminded her of some forgotten mathematics.

This is an interesting group in that they have expressed an enjoyment of mathematics but are not confident in their abilities. For two of these people, this lack of confidence may be related to the fact that English is not their first language. Their children vary in their level of confidence, while two of them enjoy it and the other thinks it is ‘ok’. All of them had tried to help their children at home but with limited success, consistent with McMullen and Abreu (2010) noted above. It is therefore understandable that these people are very keen to find a way to be able to help their children with mathematics. The two ESOL learners report having benefitted from an improved understanding of the mathematical language as well as knowledge of the school methods. They hint at an improved connection with their children through their knowledge of the school methods in this country which are different to those learned in their own countries, (c.f. Abreu and Cline 2005) although the parents in this study did not express resistance to the alternative methods. They also mention methods and resources that have been particularly helpful, such as the box method for multiplying and the 100 square for counting. The learner who also wanted to improve her own mathematics reports that her improved ability to help her own child has, in turn, improved her own confidence. Indeed, all of them have gained some confidence, want to continue learning and take qualifications in mathematics.

3. Low confidence / did not enjoy mathematics

> B – Poor maths caused her to lose her job; has language issues; gained new knowledge (UK methods) and improved understanding of mathematics language. Has gained confidence. Not helping her young child at the moment.
>
> C –Did not enjoy mathematics at school. Learner has gained enjoyment of mathematics due to teaching styles on the course matching her learning. Developed a passion for passing on her knowledge using teaching strategies from the course e.g. talking to her child about toy shapes
>
> D - This learner had bad experiences at school. But her attitude towards mathematics has improved; due to the practical teaching styles on course and links to relevance in everyday life. Has enjoyed learning current school methods and found them helpful for improving her own skills. Has gained confidence and ability to help her children with mathematics.

This group have joined for different reasons and may be the most difficult to persuade to join such courses due to their negative experiences and feeling about mathematics. For the two learners who joined to help their children, it appears that a strong desire to do so has overcome their own feelings and lack of confidence with mathematics. They were already helping their children at home but have gained confidence in their ability to do so and are very keen to use the new ideas and activities they have been exposed to with their children. One learner has not mentioned helping her child at all as a reason for joining the course and does not claim to help her child at home. Nevertheless she does help with counting although does not consider this to be mathematics. She has gained confidence and learned new things in mathematics and mathematics language. These learners point out the teaching styles and strategies on the course as being particularly attractive to them, an important issue as they did not enjoy mathematics at school. The difference in teaching methods seems to have helped change their attitudes towards mathematics and they are keener to engage with maths as a result (c.f. McMullen and Abreu 2010, where parents reported an improvement in their feelings about mathematics).

Discussion

Overall it is noted that participants on both programme types have identified a range of positive aspects of the courses and a variety of reasons for their attendance. There are parents who have their own skills higher in their sights although we also note that they feel this will help them assist their children later. There are parents who feel that the UK education system is something of a mystery to them some of whom also need significant language support. There are parents who want to learn methods that help their children learn.

From the position of a provider it seems to be important that such courses encompass a variety of aims in order to meet the range of needs of the participants. We argue that such courses should be taught in an interesting and engaging way that highlights that mathematics can be fun. It appears however, that a *more detailed initial assessment and diagnosis of learner motivations* may be helpful in responding to parents needs and in the planning and delivery of provision.

It seems that knowledge of current school methods is a priority for most but there are many other things that different people gain from such Family Mathematics courses. Our findings are consistent with Abreu and Cline (2005) although we have looked at it from the point of view of the parents' feelings about mathematics rather than the perspective of the child. Here the differences seem to be that the 'confident parents' gained more confidence from knowledge of school methods, while the less confident parents needed to develop their own mathematics, as well as knowledge of school mathematics, before feeling equipped to help. However all parents were able to express some impact on their ability to work with their children.

Conclusion

This pilot research suggests a number of possible follow up studies. There is a possible quantitative study with a relatively large set of learners identifying the mix of motivations for attending. There is a possible smaller scale, in depth longitudinal study tracking the changes in motivation and the impact of specific activities on the parents and their children.

In addition, as we noted above, there is a possible study into the composition of such family learning groups.

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A classification of questions from Irish and Turkish high-stakes examinations

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In both Turkey and Ireland entrance to third level education is determined by performance on a high-stakes examination at the end of second level education. However, the examination systems in Ireland and Turkey are quite different from each other. In order to compare the examinations we attempted to classify the types of questions asked in 2009 and 2010. We used various classification systems including the Levels of Cognitive Demand Framework developed by the QUASAR Project (Smith & Stein 1998). We will report on the use of these frameworks and the results obtained for the Turkish and Irish mathematics examinations.

Introduction

This classification of examination questions is part of a larger project that studies the effects of examinations on the teaching and learning of mathematics at post-primary level in Ireland and Turkey. We administered questionnaires to Irish and Turkish students and teachers and interviewed teachers to explore the impact of the examinations on study habits, teaching methods, and attitudes to mathematics. In order to compare the examination systems in the two countries we decided to classify the mathematics examination questions in both countries.

Several studies have been done on the classification of examination questions. A group of researchers in Australia modified Bloom's Taxonomy (1956) to classify undergraduate mathematics examination questions in order to show how an examination should be constructed to assess a broad range of mathematical skills (Smith et al. 2007). Their modification used 3 classification groups as follows: Group A-factual knowledge, comprehension and routine use of procedures; Group B-information transfer and application in a new situation; Group C-justifying and interpreting, implications, conjectures and comparisons and evaluation. Schoenfeld (1992) details a Balanced Assessment Framework which was created to help examiners set a range of questions which cover different skills. There are 7 critical dimensions in this framework: content, thinking processes, student products, mathematical point of view, diversity, circumstances of performance and pedagogic-aesthetics.

Azar (2005) compared Turkish university entrance (OSS) physics examination questions and physics examination questions asked at schools with respect to Bloom's Taxonomy. His classification showed that OSS physics questions assessed the application, analysis, synthesis and evaluation skills described in Bloom's Taxonomy. The physics teachers used questions at the application and comprehension levels in school assessments to determine students' achievements. Close and Oldham (2005) mapped the mathematics questions from the 2003 Irish Junior Certificate (JC) examination onto the 3 dimensional PISA Mathematics Framework. From the model answers provided by the State Examinations Commission, they identified the skills involved and compared them to the 3 competency classes of PISA. These competency classes are *reproduction* (performing calculations, solving equations, reproducing memorized facts or “solving” well-known routine problems), *connections* (integrating information, making connections within and across mathematical domains, or solving problems using familiar procedures in contexts) and *reflection* (recognizing and extracting the mathematics in problem situations, using that mathematics to solve problems, analyzing and developing models and strategies, or making mathematical arguments and generalizations) (Close and Oldham 2005, 187). Their results showed that most of the 2003 JC questions belonged to the reproduction class and there were no 2003 JC questions that belonged to the reflection class.

Turkish Education System

After the first year of secondary school students are streamed into 4 groups: science, social, language and Turkish-mathematics. Students in the science and Turkish-mathematics groups can take the mathematics paper in the university entrance and placement examination (OSS). The examination system has two steps. The first examination (OSS1) takes place in April and second one (OSS2) takes place in June. Students need to reach a critical mark in the first examination to get a chance to take the second examination. The second examination determines entry to third level. All examination questions are multiple choice questions and they all have the same marks. In 2010 662,894 students graduated from secondary schools. The number of applicants to the OSS in 2010 was 1,587,993 and the percentage of those placed in universities was 55.06%. Up to 2010 the second examination took place on one day and lasted 3.5 hours. The examination system changed in 2010 and now takes place over 4 days. It comprises of mathematics, science, language and Turkish-social papers. The students sit one paper in one day. The number of questions on the mathematics papers has also increased.

Irish education system

Irish students spend 8 years at primary school, followed by 5 or 6 years at post-primary/second level. Students take an examination called the Junior Certificate after 3 years and an examination called the Leaving Certificate (LC) at the end of their time in post-primary school. These are state examinations held during the month of June. The results of the Leaving Certificate examination determine entry into third level education. Students typically take 7 subjects for Leaving Certificate and because of third level matriculation requirements most students study English, Irish and mathematics. In fact 96% of students who take the Leaving Certificate study mathematics. This means that approximately 82% of Irish students study mathematics until the age of (at least) 17. There are three different levels of mathematics at Leaving Certificate – they are Foundation Level (FL), Ordinary Level (OL) and Higher Level (HL). There is some fluctuation in the numbers taking the various levels from year to year but typically 11% of students take FL, 71% take OL and 18% take HL. Students who study mathematics at FL are not generally eligible for entry to third level. Leaving Certificate examination questions are partial credit questions and the questions may carry different marks.

Methodology

Initially we classified the mathematics questions on the Irish and Turkish examination papers according to Smith et al. (2007) and to Schoenfeld’s Framework for Balanced

Assessment (Schoenfeld 1992). However, we ended up with most mathematics examination questions falling into the same categories – in the case of Smith's classification categories, it was 'routine use of procedures' (Smith et al. 2007). Recall that Schoenfeld's Framework for Balanced Assessment has 7 dimensions, which are further subdivided into categories (Schoenfeld 1992). In each of the dimensions the questions usually fell into just one or two categories. For example in 'students' product' dimension all of the examination questions fell under 'exhibition of techniques' or 'problem solutions'. So these classification systems failed to discriminate between questions both within the Irish mathematics examinations and the Turkish mathematics examinations, and between the two countries' examinations. We decided to use a different classification method called the Levels of Cognitive Demand Framework (LCD) developed by the QUASAR Project (Stein & Smith 1998). The reason for using this classification method (LCD) was that almost all questions under inspection involve using procedures and this system distinguishes different levels of procedural questions. We classified all questions on the 2009 and 2010 Irish Leaving Certificate Higher and Ordinary Level Mathematics papers and the Turkish OSS Mathematics examination papers. First, each of the three authors classified the questions independently and then we resolved our disagreements through negotiation.

The levels of cognitive demands framework

Here we reproduce the description given by Stein and Smith (1998, 349) of the four levels of cognitive demand used in the QUASAR Project (the emphasis is ours):

Lower-level demands (memorization) (LM)

The questions involve either *reproducing* previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to *memory*. They cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. They are not ambiguous. Such tasks involve the exact reproduction of *previously seen material*, and what is to be reproduced is clearly and directly stated. They have *no connection to the concepts or meaning* that underlies the facts, rules, formulas, or definitions being learned or reproduced.

Lower-level demands (procedures without connections to meaning) (LP)

The questions are *algorithmic*. Use of the procedure either is specifically called for or is *evident* from prior instruction, experience, or placement of the task. They require *limited cognitive demand* for successful completion. *Little ambiguity* exists about what needs to be done and how to do it. They have *no connection to the concepts or meaning* that underlies the procedure being used. They are focused on producing correct answers instead of on developing mathematical understanding. They require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands (procedures with connections to meaning) (HP)

The questions focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. They suggest explicitly or implicitly pathways to follow that are *broad general procedures* that have close connections to underlying *conceptual ideas* as opposed to narrow algorithms that are opaque with respect to underlying concepts. They usually are *represented in multiple ways*, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. They require *some degree of cognitive effort*. Although general procedures may be followed, they *cannot be followed mindlessly*. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully.

Higher-level demands (doing mathematics) (HD)

They require *complex and nonalgorithmic thinking* – a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example. They require students to explore and *understand the nature of mathematical concepts*, processes, or relationships. They demand self-monitoring or self-regulation of one's own cognitive processes. They require students to access relevant knowledge and experiences and make appropriate use of them in working through the task. They require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. They *require considerable cognitive effort* and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Creation of an intermediate level

While classifying questions using the LCD framework, we found some problems that we felt fell between Lower Level Demands (Procedures without connections to meaning) and Higher Level Demands (Procedures with connections to meaning). So we decided to create a new category called Intermediate Level Demands (Procedures). The description of this category follows:

Intermediate-level demands (procedures) (IP)

The questions are *algorithmic*. Use of more than one procedure may be evident from previously learned information. Algorithms with *multiple steps* may need to be used. Although there is a well-defined procedure to be used, students may need to make an educated choice of starting point. Also students may have to make *some connections from different areas* of mathematics to the underlying concepts. The questions require *moderate cognitive effort*. A complicated but routine calculation is involved in the questions.

Examples

Here we give some examples of questions in each category. The questions are taken from the Irish State Examinations Commission (2011) and Turkish Student Selection and Placement Examination Higher Education Council (2011) websites.

Lower-level Demands (Memorization) 2009 HL

Prove that the measure of one of the angles between two lines with slopes m_1 and m_2 is given by $\tan\phi = \dfrac{m_1 - m_2}{1 + m_1 m_2}$. (This is one of a very small number of proofs that students are expected to be able to reproduce.)

Lower-level Demands (Procedures without connection to meaning) 2009 OL

Let f(x) = $x^3 + x^2 - 4x - 4$. Verify that f (-2) = 0.

Intermediate-level Demands (Procedures) 2009 OSS2

Suppose α and β are the roots of $x^2 - 2x - 4 = 0$. Which of the following equations has $1/\alpha$ and $1/\beta$ as roots?

A) $2x^2 - x + 4 = 0$ B) $2x^2 + x + 1 = 0$ C) $4x^2 + 2x - 1 = 0$ D) $4x^2 + 3x - 4 = 0$
E) $8x^2 - 3x + 4 = 0$

Higher-level Demands (Procedures with connection to meaning) 2010 OSS2

How many units is the area between the $y = x^3$ curve and the $y = x$ line?
A) 1/2 B) 3/2 C) 1 D) 1/3 E) 2/3

Higher-level Demands (Doing mathematics) 2010 HL

Let f be an affine transformation. The point M is the mid-point of the line segment $[AB]$. Show that f (M) is the mid-point of the line segment $[f(A)f(B)]$.

Results

Tables 1 and 2 below show the percentages of examination questions that were classified as belonging to the five categories in our scheme.

	LM	LP	IP	HP	HD
2009 OSS1	-	40%	20%	40%	
2009 OSS2	-	23.33%	26.67%	46.67%	3.33%
2010 OSS1	-	61.54%	20.51%	10.26%	7.69%
2010 OSS2	2.5%	32.5%	27.5%	31.25%	6.25%

Table1: Classification of 2009-2010 Turkish university selection and placement examination questions

	LM	LP	IP	HP	HD
2009 HL	7.45%	39.36%	24.47%	23.40%	5.32%
2010 HL	-	50.51%	26.80%	19.59%	3.10%
2009 OL	1.55%	79.07%	9.30%	10.08%	-
2010 OL	1.75%	84.21%	5.26%	8.77%	-

Table2: Classification of 2009-2010 Irish terminal/end-of school examination questions

	LM	LP	IP	HP	HD
2009 HL	8%	34%	27%	26%	5%
2010 HL	-	42%	28%	27%	3%
2009 OL	4%	74%	10%	12%	-
2010 OL	4%	80%	8%	8%	-

Table 3: Percentages of total marks for each level of Irish terminal/end-of school examination questions

All questions in the OSS examination carry equal marks but this is not true in the Irish examination. Table 3 gives the percentages of marks in each category.

Conclusion

From Tables 1 and 2, we can see that the OSS2 examinations have more high level cognitive demand (HP-HD) questions than the Irish examinations. The OSS1 (2010) has a lot of LP questions. However, the OSS1 examination plays only a limited role in determining entry to third level. In Ireland, the OL examinations have a very large proportion of LP questions with about 80% of marks being awarded for questions with lower-level demands (Table 3). The HL examinations have less LP questions but they also do not have many HP questions and about 30% of the marks are awarded for questions involving higher-level demands. The OSS2 and HL examinations have a similar percentage of IP questions. According to these results, we can say that most OL examination questions have limited cognitive demand and little connection to concepts that underlie the procedure being used. The OSS2 questions use broad general procedures, are represented in multiple ways and have some degree of cognitive demand. The HL examination requires moderate cognitive effort and some connections from different areas of mathematics.

Azar’s (2005) results concerning physics questions in the OSS examinations have similarities with our analysis of OSS results. We too found that questions with high levels of cognitive demand were asked in the OSS examinations. Also we have seen similarities with Close and Oldham’s (2005) work, who found that most Irish Junior Certificate examination questions do not require higher order thinking skills.

Most questions on all of these examinations inspected involve the use of procedures as can be seen from the small numbers of HD tasks. For this reason the LCD classification method seems to be a good method for our situation.

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Inducting young children into mathematical ways of working in Hungary

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This paper outlines some initial findings from a small qualitative study exploring the ways in which teachers and kindergarten practitioners induct young children into mathematical ways of working in the last year of kindergarten and the first year of formal schooling. It presents a 'telling case' (Mitchell 1984) of one lesson as an example of the way in which mathematics is presented to a class in the first few weeks of formal schooling in Hungary. The mathematics involved and the ways in which the teacher works with the children are described to illustrate the centrality of mathematics in the approach and the care with which it is presented and developed with the children.

Comparative education, primary mathematics, Hungarian teaching, early mathematical learning and teaching

Background

This paper describes the findings of a small scale research project that is being carried out in a Kindergarten class (children aged 3 - 6 years) and a year 1 class (children aged 7 years) in Eastern Hungary. I made a number of visits at intervals over the course of the academic year 2010 – 2011 to look at the ways of working that the teachers adopt with young children in mathematics. The aim was to build up two cases studies over the course of the academic year and analyse the teachers' practices for comparison with established practices in England.

In undertaking this study, my research question was: how do teachers induct children into mathematical ways of working in Hungary? I chose to observe a sequence of lessons taught by one teacher in each context over the course of the academic year. Initially I observed the two classes over about a week seeing all the mathematics lessons in order to familiarize myself with both contexts and the teachers and students with my presence in the classroom. On subsequent visits I watched just one lesson in each setting.

This is in essence a small scale qualitative study and as such the question of the typicality of the observed lesson is raised. I cannot say with certainty but my experience of observing more lessons taught in other Hungarian primary schools by other teachers would suggest a strong case for typicality in some sense. My rationale for this approach is based on the case argued by Robin Alexander (2001) who suggests that presenting individual cases in comparative studies 'side-steps this problem by claiming authenticity rather than typicality for the individual case or cases portrayed in depth' and that 'any one classroom can tell us a great deal about the education system and indeed the country of which it is part, but only if ... the research methods used are sufficiently searching to probe beyond the observable moves and counter-moves of pedagogy to the values and meanings which these embody' (Alexander 2001, 266). This probing in my study was undertaken through interviews with the teachers involved about their practice, about the observed lesson and about their rationale for their practice and triangulated with additional information supplied by my translator and interpreter. This argument suggests that lessons within single countries are more similar to each other than lessons across countries and it is within this that the case study of the presented lesson is offered.

I have chosen to engage in my research and to present it using the unit of the lesson because that is the natural unit of work for primary school teachers in which each lesson forms part of a sequence of lessons but is to a greater or lesser extent complete in and of itself. In this paper I aim to offer a holistic account of one lesson in detail as a means of capturing the broader characteristics of Hungarian primary school practice. Alexander (2001, 315) talks about the lesson as a performance of a composition in a musical sense and, as a metaphor, I find this helpful in the way in which it draws out the inherent combination of attention to form and improvisation that is at the heart of each lesson. The teachers in question would teach very similar lessons each year to the ones that I observed and each of these lessons would have some similarities in terms of form and substance with one another but also some variations.

The study

The study involved following two cohorts of children over the course of one academic year. The first cohort was from a kindergarten class which included children of mixed ages from three to six years old within which were a group of ten children aged six years old and in their final kindergarten year which is compulsory in Hungary. This class was based in an ‘Ovoda’ or kindergarten setting catering solely for children aged from three to six years. The second cohort was a Year 1 class of rising seven year olds in their first year of formal schooling in a university practice school taking students from seven to eighteen years old.

Data collected through classroom observation of lessons during which an interpreter was present and a DVD and audio recording made. Each lesson was followed by a teacher interview with an interpreter present and subsequently a translation was made with the help of a translator who is fully aware of the working setting and professional concerns of the teachers involved. This translator was also able to provide relevant additional background data about the Hungarian setting. During the recorded lessons detailed field notes were made and these were used to support the development of translated transcripts of the lesson. In developing my account I have privileged the voice of the teacher and my local contacts in interpreting the lesson in order to help to support the authenticity of the account. I have also focused on the mathematics in the lesson, the development of mathematical knowledge and the way in which that is shared and negotiated between the teacher and the children.

The lesson

The lesson that is presented here is one that occurred in the seventh week of the autumn term and in which the number 6 was introduced. This lesson is presented as a ‘telling case’ (Mitchell 1984) which captures the spirit of Hungarian primary school mathematics lessons whilst at the same time providing through the detail of the single lesson some of the essence of the way in which mathematics in worked on by teacher and students in the classroom.

The mathematical presentation in the lesson involved two parts: firstly the class focused on revision of the numbers to 5 and what they knew about them and then the new concept of the number 6 formed the latter part of the lesson. The division of time for these two parts was one third (fifteen minutes) for the revision and two thirds (thirty minutes) for the new concept. The pedagogy used throughout this was teacher led whole class interactive teaching with frequent and enthusiastic contributions from the children which involved the majority of members of the class of thirty children.

The revision of numbers to 5 covered this topic in great depth and included aspects of the properties of numbers related to the number system that would help children develop their understanding of numbers on a firm foundation. All the activities presented in the lesson were described by the teacher as games and she frequently talked to the children using the language of playing. The revision emphasised the order of the numbers to 5 by engaging children in placing the numbers on a number line. It drew attention to the properties of the numbers in terms of their oddness and evenness and the pattern even/odd by writing the even numbers in red chalk and the odd numbers in blue chalk.

Different representations of numbers were used to broaden the children's understanding of the underlying essence of the meanings of each number, its symbol and the multiple ways in which it could be represented. These representations included hand signs, dotty patterns on 3x3 grid, the number line and dot patterns on a die. The dotty patterns on the 3x3 grid which are known as 'number-pictures' in Hungarian are a particularly powerful representation that builds on the human ability to subitise (recognise without counting) the numbers to five. Here is an example of the number 8 presented in this way:

●		●	●		
	●				
●		●	●		●

Figure 1: 'Number picture' of 8

This representation of numbers helps to support children in adding numbers in their heads without counting and then scaffolds the children's developing understanding of number bonds to twenty which are such an essential piece of mathematical knowledge.

In addition to this the class engaged in an activity involving solving inequalities. This was presented in a meaningful context related to the pictures which the teacher had uncovered on the blackboard. These pictures showed the houses in which farm animals lived and one was a rabbit hutch. The task was to find the number of rabbits who lived in the rabbit hutch from the inequality:

2<  < 5-1

This involved manipulating an algebraic inequality even though it did not use letters as such. The teacher led the children through this step by step identifying with them the need to solve the expression 5-1 first. The teacher wrote the answer to this on the board above the expression in a different colour and the children then went on to solve the inequality identifying three as the number of rabbits in the hutch. The final activity in the revision section of the lesson comprised presenting all the possible number sentences related to a picture of a sow with three piglets: $4 = 1 + 3$, $4 - 1 = 3$, $1 + 3 = 4$ and so on.

Having completed this thorough revision of the numbers to 5 the lesson progressed with the introduction of the new concept of the number 6. This was introduced through the story of the Ugly Duckling with which the children were familiar. There were five ducklings in the family and the ugly duckling formed the sixth. The class then looked at a picture showing the ugly duckling swimming on a pond with his 'mother' and 'siblings' and the classed counted the ducklings and other things in the picture. They also showed the teacher six fingers and clapped and knocked on the desk six times so they counted events as well as objects. They found six o'clock on clock faces which they all had on their desks and then found the six rod from boxes of Cuisenaire rods on their desks. All this equipment had been prepared by the teacher in the fifteen minute break prior to the lesson. These breaks occur after each lesson in Hungarian schools and give teachers ample opportunity to prepare resources including the elaborate board displays that are an important part of the presentation of mathematics in the lesson.

Figure 2: Blackboard display

The children went on to identify sets of six objects from around the classroom and then to count 6 counters from dice shaped boxes of counters they each have on their desks. These counters were then examined and the evenness of six was established by pairing up the counters. After this the 'number picture' of 6 was made from the counters on a 'number picture' blank which they each had on their desks (see Figure 1). Their counters fitted neatly into the spaces on the card. The lesson ended with the introduction of first the Roman number symbol for 6 (the symbol VI) – and then the Arabic symbol and the children made the symbol in the air with their whole arm, forearm, hand and finger in turn. The teacher showed the children some different drawings of the numeral 6 and they identified the correct one. There was not time for them to draw the symbol for themselves so that would wait until the next lesson.

Discussion

My original research question was about how teachers induct children into mathematical ways of working in Hungarian classrooms in the early stages of formal education and this account of the lesson draws attention to some key elements of pedagogy and presentation of mathematics in this context. This description of the detail of one lesson captures something of Hungarian ways of working in mathematics classrooms which has been described elsewhere in relation to secondary school classrooms (see for example Andrews 2003, 2007). The lessons are teacher dominated but have high levels of participation from the children. With younger children there is considerable emphasis on the use of games and familiar stories with play viewed as part of the work of the lesson. As this teacher says:

> I pay attention to the fact that these are very young children and I want to make these exercises interesting and enjoyable for them and so I want to connect the exercises they do with a story. This is playful learning.

The classroom atmosphere is friendly, happy and supportive and respectful for individuals but with a focus on success for all. There is heavy emphasis on the building up of common culture and routines in the classroom and connecting to the children's prior knowledge and everyday experience. As this teacher said:

> They also need some connection to the senses so it is easier to teach them in this way. My goal is that each and every child can take part in the lesson and this is very important to me. ... I want to see the knowledge that the children already have: to see their knowledge, their orientation, their background. ... The children also have to be instructed in a way that helps them to cope with the practical aspects of classroom life. If I have told them to put away their books then they may ask 'can I close my book?' so they have to connect these things together. I teach them how to handle these things so that when I ask them to put away their books they also know to close them. This is not so obvious for the children and they have to be taught.

Throughout the lesson the teacher models very clearly the presentation of the mathematics that she is working on and the accuracy of the mathematical expression.

> I want the children to be logical and I want to create a way of thinking in the children. It happens sometimes that a child asks something and he or she doesn't think about the conception really and he or she already knows the answer to the question and what I am trying to do is. .. I want the children to be conscious about their questions and to make good questions.

The teacher also talks about connecting everyday language to the technical language of mathematics. This careful approach is captured in her description:

> It is important to teach the technical language but it is also important to connect this technical knowledge to the everyday experiences because they learn the conceptions more easily. So especially in a Hungarian language lesson it is very useful because I teach the technical name of this (cursive letter f drawn on paper) –this is a swallow and the letter f can be formed out of it. It is important to use the child's imagination because when he or she connects the letter f to a picture of the swallow, then she learns the letter more easily. It is important to handle the children at their level and to 'translate' the technical language to the language of the age group.

Concluding remarks.

Two major themes emerge from analysis of the data gathered from the lessons in general and this lesson in particular. The first is related to mathematics and the second to the social aspects of the pedagogic practices of the classroom. In relation to mathematics there is a strong emphasis on the careful introduction of the children to an established body of mathematical knowledge in ways that link with the children's development and prior understandings. This introduction is managed with a strong emphasis on mathematics and the correct use of mathematical language and mathematical expression. The pedagogic practices of the classroom are based on the affectionate and supportive relationship of the teacher with her class and the interactions of the classroom are full of respectfulness and care both in relation to the mathematics and in relation to the children. These pedagogic practices are echoed in classrooms throughout the schools for older children as well as these young children.

These observations have led me to question the foundations that underlie these practices. From interviews with the teachers involved in the research I would suggest that their confidence in both the subject area of mathematics and the pedagogy they use contribute strongly to their classroom practice. The teacher in the reported lesson graduated in 1991 and has been teaching at this University Practice Primary School since then in the Y1/Y2 class. She trained at the local university and her final practice was in this school and she became the class teacher at this school once she qualified. She is very happy at the school and does not plan to leave until she retires. She had a really good teacher leader in her school placement who helped her with her work. The secondary school, which she went to, had a strong pedagogical tradition and specialised in training kindergarten teachers. Her college course gave her lots of teaching experience. Mathematics and mathematics pedagogy was an important part of this training.

There is heavy emphasis in the classroom on inclusivity and enjoyment for all the children in the class. The teachers have a thorough understanding of child development and what they describe as ‘typical mistakes’. As the teacher of this lesson said:

> The reason for that is that when a child discovers his or her mistake, then when he or she points it out, then in the future he or she will possibly avoid the mistake. So this is conscious and when they see it, when they see what the most obvious mistakes are, then my experience is that the children develop and they don’t make those mistakes again. In the lesson that you saw these were ‘typical mistakes’ – these are called ‘tipikus hibák’ in Hungarian. In the lesson plan these are a key points in the lesson and very important for me.

The teachers also have the security of being able to use a set of prescribed textbooks. Part of the emphasis in this initial induction period of schooling is on teaching the children to be good students in school.

This paper has sought to offer a descriptive account of one lesson as a vehicle for capturing aspects of the ways in which Hungarian teachers induct children into mathematical ways of working. Hopefully the reader is left with an impression of these ways of working that capture the central place of mathematics in the lesson as well as the careful way in which the teacher supports her students in building their mathematical knowledge. Further research comparing these ways of working with practices in England may be helpful in elucidating the significance of these descriptions for practice in an English context.

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Consulting pupils about mathematics: a straightforward questionnaire?

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We reflect on experiences of working with staff of a primary school to ascertain pupil views of mathematics. Our focus is on methodological issues. We consider the process of building on existing questionnaires to develop one appropriate for a particular school, identifying how discussions with school staff illuminated school practices. We discuss how pupils responded to the questionnaire, considering whether we can learn from the questions they found difficult to answer. A key observation is that researchers and teachers are likely to approach pupil consultation in different ways, that are likely to impact on findings.

Keywords: pupil consultation, primary mathematics, attitude scales

Introduction and rationale

We report on a pupil consultation about mathematics carried out jointly between a primary school's staff and researchers. The original reason for collaboration was concern from the head teacher and mathematics co-ordinator about whether some girls were hampered by negative attitudes to mathematics. Following discussion, it was acknowledged that the situation was more complex; the focus became consideration of the image of mathematics held by all children: how pupils perceive school mathematics and how far they see mathematics as part of life outside and beyond school. In discussion about possible methods of enquiry, it emerged that a priority from the staff's viewpoint was a method which would enable every pupil to be consulted. Interview methods were excluded and we designed and administered a short questionnaire.

Questionnaire design, development and administration

The questionnaire drew on existing studies' content and design features, enabling us to verbalise our approach, deciding for example against an anxiety rating, a theme common in the literature after Richardson and Suinn (1972), but were examining attitudes more broadly. In designing the questionnaire we consulted the Modified Fennema-Sherman Attitude scales, by Doepken, Lawksy and Padwa (2009) based on Fennema and Sherman's earlier work. We felt their theme of gender might be appropriate, though they are clearly designed for older students. These scales have a key identifying the four themes the items cover which are: personal confidence about the subject matter (C), usefulness of the subject's content (U), subject is perceived as a male domain (M) and perception of teacher attitudes (T). We decided not to include T in our study and M was also not mentioned directly, though it was implied in questions about who uses mathematics. We did want pupils' views about school mathematics, including but not limited to their confidence, so that became a theme. Our second theme was mathematics outside school, including usefulness and perceptions of who does mathematics.

For questionnaire format and for some details, we used questionnaires conducted with younger children as a starting point. We drew, for example on a study with 9 year olds by Thomas and Dowker (2000 cited in Dowker 2005 page 239) which asked pupils to rate both mathematics in general and six specific aspects of mathematics, namely written sums, mental sums, easy maths, difficult maths, maths tests and understanding the teacher. We used these categories as a starting point, but also wanted to include consideration of mathematics outside school, a theme mentioned in the survey used by Flockton and Crooks (1997) with year 4 and year 8 students in New Zealand. We also adopted the use of smiley faces as a rating scale, as used in the Flockton and Crooks study and in a survey used by Sun (2009) with children aged 3 to 6 years old in China.

In consulting these questionnaires, we became aware that the language used varied and we suspected that not all the statements about particular aspects of mathematics would be familiar to our target pupils. In particular we needed to take decisions about whether to use phrases such as ‘mathematics in books’, ‘mental mathematics’ and ‘practical mathematics’. We also debated whether the same questionnaire could be used across the 4 to 11 age range, administered in different ways, making selection of appropriate language particularly important. At this point we discussed the questionnaire’s detail with the mathematics co-ordinator to find the wording she thought most appropriate. Our intention was purely to improve question design, but the process led to extended consideration of issues such as whether pupils would recognise the phrase ‘practical maths’, what was meant by ‘problem solving’ and whether the youngest children routinely carried out calculations on paper.

A joint decision was made at this stage to include some open questions in addition to ratings, as we wanted detail, about the types of mathematics children liked or disliked and about how mathematics might be useful outside school. Finally, we decided to ask children to draw a picture of a mathematics lesson, to facilitate a different form of response. The piloted questionnaire appears in appendix 1. In the following sections we consider the questionnaire responses only from a pilot carried out with one class of seven and eight year olds. The questionnaire was piloted with the class of 12 girls and 10 boys in lesson time, with the co-ordinator reading the questions and other adults available to help with spelling.

Pilot questionnaire results

The completed pilot questionnaires were read jointly by researchers and school staff. We were interested in what this particular class had to say and whether the questionnaire appeared appropriate to use with the whole school. For the purposes of this paper, we will consider the results for this class, as a route into considering methodological issues.

How children perceived mathematics lessons

Responses to questions about school mathematics were very encouraging, with 18 positive, 3 neutral and 1 negative response to the general question about mathematics lessons. Ratings for different aspects of mathematics showed some variation, but even the least popular category, ‘hard maths’, was rated positive by 11, neutral by 9 and negative by 2. Almost all children found maths interesting as a subject and most said they usually do well at it. Gender differences were very small, though the girls appeared slightly more positive about mathematics than the boys. It is inappropriate to draw conclusions about this given the small sample size, but we nevertheless note that this is inconsistent with larger studies; Hargreaves, Homer and Swinnerton (2008) found that even when girls achieved the same results as boys their confidence in the subject was lower.

In the open question about an interesting or exciting maths lesson, the most frequently mentioned activity was playing games, which was included in the answers of all 12 girls and 6 out of 10 boys. More traditional aspects of mathematics were also popular, with nine children mentioning times tables and five including other specific aspects of number. Five mentioned shape as an enjoyable aspect. It appeared from the completed questionnaires and from discussion with the adults present, that children were able to answer all the questions dealing with mathematics in school.

Children's views on mathematics outside school

The closed questions about mathematics outside school suggested that most of the children use maths outside school, expect it to be useful to them in the future and know lots of people who use mathematics. Gender differences were again slight, with girls slightly more likely than boys to see mathematics as part of their future. However, open questions about mathematics outside school proved more difficult for the children to answer. This is evident in the completed questionnaires and was supported by the staff present. Thus, nine of the twenty-two children did not list anyone they know who uses maths outside school, despite being encouraged to answer by the adults. When answers were given, several included male and female close relatives. Other answers mentioned particular jobs or activities with shopping and shop keepers by far the most common. Many had difficulty listing jobs that used mathematics or those that did not, leaving these questions blank or listing things which did not seem to be jobs. Where names of jobs were given, shopping was again a key theme in jobs using mathematics, followed by building. It was harder to analyse answers about jobs not using mathematics, though mentions of busses or bus drivers may be due to the fact that local bus drivers no longer handle cash.

The difficulty these children had in linking mathematics to practical uses is consistent with Ashby (2009) who obtained similar responses from focus group interviews with seven and eight year olds. Another small-scale study (Cuffy and Houssart (2009) found that even secondary school pupils could not really explain how mathematics was used in employment, despite many of them making general statements about its usefulness and pervasiveness. It is interesting that they still appear to accept its usefulness. It is unclear to what extent they believe this, or whether they are simply repeating what they have been told and think adults want to hear. It would be interesting to explore this further using other methods.

Reflections on questionnaire design and pilot implementation

A key reason for piloting the questionnaire was to identify questions the children appeared unable to answer, in order to adapt or remove them. On reviewing the pilot's results, we were less certain about this. Open questions about mathematics outside school appeared difficult for the children, but this in itself was informative and the partial answers gave clues about children's understanding of the issue. In contrast, closed questions gave limited information, though using these questions with the whole school would enable some comparisons. A final issue was the validity of answers to closed questions about school mathematics. Although we have no reason to doubt positive responses, we feel that the way the questionnaire was implemented could be more likely to result in them.

Conclusions

The pilot survey told us something about the professed views of this group of children, their views of what mathematics consists of inside school and what it can be used for outside school. A more general message was that answers to questions requiring detail indicated they could not always exemplify statements made in response to closed questions.

Our experiences in trying to find a readymade survey suggest that a survey is more appropriate if it is designed, or at least a particular school's practices, a process itself likely to provide insight into practices and values within the school.

Our final reflection concerns the different views and approaches of school staff and university researchers to such an initiative. A pupil attitude survey conducted by school staff as part of their everyday activity is likely to be conceptualised and conducted differently to an apparently similar survey conducted by academics as part of research. To one group there is an expectation that all pupils will participate, that surveys will include pupils' names and be handled and discussed by teachers. Researchers work within guidelines requiring pupils to opt in and which anonymise responses. Working together in this way enabled both groups to discuss the reasons for their approaches. They are likely to lead to different findings and both are potentially problematic.

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Appendix 1 Questionnaire

Name ________________________ Class__________

Colour the face that best matches how you feel:

In mathematics lessons I feel ...	☺	😐	☹
Doing easy maths I feel ...	☺	😐	☹
Doing hard maths I feel ...	☺	😐	☹
Doing maths in my head I feel ...	☺	😐	☹
Doing maths in on paper I feel ...	☺	😐	☹
When I am asked maths questions ...	☺	😐	☹
When I am doing maths with equipment ...	☺	😐	☹
When I am solving maths problems ...	☺	😐	☹

Tick the sentences that are true for you, cross them out if they are not true for you:

Maths is one of my favourite subjects.

I usually do well in mathematics.

I am not very good at maths.

Maths is usually interesting.

Maths is useful to me outside school.

I only use maths in school or for homework.

I think maths will be useful to me in the future.

I know lots of people who use mathematics.

When I leave school I will not use mathematics.

Try to think of some maths in school that you found interesting or exciting. Write about it in the box below.

If you could design a perfect maths lesson, what would it be like? Write about it in the box below.

Do you know anyone who uses maths outside school? Use the box below to tell us who and what maths they use.

Can you list some jobs that use mathematics?

_______________ _____________________ ______________

Can you list some jobs that do not use any mathematics?

_________________ _________________ _________________

Can you draw one of your maths lessons? Please draw this on a separate page.

A representational approach to developing primary ITT students' confidence in their mathematics

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Representations of mathematical concepts play an important role in the understanding of learners (Greeno and Hall 1997), and also in the pedagogical processes involved in developing that understanding (Leinhardt et al. 1991; Brophy 1991). In this paper, we report on work with a cohort of pre-service primary teachers, with the aim of developing their understanding of mathematics and their confidence in their subject knowledge and their teaching of mathematics. This was attempted through the introduction and use of representations associated with mathematical concepts covered in primary schools. We present the results of attitude measures and qualitative questionnaire comments in identifying whether and how the use of representations supported pre-service teachers' confidence in teaching mathematics.

Introduction

Shulman (1986) identified representations as being part of teachers' pedagogical knowledge. Specifically in mathematics, Ball et al. (2008) also highlighted representations as being part of the 'specialised content knowledge' of mathematics unique to teaching. In particular, researchers have highlighted the role that representations play in the explanations of mathematical concepts by teachers (Leinhardt et al. 1991; Brophy 1991). Leinhardt et al. (1991) also identified the skill and knowledge required by teachers in considering the suitability of particular representations, as "certain representations will take an instructor further in his or her attempts to explain the to-be-learned material and still remain consistent and useful" (Leinhardt et al. 1991, 108).

Representations also play an important role in the learning of mathematics by students: "An important educational goal is for students to learn to use multiple forms of representation in communicating with one another." (Greeno and Hall 1997, 363) More broadly, multiple representations play an important role in the development of learners' mathematical understanding: "They can be considered as useful tools for constructing understanding and for communicating information and understanding." (Greeno and Hall 1997, 362) In considering the role of representations within understanding, we make the distinction between internal and external manifestations of representations (Pape and Tchoshanov 2001), or 'mental structures' and 'notation systems' respectively as referred to by Kaput (1991). Understanding of a mathematical concept is based on the internal representations of a concept, which are influenced by the external representations of the concept that are presented to learners (Hiebert and Wearne 1992). Researchers (Hiebert and Carpenter 1992; Barmby et al. 2009) have defined mathematical understanding as being a network of internal representations, with more and stronger connections denoting greater understanding.

Representations therefore play an important role in the understanding of learners, and also in the pedagogical processes involved in developing that understanding. For pre-service teachers, who are developing their own understanding and learning how to teach the subject of mathematics, their knowledge of mathematical representations is even more important. However, Turner (2007) highlighted that pre-service teachers' choice and use of representations could be problematic. In this paper, we report on work with a cohort of pre-service primary teachers, with the aim of developing their understanding of mathematics and their confidence in their subject knowledge and their teaching of mathematics. This was attempted through the introduction and use of representations associated with mathematical concepts covered in primary schools. We provide further detail on this input to pre-service teachers in the section below.

Methodology

The sample of pre-service teachers involved in this work was a cohort of 77 students on a 38-week long postgraduate teaching course (PGCE). The programme offered in mathematics is well established and helps the students to explore both pedagogy and content within the primary mathematics curriculum through lectures, seminars and workshops involving leading mathematics teachers from the local authority. However, in 2009/10, the input for students was reorganised with most of the sessions focussing on a 'representational paradigm', based on the research ideas outlined above. In sessions, a variety of representations for a mathematical concept would be introduced to the student teachers. Students would be encouraged to 'explore' what characteristics of a mathematical concept were emphasised by a particular representation; for example, considering the possibility of there being key representations which were more useful for explaining and understanding key ideas, and considering how the representations could be used to make sense of the various procedures (or algorithms) associated with the mathematical concept. In addition, as a medium for exploring ideas on representation, we used a suite of computer programmes that we had devised ourselves and which allowed the representations to be explored in a dynamic and interactive way. The programmes were created as a stimulus and as a scaffold for class discussion.

Alongside the input provided to the pre-service primary teachers, the aim of the study was to examine the impact of the input of this representational approach on the student teachers involved. More specifically, the objectives of the study were to (a) measure the change in pre-service teachers' attitudes towards their subject knowledge in mathematics, and also towards teaching the subject; (b) gain some qualitative insight into why the input incorporating representations might impact on teacher attitudes. In justifying the first objective, past research has highlighted that there is a link between teachers' beliefs/attitudes and instructional practice, although this link can be complex (Thompson 1984). In terms of actually examining teachers' attitudes towards the subject, Ernest (1989) identified the two components of teachers' attitudes towards mathematics and towards teaching mathematics. Relich et al. (1994) similarly identified two dimensions of pre-service teachers' attitudes. The attitudes of the pre-service teachers towards studying mathematics and their attitudes towards teaching mathematics were measured at the start and end of their 38-week course of study using a questionnaire. We developed measures for attitudes towards studying mathematics and towards teaching mathematics. The two attitude measures consisted of 8 and 7 questionnaire items respectively, with responses to the items elicited on a 5-point Likert Scale of Strongly agree through to Strongly disagree. The Cronbach alpha reliabilities were 0.91 and 0.89 for the attitudes towards studying and teaching mathematics measures respectively, with the unidimensionality of these measures examined through exploratory factor analysis. Using these measures, pre-service teachers in this study were surveyed at the start and end of their course. In addition to surveying this particular cohort of pre-service teachers that received the input on representations, a second cohort of students on the first year of a three-year undergraduate teaching programme was also surveyed to provide a comparison group for the attitude measures. These students were receiving input with regards to both pedagogy and content within the primary mathematics curriculum, but without a specific emphasis on representations. In total, 65 students in the intervention group and 69 students in the comparison group completed the pre- and post-measures of attitude.

In addition to examining the impact of the input on teachers' attitudes, the pre-service teachers receiving the 'representational input' were asked at the end of the course to reflect on their own learning as part of the course. To do so, they were asked to provide written answers to open-ended questions on a questionnaire. These questions asked students for areas of maths where they had deepened their understanding, and also incidents/events which had helped them learn.

Results

The results are discussed in two parts. First we explore the quantitative data from the attitude measures which show the impact of the teaching input on the attitudes of the pre-service teachers. Figure 1 shows the change in the average measures of attitudes for the input group and the comparison group over the course of the year.

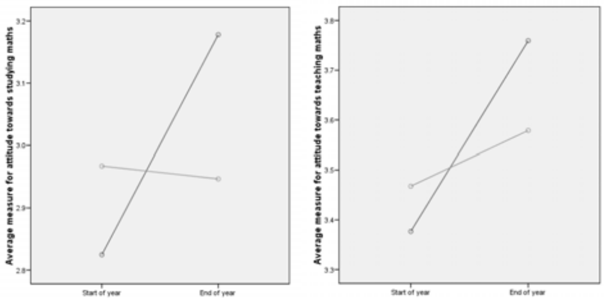

Figure 1: Change in the average attitudes towards studying maths (left) and teaching maths (right).

The average attitudes for the input group showed a greater increase over the year than for the control group for both measures (confidence in studying maths and confidence in teaching maths). Analysing the data using repeated measures ANOVA showed that the change in the attitude measures for the input group as compared to the control group was statistically significant ($p < 0.05$) for both measures. The effect size for the changes for the input group were 0.47 for confidence in studying maths and 0.64 for confidence in teaching maths (compared to -0.03 and 0.19 respectively for the control group).

In addition to the data from the attitude measures, student teachers' responses to the more open-ended questions were examined. The responses to the questions 'State a topic or concept in maths in which you feel you have deepened or modified your understanding' and 'Describe an incident or event which helped you learn' were analysed. The frequencies of responses for different topics identified, and also for different incidents/events mentioned are shown in Figure 2.

A variety of topics were highlighted by students, however two in particular were mentioned by about 30% of the students: multiplication and division, and shape. In terms of incidents and events, the most common category of response was associated with the value of discussion, mentioned by 49 (71%) of the sample. Given below are direct comments taken from individual responses.

> Discussion with university tutors and the class teacher helped to develop both my subject knowledge and knowledge of how to teach it in an engaging way.

> Talking to others to develop my understanding. For example speaking to (lecturers) in individual (or small group) sessions and talking to peers to share and compare different strategies for tackling mental maths problems.

Second in importance to the discussion, 43 (62%) of the students referred to the value of visual representations in helping with their learning in mathematics.

> Visual representations helped me understand multiplication and fractions.

> Visual representations accompanied by explanations and talking to others helped develop my understanding.

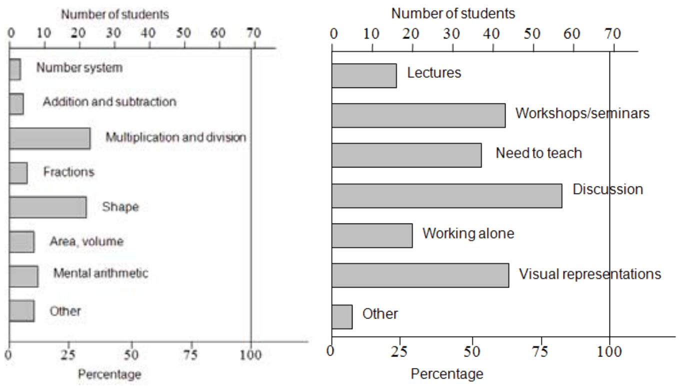

Figure 2: Topics identified (left) and incidents/events mentioned by student teachers (right).

Discussion

In this study, the ‘representational approach’ that we used with the particular group of pre-service primary teachers had a possibly significant impact on their attitudes towards studying and teaching mathematics. We identified the important roles that discussion and visual representations played in developing these attitudes to mathematics. In order to explain why these elements might be important for pre-service teachers, we first examine the issue of developing pre-service teachers’ understanding of mathematics.

Ball (1990, 458) emphasised the importance of understanding the subject for teachers: “Teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways”. We can conceptualise ‘understanding’ in terms of connections made between (internalised) representations of mathematical concepts through reasoning processes (Hiebert and Carpenter 1992; Barmby et al. 2009). Therefore, developing the range of representations (in the case of this study, visual representations) that pre-service teachers have available to them will develop their understanding of a mathematical concept. However, increasing the range of representations for teachers is not enough in itself – teachers also need to develop the connections that they have between representations, for example the connection between visual representations and symbolic representations or algorithms.

It is with regards to this development of 'connections' between representations that discussion can play a role. Drawing on Hoyles' (1985) work on discussion and learning mathematics, she highlighted three aspects to discussion: articulating ideas brings about reflection on those ideas; discussion involves framing ideas in a way that will be accepted by others; listening to others modifies your own thoughts. Interpreting these ideas in terms of our view of understanding, in discussing our mathematical ideas, we modify the representations that we have and the connections that we have made, both through our own reflection and as a result of articulating our understanding, and also through comparisons with other people's understanding. Therefore, we see the importance of representations for understanding, and the process of discussion in developing that understanding, as going hand-in-hand.

For pre-service teachers however, representations of mathematical concepts have an additional importance. As highlighted in the introduction, representations are important for the explanation of mathematical concepts in the classroom as well (Leinhardt et al. 1991; Brophy 1991). In terms of our view of understanding, we are developing pupil understanding through the introduction of representations from which they can reason to symbolic or procedural representations. Therefore, representations have the dual role as tools for developing teachers' own understandings, and also tools for explanation in developing pupils' understanding. It is for these reasons based on the research, and the qualitative comments made by the pre-service teachers, that we see why visual representations and a discussion-based approach might develop pre-service teachers' attitudes towards studying and teaching mathematics.

We have argued that a 'representational approach' with pre-service teachers may be affective for developing their attitudes towards the subject of mathematics. However, we need to also introduce a note of caution in our discussion. Firstly, we draw further on Ball's (1990, 458) ideas: "(Teachers) need to understand the subject flexibly enough so that they can interpret and appraise students' ideas, helping them to extend and formalize intuitive understandings and challenging incorrect notions". This quote states that teachers need to build on pupils' existing understanding, and the implication of this is that teachers also need to be aware of suitable representations to introduce to pupils at their level of understanding. As stated by Cobb et al. (1992, 2), "meanings given to these representations are the product of students' interpretive activity." Therefore, a further development in pre-service teachers' use of representations would be this awareness of representations for different pupils (e.g. different ages) (Barmby et al. 2009). Furthermore, as instructors on university courses, we in turn need to be aware of different levels of understanding for different student teachers and in different areas of mathematics. It is noticeable in this study that the impact on pre-service teachers' knowledge was perceived to be less in fractions (a common area of difficulty for pre-service teachers, Ball 1990; Tirosh 2000). Therefore, further work is required in using representations to develop pre-service teachers' knowledge and confidence in particular areas of mathematics.

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Children's perceptions of, and attitudes towards, their mathematics lessons

Alison Borthwick

Among the reasons attributed to the crisis in mathematics education, disaffection with pupils remains high. While there are studies that investigate this pupil disaffection at secondary school, there are few that consult younger children in order to ascertain their views of mathematics. The research study examines this issue by using drawings as the primary source of data collection, followed by interviews. It offers a view of how some children perceive their mathematics lessons and what this could mean for the future of the subject.

Introduction

Standards of mathematics have been much discussed and criticised over the past three decades (e.g. Buxton 1981, Cockcroft 1982). Adults frequently claim dislike or incompetence towards the subject, while many pupils choose not to pursue mathematics post-compulsory education. Recent reports (e.g. Smith 2004, Brown et al 2008) evidence a shortage of people qualified in mathematics in the U.K. The primary school curriculum has undergone several changes in an attempt to raise standards in mathematics. One of the most recent changes was the implementation of the National Numeracy Strategy (NNS) (DfEE 1999) across primary schools in England. However, in a climate where mathematics has continued to have a lack of support from both adults and pupils this study aims to explore the perceptions that primary pupils have about their mathematics lessons in order to understand how these lessons may be framed to enhance both academic and socials gains and attitudes.

Among the reasons attributed to the crisis in mathematics education, disaffection with pupils remains high. While there are studies that investigate this pupil disaffection at secondary school, there are few that consult younger children in order to ascertain their views of mathematics. One of the potential reasons for the lack of studies involving young children may be the difficulties in consulting them.

This study addresses this issue by using drawings as the primary source of data collection, followed by interviews. I acknowledge that using children's drawings to consider their perceptions of mathematics lessons is an unusual way to approach a difficult area on which to gather firm evidence. However, while the research is therefore speculative I also propose that this methodological tool is a catalyst to provide a forum for teacher discussion and reflection.

Literature review

The practice of consulting pupils is not new. In 1989 an international framework was introduced to change the way children and young people were viewed and treated by societies. The United Nations Convention on the Rights of the Child (1989) stated that the rights of children are that of autonomous individuals and that it was imperative that they should have a voice in matters concerning their lives. This included giving their views on education. However, Davies and Kirkpatrick observed that

> England and Wales seem to be out of line with the rest of Europe in the way that young people have no legislated and government-supported ways to participate in

> decisions about their education. There are no ways in which they can be consulted regularly about Educational policy.
>
> (Davies and Kirkpatrick 2004, 20)

In 2001 the DfES published a White Paper called '*Schools: Achieving Success*'. This paper stated that 'we will encourage students' active participation in the decisions which affect them about their learning and more widely' (DfES 2001, 28). Flutter and Rudduck (2004) carried our much research into the consultation of pupils with the aim of improving teaching and learning in schools. Another study by Arnot et al. (2004) looked at pupils' own perspectives on classroom learning and teaching. They recognised that the provisions for pupil consultation had risen over the years through such means as school councils and that there seems little doubt that 'from an early age, young people are capable of insightful and constructive analysis of their experiences of learning in school' (2004, 4). The findings from these studies (and many more) are encouraging in showing that the consultation of pupils about their learning is now being more actively sought, but there remains a lack of research material in primary mathematics education.

Just as the practice of pursuing pupils' perceptions is not new, neither is the use of a collection of drawings as a way of gathering data. Psychologists and art therapists have used drawings for years as a way of gathering information about emotional and psychological aspects of children. For example, Cox (2005) chose to observe children while they constructed their drawings while Matthews (2003) looked at why children's visual representations and expressions are important in showing adults their expressions and emotions about certain issues. Other examples include studies that have explored children's experiences of school (e.g. Prout and Phillips 1974 and Klepsch and Logie 1982) and children's perceptions of mathematicians (e.g. Rock and Shaw 2000 and Picker and Berry 2000).

Malchiodi (1998) claimed that using drawings provides a multi-dimensional view of children. I believe that it can also provide a catalyst for teachers to engage in reflection on their own teaching practices. Drawings could provide an alternative way (to current methods such as lesson observation and attainment results) to encourage teachers to adopt a more reflective approach to their practice.

Methodology and context

Four schools covering the primary age range were selected to participate in the collection of data. All of the schools were based in Norfolk. One teacher per school was identified based on their teaching experience, their interest in the use of drawings and their access to more than one year group. Using the script below the teachers asked the children to draw a picture of their mathematics lessons.

> *I would like you to draw a picture of your mathematics lessons. Your picture should show what you think mathematics is like in your classroom. I will give you a plain piece of paper on which to do your drawing but you can choose whether to use your normal pencil or coloured pencils. The drawings are going to be looked at by someone else who is interested to see how you view mathematics lessons. Try to think about the mathematics lessons you have and what your drawing could include. From looking at your drawing someone should be able to see what your mathematics lessons are like and how you view them.*

Figure 1. Copy of the script the teachers read aloud to the children

In total 162 drawings were collected. In line with other studies that had used drawings (e.g. Macphail and Kinchin 2004) I decided to reduce the number of pictures to a more manageable number to use in a more detailed way. I pursued a content/item analysis (Di Leo 1983, Gamradt and Staples 1994, Finson et al 1995) approach as the procedure for interpreting the drawings. This was used to create categories which defined and objectified content. In this way I looked for either a representational or symbolic style, or both, evident in each drawing.

15 categories were identified. Whilst many drawings contained features pertinent to more than one category each drawing was coded using the most significant and overwhelming features initially. The table below shows the number of drawings in each category.

Category	**Label**	**Number of drawings from KS1**	**Number of drawings from KS2**	**Total number of drawings**
Positive showing affection	A	2	8	10
Negative showing disaffection	B	0	17	17
Pair of pupils	C	7	3	10
Pupil with multilink tower	D	6	0	6
Counting	E	15	0	15
Several pupils but no teacher	F	0	7	7
Whole class	G	0	6	6
No indication of lesson type	H	2	7	9
One teacher and one pupil	I	1	4	5
Thinking skills	J	2	16	18
Defensive, worried, anxious	K	0	8	8
Pupils in a group with the teacher	L	1	4	5
Child with back to drawing	M	1	4	5
No teachers and no pupils	N	0	14	14
One child with examples of mathematics and or resources	O	18	9	27
Total Number of drawings		*55*	*107*	*162*

Figure 2. 15 categories that represent the collection of 162 drawings

In order to establish reliability towards the categorisation of the drawings I approached two independent reviewers. As Finson et al. (1995) point out, this helps to reduce the chance of a particular thematic content area being duplicated or missed out. Both individuals were given the collection of the 162 drawings, the 15 category titles and definitions and were asked to sort the drawings into the categories I had previously defined. Overall, despite a few minor discrepancies, both reviewers placed the drawings into the same categories as I had done previously.

Once the drawings had been sorted into the categories I chose one drawing to represent each category. These 15 drawings, I believe, are the exemplars of each category. A few weeks later after the completion of the drawings I interviewed 11 (from a possible 15) of the pupils. The interviews were carried out in two groups according to key stage. There were 5 children in one group (key stage 1) and 6 children in the other (key stage 2). The interviews were to provide further validation towards the interpretation of the drawings.

Findings and discussion

Within the two aims of the research – to explore the perceptions that pupils have with regard to their mathematics lessons and to consider how the use of drawings can support this inquiry – four themes were explored. These themes were:

1. Evidence of the children's emotions and attitudes in mathematics lessons.
2. Evidence of the children's perceptions of their peers in mathematics lessons.
3. Evidence of the children's perceptions of their teacher in mathematics lessons.
4. Evidence of mathematics in the drawings.

Whilst the first and last themes are indicative of the two main aims of this research the other two consider areas that the NNS (DfEE 1999) considered important within this phase of primary mathematics education. The first was that children should be encouraged to work more collaboratively while the second suggested that teachers should play a more dominant role in making lessons more interactive with themselves as a role model. The drawings offer potential perceptions of some of the children towards these two issues.

The following section offers two drawings under each of the four themes in an attempt to provide a flavour of the drawings and the perceptions of the children.

Figure 3. Theme: Evidence of the children's emotions and attitudes in mathematics lessons.

Figure 4. Theme: Evidence of the children's perceptions of their peers in mathematics lessons.

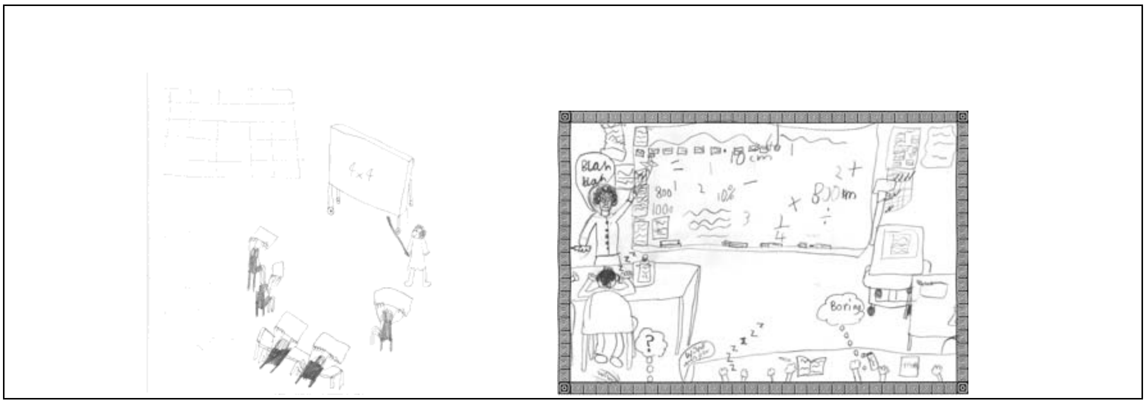

Figure 5. Theme: Evidence of the children's perceptions of their teacher in mathematics lessons.

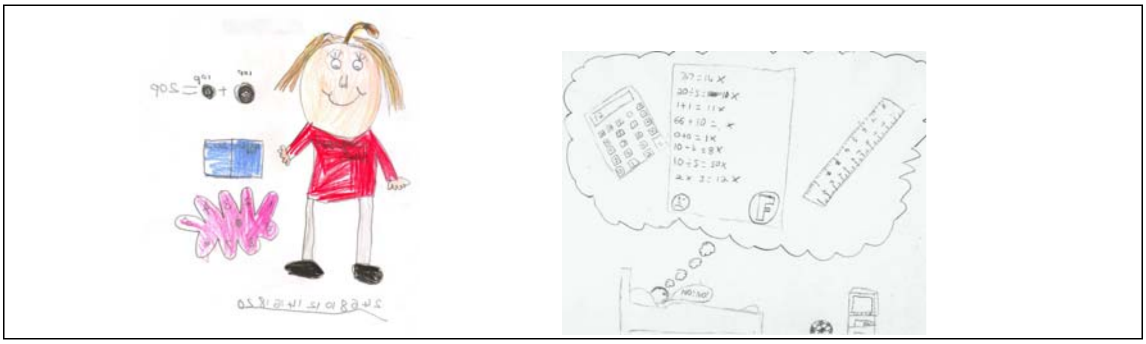

Figure 6. Theme: Evidence of mathematics in the drawings.

Conclusion

If we are to address the current crisis in mathematics education we need to find ways of consulting pupils earlier than we currently do. This study has revealed that certain elements of the NNS (DfEE 1999) are not as secure as perhaps policy makers presumed they were. Whilst a range of emotions were exhibited towards mathematics by the children in the study, already early elements of disaffection were beginning to show from the younger boys. Whilst some children drew themselves sitting in groups, they did not perceive that they worked as a group, despite their preference to (as evidenced in the interviews). Many of the drawings did not include a teacher. Whilst much could be drawn from this, these drawings offer an opportunity for reflection among teachers and the ways they present themselves in the class. Finally the drawings were dominated by number and calculations references, but little other mathematical content, yet for some children it is the other bits of mathematics that appeal and engage interest. Whilst the data revealed only potential issues and reasons for perceptions and attitudes towards mathematics lessons in primary schools, I believe it is a good starting point and may prove an effective means of influencing teachers' thinking and actions.

The results of this study revealed that we can discover what primary aged children think about their mathematics lessons. Drawings offer a unique way of discovering these perceptions. It is as Saint-Exupery (1958) writes, that grown ups cannot, on their own, understand the world from the child's point of view, and therefore they need children to explain it to them.

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The use of mathematical tasks to develop mathematical thinking skills in undergraduate calculus courses – a pilot study

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Mathematical thinking is difficult to define precisely but most authors agree that the following are important aspects of it: conjecturing, reasoning and proving, making connections, abstraction, generalization and specialization. In order to develop mathematically, it is necessary for learners of mathematics not only to master new mathematical content but also to develop these skills. However, undergraduate courses in Mathematics tend to be described in terms of the mathematical content and techniques students should master and theorems they should be able to prove. It would appear from such descriptions that students are expected to pick up the skills of (advanced) mathematical thinking as a by-product. Moreover, recent studies have shown that many sets of mathematical tasks produced for students at the secondary-tertiary transition emphasize lower level skills, such as memorization and the routine application of algorithms or procedures. In this paper we will consider some suggestions from the literature as to how mathematical thinking might be specifically fostered in students, through the use of different types of mathematical tasks. Efforts were made to interpret these recommendations in the context of a first undergraduate course in Calculus, on which large numbers of students may be enrolled. This itself constrains to some extent the activities in which the teachers and learners can engage. The tasks referred to here are set as homework problems on which students may work individually or collaboratively. We will report preliminary feedback from the students with whom such tasks were trialled, describing the students' reactions to these types of tasks and their understanding of the purposes of the tasks.

Keywords: mathematical thinking, tasks

Mathematical thinking

Many authors agree that the mathematical practices and thinking to be encouraged in learners of mathematics should mirror the practices of professional mathematicians. However, there are many different definitions and interpretations of the term 'mathematical thinking'. For instance, Hyman Bass (2005) speaks about the mathematical practices or habits of mind of research mathematicians and argues that these practices such as experimentation, reasoning, generalization, the use of definitions and the use of mathematical language can be fostered at any stage in the education system. Mason and Johnston-Wilder (2004) propose that questions posed to students draw on the following words "exemplifying, specializing, completing, deleting, correcting, comparing, sorting, organizing, changing, varying, reversing, altering, generalizing, conjecturing, explaining, justifying, verifying, convincing, refuting" (2004,109) as they believe these words denote the processes and actions that mathematicians employ when they pose and tackle mathematical problems.

Moreover, the Mathematics Learning Study Committee of the US National Research Council (Kilpatrick et al 2001) uses the notion of ‘mathematical proficiency’ to describe how people learn mathematics successfully. They believe that this has five interwoven strands which should be encouraged and developed together: conceptual understanding; procedural fluency; strategic competence (the ability to formulate and solve mathematical problems); adaptive reasoning (capacity for logical thought, reflections and justification); productive disposition (seeing mathematics as worthwhile and being confident in one's own abilities) (2001, 116).

However, there is evidence to suggest that at undergraduate level, courses often focus on procedural fluency only, to the detriment of the other strands of mathematical proficiency. Dreyfus (1991) asserts that most students learn a large number of standardised procedures in their mathematics courses but not the ‘working methodology of the mathematician’ (28). He says:

> They have been taught the products of the activity of scores of mathematicians in their final form but they have not gained insight into the processes that have led mathematicians to create these products (1991, 28).

He claims that this lack of insight means that students have knowledge but are not in a position to use it in unfamiliar situations.

Studies classifying tasks

Recent studies have undertaken work investigating the types of tasks that are assigned to students as homework or appear on examinations. Boesen, Lithner and Palm (2010) considered tasks from Swedish national second level high stakes examinations and used textbooks to classify them according to how familiar they were to students, claiming that exposure to familiar tasks alone affects students' ability to reason and so influences student learning. They found that often no conceptual understanding was needed to solve familiar tasks. They used the terms ‘imitative reasoning’ and ‘creative mathematically founded reasoning’ to characterize the types of reasoning that students might use to solve problems. Imitative reasoning involves using memorization or well-rehearsed procedures, while creative reasoning is novel reasoning with arguments to back it up, anchored in appropriate mathematical foundations.

Bergqvist (2007) also analyzed 16 examinations from introductory courses in Calculus in four Swedish universities and found that 70% of the exam questions could be solved using imitative reasoning alone and that 15 of the 16 examinations could be passed without using creative reasoning.

Pointon and Sangwin (2003) developed a mathematical question taxonomy in order to undertake a classification of undergraduate course-work questions – this taxonomy is illustrated in Table 1. Successful completion of tasks following 1-4 of the table are deemed characteristic of ‘adoptive learning’ in which students behave as ‘competent practitioners’, engaging in an essentially reproductive process requiring the application of well-understood knowledge in bounded situations. While questions in classes 4-8 typically require higher cognitive processes such as creativity, reflection, criticism; and would be characterized as ‘adaptive learning’, requiring students to behave as ‘experts’.

Pointon and Sangwin (2003) used their taxonomy to classify a total of 486 course-work and examination questions used on two first year undergraduate mathematics courses, finding that 61.4% of all questions inspected related to class 2 of Table 1 while only 3.4% of questions related to classes 6-8. They concluded that:

> (i) the vast majority of current work may be successfully completed by routine procedures or minor adaption of results learned verbatim and (ii) the vast majority of questions asked may be successfully completed without the use of higher skills (2003, 8).

Adoptive Reasoning	Adaptive Reasoning
1. Factual recall	5. Prove, show, justify - (general argument)
2. Carry out a routine calculation or algorithm	6. Extend a concept
3. Classify some mathematical object	7. Construct an instance
4. Interpret situation or answer	8. Criticize a fallacy

Table 1: Mathematical question taxonomy of Pointon and Sangwin (2003)

Frameworks for tasks

Others have created frameworks to help educators create tasks that would foster and assess aspects of mathematical thinking. Focusing on fostering conceptual understanding, Swan (2008) created a framework of five task types for use at second level. They are classifying mathematical objects, interpreting multiple representations, evaluating mathematical statements, creating problems, and analyzing reasoning and solutions. He believes that students should be encouraged to talk and write about mathematical ideas, and that teachers should emphasize reasoning and not 'answer-getting'. Earlier, Schoenfeld (1992) created a framework for balanced assessment in an NSF-funded project in which he considered the dimensions under which mathematical tasks could be measured: content (including procedure and technique, representations and connections); thinking processes; student products; mathematical point of view; diversity; circumstances of performance; pedagogics-aesthetics. The emphasis is on balance and Schoenfeld recognizes that any one task could not foster all types of thinking, for example, but that when a set of tasks is being designed one should aim to cover as many different dimensions as possible. Mason and Johnston-Wilder (2004) also advocate a ``mixed economy" (6) in which learners are given a variety of types of tasks to develop mathematical thinking.

Sample tasks for first calculus

In an effort to move away from tasks involving imitative reasoning only and following the advice of Swan (2008) and Schoenfeld (1992), the authors designed a set of tasks for use in undergraduate Calculus courses. These tasks asked students to generalize and specialize, generate examples, make conjectures, reason, make decisions, explore, make connections, and reflect. Some examples of the tasks used and their relation to the frameworks are shown below:

1. Suppose $g(x)$ is an odd function. Is $h(x) = 1/g(x)$ odd? Justify your answer.

   (Swan - classifying mathematical objects; Pointon and Sangwin - classify and justify.)

2. Determine whether the reasoning used in the following is satisfactory, giving reasons to support your answers.

   Conjecture: Suppose *a* and *b* are real numbers such that $(a+b)^2 > a^2 + b^2$. Then $a > 0$ and $b > 0$.

   Proof: If $a > 0$ and $b > 0$ then $ab > 0$. Thus, $(a+b)^2 = a^2 + b^2 + 2ab > a^2 + b^2$.

   (Swan - analysing reasoning and solutions; Schoenfeld - analyse, decision and justification.)

3. Does every rational function have a vertical asymptote? Explain.

   (Swan - evaluating mathematical statements; Schoenfeld - reflect, explore.)

4. Give an example of the following:
   a. A function *f* which is continuous at $x = 5$.
   b. A function *f* which is not continuous at $x=5$ because $f(5)$ is not defined.
   c. A function *f* which is not continuous at $x = 5$ because $Lim_{x \to 5} f(x)$ does not exist.
   d. A function *f* which is not continuous at $x = 5$ because

      $Lim_{x \to 5} f(x) \neq f(5)$.

   (Pointon and Sangwin - Construct an instance; Schoenfeld - explore, choose.)

Methodology

The tasks were trialled with first year Mathematics students at the National University of Ireland, Maynooth (NUIM) and St Patrick's College, Drumcondra (SPD) in the first semester of the 2010/11 academic year. The class at NUIM consisted of 180 first year students. These students were either first year Finance students who were strongly encouraged by their department to take Mathematical Studies or first year Arts students who chose to study Mathematical Studies along with two other Arts subjects. The SPD class consisted of 49 students. These were first year students undertaking either a BEd (Primary) or BA (Humanities) degree who had chosen to study Mathematics to degree level. All students were taking a first course in Differential Calculus – in NUIM this is a one semester course, while in SPD it runs through two semesters. There was a large variation in the mathematical backgrounds of students in both groups.

Each assignment contained some procedural questions as well as at least one question designed or selected with the task frameworks in mind. Five problem sets were assigned during the first semester at SPD, and students were asked to complete them before their tutorial session. The students were twice required to submit solutions to a non-routine problem as part of the continuous assessment for the module. At NUIM, homework was assigned seven times in the semester. Students were required to submit solutions to all questions on each problem set, however only one question per assignment was graded and students were not aware in advance which would be graded. From the seven problem sets, four traditional and three non-routine questions were corrected.

Student reaction to a selection of the tasks was collected using a questionnaire during the last week of the semester. In each institution, students were asked to comment on pairs of tasks which comprised one traditional question and one non-routine question on the same topic. They were asked to comment on the purpose of each task, whether the task contributed to their knowledge and understanding, and on the differences between the traditional and the unfamiliar tasks. Participation in the survey was voluntary and anonymous. In total 101 students completed the questionnaire - 27 at SPD and 74 at NUIM.

Students' reactions to tasks

We will first consider the students' performance on the tasks. At SPD, students submitted solutions to Tasks 1 and 2. 75% of students there gave a sound argument in response to Task 1. About half of the group realised that the conjecture in Task 2 is false but only 8.5% also realised that the proof given addressed the converse of the statement. A further 43% of students considered the proof in isolation and commented on its correctness. At NUIM, the mean score on the routine problems (65.7%) was significantly higher than the mean score on the non-routine questions (59.7%). Task 4 was one of the questions selected for grading. Almost all students were able to give examples of functions in a) and b) but about 30% had problems with parts c) and d).

At SPD, students were asked to comment on a pairs of tasks on even and odd functions. The first task was routine, the second (Task 1 here) non-routine. The majority of respondents remarked that the non-routine task was more challenging or required more thinking than the routine one. One student said:

> The first task was more basic going over the skills learnt in lectures while the second task involved us thinking more about what exactly we did, without direct examples. Had to use our knowledge to solve an unfamiliar problem. (SPD 9)

At NUIM, students were asked to comment on the differences between two tasks on rational functions. Again the majority of students felt that the non-routine task (Task 3) was more challenging and required more thinking or understanding. Some students felt that Task 3 involved opinion or theory and was not 'mathematical':

> Problem 1 [routine task] was mathematically based, and the other [Task 3] was theory based. (NUIM 26)

Students at both institutions felt that both routine and non-routine tasks deepened their knowledge and understanding but a larger proportion agreed that this was true of the non-routine tasks. The exception to this was Task 3. 41% of respondents at NUIM reported that this task deepened their understanding of rational functions while 69% said the same of a procedural task on the same topic. When asked to comment on the purposes of the non-routine tasks, students at both institutions mentioned the notions of the task as a means of gaining knowledge or understanding; as a test; or as practice. Some also spoke of working independently:

> To allow us to tackle problems without the guidance of a lecturer to test our knowledge and understanding of the topic. (SPD1)

Conclusion

The students in this study seem to have a mature understanding of the purposes of the tasks. The non-routine assignment questions have challenged them and they have reported that these questions require more thinking and understanding than the tasks with which they are familiar. Selden, Selden, Hauk and Mason (2000) investigated the ability of second year university calculus students to solve non-routine problems. They found that more than half of their students could not solve any problems even though on a separate test although they had demonstrated that they were familiar with the techniques needed. They recommended that lecturers should ‘scatter throughout a course a considerable number of problems for students to solve without first seeing very similar worked examples’ (150). We have tried to follow this advice, aided by the task frameworks mentioned earlier. These frameworks have enabled us to use a greater variety of question types. The students have recognised that these tasks are different from the traditional ones that they are used to. The tasks have helped to get some students thinking in a new way and may even have begun to change students’ view of mathematics.

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Gattegno's 'powers of the mind' in the primary mathematics curriculum: outcomes from a NCETM project in collaboration with "5x5x5=Creativity"

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In this paper I report on the outcomes of a 'Mathematics Knowledge Network' (MKN) project, aimed at developing rich tasks in the primary curriculum. The work was funded by the National Council for Excellence in Teaching Mathematics (NCETM) and carried out in collaboration with the Arts based charity "5x5x5=Creativity". The approach to the project was informed by Gattegno's ideas on the 'powers of the mind'. I report on the answers to three questions: can KS2 students access rich tasks designed for KS3, using their 'powers of the mind'?; what is the potential for KS2 students' capacities to use and apply mathematics?; what is the potential for collaboration between mathematics and arts based education?

Keywords: Gattegno, 'powers of the mind', creativity

Background

In this paper I report on the outcomes of a 'Mathematics Knowledge Network' (MKN) project, aimed at developing rich tasks in the primary curriculum. The work was funded by the National Council for Excellence in Teaching Mathematics (NCETM) and carried out in collaboration with the Arts based charity "5x5x5=Creativity". The motivation for the project came from a headteacher who wanted support in developing a more creative approach to teaching mathematics in school C, where he worked.

The project comprised twelve weekly sessions focused on developing skills of using and applying mathematics, partcularly being organised and systematic, with two classes of year 3/4 students (aged 7/8). I taught eleven of these sessions which were jointly planned with the two class teachers, who helped out in the lessons; there were reflection meetings each week to discuss, evaluate and plan the following week. In the last two sessions, following input from "5x5x5=Creativity", we gave all students a 'Learning Journal' in which to document their work, organise what they did, and take forward any questions they still had.

Research questions

We began this project with three key questions, the first one linked to my previous experience of working with rich tasks at KS3, 4 and 5.

- could KS2 students access rich tasks designed for KS3 and beyond?
- what is the extent of year 3/4 students' capacities for Using and Applying mathematics?
- what is the potential for collaboration between mathematics and arts based education practices?

Before getting to these questions, I set out the background theoretical ideas, derived from Gattegno, that informed the project design and implementation.

Theoretical framework: Gattegno's 'powers of the mind'

Through studying the early language acquisition of children Gattegno (1971) analysed four common "powers of the mind" (1971, 9) possessed by everyone who is able to master their mother tongue.

- the power of extraction - finding "what is common among so large a range of variations" (1971, 10)
- the power to make transformations – e.g., my Dad ~ my Mum's partner ~ my sister's Dad ~ my uncle's brother
- handling abstractions – e.g., any noun is a label for a general set of objects
- stressing and ignoring – e.g., focusing on one aspect of perception to the exclusion of others.

The heart of Gattegno's approach to education is to offer activities and contexts that allow learners to continue to access these four powers. I provide some more detail below on how this framework translates into practical implications for the classroom.

'The power of extraction' and 'stressing and ignoring'

These two powers combine to mean that *all* students can respond to being offered more than one example, image or action, and being asked to find what is the same or what is different (see Brown & Waddingham 1982; from where I took both activity starting points used with the Y3/4 classes). Given a pair, or wider range of items to compare students can extract common features by stressing some aspects of what they see or do, and ignoring others. At the start of one of the activities I offered the year 3/4 class, I drew on an interactive whiteboard the following image.

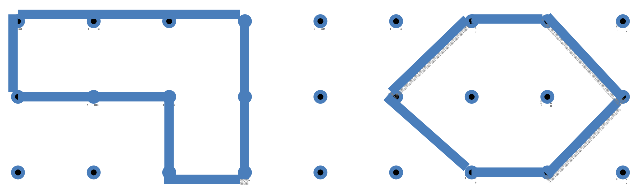

Figure 1: Introduction to working on Pick's Theorem

I began with the statement 'these are both four square shapes', and the invitation 'someone come and draw another, different four square shape'. I was aware that these students would not have been taught 'area' yet. I assumed that students would be able to use the power of extraction to see a common feature of the two shapes I drew, as was indeed the case in both classrooms. The starting statement 'these are both four square shapes' is deliberately not an explanation; the invitation is for students to begin *using* the concept of a 'four square shape'. I offer, as teacher, two examples of use; by attending to what is the same and what is different (stressing and ignoring) students will be able to continue such use. Only later, and perhaps never, will there be discussion of what it means to be a 'four square shape'. I am aware at KS3 that a slightly mysterious beginning, where there is patently some 'sense' of the situation to be made by the students can be engaging. I am also wanting to convey a message *about* our work together, that I am not going to be the one who provides explanations, and that there is work to do, which I assume everyone can do, using the powers of extraction and stressing and ignoring.

'The power to make transformations' and 'handling abstractions'

The use of notation can never be far from the doing of mathematics. We know from students' facility with their first language that there is no issue in the use of arbitrary labels (such as 'dog') to stand for a potentially infinite variety within a set of similar things. The key to accessing this power to handle abstractions is again that they are *used* rather than defined. Having set up a notation, students' use of the power to make transformations can be accessed in creating or provoking questions linked to the notation. For example, in starting the first activity I worked with students to find the number of ways they could arrange three red and two blue books on a shelf. I wrote one combination and invited students to contribute the rest. When it seemed as though there were no more different possibilities I introduced the (standard) notation $\binom{5}{2} = 10$ to mean that with 5 books in all, of which 2 were blue, the total number of combinations found was 10. With this notation we were able to talk about students trying (5,1), (5,3) or different numbers of books (4,0), (4,1), etc. Students wrote their answers on a board at the front of the class and were able to see patterns or make predictions (e.g., how many combinations if the bottom number is zero or one). I am not wanting to discuss other possible notations, as the choice is essentially arbitrary (Hewitt 1999). I want students to use the notation I offer to help organise their work and notice patterns in their answers in relation to this notation.

Could KS2 students access rich tasks designed for KS3 and beyond?

The answer to this question was an emphatic 'yes'. There seemed evidence for the universality of Gattegno's claims for his 'powers of the mind'. Students assessed as operating at level 1 were able to access both tasks we offered. A Teaching Assistant who supported one of the groups wrote (in a personal communication to me):

> The recordings that the children did, both on the paper an in the journals, highlighted the importance of allowing children to convey their own thoughts and understandings of the subject. I was able to witness the way that they explored their ideas on paper, after an introduction to the 'problem', and discover answers and patterns for themselves.

This teaching assistant worked with a table of students who had the lowest prior attainment in the group; I take her statement as evidence that these students were able to access the tasks through the 'introductions' – as is confirmed by the range of different routes students took in working on the problems. In other words the tasks, developed for students in year 7 and beyond, were accessible to the full range of a mixed ability year 3/4 class.

What is the extent of year 3/4 students' capacities for Using and Applying Mathematics?

Changes in students' organisational skills were particularly evident on the red/blue book task. By the second session almost all students had moved from writing down solutions without discernible pattern, to using some kind of system. In Fig 2, there are four cases worked on by the student, and it can be seen the student has used slightly different systems. In three cases the total is correct, but some combinations have been missed in the (6,3) case. However it is possible to observe in all cases a common strategy of keeping some aspects the same and moving others to generate successive lines. This was a typical feature of students' work.

Despite my convictions about the relevance of Gattegno's ideas and the untapped potential often claimed existed in children, I was struck by the quality and extent of the 7/8 year old students' capacity to organise their work, make and test predictions, use algebra and reason about the answers they found – all skills found at high levels with the National Curriculum strand of Using and Applying Mathematics.

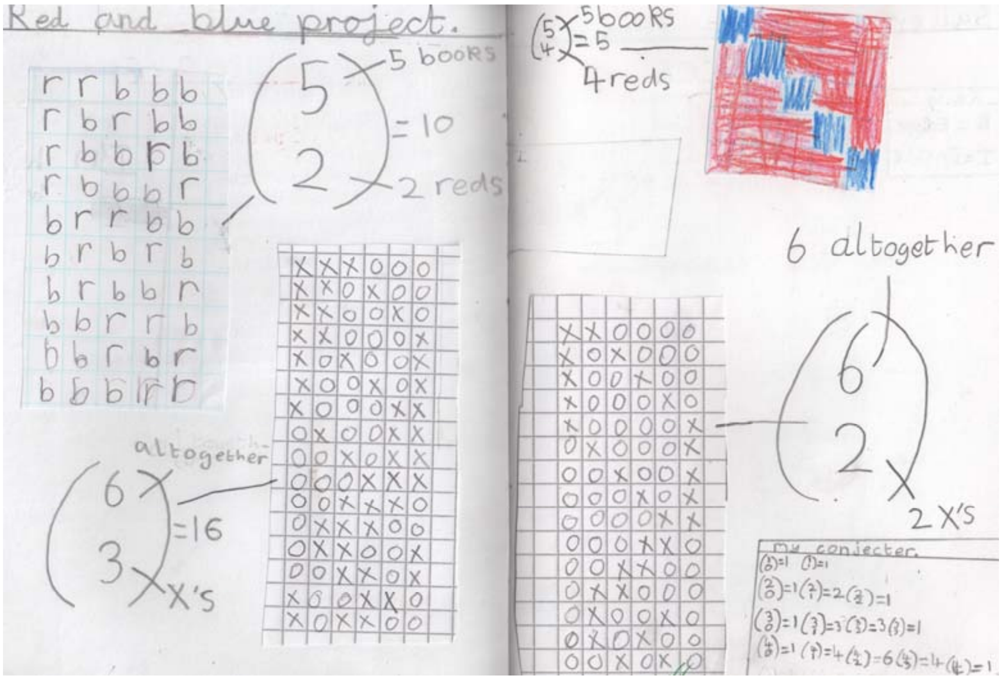

Figure 2: A student's work on the red/blue problem

There was evidence of pattern spotting in almost every student's journal. There were many examples of students making and testing predictions. The most striking (due to the sophistication of the prediction) was the following (see Fig 3).

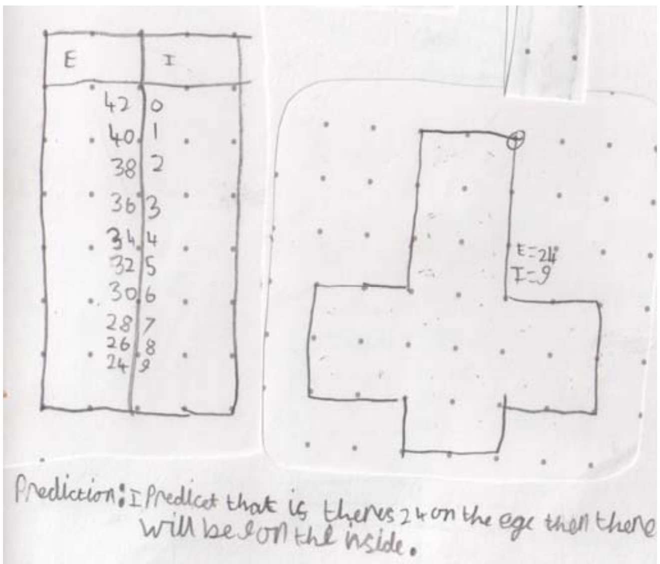

Figure 3: Student prediction and test

The student here followed a pattern they noticed that for any area shape, to get E when I=0, you need to double the area and add 2. He also noticed (as did many students) that as I increased by 1, E decreases by 2. The prediction above (which is tested and found to be correct) follows from the student choosing area 20, predicting that when I=0, E=42 (20x2 +2) and then following down the table until he got to I=9.

One student (who his teacher reported was not achieving highly in relation to the class in mathematics) articulated in discussion a prediction about the number of possible combinations for (5,5), (6,6). I encouraged him to predict for (100,100) and other high numbers and then offered him the notation (n,n) to express his idea and the word ‘conjecture’. His learning journal includes the following section (see Fig 4).

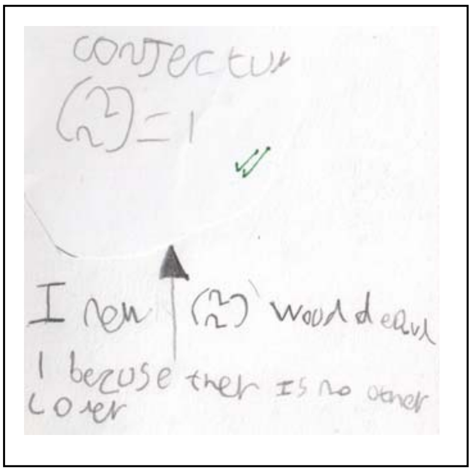

Figure 4: A student conjecture with a justification

I interpret his writing as follows: “I knew (n,n) would equal one because there is no other colour”. The reasoning was written on the journal page, the conjecture cut out from work done in the original session. There is evidence here of the students’ capacity to work with abstraction in expressing this generality with a reason behind it (with no other colour you cannot generate a new arrangement). I was left to wonder the extent to which the relative literacy difficulty this student seems to have might be contributing to his perceived lack of success in the standard maths curriculum.

The National Curriculum places at level 5 (Using and Applying) the skill of deriving a rule, expressing it algebraically, and giving a reason, which I see in Fig 4. The work in Fig 3 is, without the notation, an example of a student working with three variables (Area, E and I) – this appears at level 7 of the same strand. It seems therefore that students in Year 3/4 (and not just those achieving highly) are able to operate at a level of sophistication not normally expected until well into Secondary School, if they are given the opportunity to use the ‘powers’ of their minds.

What is the potential for collaboration between mathematics and arts based education practices?

This project was (and continues to be) a collaboration with “5x5x5=Creativity”. This educational charity places artists in schools to work with students and research the learning that takes place (all Arts based until this project). “5x5x5=Creativity” run professional development sessions for artists and teachers involved in their projects, and at one of these days that I attended with one of the year 3/4 teachers there was an input about the use of ‘Learning Journals’. Following discussion between the teachers at school C we decided to ask for a learning journal for each of our students and to offer them as a way of reflecting on their work and taking forward outstanding questions or issues. At the training day there were examples of journals kept by children and adults, which mostly took the form of collage. I decided I should start a journal about my work on the project, partly to be able to offer the students an image of what could be in a journal. We introduced the journals to students in the second to last session, and all the staff in the room were so impressed by the work the students did that we decided they would be the focus of the final session as well.

The two teachers I worked with commented to me on the value they saw in terms of getting students to reflect on their work. One teacher reported finding it hard previously to get students to be creative or inventive when asked to reflect on a piece of work in any subject. With the learning journals however the students seemed to enjoy going back over what they had done (Reflection meeting 12/11/10). One aspect the teachers spoke of was the chance for students to look at the work they had done and re-organise it. This is illustrated in Figure 5, with one student who cut up her work on 'Pick's Theorem' and organised it according to area (which it had not been on paper) and then added a table of results.

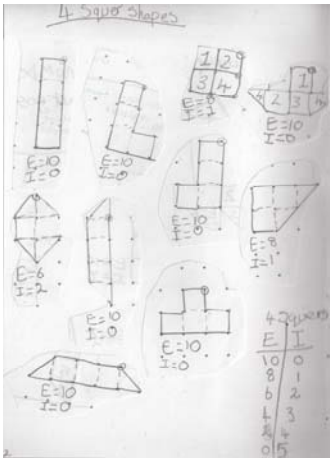

Figure 6: A student re-organising her work by area

The table of results for E and I is written on the paper of the journal and was not part of the student's original work. There is evidence here for the journal allowing the student to collect common shapes together, focus on the values E and I in creating a table – and then extend the table to shapes she has not found. In this example through reflecting on her work via a journal the student is accessing higher levels of skill in terms of Using and Applying Mathematics, that she had previously done on the same topic. The use of journals is just one technique common in Arts education that has been shown to have applicability to mathematics. The project has convinced me there is potential for further fruitful collaboration here.

Conclusion

In presenting this report at the day conference of BSRLM in London in March, several participants at the session were interested in any effect the project had in terms of the school or teachers' views of student 'ability' in mathematics. There were certainly students (e.g. see Fig 4) who surprised their teachers in terms of the quality of work they were able to do (compared to their prior attainment). The students were arranged in tables according to 'ability' in both mixed attainment classes. There was at least one case of a student moving 'up' a table following work produced in a project session; but no evidence of problematising the concept itself.

I am mindful, following a question at the conference session, not to downplay the role of the teacher (which was generally me). I tried, in the section on the powers of the mind, to offer more insight into some of the awarenesses behind the choice of starting points and initial questions, than I did in London, but am aware this remains brief and inadequate. It is of course not simple to work with students so that they access their powers no matter what the starting point. In some sense this report can be seen as an existence proof confirming e.g., the work of Cohen (1989) and Gattegno (1971), in terms of what is possible for 7/8 year olds to achieve mathematically. I hope to continue the work with the school and report findings in a future meeting.

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The application of lesson study across mathematics and mathematics education departments in an Irish third-level institution

Dolores Corcoran and Maurice OReilly, with Sinéad Breen, Therese Dooley, and Miriam Ryan.

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This presentation reports preliminary findings arising from a research project, which embodied cross-disciplinary collaboration into the teaching and learning of Mathematics. The project involved the use of a form of Japanese lesson study by colleagues from the Education Department and the Mathematics Department of a College of Education and Humanities in the Republic of Ireland. Five colleagues worked together to explore the goals of teaching two research lessons; the first of which was part of a module in the history of mathematics for BA students, and the second, a lesson in mathematics education for BEd (Primary) students. Following ethical clearance, the research lessons were videotaped using both a static camcorder focused on the teacher and a roving camera to record student participation. The research lessons were also observed in situ by the remaining participants of the lesson study group. Both research lessons were later transcribed. In this presentation we will report on our initial findings from the different perspectives of preparing, teaching, observing and reviewing the first research lesson. The potential for conducting lesson study in a cross-disciplinary fashion will be discussed.

Keywords: lesson study, research mathematics lesson, collaborative inquiry

Introduction

A call for research proposals by the Irish National Academy for Integration of Research, Teaching and Learning was the catalyst for this research project (NAIRTL 2009). Initially, six colleagues collaborated in preparing a research proposal. All members of the team - Maurice and Sinéad from the Mathematics Department and Dolores, Thérèse and Ronan from the Education Department - were interested in mathematics education research and welcomed an opportunity to explore together their own and each other's practices using lesson study as a means for doing so. Both groups approached the project with enthusiasm, interest and respect while acknowledging that they were engaged in different practices as the former taught undergraduate mathematics courses to BA and BEd students and the latter taught mathematics education courses to pre-service and practising teachers. In 2010, Ronan moved away from mathematics education and was replaced by Miriam. The use of lesson study had already been researched and theorised in the institution as a means of learning to teach mathematics with preservice primary teachers (Corcoran 2008) and members of the research team from the Education Department had some experience of working as Knowledgeable Other (Watanabe and Wang-Iverson 2005) with student groups engaged in lesson study. The inter-departmental, cross-disciplinary research team proposed a new departure, to embrace the potential of lesson study as a means of investigating and developing their own mathematics teaching, and as a model of 'good practice' for student participants.

The spread of lesson study as a protocol and a process

Engagement in the practice of lesson study has been integral to the Japanese educational system for over a hundred years (Isoda 2007). It first came to attention outside Japan from the publication of *The Teaching Gap* (Stigler and Hiebert 1999) and the work of Yoshida (1999). Since then, lesson study as a process for teacher professional development has grown in popularity in the USA, thanks to the work of two scholars in particular, Lewis (2002) and Fernandez (2005) and to their teacher colleagues. Superficially, the lesson study protocol appears to involve a group of teachers collaborating to prepare and plan a single 'research lesson' which one member of the group teaches while the others observe. The research lesson is then reviewed and possibly revised, most often with the aid of a Knowledgeable Other. Lesson study is seen as an iterative process and the preparatory and post-research lesson stages can be of varying length. Early research findings indicated that Japanese teachers brought much more than the three part protocol of prepare, teach/observe and review research lessons when engaged in lesson study. They appeared to adopt three essential lenses, namely the researcher lens, the curriculum development lens and the student lens to focus their engagement in lesson study (Fernandez, Cannon and Chokshi 2003). For groups of teachers engaged in lesson study in one school district the process was considered to lead to deep learning and teacher professional development (Perry and Lewis 2009). As the phenomenon of lesson study become more widespread in the US researchers began to ask why lesson study was so influential in improving teaching. Lewis, Perry and Murata (2006) conjectured that engagement in the lesson study process brought about an increase in teacher knowledge, built teacher community and contributed learning/teaching resources, although they also recognized its value in developing and refining mathematics lessons. Research corroborates that engagement in lesson study as collaboration between university lecturers and preservice teachers (Corcoran and Pepperell 2011) and with practicing teachers (Back and Joubert 2011) is successful in building mathematical knowledge in teaching. But the potential of lesson study is much greater than its contribution to developing mathematical knowledge in teachers, a potential recognized by Tall (2008), for example. As lesson study has become better known throughout the world, there has been a slew of research into many aspects of lesson study and its contribution to enhanced teaching of mathematics (A-PEC 2008; Hart, Alston and Murata 2011).

Establishing a community of inquiry

Preliminary meetings to discuss the research project and familiarize ourselves with the lesson study cycle and relevant research were held in July and September 2009 and the group undertook to engage in two cycles of lesson study over the coming academic year. As a group, we decided to adopt lesson study protocols (Yoshida 2005), and proposed to use the four dimensions of the Knowledge Quartet as a common framework for discussing research lessons (Rowland, Huckstep and Thwaites 2005). The notion of a community of practice (Wenger 1998) was considered as a possible heuristic for thinking about the data being generated as we negotiated the research trajectory together. However, while valuing the depth and complementarity of our educational backgrounds and recognizing the disparity in our mathematical experiences we adopted 'inquiry' as a research stance (Jaworski, 2004; 2008). Our inquiry stance embraced inquiry into lesson study, into each other's practices, into mathematics, into pedagogy and into (mathematics education) research. We set out to build shared understandings and to uncover the tacit in our practices. In consequence the research project was conceived as ‘developmental’ and the design as ‘emergent’.

Forging the research design

As with all school based research, the college and timetable course outlines constrained decisions about when the research lessons could take place, and which topics could be addressed. It was agreed that the first research lesson, reported here, would be taught by Maurice. The actual preparation phase began in December 2009, with meetings of one/two hours’ duration taking place regularly in January and February leading the first research lesson on March 8th 2010. Group members kept field notes and all relevant material was stored on a dedicated Moodle site to which all members had access. There were 8 (out of 9) BA students present at the research lesson. A schedule for observation and data collection during the lesson was agreed beforehand. For example, observers were on the lookout for moments of insight, and opportunities for increasing agency. The research lesson was of the usual fifty minutes duration and was video-recorded using a static camcorder facing the lecturer and a roving camcorder focused on students at work. The video recordings were translated into DVDs and later transcribed. A focus group of students was interviewed after the research lesson and all student participants were invited to make entries in their learning journals relating to the lesson. A second research lesson, this time in mathematics education, and taught by Thérèse, took place on November 30th 2010.

The first research lesson

The collaborative preparation phase involved planning the teaching of the research lesson and studying the course curriculum and materials to be used. The research lesson chosen was an introductory session on the history of calculus, exploring Leibniz’ conception of the calculus based on Bos’ study (1980) of Leibniz’ work of October 1675. This specialist area is one with which only the course lecturer, Maurice, was familiar. While his Mathematics Department colleague, Sinéad, could be expected to understand the underlying mathematical concepts, the material was new to the rest of us. In consequence, there was a lot of reading, questioning and explaining to be done as we grappled with building a shared understanding of the topic of the lesson, where GeoGebra would be used to explore Leibniz’ notion of the characteristic triangle. In a preliminary analysis of data of the lesson study cycle, the research team considered three perspectives: the process of preparing, of teaching while being observed and the process of reviewing a lesson collaboratively.

Preparing

As course lecturer Maurice was responsible for setting goals for teaching as he would in the normal way. However, the questions asked by lesson study team members caused him to articulate the goals for student learning during the research lesson in a more detailed and specific manner than he was accustomed to doing. His decision to have students use GeoGebra, which had been uploaded on their laptops was a new departure. Mindful of the possible effect that the presence of two video technicians and four observers would have on student participation, it was decided to prearrange the desks so that students were sitting in pairs in a U- shape. The list of nineteen questions that he proposed to ask the students was posted on Moodle. These fell into two categories: questions concerning student engagement with the prescribed text and questions concerning student engagement with the technology and the affordances that electronic and hard copies of GeoGebra worksheets would offer for student learning. While recognizing Maurice's expertise, the group felt a degree of ownership of the research lesson.

Teaching/observing

Maurice was aware of contingency opportunities in a new way as he was teaching the lesson (Rowland et al 2005). When working to understand Leibniz' characteristic triangle, one student pointed to a perpendicular bisector in the supplied diagram. The term 'bisector' was inappropriate and Maurice was aware of his attempting to deflect her attention away from the error by emphasizing the term 'perpendicular'. He later regretted this handling of the situation as a missed opportunity for enhancing her personal agency. All the observers were struck by the immediacy of observing a live research lesson of adults engaged in teaching a learning mathematics, from a perspective which is different from that of teacher, student or evaluator, roles we have all held already. One student was aware of feeling constrained by the experience:

> It kind of put pressure on you to kind of try and figure stuff out for yourself ... but then you didn't want to say it in case you sounded stupid in front of everyone else (post-lesson focus group)

Another student claimed she enjoyed it. "There is no doubt but that it was an exciting experience and I don't feel that they disrupted the lecture in anyway". Maurice expressed the affective dimension as:

> It is new terrain working with colleagues like this! Any apprehension is more than compensated for by a deep sense of gratitude towards colleagues. My self-confidence varied significantly at different stages throughout the project – from being racked with doubt to being buoyed up with confident enthusiasm.

The remaining team members expressed respect and empathy for all the participants in the research lesson, a spirit which appeared to imbue the whole lesson study process. There are resonances here as elsewhere in the data with Wenger's notions of 'engagement', 'imagination' and 'alignment' which together with his construct of 'accountability to the enterprise' appear to indicate that engagement in the practice of lesson study for the teaching and learning of mathematics is a worthwhile endeavour (Corcoran, OReilly and Breen, 2010).

Reviewing

In this study, mathematicians and mathematics educators brought many different ideas and perspectives to bear on the research lesson plan. Teaching mathematics at third level can be normally considered quite an isolated activity yet on this occasion it was experienced as a collaborative one with ideas of good pedagogic practice being brought centre-stage and shared in a structured and effective way. In the review phase, the lesson study team members appeared to hone in on different aspects of the lesson, according to how they perceived moments of insight and opportunities for increasing student agency which lead to a rich and mutually rewarding post-lesson discussion session. Space prohibits reporting these here. The video recording allowed the possibility of revisiting and reliving the lesson, enabling us to take a deeper look at interesting incidents during the lesson and probe more deeply into our initial observations. However, for the review phase to be most effective it should have been carried out without delay. Time constraints limited these possibilities.

Concluding questions

Engagement with lesson study is inherently 'work in progress'. To date the research team has undertaken two lesson study cycles each hinging on a research lesson taught in a different department. We have reported only preliminary findings from the first research lesson here. The work is ongoing and our inquiry stance in relation to preliminary data analysis throws up many questions of a generic nature. These are adapted from those posed by Breen (2004) and relate to our growing understandings of the commonalities and differences in our practices. Arising from our inquiry stance into each other's practices, we ask ourselves, what do we learn from each other? Our questions into mathematics education research relate to power and its distribution among the group. How do we work with student agency? We strive to discern whose questions are privileged? And whose theories are foregrounded? As we inquire into the lesson study process we seek personal responses to the question, how do participants cope with different agendas? Who maintains the 'project'? Who is in control of the research? Is the process of lesson study the catalyst (and the glue) which sustains the research endeavour? These questions highlight that the existing research design and/or theoretical framework will need to be augmented in future. More in-depth data analysis is required at the level of participation as colleagues who choose to collaborate in the communal space of lesson study.

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Exploring children's interest in seeing themselves on video: metacognition and didactics in mathematics using 'Photobooth'.

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This paper examines the process of interviewing five children aged 7 to 11 doing arithmetic, to begin to explore the benefits and limitations of using video of children in three main areas: the benefits to the researcher of making a video record of an interview; the use of visually stimulated recall; and the potential for the teacher, especially with children whose experience is one of failure in mathematics, to show the child they have made progress and thus to influence their future learning.

Keywords: visually stimulated recall; difficulties in primary arithmetic; metacognition.

Introduction

I have been carrying out clinical interviews (Ginsburg 1997) with five children aged 7 to 11, all of whom are in foster care, and all of whom have been identified by their schools as having low attainment in mathematics. Four of the children were also described as having very poor concentration and poor behaviour in lessons. The focus of each interview was an aspect of counting, addition or subtraction that had been identified as causing the child difficulties.

I have previously used a small digital camera to video children working, but decided instead to use 'Photobooth' software on a laptop, because the child can see themselves on the laptop screen (as though in a mirror) whilst the video is recording, and I thought that this might increase the child's motivation to take part in the interview.

Before the first interview, I had explained to each child that I was trying to find out more about how children learn mathematics, and I was especially interested in finding out about things they found hard to do, or they didn't understand, because this would help teachers do a better job. To help me, I would sometimes ask them to explain how they did something.

The benefits and disadvantages of recording the interviews on 'Photobooth'

The equipment is very easy to set up (just open the laptop) and this was an important consideration since the interviews were often in a small room in the child's school, and were sometimes interrupted and moved. I recorded sound on an audio recorder as back-up; this was helpful with one interview where the sound was accidentally switched off on the video, and on another where the child accidentally erased the interview when he tried to play it back. A 'test run' with each child enabled me to play back a short piece of film of themselves before their first interview.

Four of the five children commented favourably on being able to see themselves on the laptop screen; the fifth, Maisie, seemed uninterested during her first and second interviews, but before her third interview, when I was unpacking some counting equipment, she said anxiously, 'You haven't forgotten your laptop have you, for filming me?' All the children had watched video of other people on 'Facebook' and two had seen 'Photobooth' before. This familiarity with the medium of video on a laptop seemed helpful, and the overt nature of the filming was perhaps reassuring to some of the children: for example, Kyle commented 'I can see what you're watching when I'm doing stuff'.

Occasionally the videoing was a minor distraction – such as when Skye wanted to show the video her birthday badge, when Ronan showed his new shoes to the camera, or when Kyle noticed that the low resolution of the film meant his hand wobbled on screen. However, I felt that, overall, the obvious nature of the filming was an incentive for the children to concentrate for longer than usual. Being filmed seemed to make the children feel important.

A video recording of any interview does, of course, provide the researcher with the chance to view the material again, and to reconsider the child's body language (including facial expressions) and activity, as well as their speech. In addition, there were occasions when a child did something that I did not see when I was with them, but that was caught by the camera. One notable example of this was with Millie. We had been using plastic hundreds, tens and ones equipment to do several addition and subtraction calculations. As I looked away, she quickly picked up a plastic 'ten' strip, and counted along the markings with her finger, checking that it was ten. Without the video, I would not have known that she was still uncertain about this.

Interviewing Millie and Kyle: contrasting experiences

Millie (aged 10) was the first child I interviewed, looking at how she carried out subtractions with numbers within 100. She gave reasonably clear explanations throughout the interview about how she had completed each calculation, demonstrated what she had done, and in some cases happily said that she could not use a particular method of calculation successfully. I did not have any plan at that point to show each child their videoed interview to discuss it, and Millie did not want to view hers. Even in retrospect, I do not think I would have gained much additional information about how she felt or the strategies she used by asking her to comment on her video, because she had said so much as she went along.

My second interview was with Kyle (aged 8), who asked if he could watch it as soon as we finished, and then spontaneously gave a commentary. Initially, Kyle's comments were very general. We had been using small plastic fish that Kyle had to count out into two 'ponds' drawn on a sheet of paper, to explore number bonds within ten. His first comment on the video was to say 'You can see me. I'm doing the fish. *I've* got a shark like, you can do it in the bath, it squirts. They're good fish."

Very quickly, he began to comment on his work, sometimes commenting on activity that had not yet happened on the video: for example, he said "You wouldn't let me use the fish. I could have done it with my fingers" *before* he said on the video, "Am I allowed to use fishes as well?" and I had said, "No, not yet."

Kyle had not previously been able to give an answer to 0 + 7, so I had begun to provide a pattern of questions leading to this, using seven fish distributed between the two ponds, and asking Kyle to move one fish at a time across to the other pond. When he watched the video, Kyle said comparatively little – he leant forward, concentrating on the screen. I made notes of his comments and where they were made. His comments when he watched the original interview are shown on the right-hand side of the page on the video transcripts below.

Transcript excerpt 1:

11 RG: Very good. Ok, right, now, you can actually use the fishes, put four in there and three in there.

RG demonstrates where she wants Kyle to put the fish. Kyle puts the fish in the two ponds.

RG: Then tell me as quickly as you can how many you've got altogether. Four in that pond.

12 Kyle: They can actually stand up.

13 RG: They can, they're brilliant fish.

14 Kyle: How many in that one? *(points to empty pond)*

15 RG: Three in this one.
How many all together?

Kyle's comment: There was seven.

Kyle counts each fish.

16 Kyle: One, Two, Three, Four, … Five, Six, Seven

17 RG : Seven. Four and three makes seven.

Kyle watched the video as he successfully did 5 + 2, then 6 + 1:

Transcript excerpt 2:

Kyle moves one of the fish to make 6 in one pond and 1 in the other.

26 Kyle: One, two, three, one, one, two, three, four, five, six, seven.

K's comment: I counted them all.

27 RG: So we've still got seven. How many in this pond?

28 Kyle: (*turning away from the fish*) It equals seven! Don't it!

RG smiles at Kyle's reaction.

K's comment: **Look at my face! I got it! 7 add 0 is 7.
I learned it!**

29 RG: Yeah! So how many in this pond?

30 Kyle: One, two, three, four, five, six, and then we put that one in there and, one, two, three, four, five, six, seven.

Kyle puts the last fish into the pond on the left.

31 RG: and how many in here?

32 Kyle None.

33 RG: So what's seven and none?

34 Kyle: Seven

K's comment: I knowed that now, I knowed it, knew it, before, you know.

Kyle's realisation in the initial interview that 7 + 0 must be seven (line 28), *before* he had moved the last fish, was accompanied by him turning round in his chair, looking up at me and grinning. His excitement when he watched himself on the video was apparent, too, even though he attempted to moderate it by saying that he 'knew it before'. But knowing 7 + 0 is not the same as knowing 0 + 7:

Transcript excerpt 4:

RG moves all the fish from the pond on the left to the pond on the right.

35 RG: Suppose all these seven went over here, now we've got none and seven. How many have we got?

36 Kyle: (*Shrugs his shoulders*) Seven? I mean none?

37 RG: Look. How many have we got, we've got none there and seven there, how many altogether? (Points to each pond)

38 Kyle: Seven?
39 RG: Yeah. So what's none and seven?
40 Kyle: Seven.
41 RG: What's seven and none?
42 Kyle: Seven.

When Kyle tried the next sequence of sums, his comments showed again that he recognised he had learnt something new:

Transcript excerpt 5:

46 RG: Very good, and what's none and five?
47 Kyle: Five.
K's comment: **I'm getting good at it now.**
48 RG: And what's two and none?
49 Kyle: Two.
50 RG: And what's none and two?
Kyle leans back in his chair and stretches when he answers.
51 Kyle: Two. (grins)
K's comment: **Look at me! I'm pleased, aren't I!**

Kyle confirms this again after he watches himself completing several written sums involving zero:

Transcript excerpt 6:

60 RG: Brilliant. So now there's something you can do that you couldn't do before.
61 Kyle: I could do it, It's just I thought it was...
62 RG: It sounded silly?
63 Kyle: Yeah.
64 RG: Yeah you weren't sure about the answer were you?
Kyle shakes his head.
RG: But do you feel sure now?
K's comment: Yeah, I'm sure now.
65 Kyle: Yep.

Visually stimulated recall

Kyle made very little comment on his work during the original interview, but watching the interview gave him the opportunity to both explain how he worked things out, and to identify what he had learnt. In contrast, Millie 'talked out loud' in her interview, and after each calculation she discussed the decisions she had made and her level of success with them.

Lyle (2003) describes stimulated recall (SR) as

> an introspection procedure in which (normally) videotaped passages of behaviour are replayed to individuals to stimulate recall of their concurrent cognitive activity. (p 861)

He notes that SR has been used extensively in teaching, counselling, nursing and medical research, language teaching and sports coaching (i.e. largely with adults). He contrasts the method with 'think aloud' techniques, where the subject is asked to comment on their cognitive processes whilst engaged in the target activity, and points out that this is difficult to do in many real-life problem-solving situations. He suggests that to increase the validity of SR, 'best practice' would include making the retrospection as immediate as possible, and allowing the subject to make a relatively unstructured response.

Kyle's activity during his interview depended on him being able to notice a pattern in the sums he was doing, to convince him that 0 + 7 would be seven. SR was (albeit by serendipity!) a better method to use to explore his understanding, as asking him to 'think aloud' would have diverted his attention to talk about how he did each sum, and he may have missed the main point altogether. Millie was engaged in several separate calculations, and also seemed more practised at being asked to explain her methods, so 'think aloud' worked well for her.

Whitebread et al. (2009), in their discussion of metacognition and self-regulated learning in young children, point to the difficulty that 'think alouds' may pose for those whose verbal understanding and fluency are less developed. This may also be problematic for children engaged in SR. Skye (aged 8) watched her videoed interview without making any new comments; she laughed a great deal, and often repeated what she had said on film, but did not add any further commentary about her understanding of what she had done. However, she listened very attentively when I explained why I thought she had done well in her interview, giving her specific examples of the things I had seen her do.

Tanner, Jones and Lewis (2011) comment that pupils aged 5 to 7 in their study, where children videoed each other working, were able to recognise physical signs of thinking and concentration in others. Interpreting facial expression and body language can be difficult; it was interesting to note that one teacher felt Kyle was 'rude' in class because he sometimes leant back on his chair and stretched his arms, but in his commentary on his video interview, Kyle explained this behaviour as showing "I'm pleased, aren't I!" (line 51 in excerpt 3 above).

Potential for teaching

All five of the children I interviewed had said that they found 'numeracy' difficult, and Dylan, Kyle, Ronan and Skye had variously described themselves as 'rubbish', 'dumb' or 'no good'. In the terms of Kilpatrick et al. (2001) the children's 'productive disposition' was very poor – they did not have a "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (p 5). The remedial help each child was getting was universally based on practising skills, commonly with a teaching assistant armed with worksheets, and four of the five children were apparently making very little progress (Millie being the exception).

Shifting this resignation to failure can be a difficult task. Developing their skills at recognising when and how they have learnt something new seems to be one aspect of the children's learning where reviewing video of themselves engaged in activity could be very helpful. Kyle could see this for himself. Skye was keen for me to show her a particular place on the video where she had begun to count a pile of two-pence and penny coins successfully, and listened carefully to my explanation of why I thought it was very good. Ronan (aged 8) had not wanted to watch his first videoed interview at the time, but after his second interview he asked if he could watch the first interview, so that he could see if he had got any better. They were keen to see for themselves the proof that a video recording provides, that you can be successful.

Dylan's second videoed interview seemed unlikely to provide that proof, as he struggled with trying to answer 20 – 13. When he watched the video, at one point he asked “Can we wipe it out now? Delete it?”. The focus of my discussion with him needed to be about his perseverance – he had kept trying – and then to give him a new strategy to try. He checked that the correct answer was 7 on a calculator, then had a break, and finally thought about how he could link 7, 13 and 20, using an idea he had had before: “I could take away 10, take away 3”. It was obviously important not to leave the experience as one he saw as a failure.

Visually stimulated recall as a research method was useful with two of these five children, in providing additional data. Using the video interview as a teaching and learning tool seemed to have potential with all five, and this will be a focus of further work in the future.

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I can be quite intuitive": Teaching Assistants on how they support primary mathematics

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This paper reports on initial work on Teaching Assistants' (TAs) perceived contribution to Mathematics teaching in primary schools. Extracts are presented from interviews with three TAs who provide support to individuals with particular needs. The focus is on what interviewees say about the knowledge and understanding they bring to their work. The paper also identifies how they feel they acquired this knowledge. I show that they draw on pedagogic content knowledge and subject specific knowledge of individuals. In discussing the source of this knowledge, interviewees value experience and use of initiative.

Keywords: primary mathematics, special needs, Teaching Assistants

Introduction

It is now common practice in many primary schools for TAs to be present in mathematics lessons and recent research suggests that in many cases they work directly with pupils considered to have greatest difficulty (Blatchford et al 2009a, 2009b). This research points to the many positive benefits of TA presence, but questions whether they have any positive effect on children's attainment in Mathematics and English, a finding which is not dissimilar to earlier research focussing on numeracy (Muijs and Reynolds 2003).

This paper draws from on-going research investigating TAs own views on how they impact on pupils' progress, focusing particularly on how they work with pupils in Mathematics and English. A key follow up question is about the type of knowledge and understanding they feel they bring to their work and how they acquired this. TAs work with pupils in a range of different ways, but for current purposes, extracts have been taken from interviews with three TAs who are all employed mainly to give support to a specific named pupil with a statement of special educational needs. Extracts concerning mathematics have been selected from these interviews.

As well as presenting early findings, this paper explores whether frameworks used to represent the knowledge that primary teachers draw on might also be appropriate when considering TAs' knowledge. Such frameworks commonly draw on Shulman, including his use of the phrase 'Pedagogical content knowledge' (Shulman 1986) and his identification of seven categories of knowledge required for teaching (Shulman 1987). Three of these, content knowledge, curriculum knowledge and pedagogical content knowledge, are usually regarded as subject specific. The other four are general pedagogical knowledge, knowledge of learners and their characteristics, knowledge of educational contexts and knowledge of educational ends, purposes and values. Pedagogical content knowledge includes knowledge of ways of presenting key ideas in one's subject, an understanding of difficulties students are likely to experience and strategies for addressing these (Shulman 1986).

Researchers in primary mathematics education have shown a particular interest in Shulman's subject specific categories and the relationship between them. Ma (1999) investigated primary teachers' depth of understanding of basic mathematics, Ball and her colleagues closely examined the mathematical knowledge drawn on in teaching situations (e.g. Ball and Bass 2000) and Rowland and his colleagues (2009) offered a categorisation of the knowledge drawn on by trainee and newly qualified primary teachers when teaching mathematics. There has also been consideration of whether teachers might have knowledge of individuals which is specific to mathematics, for example by Aubrey (1997) in work with teachers of reception children and by Ball, Hill and Bass (2005). An early question considered when analysing the data which follows, concerns whether this body of research might also shed light on the knowledge TAs draw on when working with primary age children on mathematics.

Interviews

Sample and method

Those interviewed have undertaken some study at Foundation degree level including a short module on supporting pupils' mathematical learning. This is common, given the number of TAs on such courses, but the sample is clearly not representative of all TAs. Semi structured interviews were used, with TAs asked to talk about their work particularly in relation to supporting Mathematics and English, then about the knowledge that helps them and how they acquired it. Extracts are considered below that reflect these broad questions.

Supporting individuals

The three TAs all gave examples of how they worked with their individual learners in mathematics lessons. The three situations differed in many ways, though they had in common fairly longstanding partnerships between the TAs and pupils. In the first example below, Salma, a TA in a mainstream primary school, discusses her work with Harry diagnosed with Asperger's syndrome at eight years old. Salma started working with him soon afterwards, continuing until he moved to secondary school at age eleven.

> I would find that quite often the teacher would not differentiate the work and it would really be too hard for him. Because I find that if you can't do the basics, there's no point in trying to understand something, you know. So I would find that often I would have to differentiate, which would mean using visual aids like cubes and umber lines and things. Quite basic stuff, because he wasn't even able to do his times tables and things, but it just meant that I would differentiate and make sure the visual aid was there.

It was clear from Salma's interview that her preferred way of working with Harry was one to one, outside the classroom partly because of Harry's difficulty in coping with classroom noise and distractions. Because Salma worked with Harry across three academic years, he had three teachers in this time who all appeared to have slightly different ways of working with TAs. It appeared that, if necessary the SENCO would also help with negotiating what was best for Harry. Salma talked enthusiastically about one of the teachers who was happy for her to make decisions about how best to support Harry.

> ...the teacher was lovely, he was really understanding and he just kind of said, "Yeah, you can do what you want." I like it when the teachers do that, when they give you some sort of power as well, because I obviously know what I'm doing with him. So he says "Use your initiative, whatever you want to do. If you want to take him out, take him out"... a really positive year.

The next TA considered, Joyce, also preferred to work in this way. Joyce works in a mainstream school and has supported Gordon, who has a hearing impairment, for two and a half years. Joyce worked individually with Gordon for Mathematics, using specialised plans devised by a teacher with a specialism in Hearing Impairment, not the class teacher. Like Salma, Joyce used practical resources for teaching mathematics. In this case, she also talked about language issues in mathematics.

> The language in maths papers for example, 'James has got six pencils, Becky has got four more pencils than James. How many pencils has Becky got? The problem's not with the maths, because you ask him five and four and he'd do it instantly, so we quite often use that little whiteboard and draw him pictures. Or you go and get the physical resources I mean, if they're ever talking about buttons, we have a number of buttons or counters. They don't know what a counter is, then you go out and get a counter and you go, 'Look, he's got six of these'.

A slightly different type of support was mentioned by Pauline, the third TA, who discussed how she encouraged Aden to focus on his work. Aden was a Year 5 pupil diagnosed as Autistic and Pauline had worked with him for just over a year at the time she was interviewed.

> I'm actually quite strict. He could easily look out of the window and admire himself and I go, 'Right, name, date' and he'll sometimes go, 'What's the date?' I'm almost like the starter button, I say, 'Name, date, I want them down.' Sometimes, I'll count him down if he doesn't do it and he doesn't like that. He thinks, 'Oh my gosh, countdown starts.' So I start him off and once he's started and he sees other children ... and we've instigated now something called Musts, Coulds and Shoulds. So must is, he must do five, he could do eight...

Salma also had strategies for encouraging the child she worked with to engage with Mathematics. One strategy, outlined below, involved allowing Harry to write on a white board rather than in his book. Salma went on to explain that this idea of hers was now used by others in her school.

> Because he associates his maths book, I mean he scribbled all over his maths book and eventually there was nothing in there. So I said, "Okay, we'll get you a new maths book, you don't have to write in it." Because he just didn't like the squares in the book. You know, he's just got some phobia with maths. So I said, "Okay, we can do it on the whiteboard", and he just seemed to like it. So we used to photocopy all these sheets and things and date them and stick them into his maths book.
>
> When I was taking the whiteboard to the photocopier I was getting some really funny looks. They were like, "The skin will rub off." I thought, "Let's just give it a go." Now everyone does it in the school.

Knowledge used to support individuals

These TAs all demonstrated some Mathematics 'pedagogic content knowledge' (Shulman 1986). In particular they talked confidently about use of equipment and demonstrated some awareness of how to present ideas and what difficulty learners might experience with particular aspects of mathematics. The evidence is not conclusive here, as the interviews do not enable us to judge their skill at selecting appropriate equipment or representations, though they appear to do so with some confidence and success. Mathematical content knowledge is even harder to judge, partly because it appears much initial planning is done by teachers then adapted by TAs, but also because the TAs do not talk about the mathematics in detail and they do not tend to see mathematics at this level as problematic.

There is some evidence of general pedagogic knowledge, but more evidence of knowledge of individuals. This latter knowledge can be subject specific. They also have knowledge of named special needs, such as Autism or Hearing Impairment, but still distinguish between individuals with these needs. Given the time these TAs had spent with the individuals in question, it is unsurprising that they demonstrated knowledge of them. Interestingly, there was a subject specific element of the way they discussed this, with distinctions drawn between the way pupils coped with mathematical tasks, in contrast to literacy. For example, Salma considered Harry to have more difficulty with mathematics than with literacy, as explained below.

> It would depend on the lesson. With literacy he was very able ... and I used to let him ... I'd just sit back. Whereas with numeracy, numbers were very difficult for him and he really struggled with that, so I'd pull him out and I used to teach him one-to-one with numeracy ... I think it was just numbers he honestly has not grasped and he kind of gets very confused ... with everything else he's very articulate ... but he would look at numbers and he would honestly kind of panic ... He honestly, genuinely could not do them and he would really, you know, struggle.

When Pauline talked about Aden, she also contrasted his performance in Mathematics and English; in his case he found Mathematics easier.

> He's a very intellectual young man ... he likes maths, because there is an answer ... He's a very able child. The problem comes when, for instance, he's given an alternative strategy to deal with something....
>
> Probably most of the Autistic children I've had find literacy more difficult to deal with than maths ... if he's asked to use several devices within a story ... He finds that much more difficult ... whereas maths feels safe to them.

In the quotation above, Pauline started to generalise about Autistic children. Joyce talked in a similar way about children with hearing impairments, but also pointed out that she considers there to be considerable difference between individuals.

> Quite often we actually physically go and do it. Quite often you'll see a TA walking around the school with a deaf child in front of them, saying, "Walk six paces ..." You know, you actually physically do it with them.
>
> I do know this particular child very well, but he's not special. It's not special to him. Most HI kids have got this language problem. ...
>
> But everyone's got different ... everyone's got a slightly different perception of the deaf child and how they learn best.

These TAs considered themselves very knowledgeable about the children they worked alongside and felt they were in a stronger position than teachers to develop such knowledge.

> Quite often TAs are the experts on the SEN children, because they do spend more time, quite a lot of time.
>
> (Jovce)

All three TAs provide evidence that their knowledge of the children they worked alongside assisted them in trying to meet the child's needs. There was also some sense of the TA becoming an advocate for the child, as explained by Pauline below.

> I'm very concerned about that child, because I do feel they sometimes get a little bit sidetracked as Autistic children. So I want to make sure that he has as much say, he has as much voice, but also that if he's treated that way, usually that system works with all the other children too. And yeah, he does like to see justice being done.
>
> ... I just feel that I'm a voice for these children and I also would like them to have their own voice. And as a result, most of the children who work with me do tend to become more confident and do have a voice.

Sources of knowledge

When asked how they gained the knowledge to assist them in their work, all three TAs talked at some point about how they worked things out for themselves and how they used commonsense or intuition. Salma gave examples of activities she used with Harry and talked about how she had come up with these ideas herself, or spent the weekend trawling the internet for information or even bought resources such as a tables tape out of her own money.

> I used my initiative a lot and said, "I think this will work best" and after a while, they're just like. "Let's just shut her up. Let's just give her some money so she can go and get her resources and things." And it worked, he did actually start eventually learning his tables.

Salma returned to the theme of working ideas out for herself at the end of her interview when asked if there was anything she would like to add.

> I'd just like to say that I really enjoyed being plunged in the deep end of things, because I don't think I'd have it any other way. I think that's how I learn as well, by trial and error and by reading up on things. You know, especially with Harry, nobody told me "These are a set of rules and you've got to do it like this and you can't do it like this."

These three TAs had all completed a foundation degree and were continuing to study. This was usually mentioned at some point as being helpful, though more in terms of the reading they did themselves than the direct teaching. But mentions of the course still tended to follow answers about experience, as in the quotations from Joyce below.

> Most of it is experience. Yeah, most of it is experience.
>
> ... no, not training, experience. And of course, I've done a lot of reading since I've been doing this course. I know why the gaps are there now, whereas before, I knew the gaps were there, but not necessarily why.

Pauline also talked about what she had learned from her experience of working with children who need support and from her experience as a parent. She also described herself as intuitive, reflected in the title of this paper.

> Yeah, I can be quite intuitive. I feel that the more I've worked with children of this type, the more you understand.
>
> Because I've worked with those children, you get a real feel for the way that they behave. Yeah, I think it is very intuitive and actually also because I'm a parent.

The discourse of experience and intuition was strong for all three TAs, although it co-existed with their acknowledgement of academic study which they knew the interviewer was aware of. In response to follow-up questions about where they found particular ideas, they spoke of many other sources of knowledge. For example, Pauline got the idea of ‘musts, coulds and shoulds’, from a new teacher at the school. Ideas also came from staff meetings and short courses. A range of knowledge sources were therefore identified in response to questions about particular ideas, but the general discourse was about experience and generating their own ideas.

Summary and next steps

These three TAs demonstrate use of pedagogic content knowledge and subject specific knowledge of individuals, though there is little evidence of drawing on mathematical content knowledge. A next step might be to examine these two forms of knowledge more closely, perhaps considering whether beliefs are also relevant, for example in relation to use of materials and practical activities. For these TAs who work closely with individuals, subject specific knowledge of individuals is also important. Finally, the issue of the importance TAs attach to experience and intuition merits further consideration.

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Economic activity and maths learning: project overview and initial results.

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We present the overview and initial results of a project that explores the links between the out-of-school economic activities of UK children (8 - 16) and their learning of mathematics. The aim is to inform and enhance children's classroom mathematics experience to make it meaningful and engaging, especially for underachievers. Economic activity is interpreted and researched in a broad sense, covering activities around money (e.g., paid work, pocket money, gambling) but also other non-monetary transactions (e.g., swapping, collecting, giving gifts). The project methodology has three stages: A survey of children' seconomic activities, Qualitative studies including diary studies and focus groups and task-based interviews. The initial results of the survey have both similarities and differences with previous studies of children's money usage, and revealed a wide range of economic actitivies in which non-monetary goods are central. Moreover, some of these activities are likely to involve matehmatics more complex than simple arithmetic. These findings are discussed in relation to the anticipated results of the project.

Informal learning, transfer, economic activity, maths learning

Introduction

There has been a debate in the mathematics literature since Carraher, Carraher and Schliemann (1985) regarding the relationship between out-of-school experience and classroom mathematics. This research showed that children (in this case Brazilian street traders) can develop sophisticated strategies in response to arithmetic problems. However, there is evidence to suggest that children are unable to draw upon these strategies when similar problems are presented outside of those children's usual frame of reference. This research, combined with findings from similar studies, has contributed to the development of situated learning theory (e.g., Lave and Wenger 1996), built on the assertion that learning is grounded in the context in which it occurs. In response to the development of situated learning theory, cognitivists have provided evidence that children are in fact able to transfer learning from one context to another (e.g., Anderson, Reder, and Simon 1996). Research has also shown that it is easier for children to transfer learning from abstract to concrete contexts than the reverse (Mevarech and Stern 1997).

Despite the mixed evidence regarding the transferability of informal learning, there is certainly evidence that the incorporation of realistic context in mathematics education can be beneficial for students. For example the Realistic Mathematics Education project (RME, e.g., Gravemeijer 1994) has had great success in improving outcomes of mathematics students in the Netherlands, by making efforts to ensure that classroom mathematics is made relevant and meaningful to children (not only by incorporating context). The work on RME is related to research on children's responses to word problems (Verschaffel, Greer, and De Corte 2000), showing that children often fail to take account of context when giving an answer to a word problem.

If there is a consensus to be drawn, then it is likely to be that realistic context is essential for learners, especially low achievers, to become interested and engaged in the curriculum, but that there is no clear understanding at this point regarding the best ways to incorporate realistic context in classroom mathematics. Much of the empirical research that has been done has either looked at what children can do in informal situations that they can't do in formal contexts or the mistakes that children make when working on 'realistic' problems. Very little work has been carried out that actively explores ways in which UK children's out-of-school practice could impact on their mathematics classroom experience. Some recent work has made some headway in this area. Gonzalez, Moll and Amati (2005) use the term 'funds of knowledge' to describe the knowledge students gain from their family and cultural backgrounds. This concept was drawn upon during the Home-School Knowledge Exchange project, part of which involved an investigation of ways in which schools could stimulate the transfer of knowledge both from school to home and from home to school, in order to encourage children's numeracy development (Winter et al. 2004). The project described here aims to build a bridge between the economic psychology literature and research on the relationship between children's mathematics practice in and out of the classroom. The main aim of the project is too explore ways in which the development of such a bridge can contribute in a positive way to children's classroom experience.

Children's economic activity

Children's economic activity has the potential to impact on their experience of classroom mathematics learning. In this project, economic activity is interpreted in a broad sense, encompassing different sorts of transactions (e.g., exchanging, buying, saving) with adults (e.g., parents) and peers (e.g., siblings, friends). There is research about children's usage of pocket money (e.g., Furnham 1999, 2001) and their understanding of economic concepts (e.g., Leiser and Beth Halachmi 2006), but only a small number of studies have studied the mathematics involved in children's economic activities. For instance, Taylor (2009) documented strategies employed by low-income American children during everyday purchasing activities, such as their misuse of whole number when dealing with coins of different value. Similarly, Saxe (1988) described that Brazilian children selling candies on the streets employ strategies such as grouping small numbers into larger units to handle a changing price ratio, and Guberman (2004) reported that children need to compare and count sets and make multidigit operations (e.g., summing and multiplication) in out of school economic activities such as shopping. This is the sort of activities and mathematics that this project aims to discover. However, we want to expand the view of economic activities to include not only monetary transactions, but also activities around non-monetary goods. Moreover, we are not only interested about how children develop an understanding of adults' economy, but also about the conventions and mathematics of their own economies.

Overview of the project

Participants in the research will consist of English primary and secondary school students in three age groups, 8-10, 11-3, and 14-16. These groups fall into each of the three Key Stages in the English National Curriculum (Key Stages 2, 3, 4). The research questions are:

What economic activities do children engage in out of the classroom?
- How does children's engagement in economic activity contribute to their development of economical and mathematical knowledge and concepts?
- How far does children's out of school economic practice correspond with classroom mathematics practice?
- What opportunities are there for classroom teachers and curriculum developers to link class mathematics teaching and learning with children's informal economic activity?

The project employs a mixed-methods approach and consists of three stages:

1. *Survey.* Questionnaires for the three age groups will be designed to define a contemporary taxonomy of children's economic activities.
2. *Qualitative studies of children's economic practice.* Diary studies will be conducted to make an in-depth exploration of how often children engage in economic activities, and in what particular contexts. Children will be asked to document their activities with mobile technologies such as cameras and mobile phones. Following this, focus groups will be conducted to discuss the economic activities identified both through the diary studies and the survey.
3. *Task based interviews.* These will be conducted to determine the extent to which the knowledge of children's out of school mathematics can be integrated with the formal mathematics curriculum.

Surveying children's economic activity

The survey stage was initiated with the design of different questionnaires for each age group (8-10, 11-13, and 14-16). Some topics inappropriate for younger children (e.g., gambling) will be covered only amongst the older groups. However, the questionnaires for all age groups cover the same three types of economic activities:

1. *Producing.* Any effort to create value, like doing things for pocket money, working, or borrowing money and things (e.g., toys and clothes).
2. *Consuming.* The acquisition of goods and services, mainly through shopping.
3. *Distributing.* Transfers of value in transactions such as swapping, sharing and giving gifts.

150 children in each age group will be surveyed. Here we present initial data from the younger group (8-10). 53 Year 5 children (29 girls) answered the questionnaire. Most (34, 64.2%) were white and the rest represented a range of ethnicities including British black and British Asian. The majority (31, 58.5%) live in families where at least one parent or carer works in a high-level occupation[1]. In spite of its preliminary nature, the data already describes a wide range of economic activities. These results are summarised in three sections: pocket money, economic activities around non-monetary goods, and the mathematics of children's economic activity.

Pocket money. There are both similarities and differences between our survey and previous studies of pocket money. Our participants regularly get pocket money in relatively small amounts and they do not spend it frequently. The minority of our respondents get no pocket money at all (3, 7.9%) or only on special occasions (6, 11.3%). Most children get money regularly, every week (19, 35.8%), every two or three weeks (8, 15.1%) or monthly (17, 32.1%). This is partially consistent with Furnham (2001; 1999) who found that most parents believe children aged 5 or above

[1] According to the National Statistics SEC

should receive pocket money. However, in our survey most of those who receive pocket money weekly get up to £1 (14/19); which is less than the £2 that parents in Furnham's studies reported as appropriate for 10 years olds, and much less than the £4.75 reported as last year's average UK amount of weekly pocket money amongst children 8-11 (Halifax 2010).

Some respondents do not have to do anything to get pocket money (14, 21.4%), whereas about half are asked to do chores (23, 43.3%), which is in line with the 51% of parents who said that pocket money should depend on chores in the study of Furnham (2001).

Our survey also suggests that 10 year olds do not use money frequently. The majority of participants did not remember having spent money during the last seven days (38, 71.7%). In fact, many reported not spending money at all or at least not regularly (22, 41.5%). Only a few spend money every 1 to 3 days (5, 9.4%); and most do it once every 1 or 2 weeks or once a month (26, 49%).

Activities around non-monetary goods. Children's economic activities are not only about money. They might also work around non-monetary goods, such as food (Nukaga 2008), and marbles (Webley 1996). In line with this, our data revealed that non-monetary goods might be as important as money in children's economic activity. In fact, few of our respondents included money amongst their three most valued possessions (6, 11.3%). The objects listed as most valued possession were technologies, including videogame items (26, 49%) and devices such as mobile phones and mp3 players (19, 35.8%). Rather than money, these and other things tend to be the object of economic activities such as borrowing, swapping and collecting.

The majority of children borrow things (40, 75.4%). These children borrow books and comics (21/40), clothes (10/40) and toys and videogames (8/40). They borrow more from siblings (16/40) and friends (12/40) than from their parents (5/40), which seems seems natural: what a child likes is more likely to be owned by a peer than by an adult.

Half of the children swap things with their friends (27, 50.9%). They swap a variety of things including toys or videogames (7/27), comic books and books (5/27), cards (4/27) and clothes (4/27). Similarly, about half of the children collect things (22, 42.15%). They collect things such as toys and videogames (10/22), clothes (2/22), cards (3/22), books or comic books (3/22). Thus, swapping and collecting seem to be fairly common activities amongst children. However, a wide range of things are swapped and collected by small numbers of children, so what is swapped and collected might depend on individual preferences rather than on collective trends.

The mathematics of children's economic activity. Children reported a number of activities that one would expect to require mathematics. Many of these occur at home with the family, such as planning birthday parties (37, 69.8%), shopping for clothes (35, 67.9%), playing board games or card games (64.2%), playing video games (33, 62.3%), buying in the supermarket (33, 62.3%), cooking (31, 58.5%), and planning holidays (26, 49.1%). Some activities might require basic arithmetical operations, for instance shopping for clothes or buying in the supermarket. But other activities might demand more complex operations. For example, children may have to estimate proportions whilst planning the 'goodies' for a birthday party, or calculate probabilities when playing card games. Other sorts of mathematics are likely to be employed in activities that children perform with their peers. For instance, children who collect may have to estimate the value of an object (a card for instance) in relation to the wider 'market' (the cards of others). They will probably have to use this information to bargain during swapping activities (card exchanges for example).

Discussion

The presented preliminary results of the survey show that children as young as 10 years old engage in a range of economic activities where non-monetary goods seem to be equally (or probably more) important than money, and that some of these activities might involve mathematics more complex than basic arithmetic. These findings are promising and we expect to have a wider picture of children economic activity once the survey stage of the project is completed. For example, the data presented reflects the activities of economically advantaged children. Perhaps children in lower socioeconomic strata will engage in other sorts of activities For instance, they are probably more likely to do things such as selling or working informally to get money. Or perhaps they will swap, borrow and collect following different conventions and with other non-monetary goods. As for older children, we expect a wider range of activities. They will have more money and autonomy than 10 year olds and therefore, their economic activities should be more abundant and complex, involving a richer variety of mathematics.

We anticipate that older children will handle more money, and that they will use it more frequently and in a wider range of goods and services than 10 year olds. Moreover, older children should participate more in institutionalized activities such as having a bank account to administrate and save their money, or even using credit. All these aspects might involve complex mathematics, for instance to understand formal economic concepts such as interest rate. Also, the economic activities of older children might involve a wider range of non-monetary goods. Thus, they are expected to report more borrowing, selling, swapping and collecting. These activities might occur over the Internet (e.g., using ebay), especially using their mobiles, something rarely observed in our preliminary data.

When the survey is completed we will be able to see the taxonomy of economic activities. Those activities with the most potential to involve mathematical operations beyond arithmetic will be the subject of in-depth investigation with diary studies and focus groups. This methodology will give us insights about children's economic activities from their own point of view, including aspects such as why they value certain things and not others; their understanding of the rules upon which their economies work (e.g., what gives value to a card or a toy in the economy of their schools or neighbourhoods) and their awareness of the mathematics involved in their economic activity.

To conclude, our preliminary data tell us that children might be learning and using a variety of informal mathematical operations whilst taking part in economic activities. But to link this sort of activity with the formal learning of mathematics in school, we need both an abundant documentation of activities and a deeper understanding of its conventions and the mathematics they involve, especially from children's point of view. This is what the methodology of this ongoing project aims to do.

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Imperative- and punctuative-operational conceptions of the equals sign

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At the British Society for Research into Learning Mathematics day conference held at the London Institute of Education in March 2011 we presented evidence for the existence of a substitutive conception of the equals sign. During the session, Jeremy Hodgen (Figure 1) questioned the use of active and passive items in the instrument, suggesting our results demonstrate not a substitutive conception but rather children's preference for passive items. This is an astute observation, and one we have investigated albeit within the context of operational rather than substitutive conceptions. Specifically, we hypothesised and tested whether Year 7 children (*N*=99) distinguished between imperative (active) and punctuative (passive) formulations of the operational conception. We found no difference, thereby refuting both our own hypothesis and Hodgen's suggestion. In this paper we present these previously unpublished findings.

Keywords: equals sign, substitutive conception

Introduction

A well defined and coherent body of literature exists on children's conceptions of the equals sign (Behr, Erlwanger, and Nichols 1976; Denmark, Barco, and Voran 1976; Kieran 1981; Knuth, Stephens, McNeil, and Alibali 2006; Li, Ding, M. M. Capraro, and R. M. Capraro 2008; McNeil et al. 2006; Molina, Castro, and Mason 2008). A key finding is that many young children, in Western countries at least, tend to view the equals sign as a "do something signal" (Behr et al. 1976), or operator, akin to "+", "×" and so on, rather than as symbolising an equivalence relationship. As children develop, many come to a relational conception of the equals sign as indicating numerical sameness, and are accepting of a wider variety of equation types. However, this happens to varying extents and even those that develop a sophisticated understanding of the equals sign readily revert to operational views of symbolic mathematics (McNeil, Rittle-Johnson, Hattikudur, and Petersen 2010). By contrast, in China the relational conception is taught from the start of schooling and this avoids many of the difficulties with arithmetic and algebra experienced by Western children (Li et al. 2008).

At BSRLM in March 2011 we presented evidence that in addition to the operational and relational conceptions there is a distinctive substitutive conception. We also demonstrated its endorsement by Chinese children but not by British children (see Jones, Inglis, Gilmore, Evans, and Dowens submitted). The evidence came from a definitions-based instrument adapted from the literature (Rittle-Johnson and Alibali 1999) in which children (*N*=243) rated the "cleverness" of fictitious definitions of the equals sign on a three-point scale. The items corresponding to relational, operational and substitutive conceptions are shown in Table 1. (Three distracter items were also included and are not shown.)

Figure 1: Jeremy Hodgen.

	The equals sign means…	Conception
R1	…the two amounts are the same	Relational
R2	…that something is equal to another thing	Relational
R3	…that both sides have the same value	Relational
O1	…the total	Operational
O2	…work out the result	Operational
O3	… the answer to the problem	Operational
S1	…the two sides can be exchanged	Substitutive
S2	…the right-side can be swapped for the left-side	Substitutive
S3	…that one side can replace the other	Substitutive

Table 1: Equals signs definitions presented at BSRLM, March 2011

Hodgen's challenge was that some items in the instrument shown in Table 1 are active while others are passive. For example, the operational items "the total" and "the answer to the problem" are passive, while "work out the result" is active. Moreover all the substitutive items suggest making an active replacement of notation, whereas all the relational items are passive descriptions. On this basis our conclusions can be called into question because children might have discerned between passive and active items rather than substitutive and relational conceptions. The distinction between passive and active items presents a valid concern, and one we have previously investigated. In this paper we present evidence that children in fact do not discern between passive and active items, and therefore the original conclusion that there exists a substitutive conception of the equals sign stands.

The operational conception of the equals sign

There are two main types of evidence that young children view the equals sign as an operator. The first is children's rejection of arithmetic equations that do not possess an expression on the left-hand side and a numerical result on the right. This is usually interpreted to mean children consider the equals sign to be an instruction to perform a calculation and write down the answer (Behr et al. 1976). The second is children's definitions of what the symbol "=" means, to which children typically respond "add the numbers", "the answer" and so on (Knuth et al. 2006).

However, on closer inspection, there are subtle inconsistencies in how scholars interpret this evidence (Jones 2008). If the symbol "=" is a "do something signal" then perhaps we should expect pupils confronted with "2+4=" to perform a calculation but pupils presented with "2+4" to do nothing. In fact this is not the case. As Behr et al.

(1976) observed, "even in the absence of the = symbol … 2+4 serves as a stimulus to do something" (13). Frieman and Lee (2004) suggested that in such cases the symbols "+" and "=" might act as a composite operator with "+" signifying the type of operation required. Some authors go further, arguing young children view the equals sign as an indicator of where the result should be written rather than as an instruction to perform a calculation. For example, Renwick (1932) observed that children "use the '=' sign simply to separate an expression from its answer" (182). Similarly, Kilpatrick, Swafford, and Findell (2001) argued that for pupils "8+5 is a signal to compute" and the equals sign is "a signal to write the result of performing the operations indicated to the left of the sign" (261). Some scholars have cited children's use of "running equations" as computational aids, as in "1+2=3+4=7", as evidence they view the equals sign a kind of punctuation mark, performing a role analogous to a full stop in written language (Hewitt 2003; Renwick 1932; Sáenz-Ludlow and Walgamuth 1998). Sáenz-Ludlow and Walgamuth (1998, 166) argued running equations demonstrate the equals sign is "only used as an index to represent a final point of each binary state in [a child's] over-all additive process". Similarly, Hewitt (2003, 65) said the equals sign "breaks up the flow of left to right by creating a new beginning after [the equals sign]".

Moreover, not all the definitional evidence quite conforms to the operational interpretation. For example, Knuth et al. (2006) asked sixth to eighth graders to define the symbol "=" and offered five responses as representative of the operational conception: "What the sum of the two numbers are" (2006, 302); "A sign connecting the answer to the problem"; "What the problem's answer is"; "The total"; "How much the numbers added together equal." (2006, 303). However, these arguably suggest the children viewed the equals sign as a punctuation mark than rather than as a do something signal. Similarly, McNeil and Alibali (2002) coded as operational children's definitions of the equals sign that could as readily be interpreted as punctuative. An examination of the fictitious operational definitions we used in the study presented at BSRLM in March 2011 (Table 1) shows that one is imperative ("work out the result") and the other two are punctuative ("the total", "the answer to the problem").

In light of the above we hypothesised that the operational conception might in fact be two distinct conceptions. One corresponds to viewing the equals sign as an instruction to perform an arithmetic calculation (imperative) and the other to viewing the equals sign merely as a place-indicator for writing down a number (punctuative). Evidence on this issue would also inform Hodgen's critique that the substitutive and relational items used in our instrument are active and passive respectively.

Method

The instrument shown in Table 1 requires pupils to rate each item as "not so clever", "sort of clever" or "very clever". For this study the relational, substitutive and distracter items were retained, and operational items were separated into imperative and punctuative forms, shown in Table 2. Each of the three punctuative items had an imperative correlate. Items 1 and 4 both contained the term "total", items 2 and 5 both contained "result" and items 3 and 6 both contained "answer … to the problem".

The participants were 99 Year 7 pupils (ages 11 and 12) in a school with above average socioeconomic intake and GCSE results. The instrument was administered by the children's mathematics teachers in class under test conditions.

Teachers were asked to encourage pupils to complete all items. Randomised hardcopies of the instrument were sent to the schools to avoid ordering effects.

	The equals sign means…	Conception
P1	…the total is written here	Punctuative
P2	…the result is found here	Punctuative
P3	…this answer is connected to this problem	Punctuative
I1	…get the total	Imperative
I2	…work out the result	Imperative
I3	…calculate the answer to the problem	Imperative

Table 2: The operational items from Table 1 separated into imperative and punctuative.

Analysis and results

The data were first prepared for analysis. Responses for each item were coded as 0, 1 or 2 for "not so clever", "sort of clever" and "very clever" respectively. Six participants had not completed all items on the instrument and were removed from the analysis leaving a total of 93 participants. The three distracter items were removed.

To determine the children's conceptions we subjected the data to Principle Components Analysis (PCA). The responses to the items were ordinal and accordingly we conducted PCA on a matrix of polychoric inter-item correlations (Holgado–Tello, Chacón–Moscoso, Barbero–García, and Vila–Abad 2008). First, the appropriateness of the data for PCA was checked. The Kaiser-Meyer-Olkin test produced a value of .710, exceeding the recommended .6, and Bartlett's Test of Sphericity reached significance ($p < .001$) demonstrating the suitability of the data for analysis. We found four components explaining 27.2%, 15.2%, 12.4% and 8.1% of the variance respectively. The screeplot revealed a clear break after the fourth component and accordingly four components were extracted. The loading matrix was subjected to varimax rotation to aid interpretation. This revealed a number of strong loadings.

Relational items loaded strongly onto Component 1 and substitutive items loaded strongly onto Component 4. This confirms our previous findings that the substitutive conception exists independently of the relational and operational conceptions (Jones et al., submitted). We calculated the children's cleverness ratings for the relational and substitutive conceptions on a scale of 0 to 6 by totalling the scores for each item. The mean relational rating, 3.04, was significantly higher than the substitutive rating, 1.68, $t(93) = 6.68$, $p < .001$, again confirming our previous findings. However, this did not address Hodgen's critique because the substitutive items were active and the relational items passive (Table 1).

The imperative and punctuative items all loaded onto Component 2, with one exception (imperative), which was the only item that loaded onto Component 3. This refuted our hypothesis that there are two distinctive forms of the operational conception, active and passive. It also addressed Hodgen's concern that our data in support of a substitutive conception is in fact due to children preferring passive to active items in the instrument. It is unclear why one imperative item (I6: "calculate the answer to the problem") loaded onto Component 3 whereas the other five items loaded onto Component 2, although its passive correlate (P3: "this answer is connected to this problem") loaded only weakly onto Component 2. A reexamination revealed I6 also loaded weakly onto Component 3 (.325).

	Predicted conception	Component loadings 1	2	3	4
R1	Relational	.855			
R2	Relational	.732			
R3	Relational	.635			
P1	Punctuative		.741		
P2	Punctuative		.637		
P3	Punctuative		.496		
I1	Imperative		.540		
I2	Imperative		.553		
I3	Imperative			.937	
S1	Substitutive				.714
S2	Substitutive				.819
S3	Substitutive				.866

Table 3: Component loadings for the items.

Discussion

Jeremy Hodgen challenged our evidence presented at BSRLM in March 2011 that a substitutive conception of the equals sign exists independently of the operational and relational conceptions. His challenge was based on the passivity of the relational items and activeness of the substitutive items used in the instrument. He proposed that we had detected a preference for passive over active formulations rather than an independent conception of the equals sign.

In this study we have presented a previously unpublished result that addresses Hodgen's challenge. Our data show that within the context of operational conceptions of the equals sign children do not discern between imperative (active) and punctuative (passive) items. As such our hypothesis that the operational conception as reported widely in the literature is an amalgam of two distinctive conceptions is refuted. Furthermore, it is reasonable to assume that this applies to all items on the instrument and that relational and substitutive items load strongly and uniquely on to different components because they genuinely measure different conceptions. We therefore conclude that Hodgen's critique, while astute, is incorrect.

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‘Ability’ in primary mathematics education: patterns and implications

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Ability is a powerful ideology in the UK, underscoring many educational practices. We have extensive evidence pertaining to the impacts of these, particularly setting, in secondary mathematics, but there is relatively little research into the impacts in primary schools, despite an increase in ability-grouping practices at this level. This paper begins to address this gap, discussing some of the results from my doctoral study. It explores the pervasive nature of ability and the strength of young children’s convictions in innate ability. It also examines the role of assessment in perpetuating an ability ideology, suggesting that many of the implications seen in secondary education are also issues for primary mathematics.

Keywords: Ability, Primary Mathematics, Setting, Grouping

Introduction

The research reported here presents results from my doctoral study into ability in primary mathematics education. It extends our understanding of the implications of ability in mathematics, considering the transferability of the secondary literature into the primary context. The UK has a long history of segregated education built on a strong ideology of ability. Until the 1960s, almost all pupils were educated within a tripartite system, assigned to schools on the basis of 11+ results. Whilst subsequent comprehensivisation brought a greater degree of mixed-ability teaching, mathematics has always retained a belief in the need for ability-grouping. Setting has increased in recent years, being an organisational method promoted by the Government (e.g. Gove 2007).

The impacts of ability-grouping are often considered in terms of attainment and attitude. Whilst the studies are not fully in agreement, the overall picture is of negligible overall effect (Hallam 2002). However, ability-grouping has the potential to create and extend existing achievement gaps and a number of studies have found that it is assignment to higher sets and / or enriched curricular that makes the difference in terms of attainment gains (e.g. Wiliam and Bartholomew 2004). These differences may be the result of different expectations and pedagogy in higher sets. This underscores Boaler’s assertion that “the set or stream that students are placed into, at a very young age, will almost certainly dictate the opportunities they receive for the rest of their lives” (1997, 142).

As with attainment, the impact of ability-grouping on pupils’ attitudes is also contested. Studies (e.g. Ball 1981) have shown the potential of grouping practices to polarise attitudes, although Boaler’s (1997) study suggests this to be more complex. Despite these known impacts, the use of ability-grouping continues to grow. 52% of primary schools began ability-grouping in the first year of the National Numeracy Strategy (Hallam, Ireson, and Davies 2004) and this appears to be increasing, hence the need to develop our understanding of the impacts of this on primary pupils.

'Ability' in UK education

Ability is a contentious notion in educational research, yet is a powerful ideology in the UK. Within UK society, ability, segued with intelligence, is seen as a fixed, hereditary quality, genetically determined and characterised by upper limits and a sense of inevitability (Howe 1997). The historical underpinnings of this ideology are long and complex. We have reached a stage, White (2006) argues, where accounts of intelligence arising from the work of Galton have so influenced everyday understandings, that we no longer have the capacity to see them as in any way peculiar. Ability is "one of the dominant discourses in schools and policy" (Stobart 2008, 32) and the foundation of many forms of UK classroom organisation. Adey et al. (2007) argue that teachers lack any model of cognition containing plasticity, evidenced by Kovas et al.'s (2007) finding that over 90% of teachers believe that genetic influences on ability are more, or at least as, important as environmental ones. This is important, for whilst such beliefs pervade, associated practices and their implications will continue to be legitimised.

Research design

This mixed-methods research took the form of a multiple case study. To explore the impact of different discourses and practices of ability, the research included two diverse school environments, one teaching mathematics through a strong philosophy of setting and the other employing mixed-ability teaching. Within each school, focal sets, classes, teachers and pupils were followed over the course of one academic year to explore their experiences with respect to ability.

Sample

Avenue Primary and Parkview Primary (both pseudonyms, as are all names in this report) were both 3 - 11 schools in Greater London local authorities. The schools were matched by Contextual Value Added scores. Avenue Primary was a three-form entry school. They set pupils in mathematics into four sets in each year group from Year 2 (ages 6-7). Movement between sets was very limited. Parkview Primary was a two-form entry school. This school had a strong inclusive history, having provision for pupils with Special Educational Needs, with mainstream integration. Pupils were taught in mixed-ability classes except in Year 6 where they were set for mathematics.

Within each school, all classes / sets in years 4 and 6 were included in all quantitative elements of the study, totalling 284 pupils. For the qualitative elements, top and bottom sets at Avenue and all classes at Parkview were used as focal classes. Within each focal class / set, three focal pupils were chosen by the teacher to reflect the range of attainment within the class, totalling 24 focal pupils.

Research methods

A variety of research methods were employed to gather data at different levels. Quantitative methods included attainment testing and attitudinal questionnaires, whilst qualitative methods involved classroom observation, group and individual interviews and data gathered through day-to-day observation in the schools. This approach brought with it many benefits, using many types and sources of evidence to give a rich account of events as experienced by the research subjects whilst at the same time allowing for analytic theory and generalisations to be drawn from the data. The data presented in this report all arise from the qualitative elements of the study.

Findings

The findings presented in this report represent a small selection of those discussed in the full study. The themes have been selected to provide an overview of the key issues arising across the research, but also to tell a coherent story. Data extracts are used to illustrate the findings discussed; these are selected as typical rather than extreme examples and are generalisable across both schools and year groups.

Pervasiveness of 'ability'

One rationale for including a school using a mixed-ability organisation was to give access to an understanding of the impacts of ability beyond the obvious practice of setting. A key issue to emerge early on in my school visits was that ability impacted on practices across the schools in explicit and implicit ways. In many cases, in the same ways it is argued we have lost the capacity to see our everyday use of intelligence as in any way peculiar, there seemed to be a lack of awareness of just how pervasive ability is and just how much it invades the everyday issues of teaching and learning.

My first confrontation with the pervasiveness of ability came prior to the main research during the process of school recruitment. Parkview primary had been recommended as having a very strong mixed-ability ethos. I was told the school was mixed-ability throughout with no use of between or within class grouping with the exception of Year 6. During my first meeting with the head-teacher she emphasised her position, that no ability grouping was used, and to solidify this, asked a 6-year old pupil in the room with us to confirm that there was no movement into groups for mathematics. Unfortunately, the pupil's response did not confirm this, but instead came as a shock to the head-teacher:

> No Miss, Miss Mason makes us go and sit in our maths groups, there's the green table, the purple table, the blue table, the yellow table and the red table. The green table are the best at doing maths; I'm on the red table. (Adina, Year 2, Parkview Primary)

Whilst this suggested that classroom organisation at Parkview was not as mixed-ability as first assumed, I included the school within the study as it demonstrated firstly the extent to which ability grouping practices are part of UK primary education and, secondly, how these may have become so normalised that we do not notice them. Across the study many focal pupils spoke in similar ways to Adina, demarcating tables and groups by their names, labels and / or National Curriculum levels with a full awareness of the meanings and implications of these groups.

Within the study I interviewed the teachers individually. This was very revealing, not just for the study, but also for many of the teachers who, for the first time, were given the space to think through and question the ability predicated practices they were engaged in. These teachers talked about practices which, when asked to discuss further, they could not logically defend and in many cases talked about doing things in a particular way because that was how it had always been done. Within these interviews, comments were made such as: "I don't know … it's odd, isn't it?" (Mrs Jerrett, Avenue Primary) when asked why they set pupils for mathematics and not for other subjects, whilst another teacher concluded her interview by saying: "It's freaked me out now" (Miss Barton, Parkview Primary). These teachers, involved in reproducing many of the same experiences they had as pupils (Hodgen and Marks 2009), had never been given the space, time or resources to think through the pervasive ability-predicated practices they were engaged in or the implications of these for their pupils.

Children's beliefs about 'ability'

There is much in the literature to highlight the erroneous beliefs that underscore the ideology of ability dominant in UK society and in education. Within this study I was interested in exploring the extent to which primary pupils held these beliefs and the potential impact of any beliefs they held on their learning in mathematics.

The beliefs pupils conveyed in their individual interviews were virtually unanimous with all pupils making some reference to the dominant beliefs. Pupils expressed the same views during group interviews suggesting a perceived shared understanding and a willingness to be seen to hold such views. The predominant views centred on ideas of heritability and genetics. Pupils expressed beliefs that only some individuals could be good at mathematics and that this was dictated by being born to be good at mathematics or not:

> Rachel: So what makes someone good at maths?
>
> Wynne: Their brain's bigger. And they're cleverer and better … it just happens. They were born like that. They were born clever.
>
> Rachel: And what might make someone not good at maths?
>
> Zackary: Some people are just not born clever.
>
> (Wynne and Zackary, Year 4, Avenue Primary)

In holding a belief that mathematical ability was something individuals were born with, pupils also expressed a conviction that any individual, including themselves, would not be able to improve on their attainment in mathematics. As one pupil put it, "you can only do so much can't you?" (Peter, Year 6, Avenue Primary). Pupils at Avenue Primary tended to stay in the same sets throughout their primary careers. A number of pupils appeared to use this as evidence of the 'fact' that individuals could not improve, talking as if such practices were legitimate given their understanding that individuals only possess a certain quantity of mathematical ability:

> I know I am worst out of everyone … I'll just be low now in my next school too. (Samuel, Year 6, Avenue Primary)
>
> I think I would not move. I think I would normally stay in the same place. I don't think there's anything I could do to make myself better. (Zackary, Year 4, Avenue Primary)

Sam and Zackary were both bottom set pupils. They had always been in the bottom set and both expressed the view that they felt this would not change. The implications of this are important. Zackary was eight years old at the time of this interview and so had at least another eight years of fulltime education ahead of him. However, in holding the view that he was powerless to effect group movement, one outcome may be that any effort he puts into change is limited, with his set placement effectively being self-fulfilling.

'Ability', assessment and selection

Whilst setting is a clear explicit practice of a dominant ability discourse, many other practices are tied to ability in complex ways. These both rely on the existence of a stratifying discourse, but in turn also feed into it. A number of these practices revolve around issues of assessment and selection. Both Parkview and Avenue Primary were in the catchment areas of academically selective secondary schools. The impacts of this on the pupils, even at Year 4, were high and, as with the lack of movement between sets, legitimised their beliefs about ability. Pupils talked about grammar schools with a sense of awe, citing them as where "everyone who is really really clever goes" (Thomas, Year 4, Avenue Primary) and perpetuating the belief that selective processes, such as verbal and non-verbal reasoning tests, give a reliable indication of what someone is able to do now and in the future. Further, local authority allocation procedures for secondary education served to strengthen pupils' beliefs about innate individual differences and a lack of opportunity for change:

> SATs are more like year 6 exams, and in year 5 they're like banding exams, that bands you like, some people are 1A and 1B and to get to, well some state schools, they get a certain amount from 1A, a certain amount from 2A, some from 1B, and the others. (Abbie, Year 6, Parkview Primary)

Here, Abbie was talking about the different assessments she had undertaken in mathematics. She reproduced the language of banding, understanding this, as many do, as an assessment of innate ability. Believing that each pupil fits a particular category and can be labeled thus – it was not uncommon to hear pupils referring to themselves by their National Curriculum level or band identifier – intensifies a belief that mathematical ability is a fixed given quantity that cannot be changed.

In addition to maintaining beliefs, assessments could act as a gatekeeper, ensuring the perpetuation of practices and keeping individuals where they had been placed. This was particularly the case for one pupil at Avenue Primary who was disapplied from the SATs at the end of Key Stage Two due to his attainment level:

> My friend thinks I'm dumb and so dumb that when it comes to the tests they think, they don't even give me the test, the teachers say I can't do the test and my friends think I'm dumb for not being allowed to do the test. That's how it works, I won't do the test, it makes me unhappy and I can't get better to get the tests to go up. (Samuel, Year 6, Avenue Primary)

Samuel understood the importance of the tests in having the potential for allowing him to demonstrate his attainment and hence move up through the sets. Without access to the tests, Samuel had always been, and as he saw it, would always be, placed in the bottom set. He talked vividly during our interviews about his lessons and understood the limitations that were being placed on him through the teaching and learning in the bottom set. Dominant beliefs about ability led the teachers to believe that they were doing the best thing for Samuel in protecting him from the SATs, but as Samuel identifies, this becomes reproductive, with ability beliefs serving to limit mathematical experience.

Discussion

The findings presented in this report represent only a small part of my study, yet they highlight a number of important issues arising from this research. Despite the different cultures of primary and secondary mathematics education, many of the issues we are aware of in the secondary mathematics education literature on ability appear also to be issues for primary mathematics. This is an important finding given the rise of ability-grouping in primary mathematics and may suggest that Boaler's quote concerning the early stratification of life-opportunities can be brought down and applied from a much earlier age.

Children as young as eight years old demonstrated a strong acceptance of and belief in the mathematical ability myths that pervade education, legitimising resultant practices in the same ways teachers, policy-makers and wider society do. Given the strength of these beliefs it seems likely that even younger children may be holding and forming similar views. It is important that we understand the views pupils are holding as they may lead to many pupils feeling that mathematics is something they cannot, and never will be, able to do, with potential impacts for their attitudes towards and application in, mathematics lessons.

There is some hope for change. Although many of these practices take place with very little awareness, hence the pervasiveness of ability, teachers, when given the space to think, were genuinely interested in questioning these practices. It seems feasible to suggest that practitioner reflection may be one way into addressing the inequity that is currently legitimised through our discourse of ability. However, whilst ability truths continue to dominate education, change will be very difficult.

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Topologic and topographic features of parameters of functions and meaning transitions within a microworld–microidentity interaction

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Two year 10 English students, one boy and one girl, worked together in a task developed using a GeoGebra software (Dynamic Geometrical System) to promote the development of ideas approaching to the notion of function. During the task, students' voices and the computer screen were recorded to assess how this task promotes the developmental process of what Mason call mathematical being through a perceptual guided activity regarding to parameters notion.

In accordance with Mason, who thinks that teacher can not do the learning for their learners, we assume that learning come about from the activities students develop in a learning episode. Here we will discus the capabilities of the micro-world to direct students' attention to some general topographic and topological features of the geometric enactment of algebraic expressions; and the meaning transitions of the micro-world elements experimented by students regarding to the specific manipulation type (dragging points, sliders, or typing).

The micro-world

Armando Landa Hernández developed the task through GeoGebra, a computer program that is defined as a Dynamic Geometrical System. The task includes a compendium of functions such as $\mathbf{f(x)= a\varphi(x\text{-}b)^n+c}$**,** and it is designed to presents graphical representations of specific randomised functions. This micro-world and others may be freely downloaded via the Parameters of Functions link from the GeoGebra site: http://www.geogebra.org/en/wiki/index.php/Parameters_of_Functions

Two graphics are presented as the main objects. One of the graphics (coloured in green) is taken as the graphic target, the second one (coloured in blue), is a graphic that can be manipulated by students' direct actions in three different ways: 1) dragging points, 2) through slider manipulation, and 3) typing directly sliders' values (see figure 1). The student task was to match the blue graph with the green one. It has also been included into the design of the micro-world the label 'go it', which appears when blue and green graph match. The main activity during the task was the students' direct manipulation of variation in the corresponded graphic representation of every parameter of the algebraic expression. At the end the students were asked to identify and write down the algebraic expression regarded to the graph.

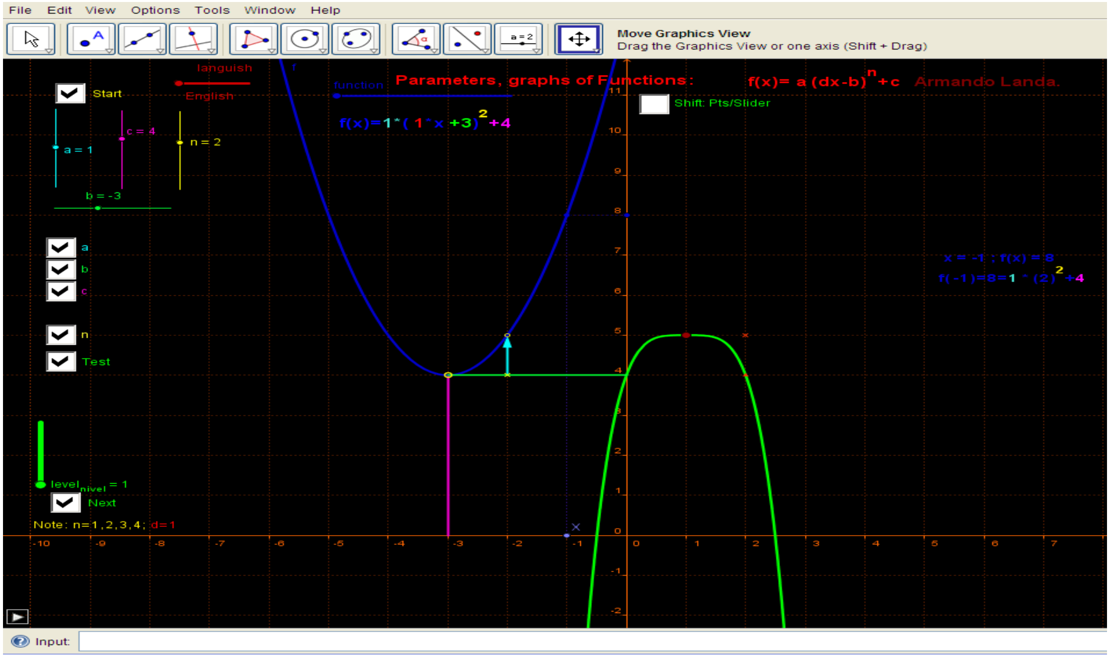

Figure 1. The microworld in ‘crane’ mode

We use the term “micro-world” as we consider this activity in the way Varela (1999) does, interpreting students’ interaction with the task as the couple microworld-microidentity. The assumption that the action is perceptually guided is the central idea supporting the micro-world design.

As we agree with Stewart (2011) when saying, “that “learning” can only be a modification of the developmental process; this means that what can be “learned is both enabled and constrained by the epigenetic landscape”, we consider the complexity of the micro-world a feature to be highlighted. The micro-world includes a compendium of functions distributed in 6 classes and five levels, with two regarding sliders to change between them. All of the elements interact directly between each other. Students’ activity in the micro-world can be considered a *visual and kinesthetic experience* (Nuñez 330) about the variation of the parameters on graphs. Every parameter (a, b, c, and n) has visual elements (colours, lines, sliders, numbers, points) that allow the students’ identification of interaction between screen elements.

Parameter sliders, points, lines and colors are visual stimulus where students’ perceptions and actions are directed. Different positions and colors of sliders allow students to differentiate its effects on blue graph in a way that perception of properties of blue graph came about from dragging sliders, points or typing numbers and its correspondent effect on graph.

By clicking on the check box ‘Start’ a set of different objects appears on the screen of the computer: a blue graph, sliders in different colours and check boxes among others. By clicking on the check box ‘Test’ a green graph appears and also a check box labelled ‘Next’. In this paper we make reference only to functions $f(x)=a(x-b)^n+c$ which correspond to the ‘$level_{nive}=1$’ one of the ‘Test’ part of the microworld.

As it has been said, we have assigned different colours to the labels of check boxes in correspondence to the colours of the sliders and objects related to them (segments, vectors, parameters in algebraic expression related to the blue graph).

Written instructions (which disappear when check box ‘Test’ is clicked on) are: ‘Match blue graph with green graph by typing the values for a, b, c, n. into the window Input (ex: a=3 <Enter>) or by dragging sliders a, b, c, n.’

By clicking on the check box ‘Shift: Pts/Slider’ another blue graph appears and sliders disappear (except slider n). The correspondent written instructions are ‘Match blue graph with Test graph by dragging points V, A and slider n’. (Figure. 2)

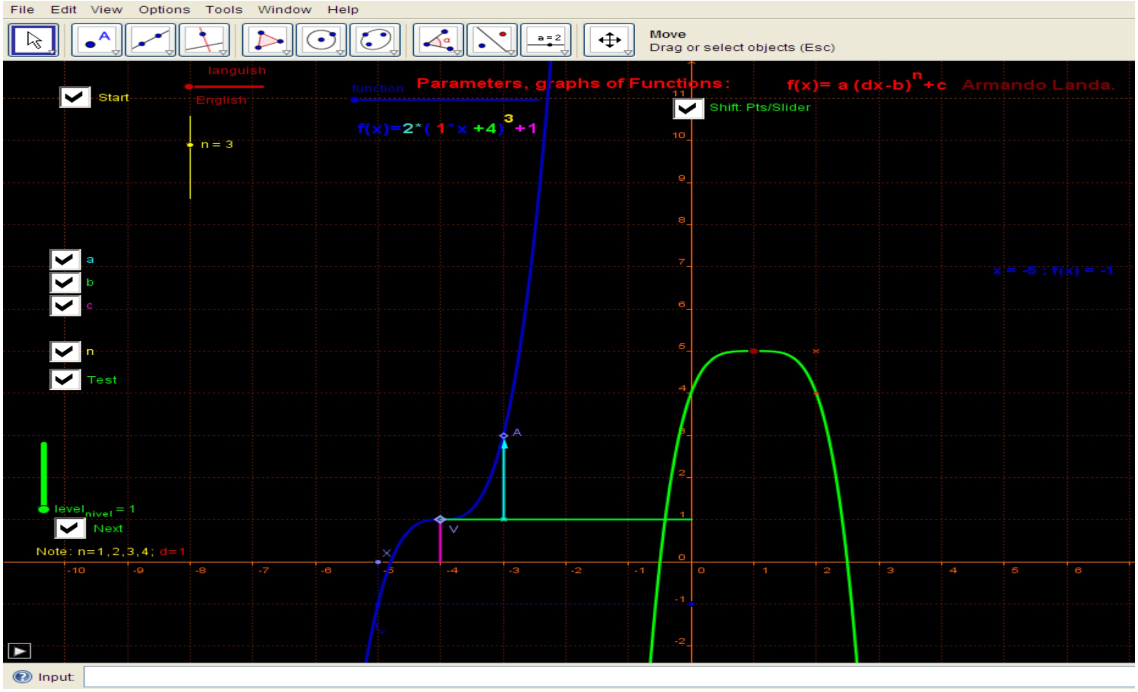

Fig. 2. Micro-world in ‘superman’ mode

The students task

Screen elements, roles and meaning

In order to analyze the learning episode with the two students, we consider the assertion of Spencer-Brown (1969) that mathematics can be derived from the activity of drawing distinctions. As our claim in this paper is that one approach to the notion of function from a geometric perspective may help to make sense of the parameters significance within the algebraic expression and their impact in the graphic enactment, for this paper, we take into account five objects types and their respective identification of topology and topography properties in the task: (1) general and specific algebraic expressions; (2) graphs; (3) modes of manipulation (sliders, points and typing); (4) ‘x’ and ‘y’ axes; and (5) *got it* message

The first activity taken into account in the micro-world is fitting the correspondent parameters identified in theirs graphical representations, in order to match both graphs each other. It is important to realize that the objects distinctions in the micro-world are made from an interdependence perspective that students may or may not draw in the same sense. This means that, as drawing a distinction is an act of absolute freedom, it could be grounded in anything else. Thus, identifying elements in screen is likely linked to perception of properties (Mason 2008) of algebraic expression regarding its geometrical presentation. Having in mind this considerations, lets explores what happened during the beginning of the episode:

Students work with the micro-world around one hour and twenty minutes. Initial instructions given to the students by the teacher were ‘…just play a few minutes with it…’ No names were given to the students about objects on the micro world (sliders, check boxes) or indications how to use those objects or what was the task to be accomplished or the aim to 'play with it'. Moreover the micro-world was given to the students with the software GeoGebra not customized. All tools, commands and properties of the software were left. So one of the remarkable things happened was the fact that some objects were identified and used in the way we expected. Just some seconds had passed after they sat in front of the computer and the girl press the start check box; They began to read instructions, press check box for parameter 'a', familiarity with computer use was evident when he tried to drag the point at the end of the vector –which we name V-. Nevertheless, they could not drag the point, because it was in the sliders mode, this evidence that they did not know what a slider was, because even when the instructions said drag sliders they do tried to drag but the points. After a couple of tries they read again the instructions (53 seconds had passed). She tries to drag point V again, he realize the objects we call sliders and ask her to try one, in trying to do it, she moved the mouse in a way that screen zoom in, then zoom out to the original position and then drag slider; as they noticed the simultaneous movement of the graph he says 'here we go'. Two minutes and forty seconds since they began the task they read again the instructions and press the 'test' check box, a green graph appears. Then they started to manipulate the sliders and took them about 2 minutes to match the graphs by the first time, they realised the 'got it' label appearing when blue graph match the green one. Immediately, they press 'next' check box. During the next 12 minutes they match 14 green graphs.

In the other hand, every element on screen is taken as a possible signifier (Jay 2011) but each one of them related with the others in a different dependence interaction determined for the task. And it is the task, which allow directing attention to different properties when they are relevant for the task. This mean, that distinguishing a slider from a graph in a mathematical sense includes at the same time taken them as different elements but interdependent (E.g. the blue graph is manipulated by means of the sliders). So, after twenty minutes of free activity, students were asked, this time in an explicit verbal way, to match graphs by using the three different modes of manipulate parameter values. This instruction were told using metaphors: for points direct dragging, they were asked to imaging superman carrying the blue graph directly to match the green one; for sliders, teacher asked them to imaging that without superman they had to learn to use cranes; and for typing, they were asked to imaging being programmers and that they need to identify the values and introduce them to change the graph shape to match the graphs. During the next thirty minutes they use, in an alternate way, the 3 modes for varying parameters values, they got 19 matches.

We highlight the fact that in the first moment the cognitive process is guided by the perceptual identification of the reference in the Cartesian plane. Thus, we thing the'X' and 'Y' axes represent the main topological reference, and plays an important roll in the developmental process. As position of graphs is defined by parameters b and c, which taken as the point with position (b, c) (topology), denoting point V (see, figure 2), and parameter 'a' and 'n' determines the shape (topography), we found that perceiving the topological properties of parameters 'b' and 'c' and the topography regarding to 'n' parameter occurred sooner during the manipulation of parameters than perceiving the topographic properties regarding their values and its correspondent effects on the graph of parameters 'a'.

Points cannot do the graphic enactment of 'n'; because of every change in it represent a new shape. That is way there is only two modes for variant the 'n' value. Perceiving 'n' properties depends on identify the shape variation with its correspondent value. But the position of the arrow related to slider 'a' (and its initial and final point) is updated according to the values for ‘b’, ‘c’, for the position, and ‘n’ value, for shape, then its topological properties come about in a latter moment during manipulation. By dragging slider ‘a’ the shape of blue graph is modified in different way than by dragging slider ‘n’. Of the 19 graphs that were matched, five matched by dragging points, 8 by dragging sliders, and 6 by typing directly the parameter values. It is remarkable that in all the trials they start by ‘b’ and ‘c’ (defining the position) or by ‘n’ (defining the shape), but never by defining ‘a’.

Another interesting aspect of the way the task direct the attention of students was when in a third stage, they were asked to find the parameters values when the blue graph were hidden; they had to introduce the values of parameters just identifying them in the green graph, without the guide reference of the blue graph. The students by then had got a notion of the coordinates’ system used, we think that this is evident when in the first trail, without the blue graph reference, they got match in a single try for each parameter. But the second trail show that the parameter ‘a’ was not been understood. They started to look for understand what was ‘a’, he aloud said “but what is ‘a’? And then they show the first reference to the blue algebraic expression, the girl point the light blue number in the expression and said: “This is ‘a’”, the boy answered “yea, I know, but how is that related to the graph? Till this point they had related many objects in the micro-world, even the graph; but the ‘a’ parameter remained being an incognita. And it is now, when there is some thing that was passed unobserved, when the need took them to reconsider and observe all those properties they had not observed and more. But it took them more than one trail to get it. He show evidence of being referring the ‘a’ value to the X axe when he said: “I am sure is minus five” pointing the minus five in the axe with the correspondent point for ‘a’ parameter in the green graph. They were told to use the blue graph reference again but at the end they decide to give up on this graph, and change to the next one. In the third trail the green graph ‘a’ parameter coincide, as in the first trail, with the distance to the X axe again, they again did not know what ‘a’ was. This time the first reference they took into consideration it seemed to be were the graph cross the Y axe, because the firs try is 3 and here is where the graph cross Y axe. He showed being disappointed when probing writing 3 and having not got the matching, what happened after show us how he had the idea that ‘a’ was referred to the axes: She suggested and wrote minus two –the distance to the X axe-, but he said “no, no because all you do with it is move it along that, vertically”. Again they were told they might use the blue graph reference and sliders, this time they did, when examined the graphs relation. He seems to return to the axes reference idea because of the relation to the specific position of the graph. The third and four trail contributed to maintain this idea for the relative position of the graph, this time again, coincided with the X axe but now the V point over the X axe in both case. They got matched without more problems, till now they, without the blue graph assistance, had matched 4 graphs, recurred to the blue for a short glance in one, and give up one. The next graph not coincides any more with any of the axes. Now as they could not find ‘a’ value again they decided to use the blue reference and realize what is ‘a’, when he done it he said: “Now I see, that’s way ‘a’ goes down from, it’s from that line to where it crosses, and how many down is, so ‘a’ is minus three”

Conclusions

We conclude that manipulating the variation of parameters values attached to immediate perceptual feedback of their consequences in the graphical representation, allowed students to direct their attention to parameters defining the geometrical characteristics of the function of graphs of the type $\mathbf{f(x)= a\varphi(x\text{-}b)^n+c.}$ As data shows the micro-world arise as a powerful mean to promote the perception of topographical and topological properties of the parameters; also showed us how the kind of manipulation direct attention to different properties of the parameters.

By dragging point V to the position of the red dot allowed students to identify the role of parameters in the geometrical representation; with sliders, sliders 'b' and 'c' separate the movements of blue graph on the screen in two differentiate movements and allowed students to differentiate between parameters. But in both cases, dragging points and sliders, matching the position of the blue graph on the position of the green graph requires only the need of being aware of some visual properties, but it is not required to be aware of its coordinates so no need to pay attention about its numerical values. Typing directs their attention to the numerical properties; and when the blue graph is hidden properties became significant in a different mathematical sense.

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The extent to which a primary maths teacher's success in the classroom is dependent on subject knowledge.

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This paper tracks 5 Primary PGCE Trainee Teachers through their Course: In particular it considers their Subject Knowledge (as measured through exam results and the PGCE mid- course Audit) and explores the extent of its significance in helping children to understand mathematical ideas and to make connections. It analyses the choices the trainees make prior to, and during, the 10 lessons observed. The Trainees reflections are heard in their post lesson discussions and in their focus group discussion at the end of the course. This is evaluated in terms of the balance between Subject Knowledge and Pedagogic Content Knowledge alongside Generic Teaching Pedagogic knowledge. Consideration is given to the need to create teachers who help children to make connections in maths when many trainees have not experienced such teaching themselves and are often fearful of trying to teach in such a way.

Keywords: GESK(Good Enough Subject Knowledge), Subject Knowledge (SK) , Pedagogic and Content Knowledge(PCK)

Introduction

The desire to clarify a blue print in terms of teacher subject and pedagogic content knowledge has led to an impressive collection of material. Shulman's (1986) PCK has been supported by, among others, Rowland and the Cambridge Team (2009): They made a significant attempt to define the blueprint for the Primary Maths Teacher. Askew (1997) referred to connectionist teaching that allowed more active involvement of children than most previous didactic, transmission- based approaches to learning. His references to the importance of good quality, ongoing, Professional Development for Classroom teachers seems to have been supported by the recommendations for deeper rooted subject knowledge proposed by Williams (2008.) These works are only part of a trail that has received many relevant additions from contributors to these informal conference papers.

Skemp (1976) was clearly identifying the need for interconnected teaching and learning. Whilst acknowledging the need for learnt knowledge, his distinction between "instrumental" and "relational" understanding seems at the heart of the trail so many of us have been following since then. The need to help develop trainee subject knowledge (often learnt through instrumental teaching) has challenged ITE Maths Trainers. The need to explore the degree to which students own learning affects their effectiveness led me to analyse a small group of trainees on the Primary PGCE Trainees to analyse the degree to which their knowledge assists them. I am indebted to Shulman (1987), whose discerning distinctions between subject, content and pedagogic knowledge I allude to in this paper.

Methodology

A questionnaire sent to PGCE Students in October 2009 allowed feedback from trainees regarding their hopes and fears about teaching maths. It revealed wide variations in trainee performance and considerable apprehension about explaining concepts to children when some of their own knowledge felt insecure. Many, however, responded positively to the modern curriculum (Strategy and Framework), particularly the emphasis on mental calculation. The pressure that mental calculation can bring worried some. Others wished they had been taught in such a way.

The five trainees selected were observed twice in 2010 (in February and again in June). Trends relating to Audit scores across the course were analysed. Finally all four of the five trainees who completed the course took part in a focus interview using an agenda I had prepared. I was present but didn't contribute to the session. The observations of the trainees do not themselves constitute meaningful evidence about how subject knowledge is used in significant way. However, by outlining a little of their teaching and the subsequent focus interview, the issues raised do merit analysis. The findings are evaluated and the conclusions are a contribution towards the relentless pursuit to one very large question. How do we develop effective primary maths teachers, when so many have been taught in a didactic way, that doesn't make it easy for them to stimulate, support and respond to children? These trainees default position often involves repeating their own experience and developing isolated understanding that follows only established rules.

Reflections on trainee observations

Teacher A (Maths GCSE – Grade C) didn't feel her teachers taught to allow understanding. She could see maths was more enjoyable when the purpose is clear. She feared she would be slow to make connections. She was keen to hear children's ideas but struggled to decide when to tell and when to scaffold when children were explaining. She appeared unaware of a lot of the difficulties the children experienced in their work and chose a child to talk in the plenary, based on his behaviour, not the content of his work.

Teacher B (A-Level Maths– Grade A). His questionnaire indicated that he wanted to be able to teach different abilities simultaneously; to get different levels of learning going within his classes. He wanted maths to be enjoyable. Both observations related to work he had prepared in the light of weaknesses he had noted in the children's previous lessons. His learning intention for the second observation was:

> To learn to solve word problems and establish secure written methods of calculation

As the title indicates, the content matter was vast. Although the children's voices were heard in response to being asked to verbalise some methods, Teacher B's voice was very much to the fore. He knew he was talking for too long to be effective:

> I have been talking for a long time and you've coped much better than I could have envisaged.

His high audit and exam grades are reflected in his confident outlining of mathematical algorithms (and awareness of Framework and Strategy methods). This is suggestive of good Subject and Content Knowledge. However his lessons were not successful in terms of learning. Children lost focus; the sheer baulk of issues being thrown at them was too daunting. He couldn't cue in to the comments of the children to bring the lessons to life. His timings were inappropriate to the children's needs. His Foundation knowledge (Rowland 2009) was sound. However, a limited capacity to transform the lesson was not supported by appropriately manageable connections for the children. Rowland's fourth quality, Contingency, was also absent.

It appeared that the ability to respond to children's comments was marginalised by two things. Firstly, his own agenda seemed to take priority. Thus, the children's comments in some ways were seen by him as slowing his delivery. Secondly, he seemed to feel burdened by the gulf between what he knew and all the things he could detect that the children didn't. He couldn't provide manageable learning experiences in a way that addressed misconceptions to help the children learn.

One might argue he may move from a "transmission" based approach to a "connectionist" (Askew 1997, 31) one. His own reflections suggest this moment is not imminent.

Teacher C (Maths A Level with Statistics- Grade A). She remembers very little dialogue from her own schooling. She feels enthused by the modern emphasis on mental calculation. She feels lots of different, connected experiences may help children to make connections. She was seeking to use open ended questions to develop children's understanding rather than just teaching to achieve the right answer.

Such a willingness to hear Year 1 children's voices and allow them to make connections came through in her teaching in a number of ways. The style of a number of her questions was very open

> What do you know about money? (Lesson 1).
>
> How might any of this equipment help us in our work? (Lesson 2).

She invested a lot of time hearing children's different responses. In her summer lesson in number work she had located every conceivable resource the school possessed (some of which the class teacher had never seen before). She was in the middle of allowing the children to access all of it over a 3 day period. She was aware of what came to pass, namely that the children needed time simply to explore the equipment for a period of time on their own terms. She allowed this to happen before refocusing children on the task.

She already possessed, in part at least, Ma's (1999) notion of "embedded knowledge"- an awareness of what children need to know and questions and experiences that might help them get there. She also had one other notable quality. Even on the first observation she revealed that this research observation "liberated her". She trusted the process. I wasn't there to judge but as an interested (and maybe knowledgeable) observer. She wanted to experiment, to try things out and to engage in dialogue. This again conjures up images of Ma (1999) defining the ongoing interest in debating mathematical issues , present in so many Chinese teachers but comparatively few American or British ones. She mentioned the constraints of school policy, the need to avoid embarrassment when being observed formally, as constraints. Indeed, it would take a confident trainee to digress too far. Overall I felt Teacher C was confident. Her knowledge and experimentation would, I felt, bare fruit soon.

Teacher D (B Grade- Maths GCSE) Her mid course audit placed her as the weakest of the five. However, in terms of generating engaging lessons and facilitating learning to develop relational understanding I felt she was much higher. What did she do to make me feel that real connections were being made by the children?

She researched thoroughly. She had made herself enough of an authority on the content knowledge of the areas being taught. She allowed enough time to be spent clarifying understanding and addressing misconceptions.

The pace of her lessons meant children were stimulated, receptive and engaged through clarity and appropriateness of task. Her knowledge had limits. She had been unaware of all the connections possible in the task for her most able children and yet, even in her first teaching practice had been smart enough to buy herself two minutes to explain the task after she had sent the rest from the carpet to work.

By the summer her work in a challenging year 2 class also bore fruit. The main lesson developed children's understanding of arrays. They had tackled word and number problems after her modelling. Yet Teacher D wanted to allow further connections to be made through a different plenary experience. Her Plenary focused on a solution to a problem that needed to be explained so that the children could become used to starting at different points in a problem. She had excellent pedagogic teaching skills as well as excellent awareness of factors affecting achievement in maths. This differs from SK.

Teacher E (GCSE Grade D Maths) She developed a negative attitude to mathematics at school, and low self esteem. Despite improvement she fears what she doesn't know

A weak first observation in Year 1, (Data Handling Block Graphs) revealed a lack of confidence in behaviour management and planning as well as a lack of understanding as to how to set up experiences where learning by discovering was possible.

By June she was composed, organised and had made clear strides forward in her behaviour management strategies. She modelled visual representations of methods outlined by year 5 children, solving addition problems.

Progress

Such teacher progress in terms of, particularly, generic teaching strategies used, is not uncommon on the PGCE course. In addition, managing learning experiences for the children in terms of pitch, style and content are areas where a lot of students make pleasing progress. Teacher E mirrors this. By the Final Teaching Experience the majority of students were making connections, learning from doing (and through making mistakes). Trainee teachers learn to understand what learning is rather than providing activities. They may have started to assess what children do and don't know.

Focus interview – July 2010

The agenda relate to trainees experiences of teaching maths, what had helped or hindered and what they thought was important. Several notable points emerged from the trainees discussion.

They spoke of a growing understanding of children's physical and intellectual needs mentioning visual and concrete support and basis for learning. In addition to active learning they also spoke of the need to develop thinking through questioning children; allowing children to discuss work and to articulate their thoughts. On a practical note they identified the need to find time to research, prepare and plan both the lesson and its' intended goals. They also discussed the impact of differing attitudes among the class teachers whose classes they had been in. It was felt school policies, ethos and class teacher insecurities had meant different levels of encouragement and expectation had been present in the 4 different school classrooms.

Finally they identified the ability to respond to the unexpected comment or discovery, from a child, as a skill to strive for; both to help children make links but also to allow them to become actively involved in their learning. This has clear links with “Contingency”(Rowland 2009). Teacher E saw this as a long term goal to aim for. She would sometimes divert the conversation if children were commenting in ways she, personally, found difficult to evaluate or link to her planned lesson.

Points emerging from research

Pulling together the threads from this research, trainee mathematical knowledge, and received teaching style, impacted on teaching. Those who learnt instrumentally had to adapt more.

Not all trainees have weak subject knowledge (witness Teachers B and C). In addition their general, pedagogic teaching skills often develop quite quickly in the second part of their PGCE (witness Teachers D and E).

However, it appears to take Trainees with more “instrumental” (Skemp 1976) understanding longer to prepare for lessons and to create meaningful learning situations where children can learn to make connections. Even among teachers with good subject knowledge, if it is not accompanied by reflective teaching skills that embrace child development then some of the impact may be lost.

At this point I am left with the idea that very effective and reflective teaching skills could be more effective than, say, good or very good subject knowledge if the teacher has what I am terming Good Enough Subject Knowledge (GESK) to scaffold their own development from: the willingness and confidence to advance less developed understanding through research, discussion with children and clearly through their own general professional development, highlighted by Askew (1997).

However quality professional development in maths is not an easy area to access. The newly qualified teacher also has many demanding transitional challenges to make. Insecure subject knowledge is often addressed through safe maths teaching, which targets transmitted teaching, and often develops instrumental understanding.

Supporting trainee teachers

The emphasis (in maths teacher training) to develop awareness of efficient mental calculation and models and images, that can scaffold the transition from concrete to abstract thinking, is often helpful to less confident trainees. It models and clarifies the areas they will seek to support children in. Time is invested in focusing trainees on the area of assessing children’s responses and linking mistakes to misconceptions that can be addressed and overcome.

A wider range of maths lessons available on video would also help. Analysis of good teaching that engages, clarifies understanding and facilitates active thinking would model effective teaching and provide a basis for effective group planning and discussion, including the use of misconceptions. This would support the Post Williams (2008) MAST Math Teaching Programmes that are currently supporting existing teachers to possess deeper mathematical understanding to support colleagues in schools.

Relevance of research

Thus, this research aims to support previous attempts to define the attributes and style of effective maths teaching. Instrumental (Skemp 1976) teaching and learning still dominates most primary classrooms. The coining of the term GESK is relevant for two reasons. Without defining the extent of subject knowledge needed it acknowledges its' relevance to other content and pedagogic skills. It also gives hope to a significant percentage of anxious trainees who are all too aware of their insecure and underdeveloped knowledge. This knowledge is, in the main, understood in an instrumental way. Trainees can see their own mathematical understanding as a work in progress: This means that the possession of "Good Enough" understanding can be either a goal that they can work towards (as quickly as possible in some cases) or it can be a starting point from which to deepen their understanding further.

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Women's stories of learning mathematics

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In this session we looked at women from three generations of one family. All three women have no formal qualifications in mathematics and all left education at the minimum school leaving age. They were video recorded talking about their experiences of learning mathematics at school and their current levels of confidence. Given their different ages (82, 64 and 44) we might expect their stories to be different, but there are surprising similarities. I am at an early stage in this 'grounded' research, approaching the topic with no intended preconceptions of what I might discover.

Theoretical background and significance

There is a substantial body of research into attitudes to mathematics and closely related themes. These studies use as their subjects school students (eg Hodgen et al 2009, Tapia and Marsh 2005), university students, training and practicing teachers, and adults returning to learning (eg Ward-Penny 2009). These could be described as a 'captive audience' for researchers. Adults who are not in education are unlikely to be the subject of surveys on attitudes to mathematics. Given the scarcity of research evidence, we know little of the long term impact of school mathematics on people's attitudes to, confidence with and use of mathematics. We know little also of the emotional legacy in the general population of learning mathematics.

We know that attitude is important to achievement from the plethora of research on this topic with school students. For example in a paper for BERA Conference 2007 Brown, Brown and Bibby (2007) claim that perceived difficulty, lack of confidence, disaffection and perceived lack of relevance relate to predicted GCSE grade. Another example comes from the USA (Singh, Granville and Dika 2002). I make the assumption that in teaching children mathematics we are not solely aiming to affect their attainment while they are children, but that we are intending to have a long term positive impact on their attainment in and use of mathematics. Research that throws some light onto the long term impact of learning mathematics is therefore useful.

The focus on women from poorer backgrounds is justified by the evidence that both gender and socio-economic circumstance are factors in attainment. Much research into achievement has analysed the differences in the findings for girls and boys, for example APU (1982) in this country and Bachman (1970) and House (1975) in other countries.

> The findings were broadly similar: lower achievement by girls and more negative attitudes at secondary level. (Onion 1998, 8).

Despite efforts by the government in England and its agencies, the relationship between socio-economic status and achievement in mathematics persists, for example as shown in the longitudinal study by Demack, Drew and Grimley (2000). This is also an issue in Australia (McConney and Perry 2010) and in the USA (National Center for Educational Statistics. 1995). Furthermore Nunes et al (2009) show that the educational achievement of mothers has a high correlation with the attainment of their children in the early years of schooling.

Finally, an aspect of the significance of this study for me is one of personal significance. My first participants are drawn from my own family of origin. The three women interviewed so far are my mother, my older sister and her oldest daughter, my niece. I am interested in them as people and curious to hear what they have to say about their experiences of mathematics, partly because they differ so markedly form my own.

Methodology

The choice of interview as a technique for this research is rather obvious. If I am interested to find out about the legacy of women's mathematics education, what better way to find out than to ask them. I recognise that in doing this I am not seeking historical truth about what occurred in the past; I am instead seeking to find out what people remember of learning maths and what impact it had on them. I am hoping to hear about critical incidents, those events charged with emotion that are more likely to be remembered, as well as about the possibly more mundane setting and narrative drive through the years of schooling for my participants. In his seminal work *The Psychology of Personal Constructs* (Kelly 1955) tells us about the importance of the meanings that individuals attach to events and that there is no absolute truth to be had about events in the past.

Moving onto the issue of choice of subjects, according to Cohen and Manion:

> A good informant is able and willing to establish and maintain a close, intimate relationship with the researcher. (Cohen and Manion 1994, 60).

As I have a pre-existing close relationship with my relatives they are more likely to be good respondents. Starting with my relatives, I intend to use a 'snowball' sample, drawing on their friends and acquaintances. I hope that this personal connection will make it more likely than otherwise, that the respondents will be candid with me. Participants were asked to tell their own story of learning maths at school, with prompts where needed. This was followed up with a question about current state of feelings towards mathematics and use of mathematics in every day life.

The methodology presented at the BSRLM Conference in March 2011 mentioned 'voice' and 'personal life history' (eg Kelchtermans 1994, Nelson 1992)), although I do not elaborate on these here. Aspects of 'Grounded Theory' are key to the way I intend to proceed with this research so some basic information about 'Grounded Theory' is given in the following passage.

Grounded theory

In 1967 Glaser and Strauss published the first book on Grounded Theory (Glaser and Strauss 1967). Essentially grounded theory is a method of research and analysis that allows the on-going collection of data to influence both the choice of subsequent samples and the way in which the data are categorised. So for instance, I may begin by asking participants in my research only about their mathematics lessons and not be alert to information they give me about the wider context of their education. It then emerges that factors outside the mathematics classroom, indeed outside school, are crucial to success in mathematics. The notion of continuing the iterations until new data provokes no change in categorisation method or suggested sample is called 'saturation'.

Not long after the publication of their book there was a divergence between Glaser and Strauss and each developed a different strain of grounded theory. Strauss and Corbin's book (1990) sets out the delicate and complicated process of allowing the data to influence subsequent research. Glaser suggests that aspects of the Strauss and Corbin method contaminate the data (Glaser 1992). The criticisms given by Glaser (1992) of Strauss and Corbin's ideas on grounded theory were critiqued among others by Kelle (2005). Kelle leans more towards a Straussian approach but essentially tries to reconcile the two views.

> …….basic problems of empirically grounded theory construction can be treated much more effectively if one draws on certain results of contemporary philosophical and epistemological discussions…… (Kelle 2005. 1)

More recent work on grounded theory includes a description of Constructivist Grounded Theory Method (eg Charmaz and Bryant 2011), hence taking into account one of the key contemporary philosophical positions that Kelle suggests. Constructivist grounded theory moves on from the somewhat positivist assumptions of early grounded theory that the process of finding something out does not affect what is found out. It takes into account that who the researcher is and how the researcher inter-relates with participants will have an effect on the data. These more recent views on grounded theory method will be of more use to me in my research than the earlier approaches.

Outcomes from interviews

At this stage of the research, three interviews have been undertaken. I have begun to transcribe and analyse the interviews, but none of the transcription or analysis was shared at the BSRLM Conference session. At the session I showed short clips from each of the interviews, preceded by a short introduction about each respondent I asked participants to note down what they observed and to agree their observations with one another. In the latter part of the session we considered similarities and differences between the interviews with the three respondents. For the purposes of this report, for each respondent I include the thumbnail sketch that was shared with participants at the session, and, in lieu of video clips, a description of what was shown. This is followed by the identification of some similarities, which emerged.

Case 1: Joan

Joan was born in 1927. She was evacuated during WWII which disrupted her schooling. She left school aged 14 and spent most of a year as an apprentice cutter in a fashion workshop. She worked in various clothing factories until she had children. While raising her children she worked as a cleaner, took in mending and did piece-work at home, for example painting Christmas cards and packing Easter eggs. She returned to work in a clothing factory when the children were older and worked briefly in manufacturing before retiring in her 50s. The interview took place in March 2010 when Joan was 82 years old.

Joan begins her account with times tables and she chants or sings the beginning as she (mis) remembers it "Once one is one, two twos are two and so on". "and then three times and you were getting on by then.…and up to twelve times and that's as far as I got." In the next clip she tells us about learning about money. "And then we got onto pounds, shillings and pence. And I was alright with them; I was never very good with decimals" "I was never very good at maths" In the next clip she talks more about decimals and tells us that she did not want to go to arithmetic lessons at school. In the next clip Joan is talking more comfortably about measurement in a practical context when she was an apprentice cutter. In the final clip shown, Joan returns to the theme of written arithmetic and use of the decimal point. She mentions that her husband is good at maths and does the household accounts.

Case 2: Lucille

Lucille was born in 1946. She is the oldest of Joan's three children. She left school aged 15 and worked as a punched card operator, then did various office jobs until she stopped work to raise a family. While raising her children she worked as a cleaner and continued in cleaning work until ill health prevented her working. She is now registered carer for her husband Charlie who has multiple sclerosis. The interview took place in March 2010 when Lucille was 64 years old.

Lucille tells us at the start that she is "no good at maths". She talks about primary school and says she was OK with learning tables, but not with long division. She elaborates on this, "I remember the teacher doing it on the blackboard and by the time she'd finished she'd lost me at the first bit." She laughs. Her husband who is present for the interview introduces the topic of algebra as something he struggled with. Lucille adds "I never even got taught algebra at my school; we never did it." In the next clip I ask Lucille about decimal currency. She replies "Woowh, don't talk to me about that; when it first went decimal in 1970 I couldn't cope with it at first. I didn't know what they were talking about." And then she goes on to say, "I got the hang of it after a while. In fact it was the kids who helped me, 'cause they was at school and they knew". When she asks Charlie if he was OK with decimals he said he was. In the next clip Lucille is asked about learning pounds shillings and pence at school. She explains, "Just the same; everything was written on the blackboard and you copied; the teacher would do it and then say, you know, ..then she'd rub that off and write up what you should do and get on with it." In the next clip Lucille tells us about avoiding maths lessons "I was in the toilets having a fag. I didn't like the teacher; she was horrible. She hit me with a duster: one of those brush dusters, not a duster. She threw it at my head and it hit me there" (touching temple). In the final two clips shown Lucille goes on to say that she is still not good at adding up, She reiterates that "the kids" are better than her and that she does not do any arithmetic now.

Case 3: Angela

Angela was born in November 1965. She is the oldest of Lucille's five children. She grew up in London and moved to Harlow with her family when she was 15. She left school aged 16. She had two children, a boy and a girl, with her first partner and three boys with her second partner. She now lives in Harlow with her third partner. The interview took place in May 2010 when Angela was 44 years old.

Four clips were shown from the interview with Angela. Angela mentions times tables and finding maths easy. She says she had no problem with maths until senior school. She talks about many changes of school. In the second clip she talks about two of the schools she attended. She talks about the teachers writing on the board. There is a memory of a teacher "throwing things at the boys". Although not herself a victim of this violence it stuck in her mind. The final two clips are a mixture of positive and negative recollections. Angela tells us that she did not like “maths in stories” and that she was not good at presenting her work; she gives the example of long division for this. She says that she was OK with algebra. The final clip finishes with Angela talking about truanting from school and how this led to her being in a ‘special class’.

Similarities among the three cases

The similarities that were identified between the extracts from the three interviews shown in the BRSLM session fall into two categories: those that relate to mathematical content and others that are more general. The mathematical content areas are: multiplication tables (all three respondents), long division (Lucille and Angela), pre-decimalisation currency (Joan and Lucille), decimals (Joan and Lucille) and algebra (Lucille and Angela). There is a question here to be asked about the significance of these findings. It would be possible to ignore details about mathematical content and to focus the research as it continues on affective reactions to mathematics in more general terms. However, at this stage, I am inclined to keep an open mind about the significance of particular mathematical topics.

The non-mathematical similarities identified are: teacher violence (Lucille and Angela), changes of location (Joan and Angela), truanting (Lucille and Angela) and gender references (Joan and Lucille). Both Lucille and Angela remember a teacher throwing things in a maths lesson, in Lucille’s case at her, and in Angela’s case at the boys in the class. These are significant negative recollections. The changes of location for Joan were brought about by World War 2. For Angela the moves were related to firstly her parents’ divorce and secondly a family move out of London. Furthermore Angela truanted from school, for various reasons. Here we see events well beyond the maths classroom impacting on these women’s education in general and hence on their learning of mathematics. Finally, on gender, both Joan and Lucille make reference to their husbands being better at maths than them. I shared with participants at the session that Angela has referred to her sons begin good at maths; she did not mention her daughter. The part of the video that included this was not shown at the session.

It will be interesting to see whether the particular mathematical topics that have emerged so far and the themes of teacher violence, disruption and absence, and gender come up in subsequent interviews and also to see what new themes emerge.

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Tablets are coming to a school near you

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Improving mathematics learning is a major educational challenge. It is predicted that schoolchildren across the developed world, will soon have personal Tablet computers with the potential to support learning. The scope for improvement in mathematics learning support is examined from several related viewpoints: previous contributions of Information Technology, including PC labs for mathematics classes; IT innovations children themselves adopt; an analogy between office work and classroom learning; individualised learning environments such as SMILE; and alternative classroom configurations. The potential of personal Tablet computers as a learner's interactive textbook, notebook, test-paper and progression-record and as a teacher's class management tool is outlined

Keywords: Mathematics learning, learning environment, classroom configuration, personal tablet computer, interactive textbook, class management tool.

Introduction

Maths Education has an elephant in the room: by lower secondary half our schoolchildren "can't do maths". But it gets worse- because, after GCSE, most of the half who can do maths won't (Brown 2008).

This paper considers the possibility of improving the mathematics classroom as a learning environment from several viewpoints and relates these to the potential contribution of Tablet computers. These viewpoints are: earlier contributions of information technology (IT) to education; IT innovations that children adopt; an analogy between office work and classroom learning- comparing a paperless office with a paperless classroom; comparing mathematics learning with learning in laboratory subjects such as physics; some pros and cons of an individualised learning environment; alternative classroom configurations; and a Tablet's potential as an individual learner's interactive textbook and notebook and test-paper and progression-record.

Information technology in education

This paper predicts yet another wave of IT innovation is about to wash over our schools (there have been so many- including calculators, graphing calculators, PC labs, programming in Basic or logo, Excel spreadsheets, Interactive White Boards (IWBs), interactive maths learning software such as Geogebra, Integrated Learning Environments) and considers whether this one could make a significant difference to maths learning. IT has permeated education at all levels and the easy victories have used software technology developed outside education- for administration of student records and Internet access to stored knowledge. Technology specifically for education is expensive (because relatively small numbers are sold) and sometimes counter-productive- thus automated testing can dumb-down learning and IWBs can encourage teachers to do presentations rather than teach (and maths is essentially a learn-by-doing subject). Networked PC laboratories (labs) are now common in schools but, because of their cost, they are scarce resources typically with limited timetabled access for each class. Can we say with any confidence that any of these IT innovations have helped improve mathematics learning significantly? So should we be optimistic about the prospects for yet another wave of technological innovation in the mathematics classroom?

Mobile phones now and Tablets soon

The current generation of schoolchildren love their mobile phones. For many they are their most precious possession and the centre of their social lives. Schoolchildren across the developed world have wholeheartedly adopted mobile phone technology and the whole culture of rich inter-person intra-group communication at-a-distance that has been developed on top of this technology- replacing those sparse at-a-distance communication practices: note and letter writing and landline phone calling. It is striking what accomplished electronic communicators and users of “apps” children have become in a rather short time.

Tablet computers, like the iPad, are currently interactive communicator and book-reader toys for adults. But they are an imminent second wave of “must-have” technology for schoolchildren. Their large screens (about 10 inches diagonally) and large memory (16 GBytes minimum) and similar processing power to Netbook PCs differentiate them from mobile phones and give them a different IT dimension: they are not just communicators, they are computers- with all that fact implies.

iPad the market leading Tablet is predicted by Gartner forecasts, quoted by Halliday (2011), to sell about 48 million iPads worldwide in 2011. This is a near fourfold increase on 2010 and about 70% of the market. The main competition will come from Tablets running Google’s Android operating system. In 2015 sales of Tablets like the iPad are predicted to be about 300 million units- half iPads and half running Android. As more companies offer iPad clones, and assuming production can keep pace with demand, prices will surely fall from the current level of £400+ towards £100+ and at this level every schoolchild in the developed world will want one and probably get one.

The cost of ownership of Tablets like the iPad should be lower than for PCs, since Tablets are inherently more reliable than Laptop PCs because of the absence of moving parts (the major source of hardware unreliability): no hard drive, no physical keyboard, minimal connectors and because, hopefully, of a lesser vulnerability to malicious software. Without a hard drive they rely on access to data stored elsewhere- a networked server or remotely “in the Cloud” but for mathematics learning at least, their memory should be much more than adequate.

Are classrooms like offices?

Nowadays most secondary schoolchildren are familiar with computers and know how to use them. Experience elsewhere in education shows that all learners having their own PCs is not a sufficient condition for learning to occur. Thus, for several years now, university business school students have all had their Laptop PCs- and proudly carried them everywhere around the campus. I asked a colleague what they use them for and he replied “email, games, and social networking, although of course they do use them for Internet research and word processing for their essays”, but the PCs are not used to support learning in any more direct or organised or intensive way than this. Why is this?

Compare this situation with the *organised* use of PCs in office work in the world at large, where nowadays there is a PC in front of every worker and every manager. The software on each worker's PC presents her with a succession of tasks and affords her some autonomy in carrying them out, and records progress. The manager's software monitors workers' progress and alerts her when difficulties- either particular to one of the workers or more generally across her team- occur and need her attention to resolve. An analogy between work in an office and work in a classroom is obvious: *Schoolchildren are learning workers* and *Teachers are class managers.* The change, from a more traditional paper and voice-communication-based office to the modern office organisation with its large scale adoption of IT- a PC in front of every worker and every manager, has occurred quite recently and was achieved across the developed world in a very few years. The outcome has been greatly increased productivity. The roles and interactions of managers and workers have changed significantly but not out of all recognition. There is considerable variation in tightness of management control and degrees of worker autonomy and having some workers based at home, at least for part of each week, is not unusual.

Despite quite heavy investment in IT, the organisation of the traditional learning environment- the secondary school classroom particularly- has changed very little: desks in rows, children writing on paper, teacher up the front- although maybe with an IWB. The driver for reshaping office work was improved productivity. The analogous benefit in education would be improved learning. It is tempting to assume that given the right conditions it would occur.

Is mathematics a laboratory subject?

Mathematics education has been willing to expand its comfort zone and see itself as a lab subject, rather like physics, with classes timetabled as theory, held in an ordinary classroom, or practical held in the PC lab. How valid is the physics analogy? In physics the purpose of the labs (besides teaching laboratory techniques) is for the students to perform milestone experiments: no sound in a vacuum, a prism splitting white light into a spectrum of colours, etc. By analogy a mathematics lab can demonstrate Pythagoras theorem or the graphical solution of simultaneous equations. But the analogy is false. Mathematics is a thinking-and-doing subject with the two actions intimately bound together, whereas in science education a separate presentation of theory and experiment is appropriate: the experiments show the theory has been tested and hence validated. School mathematics has its theory too- its collection of rules- like the associative and commutative laws- but it is much more about developing practical knowledge of mathematical language: how to write it and how to use it. (A mathematics lab has more in common culturally with a modern language lab than a physics lab- except that maths is a written language and language labs have, hitherto, emphasised the aural form.) Because mathematics is a thinking-doing subject, timetabled mathematics laboratory classes are counter-cultural: every maths class ought to be a laboratory class, but unfortunately the technology has so far been too expensive for this to be a reality.

A non-traditional mathematics classroom

SMILE: Secondary Mathematics Independent Learning Experience (Gibbons 1975) was widely used in many schools, mostly in the London area the from the 1970s to the1990s, and is still used, at least partially, in a few schools. SMILE is a system for management of whole-class mathematics learning, while accommodating the needs of individual learners. Learners perform a series of individually allocated mathematical tasks, organized by topic and attainment level, and individual progress is recorded by the class teacher on a grid. Individual allocation of tasks allows children in a class, who have a range of levels of attainment across different topics, to learn mathematics concurrently. The individual tasks have been validated, and in many cases developed, by a generation of dedicated mathematics teachers, who effectively formed a SMILE development cooperative. For each SMILE task there is a printed Card describing the task and a Box containing the materials needed- playing cards, dice, or whatever. Detailed accounts of SMILE in operation at two schools are recorded in (Bartholomew 2001). The SMILE archive (STEM 2011) contains descriptions and materials for about 2,000 distinct Tasks.

SMILE has five components. Two of them: the Database of Tasks and the Grid or Matrix for recording the progress of each child in the class are paper-based. The third is the provision of physical resources- “SMILE Boxes” for tasks. The fourth is a “next-task allocator”- not customarily identified so explicitly: it is expert knowledge stored in the heads of experienced SMILE teachers. The fifth is provision for the child to talk about her task- to the teacher or other learners- to reinforce and extend what is being learned. The strength of SMILE is it treats learners as individuals, its weakness is the substantial learning curve for new teachers while they acquire next-task allocation experience.

Alternative configurations for the maths classroom

The modern version of the traditional arrangement of a mathematics classroom has the teacher in front of an IWB facing her class seated in rows of desks. A class teacher’s time is a scarce resource and so the class is likely to be a set of children of similar maths attainment level, thereby ensuring a small spread of attainment so that whole-class teaching, aimed at the median attainment level, will (hopefully) result in the bottom quartile keeping up and those in the top quartile not getting bored. Whole-class teaching apparently makes efficient use of the teacher’s time. But where this means the teacher is talking and the children are (hopefully) listening, it may not be the optimal way to promote mathematics learning. Learning has a social dimension which can be harnessed in various ways, for example by group projects and by encouraging the children to talk to one-another about their work. The traditional classroom configuration favours the whole-class working in concert, and individuals getting occasional tuition, but it hampers small group interactions.

Ideally a class teacher should be able to optimally allocate the scarce resource which is her time, between the whole-class, small groups, and individual learners. SMILE includes small-group tasks and, in schools where SMILE was established, mathematics classrooms were likely to be furnished with tables- the children working facing inwards- rather than sitting at the traditional rows of front-facing desks (Bartholomew 2001). The SMILE experience shows teachers are willing to adopt a non-traditional classroom arrangement, to facilitate all forms of teaching, if this is approved in their school.

A number of universities (City and Durham for example) have installed IT-equipped facilities for small group interactive working- semicircular tables so that a group can cluster round a laptop PC and with a larger screen display along the straight edge. A networked classroom version could be configured as a hollow square with these small groups around three sides and the teacher's table and IWB on the fourth side: a form of PC lab optimised for small group working.

Tablet mathematics workbook: communicating, storing and interacting

A wireless networked Tablet in front of every child in every mathematics class would be more affordable than PC labs, especially if the children used their own Tablets (and why wouldn't they want to?). But even if the school had to fund the Tablets- at about £100 for the whole of each child's time in secondary education- together with a wireless intranet for the school and a PC for the school server, this would be significantly cheaper than conventional PC labs.

Hitherto, much learning software has been expensive. In contrast with hardware, software costs are all in the development- replication and distribution cost is negligible. Thus the cost of software is amortised over the numbers sold. The annual student cohort in this country exceeds half a million and is an order of magnitude more across Europe and similarly across the English speaking world and more again across the rest of the developed world. This is relevant because the problem of adequate mathematics learning is trans-national and so is the language of mathematics. The annual student numbers are enormous, so if standards for mathematics learning software for Tablets were developed, the prices could be essentially zero and the cost of lifetime-ownership of a Tablet for mathematics learning would be no more than £100 for the Tablet hardware and operating system plus a share of the networking costs.

A Tablet's functionality potentially allows it to behave as a paperless combined textbook/notebook/test-paper/progress-record. Because of the Tablet's communicating and storage properties the textbook can be downloaded from the school server and then stored for use as needed. Because of the Tablet's interactive property, the textbook can also function as a notebook- the learner's working being entered in the appropriate place in the textbook in response to the latter's prompting. Because of Tablets' communicating ability, at a time when a test is due it can appear on the screens of the whole class- to be whisked away to the teacher's or examiner's machine for marking at the end of the test period. The progress/recorder allows all work attempted to be automatically logged and all marks awarded to be automatically entered from the teacher's machine.

With children's Tablets in front of them all the time, all mathematics classes are laboratory classes, and with wireless networking there are no constraints on classroom organization. So much for the learners' Tablets. Teachers Tablets would have class management software, would be able to monitor the progress of every child in the class and could include a task allocator to facilitate support of individualized learning as in SMILE. And the classrooms would be paperless.

Conclusion

The issue addressed by this paper is whether Tablets can improve maths learning. We have seen that a Tablet PC in front of every child in maths classrooms should be affordable and could make classroom learning more, but differently, organized and more intensive, just as modern office work is. Whether such improvement in efficiency in the classroom together with other kinds of support for learners and teachers, potentially offered through the interaction, communication, display and storage properties of Tablets, will help overcome the barriers to learning mathematics so many children seem to have, remains to be seen. We should be optimistic. There is a little time before the wave of Tablet-owning children arrives in our schools. It would be good to get ahead of the game- by using this time for some trials.

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What is 'mathematical well-being'? What are the implications for policy and practice?

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This project will attempt to investigate the 'usefulness' of the capabilities framework as a means to empower adult learners to identify and reflexively consider the mathematics that holds intrinsic value to them. The discourse terrain offered by the twin concepts of capabilities and well-being will be used to sketch a theoretical landscape to show how a learners approach to learning mathematics can either impinge or capitalise on the substantive opportunities that present for improved mathematical wellbeing.

Literature review: background to the capabilities framework

Amartya Sen, Nobel prize-winner, economist, and political philosopher is one of the twenty-first centuries key thinkers and is the architect of the multi-dimensional capabilities framework; a broad analytical model that makes normative (moral) assessments about the quality of life and equality of social arrangements. The approach lies within the liberal school and forms its roots from a sometimes curious plurality of perspectives including Aristotle, Adam Smith, John Stuart Mill and Karl Marx (Sen 1999), whereby the heart of the paradigm revolves around two core principles

> "first, that the freedom to achieve well-being is of primary moral importance, and second, that freedom to achieve well-being is to be understood in terms of people's capabilities, that is, their real opportunities to do and be what they have reason to value" (Robeyns, 2011).

Sen (1999) posits that in contrast to the contemporary Utilitarian evaluative indicators (with a focus on income and wealth), social arrangements should primarily be evaluated through a focus on the actual living that people manage to achieve; the obstacles that they face but more importantly, on the freedoms to achieve the types of lives they want to lead. Sen describes well-being as the substantive freedoms people have to achieve the functionings that they value; a recognition of freedom and opportunity that is independent of whether or not translated into achievement. Well-being is developed through capability vectors, which Sen describes as

> "the various combinations of functionings (beings and doings) that the person can achieve (or do) ... reflecting the person's freedom to lead one type of life or another ... (or) to choose from possible livings" (Sen, 2009)

Advocates of the Capabilities framework in an educational setting argue that traditional indicators fail to adequately consider the cultural, social and political barriers that can serve to perpetuate inequality through the educational system. I hope to use the Capabilities framework as the conceptual means to collapse and disaggregate the notion of agential freedom to evaluate the extent to which a small sample of adult learners have been able to distinguish between the mathematical competencies (for qualification purposes) and the capabilities for well-being. This research also intends to investigate the usefulness of the notion of well-being as an appropriate proxy for learners, practitioners and policy makers to reflexively evaluate an individual's response to *real and lived* opportunity as they present throughout the learning experience.

The capabilities framework, however is a tool to measure (in)equality and does not attempt to provide an evaluative spotlight to gather insight into how an individual comes to navigate their life experiences. In realising that I wanted to ask the learners about their own learning experiences, I became keenly aware of the apparent limitation of the discourse to embrace the particularities brought about by individual voice. Elaine Unterhalter (2006) provides a useful framework from which to start to interweave the theoretical purposes that may traditionally, be regarded as threads that should not be readily stitched together.

She advocates that it is essential for researchers to continue to challenge universal definitions, with particular regards to gendered assumptions about the ways in which learners approach education. She argues that although normative guidelines are necessary to make evaluative statements about the equality of social arrangements; she also asserts a continued need to focus on the fluid and often shifting processes that create identity. Especially, she argues, for those more likely to experience marginalisation and she posits an and/both approach that views different purposes as opposite sides of the same coin; as opposed to theoretical tensions that need to be resolved.

With this in mind, I intend to supplement the capabilities framework by drawing from a variety of educational discourses to capture the sense making processes and the concurrent representations (Squire, 2009), as the individual participants position themselves with regards to mathematics and wider learning. Three perspectives will shape my interpretations including Bourdieu's understanding of *habitus* (1977) and *field* (1993), a Vygotskian focus on language as a cultural tool and a Foucauldian interrogation of power dynamics through discourse analysis.

Gap in knowledge

Given the size of the sample and the methodological approach that I have undertaken, I feel it would not be useful to attempt to construct, at this stage, a multi dimensional measuring device for evaluating the impact of learning mathematics on an individuals' freedom to live a life that they value. Instead, drawing from strands of critical mathematics pedagogy and post-structural literature on voice, I intend to use the interview space to explore difference by encouraging the participant to problematise their approaches to mathematics with particular reference to the structuring factors of class, race and gender that have impacted on their opportunity to learn. I then propose to use the richness of the multiple horizons (Rapley 2004) to pool the emergent themes in order to investigate the potential usefulness of the twin pillars of capabilities and well-being as a powerful tool for learner self assessment.

Outline of research

Research questions

- To what extent do values and beliefs influence the ways in which learners approach classroom mathematics?
- How do learner perceptions of formal mathematical structures affect their learning progress?

- What are the implications, for policy and practice, of using the concepts of capabilities and well-being for improving the experience of learning mathematics?

Sample

The sample is non-probability based and has been purposively constructed to capture a breadth of learning contexts. In total there is a sample of 11adult learners (19+) from a variety of educational settings including:

- discrete numeracy settings including an adult education college, a residential women's college, family learning provision within a primary school and work based learning (classroom assistance)
- embedded numeracy provision including foundation tier (business), ESOL learners (IT) and an access to HE programs (nurses and teachers)

The learner participants have been drawn from a small sample of experienced and specialist mathematical teachers who, to a varying degree, interweave mathematical discourse (as a pedagogic approach) into the learning of mathematics.

Methods

The data has been collected through a variety of techniques drawing from a narrative approach to capture learners' voice, non-participatory observation for discourse analysis and then followed by a semi-structured in-depth interview. In taking a flexible methodological approach, I have attempted to ensure that the data collection tools serve to complement rather than restrict participant voice.

Oppenheimand (1992), Rapley(2004) , Ritchie and Lewis (2003) all concur that with preparation and reflexivity, the researcher should aim to humanise the interview process. Whilst Rapley (2004) warns against the interviewer falling into the binary trap of attempting to choose between neutrality and opting for personal disclosure prior to meeting the individual participants, Oppenheim (1992) jutaxposes this stance advocating a more determind line. He argues that the processes of interviewing are "inextricably and unavoidably historical, political and context bound" stating that it is simply not useful for a researcher, particular an inexperienced one, to attempt to situate themselves as neutral tool. Instead, he likens the use of personal disclosure to that of a walking stick; that is as a tool that can help some participants to find their feet during the interview process. It is with this view in mind that I have undertake a narrative approach to collecting the learners' life educational history.

I designed an open interview guide, to create space for the participants to talk about and to prioritise their learning experiences. Throughout this phase, I asked the participant to identify their own personal as well as mathematical strengths, which I then used as a platform to interpret how they construct and respond to the obstacles that, thus far, have prevented them from becoming the mathematician that they feel they could have achieved. In taking this approach, I held two purposes in mind: The first is to co-construct a positive (rather than a deficient) space for the participants to discuss their approaches to learning and the second, to analyse the role of agency, capabilities, and well-being, in overcoming complex (material, personal and cognitive) barriers to learning.

However, asking a participant to relay (with examples) what mathematics means to them and to describe the sort of a mathematician they aspire to be, is abstract and almost impossible to visualise let alone answer. If I am to ask the research participants such abstracted questions, I need to do so in a meaningful way. I want to be able to discuss how they approach learning through the theoretical lens of analysing how they 'do' mathematics and how the solution was negotiated within the group and I intend to use the second interview to achieve this focus. I will use observations from a mathematical class as the tool to conduct a semi-structured interview from which to co-construct an account of the learning experience.

Interpretative paradigm

I am for the moment, using grounded theory as a theoretical guide to allow the learner voice to generate theory however, I remain keenly aware that even with a more fluid Glaser or Charmezque (Charmaz, 2010) framework; I am not undertaking grounded theory research. There are many points of similarity which has allowed me to use the initial part of the framework as useful structure as I learn to interpret voice. Nevertheless I am coming to this research as an experienced practitioner and if I were to view my teacher self through a grounded theory lens, I would have to create a veil of ignorance to cover my professional knowledge in order to remain open minded to participant voice. I can see the dilemma but would prefer to use a reflexive biographical approach to identify and reconcile my preconceptions so that I can use my knowledge professional and personal strengths to guide my findings. Once I am confident that I have listened to (rather than inferred from) participant voice, I will then move towards a more thematic approach to investigate the root causes of the barriers that have thus far prevented this small sample of adult learners from becoming the sort of mathematics that they feel they should have achieved.

Findings

Thus far I have completed all of the initial narrative based interviews and am about to start the non-participatory observation and subsequent semi structured interviews. Although I have by no means analysed the data, there appears to be some promising emergent themes.

… on relationship with mathematics

According to Heather Mendick (2005), learners often hold a fluid, fragmented and often-contradictory identification with mathematics where mathematicians tend to be characterised as independent thinkers who are separated from, rather than connected, to the rest of the world. 'Real' mathematicians tend to be different to other people and the preliminary result of this research shows similar patterns of identification. Within this small sample, many of the learners have constructed their relationship through the lens of an idealised often masculine vision of a 'mathematician' and this tendency to view mathematics through a lens of 'otherness' is nicely captured by J's remarks in her initial interview

> ... So it's almost like hands off, I don't know. So I think I have been brought up in that sort of environment although my father worked as an engineer so it's almost like, well he's the one that knows it all so ... yeah. (J 2011)

... on the narrative of the ideal learner

M studying mathematics on a foundation learning tier course in Business, like J, she voices her (in)ability to 'do' mathematics in relation to 'otherness' generated by her vision of an 'ideal' mathematician. M is not alone in her thoughts; according to Boaler (2009) speed and memory are often cited by learners as the key mathematical skills that must be learned in order to succeed.

> “Yeah it takes me so long and everyone will do it like that (clicks her fingers) and do it in there heads and stuff. I mean there is this guy who kind of knows everything and so he will teach me it but it will be like derrr and then it’s gone”

... on capabilities and well-being

H studying mathematics through IT is very frustrated by his performance in mathematics. H has lived in the England for 2 years and has achieved a very high standard of English. H has a very successful academic history and despite an impoverished childhood, the first in his family to gain an undergraduate qualification, he too told a story of his struggle with mathematics. Throughout his interview, he demonstrated very effective and critical learning capabilities but was frustrated and angry with himself (and the curricula) for not being able to transfer his generic learning capabilities into the mathematical space.

> All this numbers you know it was like some kind of magic ... all these things I learned before but now they slipped and I couldn’t answer the questions. I was thinking this is ridiculous to forget all these things but I don’t ... (but) this kind of exercise it give you the possibility to go around ... this curriculum ... for example if it doesn’t make me learn these things that I don’t know ... it doesn’t help me, he (it) destroy me little bit.

.... On agency

It is also evident that although the notion of agential freedom needs to be disaggregated in order to gain a better understanding of the learner and their relationship with mathematics, it remains integral to the learning story. D another ESOL learner, but this time with a poor experience of learning in school, demonstrated a strong sense mathematical capabilities and an emerging sense of mathematical well-being.

> “Yes as long as the problem is give you this liberty, yes I like it because ... on your own you can say, you know, this is my result because ... but I have been taught (in Romania) and I got used with problems that give you something strictly asking (questions) so you look for the answer on thinking of those askings."

... But in recognising his emerging sense of the self, throughout the interview he made statements that suggest a poor sense of agency. The teaching of mathematics is changing and there is according to Swan and Swain (2010), some progresses towards mathematics teachers adopting the kind of pedagogic principles that require learners to reason rather than to recall answers. This sort of approach could frustrate D unless he is giving the curricula space to consider reflexively his changing approach to learning mathematics

> ... but you tell me exactly what you want me tell me exactly how you want it to be done ... If you want something you have to say what the askings...

In summary, this research intends to capture learners’ voice through collating narratives to compile a mathematics and/or educational history to be interpreted through Bourdieu’s notions of habitus and field. Participant histories will then be used to identify the key themes and subsequent, more targeted, in-depth interviews will provide the thick descriptors for interpreting their learning experiences. A Vygotskian lens will provide the theoretical framework for the discourse analysis on the structures that underpin the mathematical discussions and a Foucauldian lens to analyse the sequences and development of conversation and the power structures that create and maintain the norms of the learning environment.

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Early entry in GCSE Mathematics

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The change in school accountability measures at KS4 to include GCSEs in mathematics and English at grade C or above has led to increasing use of early entry to ensure that performance targets are met. We discuss the evidence around school entry practices from two surveys completed as part of the independent evaluation of the mathematics pathways project and discuss the need for a quantitative research study into the impact of early entry on participation and attainment.

Assessment; accountability; GCSE; measures of performance; early entry

Introduction

Since 2006 school accountability measures for KS4 (age 16) have included five GCSEs at grade C and above including English and mathematics (DCSF 2005). Performance in this measure (see Table 1) has increased and the data suggests a simple and positive trend, during a time of significant change for GCSE mathematics.

	2006	2007	2008	2009	2010
5+A*-C including English and mathematics (DfE)	45.6%	46.3%	47.6%	49.8%	53.4%
A*-C mathematics (JCQ)	54.3%	55.3%	56.4%	57.3%	58.5%
Average point score for mathematics	4.5	4.531	4.575	4.639	4.686

Table 1 GCSE performance in England 2006-2010

As the data in Table 2 shows, in 2008 when the first awards were made on two tier GCSE there was a drop in the proportion of grade Bs by 1.5% (9000 candidates) and an increase in grade Cs by 2% (12000 candidates) and in 2009 the first awards on GCSE without coursework saw boys' achievement overtake that of girls (by 1.5% from 2008 to 2009).

(Overall/female/male%)	A*	A	B	C
2006 3 tier with coursework	4.1/4/4.2	9/9/9.1	17.6/18.2/17	23.6/23.7/23.3
2007 3 tier with coursework	4/4/4	9.7/9.7/9.6	17.7/18.3/17.1	23.9/23.9/24
2008 2 tier with coursework	4.5/4.7/4.3	9.8/10/9.6	16.2/16.3/16.1	25.9/25.8/25.9
2009 2 tier w/out c'work	4.6/4.5/4.7	10.7/10.6/10.8	15.7/15.6/15.7	26.3/26.3/26.4
2010 2 tier w/out c'work	5/4.8/5.1	11.2/11.2/11.3	16/15.8/16.1	26.3/26.6/26.1

Table 2 GCSE mathematics results in England 2006-2010 (JCQ)

Anecdotal evidence suggested that as a direct consequence of the change in school accountability measure, schools were adopting a range of practices to secure grades C in GCSE mathematics, that were not necessarily enhancing learners' experience of mathematics. In particular early entry to GCSE was being used with a wider range of students, particularly by schools identified as 'challenge schools' where early and multiple entries to both GCSE English and mathematics were being encouraged as a means of ensuring that government 'floor targets' were met. Early entry had been the preserve of high attainers and this had increased as local authorities used it as an indicator for schools meeting the needs of 'gifted and talented' students.

In 2009 Ofsted reported the prevalence of 'teaching to the test' in school mathematics and QCA developed a factsheet distributed to all schools and local authorities in England through the National Strategies and Awarding Bodies about the forthcoming changes to GCSE from 2010 (following changes to the National Curriculum implemented from 2008) which stated

> Candidates who take GCSE early and achieve a lower grade than A* are less likely to continue their study of mathematics post-16 than students who achieve their full potential in mathematics at age 16. In other words, for candidates who may achieve lower grades through early entry, it would be better to delay entry and give them a richer experience of mathematics and the opportunity to achieve a higher grade.

During this time (2007-2010) QCA was managing phase 2 of the Mathematics Pathways Project which included the trialling and piloting of two GCSEs in mathematics, one of which incorporated functional mathematics and the other which had a greater emphasis on mathematical thinking and problem solving. This pilot involved approximately two hundred centres and was subject to an independent evaluation led by academics from Nottingham, Manchester and Sussex Universities (the Evaluating Mathematics Pathways project (EMP)). During summer 2009, the EMP team conducted a survey of these centres to explore emerging entry practices. With responses from just under 100 centres, the report concluded that

> The inclusion of mathematics in the 5 A* - C GCSE count in the headline performance measure for schools is resulting in a substantial proportion (about one-quarter) of schools entering students early for GCSE mathematics. There are indications that schools in which post-16 education is not possible are adopting this strategy more vigorously than those with post-16 provision. Such strategies are used alongside targeted interventions for students identified as being at risk (ie on the C/D borderline). For students who achieve their goal at GCSE before the end of year 11 their continuing study of mathematics is handled very differently from school to school.

In summer 2010 the EMP team completed a national survey of schools to investigate entry practices more widely. When exam results were published in August 2010, it was reported that 10% of mathematics GCSE results were for candidates aged 15 or under (BBC 2010). Andrew Hall, CEO of the awarding organisation AQA, raised concerns around increasing levels of early entry (more than 150% over two years, from 33000 in 2008 to 83000 in 2010) in his speech to the ACME conference in March 2011. In this paper we discuss the outcomes of the summer 2010 survey and a possible research study into the impact of early entry on participation and attainment.

EMP 2010 survey

The national online survey was widely publicised by subject associations, the National Strategies, mathematics advisers and consultants and awarding bodies. There were 368 responses which represents more than 10% of all secondary schools in England. Approximately 66% of respondents were from 'through 16' schools. The

GCSE achievement from respondents was higher than the national average. Nearly all schools used one awarding body and most (62%) entered candidates at the end of KS4, this was particularly the case for independent and selective schools.

20% of respondents entered high attainers before the end of year 10, and a third of respondents had entered a wide range of students before the end of year 11. This was an increase over the findings in 2009. In preparation for entry about two-thirds of centres reported targeting particular teachers at C/D borderline students and providing additional support at lunchtimes or after school. In line with the 2009 survey these strategies are used more in 'up to 16' schools, than in 'through 16' schools, and by schools with lower GCSE performance. Some schools had introduced additional time for teaching mathematics to all students, most of these also use the other two strategies as well.

Many early entrants are re-entered if they fail to get a grade C or the grade expected (through, for example, Fischer Family Trust data). Respondents were asked what mathematics provision was available to early entrants. Figure 1 shows that a quarter of those achieving a grade C and 11% of those achieving grades A and B stop studying mathematics and only a substantial proportion of those who achieved a grade A had access to more mathematics at a higher level.

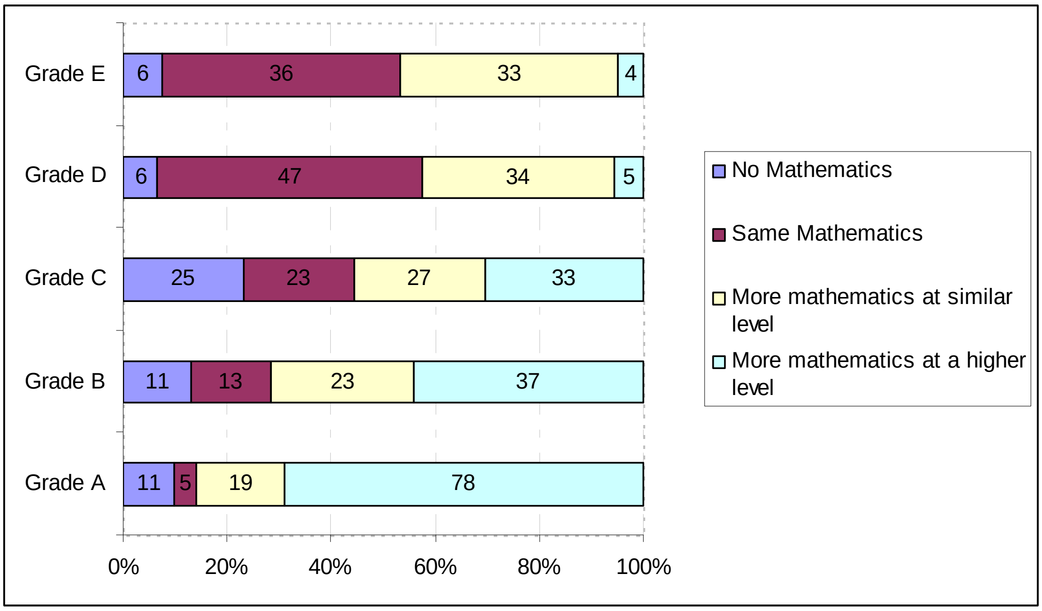

Figure 1: Mathematical provision for students who take GCSE early by achieved grade

In contrast to the 2009 survey there was a slightly more positive disposition towards two tier GCSE, although those who were opposed to two tier were more vociferous, and the proportion of students being entered for higher tier was greater. The increase in higher tier entries may be a consequence of the nature of the respondents. Many centres that are adopting early entry tend to enter most students at foundation tier initially and then re-enter those they think might be able to get a higher grade at higher tier.

What is the impact of early entry?

There is no doubt that not doing any mathematics from the age of 15 will ensure you are ill-prepared to take up the study of mathematics post-16. Like playing music or sport, skills will atrophy without practice. Given the focus on 'grade C' many students think that a grade B will do. They do not realise that when they apply for a popular undergraduate course at a prestigious university their GCSE results will count and a grade B a year early will not be good enough (Daily Telegraph 2008). Many centres believe that once students have achieved a GCSE in mathematics their statutory entitlement of a broad and balanced curriculum that includes mathematics up to the age of 16 no longer exists.

For some students, who may 'opt out' during Y11 or be eligible to leave before the end of Y11, taking GCSE early may help to ensure they have a grade which they might otherwise not have achieved. This is clearly advantageous to both the student and the school. However, there is considerable anecdotal evidence that in schools where early entry is adopted for the entire cohort and 'students are offered a chance to improve their grade', many centres find motivation a significant issue. Anecdotally, this was often the case when 'top sets' were entered at the end of Y10 and began studying A level in Y11. Students were often more concerned about their other GCSEs than the A level course and the experience could make them less inclined to continue studying mathematics post-16.

The need for a quantitative study

Given the serious impact upon students’ experiences of learning mathematics and the probable longer term effects of early entry on participation, further research is needed in this area. A quantitative study would enable answers to the following questions:

- What is the extent and nature of the increase in early entry to GCSE?
- Is there any evidence that early entry results in underperformance?
- What is the impact of early entry on participation and achievement in AS and A level mathematics and does this differ depending on the type of institution?

It would provide a baseline for examining future changes and would also help ensure that any policy in respect of early entry practice is appropriate and well informed. It is worth noting that in Scotland 'early presentation' is strongly discouraged by the Scottish Executive (2005).

Through using matched candidate data for a large age-group cohort, the real impact of early entry on subsequent participation and achievement can be investigated. Identifying all students who achieved GCSE in advance of the summer in which they completed KS4 and tracking their subsequent mathematical achievements, allowing for school type, prior attainment, gender etc. and comparing that with the progression of those who completed their GCSE at the end of KS4 will enable a thorough investigation of the questions identified above.

One of the challenges of this approach is the longitudinal nature of the data. Data is now becoming available for students completing A levels in the summer of 2010. This is the cohort that might have been entered early for GCSE, as year 10 students, in the summer of 2007. Although this analysis would provide a baseline the research would need to be repeated with subsequent cohorts in order to get a sense of the evolving impact of the performance measures and early entry practices.

Conclusion

There is a general consensus in the UK mathematics community that enrichment through breadth and depth of study is generally better than acceleration in meeting the needs of 'able' learners (UK mathematics foundation 2000). The performativity culture in England has seen acceleration being more widely adopted for a wider range of learners. Indeed, the government decision in 2008 to abandon National Curriculum tests at the end of Key Stage 3 has led many schools to start KS4 early and enter students for GCSE units in Y9. This could lead to an even greater proportion of centres entering students early.

Evidence from the EMP surveys (2009 and 2010) suggest that early entry is on the increase and that the mathematics provision pre- and post-entry is not necessarily what might be hoped for in terms of a broad and balanced curriculum with opportunities to learn for understanding and develop confidence as a learner and user of mathematics.

Does early entry result in underperformance? Does it undermine participation in post-16 study? A quantitative study has been proposed that will enable answers to these questions to be found.

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Mathematics but and yet: Undergraduates narratives about decision making

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Abstract: This paper draws on research from the ESRC-funded project Understanding Participation rates in post-16 Mathematics And Physics (UPMAP) and draws on the strand of this project that has interviewed undergraduates about their choice of university course. These interviews were conducted in a 'narrative-style' and their construction and analysis were informed by psychoanalytical theory and practice that acknowledges unconscious influences on decision-making. The focus here is on narratives from undergraduates who are reading mathematics with other subjects of study. The questions 'why is this person studying mathematics?' and 'what is the role of the minor or joint subject?' are both considered. The observation is made that while mathematics functions as a place where results are definite – a notion cited by many students in the study – the minor or joint subject functions as a place for fantasies or is used as defence.

Keywords: Undergraduate, affect, participation

Introduction

From the perspective of mathematics education, it is intrinsically interesting to consider reasons for participation in mathematics at all stages of education. Of course, the average child throughout the UK is required to study mathematics until 15 or 16 as part of their compulsory education and in most other 'developed' countries there is a requirement to study mathematics for longer (Nuffield 2010). Within UK contexts, there is a decision point for students with mathematics-gate-keeper qualifications in post-compulsory education as to whether to continue the subject or to 'drop' it. From this even smaller pool, some students again opt for mathematics to study at university some of whom are also studying a joint or minor subject. This paper presents stories from students with an aim of getting closer to why 'mathematics but and yet'.

To ask 'who chooses maths?' could prompt perusal of applicant data on the UCAS site and reply with a statistic. For a prospective undergraduate, 'choosing a course' in that context would be to follow the directions through ucas.com\students. But there is much involved in course selection that is particular to the life of the prospective undergraduate before this actual application procedure is enacted and there will be influences that operate beneath his/her conscious awareness. To attempt to make sense of such influences, this paper draws on concepts from the psychodynamic field (mainly from Melanie Klein and associates (Waddell 1998)) as psychoanalysis provides tools to detect and analyse unconscious forces.

The research is part of a longitudinal (2008-2011) project, Understanding Participation in Mathematics and Physics (UPMAP), that has been investigating participation patterns in both mathematics and physics using a range of methods (Reiss *et al.* 2011). In this part of the UPMAP study interviews were conducted with about 50 first year undergraduates – who were qualified to read either mathematics or physics – at one of four UK universities. Of these undergraduates, nine were reading mathematics with a joint or minor subject; these students were distributed over the four universities involved. Their minor or joint subjects were: computer science, French (two undergraduates at different universities), physics, management, statistics with finance, finance, accounting, Spanish and Arabic.

The outline of the rest of this paper is as follows: firstly, initial extracts from interviews that are examples of 'mathematics but and yet' are presented (together with brief biographical details about the undergraduate who spoke the words) and methods of interviewing and approaches to analysis are explained briefly. Then, using both the initial and also further data extracts, the psychoanalytic notions used in this particular paper are outlined and exemplified together with analyses that reflect on why these undergraduates are on a mathematics with joint/minor subject course.

Interview extracts

In this section, illustrative extracts are presented that come from interview transcripts from five undergraduates who are reading mathematics with another subject. These extracts have been selected in order to clarify the observation that when mathematics functions as a place where results are definite or the relationship with mathematics is secure, minor or joint subjects function as places for fantasies or are used as defence.

Evan[2] is reading mathematics with Spanish and Arabic at a Russell group university. He went to a co-educational private school in the North West of England.

> Evan – I just think there's this image that people have of mathematicians and they think they're all really nerdy and don't have fun and they are quite intense students. *I – where does this image come from?* E – not sure, the media but yeah I'm not sure, erm, I have to say I was worried that's why I chose to do languages as well cos I thought these people are going to be intense students and I'm going to need some sort of release from the intensity and I'm not sure where the image comes from I just think, I don't know, they are definitely determined people, a lot of people I have met just do work really, really hard and cos it is a difficult subject I think and there's less freedom to have your own creativity which if you compromise on creativity then that just blows your personality in a sense I mean I'm not sure but.

Trish is reading mathematics and management at another Russell group university. She was a scholarship student at a small private girls' school in the North East of England.

> Trish – I didn't just want to do straight maths cos I wanted to do something different. I've always done 101 things and I didn't just want to come and do one and I thought it would be harder as well and I like the sort of businessy thing although I didn't like my [business studies] class [at GCSE] that much and I didn't like the syllabus it was the management part I preferred and I don't know, I've no idea. I had maths with Artificial Intelligence on one of my [UCAS] options, I had maths with criminology as another one.

Lee is reading mathematics and statistics and finance at a pre-1992 university. He went to a boys' 11-16 comprehensive and a catholic sixth form college in London.

> Lee – I thought with the maths and stats and finance degree maybe become a banker or some sort of trader, stock trader or something like that. And it's a lot of money if you know what you're doing. I come from a kind of under-privileged background and I just want to make myself a better person you know, just have a better lifestyle. So, you've got to work hard for it and you know, earn money, earn money's important, very important. So, you know I thought maybe that's why I

[2] Pseudonyms that preserve gender are used.

> chose this degree as opposed to something that's straight maths or something. Yeah. A lot of money.

Chloe attended a selective maintained girls' school in the midlands and is reading mathematics and French at a Russell group university:

> Chloe – I like the logic that goes with maths, where you can see there's a proof, you're working its way through and you get to English and its all subjective ... My mum's a French teacher but I don't think that that really affects why I am doing French erm although I don't know cos obviously my Mum's always been a French teacher so I wouldn't know what it'll be like if she wasn't.

Laura, who comes from the same region of the country as Chloe and also attended the same type of school as Chloe, is reading mathematics and French at a post-1992 university:

> Laura – Essay writing, you can be kind of correct but still not do very well. Like, it's too subjective for my liking. Em, like obviously I do maths cos it, it's either right or it's wrong and I find that better than something where I have to like, argue or, yeah, just think like, essay writing. ... [*]em, just cos I wanted, I, like a language is useful and I probably want to do another ski season when I'm done em, and if I had, if I could speak French it would be easier. Obviously in the Alps and things.

Undergraduates were introduced to the UPMAP project through electronic communication from their university. From those who expressed interest in being interviewed, undergraduates forming a purposive sample - covering a range of subjects, backgrounds and universities - were invited to come to talk with one of the interviewing team about their "choice of course" (the quotation is from the invitation's email header). The audio-recorded interview was conducted in a relaxed and conversational manner in a comfortable environment in the undergraduate's university. The transcription and the audio file were used as texts for analysis of the 'narrative-style' interview co-constructed between undergraduate and interviewer. Further details about the interviewing methodology has been presented and discussed elsewhere (Reiss *et al.* 2011). There were three different interviewers (including the author) for the interviews that have been used in this paper.

Defences and phantasies: concepts from psychoanalysis

In a short paper such as this, there is insufficient space to discuss psychoanalytical concepts in any detail. So to give a flavour both of the theory and of how it has been used, a very brief and limited introduction to the particular psychoanalytic concepts of 'defence' and 'phantasy' is given through interpretation of some of the interview data. Firstly, data illustrating Evan's and Trish's 'defences' are presented then Lee's, Chloe's and Laura's stories are used to communicate how fantasy/phantasy has been understood.

Defence mechanisms were originally posited by Sigmund Freud as the way the psyche deals with anxieties by protecting the self and the notion of defence employed here follows Melanie Klein's understanding which was developed from Freud. Well-known mechanisms of defence include denial, repression and displacement and these operate from the unconscious. While there is a sense of negativity that surrounds the word 'defence' in everyday language this is not the case in psychoanalytic contexts including the discussion here. In particular, Klein's

[*] '…' indicates a portion of transcript text has been cut.

approach to defence mechanisms was based on her work with very young children and Klein theorised that defence mechanisms are a central part of mental development: mind comes into being in infancy by defending and coping with consequences of defending (Wadell, *op. cit.*).

What defence mechanisms can be discerned in the undergraduates' interviews that are relevant to their studying mathematics and another subject? Jacques Nimier (1993) posited that mathematics – as a discipline – can be used as a (manic) defence which can have an influence on subject choice (Rodd 2011). In Evan's extract given above, where mathematics itself is positioned as the potential destroyer of "personality", Evan articulates that he perceives a psychic danger of "blowing your personality" and also of a social danger of perceived "nerdiness" and this is the reason he cites for doing languages as part of his degree. But, and yet he does do mathematics. To get an idea of why mathematics is still useful to his psyche – despite the risks he can voice – it is useful to know that Evan is the younger of two brothers and his elder brother went to – in Evan's words – a "state school"

> he didn't really reach his potential and yeah he would have done a lot better in private school … I think he could have done a lot better but he just didn't have the enthusiasm he wasn't pushed at school whereas at the school I went to you were pushed the whole time, not threatened but they say if you don't get your GCSEs if you don't get straight As then you won't get in to university and if you don't do A levels you won't get anywhere in life and you'll fail miserably but I don't think that's the case.

In a subsequent part of the interview, Evan explains that he did further mathematics A level in six months and received A grade – a course that will indeed have entailed his being 'pushed'! In the passage directly above, Evan expresses his uncertainty as to whether or not "intense" (his word from the first extract) pushing is what is needed for him not to "fail miserably" as his older brother did. An interpretation is that Evan's loyalty to and identification with his brother both allows and necessitates the release from the intensity that is experienced when the self is manically defended with mathematics (Nimier, *op. cit*). In other words, to defend his young self who looked up to a big brother as well as his family's integrity, an outlet from intensity is needed. The conduit of release in languages, Spanish and Arabic, is a psychically skilful route: Evan had already excelled in Spanish at school and Arabic is a new venture.

Quite a different story comes from Trish. Trish is the elder child of a single mother who is a fulltime carer of Trish's half-brother (who has disabilities associated with ADHD and autism). Trish says of her biological father:

> It just kind of drifted off and I got to the stage where I was just, well if he's not bothered I'm not bothered. But my brother has a different Dad and he's my Dad basically but he doesn't live with my Mum he lives separately. He's who I class as Dad. My brother goes to his dad's every weekend and I might go see him.

And generally positions herself away from her family:

> I'm the first person to go to university which was quite difficult because they didn't know anything about it, nothing and didn't know what to expect. Other friends and things their parents know all about it cos they've all been there, especially at a private school, the majority have degrees and even PhDs so I'm a little bit different in that respect, my Mum still doesn't have any idea.

Her school has supported Trish not only financially through scholarships, but also by providing reliability and constant 'feeding':

> My school was brilliant, especially my maths teachers and even like when the exams were on and stuff they were always like there and they were always like come and see me and I'll go through this or something like all the time and I would not hesitate to in lunch time or something if I was doing some maths questions and I was stuck on something I'd go to the staff room and they would be there.

There is a sense that the school is Trish's parent, that school contains and protects her. In particular, her maths teachers were available whenever she wanted and her maths attainment kept her buoyant there. In her final year (Y13), the school provided board and lodging for Trish in exchange for her having responsibilities for younger girls in their boarding house. When Trish refers to 'we' it is the school. Trish's choice of mathematics is consonant with defending the relationship with her 'parent-school' and she defends herself by repressing the importance of her actual parents. What does her choice of management signal? Recall the previous quotation from Trish's interview that she had put a different minor subject down for each of her UCAS choices; later in the interview she says:

> I didn't really just want to do straight maths so that's why I chose management cos I don't know why I chose management, just for fun!

Such utterances raise the possibility that the minor subject is serving as a place for imaginings, or, in psychoanalytic terms fantasies or phantasies (Wadell, *op. cit.*).

The dual spelling of fantasy/phantasy signals the distinction between conscious *fantasy* like Lee's "lot of money" and unconscious *phantasy* that can only (if ever) be read 'between the lines'. Using Lee's story to illustrate this distinction, it is useful to know that when Lee was five, his biological father left Lee, his brother (two years older) and his mother who had come to London from mainland China at the age of 18 less than ten years before; Lee's mother soon remarried a social worker, originally from Hong Kong. The following extract is early in the interview:

> Lee – Maths, I've always been good at it. I remember back in year 1 or something like that, I was quite stupid, not stupid, but somehow all the other kids seemed to shine. Then my Dad made me learn my times tables and then after I did, suddenly maths, anything about maths seemed to make sense, anything, and then that's how I got better and better …. I'm not sure why he made me learn the times tables. … I guess my Dad just wanted me to just be on the ball. That's it, that's why. And it paid off really well.

Much later in the interview, 'Dad' turns out to be Lee's Stepfather who:

> does as much as he can to encourage me. He is quite, he is a decent man, he is. He's quite a well-paid social worker, probably a high senior manager for H. Council. So, yeah he's definitely qualified anyway, he's probably got a Masters, I'm not quite sure, but something … *I – And your Dad?* L – My Dad, my real Dad? I haven't seen my real Dad in years so I don't know anything about him right now. I live with my Stepdad. My Stepdad's the social worker. So, yeah.

While there are always multiple interpretations for any given text, the quotations above have been chosen to encourage the reader to imagine Lee's unconscious motivation to please his Stepdad, to receive his teachings and even to be in competition with him. We cannot fully know even our own phantasies, but part of the human project is to mind-read others and imagine their deep desires and fears. Lee's conscious motivation to study mathematics, statistics and finance is money, but even in these brief quotations it is possible to sense unconscious motivations, that surround his relationships with his family, that subliminally direct him to finance, to '*fa cai*' ('create wealth' – the standard Chinese New Year greeting) and to be grateful for his Stepfather's giving him an entry in mathematics.

The two young women reading mathematics and French both like the definiteness they believe is characteristic of mathematics yet their stories around their option of French are quite different. Chloe seems to be still attached to her mother and possibly had been unconsciously acting out her mother's wishes:

> Chloe – Mum wanted to do maths like at her A level thing but then she couldn't because of how her timetable went out and she said she was always annoyed that she could never do maths to A level. Umm and I don't know whether that's sort of, so Mum definitely likes maths.

Whereas Laura enlivens a fantasy that French opens up skiing and a world of holiday!

Conclusion

Making a choice of university course is not an easy step to take in a young person's life so it is reasonable to conjecture that a prospective undergraduate might spread their risk over more than one subject. Analysis of the small sample of interviews with first year undergraduates gathered as part of the UPMAP project does not support the conjecture that reading a second subject is to hedge bets of because of lack of commitment. Analysis suggests that while there were articulations that single subject mathematics would be too "intense" (Evan and Chloe), undergraduates' relationships with mathematics were rather secure, either because of attainment, or because of perceived lack of subjectivity in mathematics as a discipline. With this security in the relationship with mathematics, the undergraduate used another subject for fantasy or phantasy or to defend themselves in some way.

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Primary school teachers in Seychelles reporting on their impressions of a mathematics teaching reform

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The Mathematics Lesson Structure reform in Seychelles is stimulating debates on the way mathematics teaching can be improved in the context of a small island developing state. Due to limited research on the reform, to date, its impact in schools has not been established empirically. Informal evidence suggests that teaching is gradually changing to accommodate the major ideas of the reform. A systematic inquiry is needed to develop an understanding of its impact on teaching and learning. This paper is a first step towards documenting the impact of the reform. Analyses carried out on data collected during the early years of implementation show that the teachers have been well sensitized to incorporate MLS in their practices even if they find some elements of the structure difficult to apply. The study suggests implications for in-service teacher education.

Keywords: Mathematics Lesson Structure, instructional reform, Seychelles, teacher change

Background

In 2003, following claims of weak pupils' performance in mathematics, (Khosa, Kanjee, & Monyooe 2002; Trencansky 2002; Valentin 2003) the Minister of Education commissioned a Mathematics Working Group (MWG) to study the situation and propose suggestions. The work of the working group converged into a project – the IPAM Project – with mission to work with schools to improve the quality of mathematics education. IPAM is an acronym formed from initial letters of words in the phrase: Improving Pupils' Achievement in Mathematics. Evidence from the main research done as part of this quest suggested teaching as the main activity needing improvement (Benstrong, Theresine, & Albert 2004). Most lessons were disorganized and unstructured. Of concern was the need to improve the pedagogical characteristic flow of lessons (Schmidt 1996) – the general pattern which lessons within a country tends to follow.

The quest to improve the instructional practices was inspired by the National Numeracy Strategy, mainly, the three part lesson structure (DfEE 1999) which was in place in the UK schools. NNS was commended for its three-part lessons which provided a framework for teachers to organize contents of a lesson. Pupils' achievement after the implementation of the structure showed slight improvement (Brown, Askew, & Millett 2003) and their confidence in mathematics improved during the first years of implementation (Kyriacou 2005). Since mathematics lessons in Seychelles, then, generally lacked a proper structure, it was thought that an adapted version of the UK model could serve as a starting point. A Mathematics Lesson

Structure (MLS) reform was introduced and made mandatory in 2006. The reform prescribes the way teachers should structure their daily maths lessons.

Time scale	Expectations
00 – 05	Stimulate the mind with a mental activity
06 – 10	Review last lesson for relation to current lesson
11 – 13	Establish focus of the current lesson
14 – 29	Make provision to develop pupils' conceptual understanding
30 – 37	Engage pupils in consolidation tasks
38 – 40	Conclude to bring out the gist of the lesson

Table 1: The progression of MLS lessons

Methods

This paper is drawn from findings of a survey carried out in 2008 about the teachers' instructional practices and their impressions of the reform. The discussion relates to four items of the questionnaire on which the teachers reported on their a) frequency of using MLS, b) motivation to use the MLS, c) difficulty of incorporating MLS in their actual lessons, and d) overall impression of the reform. The questionnaire was administered to all teachers who teach mathematics in the state primary schools (n = 430). The teachers were asked to complete the questionnaire on their own. The outcomes are discussed in this paper.

Results

Frequency of using MLS

Teachers' responses to how often they use the MLS structure as a basis for planning their daily lessons are shown in Table 2. Eighty five percent of the teachers reported "Very often" and 15% reported "Rarely". These values demonstrate teachers' strong allegiance with the call to adopt the structure. Even if these teachers were positive about the use of the structure, the 15% who responded rarely to the frequency of using a mandatory strategy creates some concerns.

	Frequency scales	
Motivation to use MLS	Very Often (%)	Rarely (%)
To facilitate planning	36	3
To improve the conduct of the lesson	29	
To improve some aspects of the teaching	9	1
To satisfy the recommendation	3	8
To organize pupil learning activities	4	
To improve pupils' learning of mathematics	4	3
Total	85	15

Table 2 Teachers' motivation to use the reform against the frequency of using it

The second item sought to ascertain the teachers' motivation for using MLS. They were to select one from six given reasons. These were the reasons which emerged as most popular among teachers as we participated together on activities in relation to MLS. The findings are reported in Table 2. To facilitate planning, and to improve the conduct of lesson, appeared as the most popular reasons why teachers were using MLS as part of their instructional practices. About 11% chose the option "to satisfy the recommendation" as their top reason for using MLS. Four percent (4%) chose "to organize pupils' activities" as their main reason. Surprisingly, just 7% of them chose "to improve pupils' learning in mathematics" as the main motivation for using the reform idea in their teaching.

Teachers' compliance

There are indeed several pedagogical ideas that the MLS reform is aiming to promote. During the development and monitoring of the reform many teachers reported that it can be difficult to include all the elements in one lessons even during one week. Some of these reform ideas, they said, are inappropriate for a variety of lessons. So teachers were asked how often they include the various components of MLS in their actual lessons. Table 3 shows the results. The MLS requirements, begin the lesson with a mental activity, and provide key examples for learners to follow were "most of the time" included in the lesson by all the teachers (100%). More than 80% of them selected "most of the time" to all the basic requirements of MLS lesson parts. However, the two requirements which relate to varying the learning and teaching experiences were less popular in lessons. The percentages of teachers reported "rarely" to 'use all the learners' organization' and 'use all the teaching strategies recommended' were relatively high – 43% and 47% respectively.

Implementing the MLS requirements

Some elements of MLS are easier to be incorporated in lessons than others. Some strategies which are uncommon in everyday teaching require greater support to be applied in lessons. In a fourth item, the teachers were asked to indicate the extent to which they find it easy or difficult to incorporate each element of MLS in their teaching. The results are reported in Table 3. All teachers (100%) found it easy to begin their lessons with a mental activity. More than 90% found it easy to review last lesson, tell the pupils the focus of current lessons, and provide key examples for learners to follow. On the other hand more than two thirds of the teachers found it difficult to: allocate consolidation tasks, use all the suggested learners' organization or teaching strategies.

	Frequency scales		Difficulty level	
	Most of the time	Rarely	Easy	Difficult
Begin lesson with a mental activity	100	-	100	-
Provide key examples for learners to follow	100	-	91	9
Review last lesson	97	3	94	6
Incorporate real life examples	96	4	83	17
Create space for consolidation tasks in lesson	95	5	14	86
Tell pupils the focus of current lesson	91	9	98	2
Provide a concluding activity in every lesson	80	20	36	64
Use all suggested learners' organization	57	43	23	77
Use all suggested teaching strategies	53	47	30	70

Table 3 Teachers' responses to their actual application of MLS in their teaching

Teachers' general attitude to MLS

Another item of the questionnaire required the teachers to indicate their overall attitude to MLS. Teachers' attitude to MLS was measured by the percentages of those who agreed to positive statements about the reform. Table 4 reports the findings. The data indicates that the teachers were overwhelmingly positive about MLS. More than 80% of them were agreeing with statements such as: It was a good idea to introduce MLS in schools; there are changes in the quality of teaching since MLS has been introduced (for the latter item I suspect that they were referring to positive changes); and MLS is being useful. The low percentages of teachers who agreed to negative statements such as, MLS restricts teacher – creativity, and MLS limits teaching and learning, show that generally the reform has been well received in the primary schools in Seychelles at least by the teachers who were using it.

Items	Percentages (%)
The IPAM Project is having an impact in my schools	91.6
It was a good idea to introduce MLS in schools	89.8
A remarkable change in the quality of teaching since MLS	82.0
I find MLS very useful	81.7
Teachers find MLS as a vital teaching tool	77.4
MLS is not necessarily applicable at all levels	32.2
MLS restricts teachers' creativity	29.5
MLS limits teaching and learning	20.9

Table 4 Percentages of teachers who agreed to each items

Discussion

The MLS reform represents a significant development in the teaching of mathematics in Seychelles although it has not been adequately researched or evaluated. There is only one empirical study which has been carried out so far on the reform (Valentin 2007). This limits what is known about the effect MLS is having in schools. Subsequently, it is becoming difficult to continue to persuade teachers to use the structure as there is no substantial case built on it. This paper reports on a study that is addressing this gap. A new empirical perspective about the reform is being generated.

During the early implementation years, teachers were positive about MLS even though it required them to make drastic change to their fundamental methods of teaching. This overall positive reaction to MLS may be due to their overall perception that the mathematics education reform was having an impact in their schools. Almost 92% of them agreed to the later perception. Eighty two percent felt that introducing MLS was a good idea. The notion of teacher receptivity (Ma et al. 2009; Waugh & Godfrey 1993) can be used to make sense of this positive impression teachers had on MLS. The teachers welcomed the reform and were ready to try it out. Receptivity is a key factor for the overall success of classroom change initiatives (Waugh & Godfrey 1993). Moreover, the idea that teachers are "active brokers" in the quest to reform teaching (Spillane 1999) is strongly supported by data of this small investigation. By referring to the teachers as active brokers, I want to emphasize that the success of the MLS implementation is due in part to the effort and fidelity of the teachers about the reform quest. Even if the introduction of MLS required them to make significant changes to the core of their practice, their will to improve their teaching motivated most of them to embark on the reform. Just as (Fullan, 1982) argues teachers' motivation to fully implement the reform ideas depends on the extent to which they feel the intents of the reform will work. On the basis of evidence reported here, the teachers in this study appeared to strongly believe that MLS would work, at least would improve their teaching.

The development of MLS was part of a bigger project to improve the quality of mathematics education which the teachers themselves agreed upon. Furthermore, school representatives participated actively in the development of the lesson structure. Perhaps, another reason for their high compliance may be due to the fact that they were part of the decision to improve mathematics, and they could have realized that such effort to improve their teaching in the first instance will have positive impact on attainment. Teachers' belief that MLS is having an impact in their schools is high, although it is somewhat surprising that only 7% were motivated to use the reform mainly on the ground that it will improve the pupils' learning. On the other hand the proportion of teachers who reported that they use MLS primarily to satisfy policy requirement is high (11%). Since the teachers' main motivation to comply with the reform idea can be, to satisfy recommendation, then policy makers who monitor the implementation of reform activities should be careful when they interpret compliance figures. Since only 7% of the teachers indicated that their main motivation to use MLS is to improve pupils' learning may be suggesting that the teachers viewed MLS more like a pedagogical improvement tools than a learning improvement tool.

Unfortunately, teachers' responses to the difficulty to incorporate the various elements of MLS in their practices suggest that fundamental features of mathematics teaching were still under developed in the primary schools in Seychelles during the early years of implementation. It shows that many of them were using a limited range of teaching strategies. In fact these conditions were imposed on teachers to increase variations so as to cater for mixed ability classes. The fact that the various teaching and learning strategies were largely not being implemented suggests that mixed ability teaching might be a challenge to achieve in the Seychelles' schools.

The data illustrates how teachers' perception of a reform initiative being effective may lead to a widespread application. Moreover, this study provides hope to policy makers in reform-minded education settings that the pedagogical characteristic flow of a country can be altered. More importantly, whatever strategies that one applies towards reforming teaching, should convey the message that the reform will work, and should contain indications *how* and *why* it may work. The evidence emerging from this study indicates that the majority of the teachers began to implement MLS with a strong positive attitude – a prerequisite for success of instructional reform. This initial attitude may has led the teacher to comply, although they have had difficulty to use some of the non routine practices such as varying strategies, pupils' organization, and in terms of lesson structuring, formulate a conclusion for their lessons. Unfortunately, due to limited research on the reform, it is not possible to formulate a picture of how teaching of mathematics in Seychelles has changed over the years as an effect of this reform.

Conclusion

The self-reported data generated by this study suggest that changing the culture of mathematics teaching in Seychelles is possible. This approach to reforming teaching – prescribing instructional practices – may works in disadvantage settings especially in situation where traditional long term training is difficult. The mere fact that teachers who used MLS on a daily basis continue to hold a strong positive impression about the reform suggests that such model to teaching mathematics has scope to develop into a model of effective teaching for teachers with inadequate or no training in mathematics education. Additional research inquiring on the cognitive aspect of the model is now imperative.

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