Is Household Heterogeneity Important for Business Cycles?*

Youngsoo Jang Takeki Sunakawa Minchul Yum

August 2018

Abstract

This paper explores how the interaction of household heterogeneity and progressive government transfers shapes aggregate labor market fluctuations. Based on the key insights from analytic results in a tractable static model of extensive margin labor supply, we build a dynamic general equilibrium model with both idiosyncratic and aggregate productivity shocks and show that household heterogeneity shapes the dynamics of macroeconomic aggregates substantially when interacted with progressive transfers. Specifically, a notable feature of the performance of our baseline model is its ability to reproduce moderately procyclical average labor productivity while retaining the success of the representative-agent indivisible labor model in generating a large cyclical volatility of aggregate hours relative to output. Using the panel structure of the PSID, we document that, among low-wage workers, (i) the individual-level probability of adjusting labor supply along the extensive margin is significantly higher; and (ii) the fall in employment rate is considerably steeper during the last six recessions, both of which support the key mechanism of our model.

Keywords: Heterogeneity, progressivity, government transfers, labor supply, business cycles

JEL codes: E32, E24, E21

*We are grateful to Eunseong Ma and Yikai Wang for their constructive discussions. We also thank Björn Brügemann, Pantelis Kazakis, Aubhik Khan, Sun-Bin Kim, Tom Krebs, Matthias Meier, Claudio Michelacci, Benjamin Moll, Shuhei Takahashi, Michèle Tertilt, Julia Thomas and seminar participants at 14th Dynare Conference in European Central Bank, Asian Meeting of the Econometric Soceity in Seoul, 9th Shanghai Macroeconomics Workshop, 2018 KAEA-KEA International Conference, ZEW Conference on Macroeconomics and the Labour Market, the University of Mannheim, the Workshop for Heterogeneous Macro Models in Kobe University, the Ohio State University, Queen Mary University of London, and the University of Kent for helpful comments. Jang: Institute for Advanced Research, Shanghai University of Finance and Economics, China, e-mail: jangys724@gmail.com. Sunakawa: Graduate School of Economics, Kobe University, Japan, e-mail: takeki.sunakawa@gmail.com. Yum: Department of Economics, University of Mannheim, Germany, e-mail: minchul.yum@uni-mannheim.de.

1 Introduction

There has been great interest in incorporating rich micro-level heterogeneity into macroeconomic models over the recent decades (see e.g., Krusell and Smith, 2006; and Heathcote, Storesletten, and Violante, 2009 for literature reviews). It is of great importance to incorporate household or firm heterogeneity when studying distributional issues within a macroeconomic framework. However, what is less clear is whether heterogeneity at the micro level matters for aggregate business cycle dynamics at the macro level. Despite the extensive studies showing the importance of heterogeneity in macroeconomic aggregates and equilibrium prices in the absence of aggregate risk (e.g., Huggett, 1993; and Heathcote, 2005), the recent quantitative macroeconomic literature with aggregate uncertainty has suggested that incorporating micro-level heterogeneity has only limited impacts on the business cycle fluctuations of macroeconomic aggregates (e.g., Krusell and Smith, 1998; Thomas, 2002; Khan and Thomas, 2008; and Chang and Kim, 2007; 2014).

This paper explores how household heterogeneity shapes aggregate labor market fluctuations in the presence of progressive government transfers both theoretically and quantitatively. We first present a simple tractable model of extensive-margin labor supply. We analytically show that the interaction of household heterogeneity and the progressive transfer system can increase the degree to which aggregate hours vary over the business cycle and make average labor productivity less procyclical. Based on these key insights, we construct a standard dynamic general equilibrium model with idiosyncratic and aggregate shocks, augmented with progressive government transfers. We find that our quantitative business cycle model delivers moderately procyclical average labor productivity and a large cyclical volatility of aggregate hours relative to output, both of which are known to be hard to explain in standard real business cycle models. In particular, our result is distinct from the existing literature since our heterogeneous-agent model dampens the strong link between average labor productivity and output without relying on additional source of exogenous aggregate shocks.[1] At the same time, our model retains the success of the canonical representative-

[1] The existing quantitative theoretical explanations for lowering a highly procyclical labor productivity in the model rely on the introduction of additional shocks. More specifically, Benhabib et al. (1991) consider home-production technology shocks; Christiano and Eichenbaum (1992) suggest government spending shocks; Braun (1994) introduces income tax shocks; and Takahashi (2017) incorporates idiosyncratic wage uncertainty shocks into a real business cycle model.

agent indivisible labor model in generating a large volatility of aggregate hours. This is particularly notable since our result does not require a low curvature of the utility function along the leisure margin, which is inconsistent with the micro estimates of the labor supply elasticity (see e.g., Keane and Rogerson, 2015).[2] We show that the key to our quantitative results is different labor supply responses at the micro level in the presence of progressive government transfers, as is consistent with both our theoretic results from the static model and our empirical evidence at the micro-level data.

Our main result suggests that household heterogeneity at the micro level is important for the dynamics of macroeconomic variables. This result is broadly in line with recent papers such as Krueger, Mitman and Perri (2016) and Ahn, Kaplan, Moll, Winberry and Wolf (2017), both of which find that heterogeneity at the micro level is crucial in shaping the impact of aggregate shocks on macroeconomic variables.[3] Although the distribution of wealth plays an important role in these studies, it is important to note that Krueger et al. (2016) and Ahn et al. (2017) focus on the consumption-savings channel whereas our paper focuses on the labor supply channel as a key mechanism through which micro-level heterogeneity matters for the business cycle fluctuations of macroeconomic aggregates, such as total hours and labor productivity.

Our baseline model economy is based on a standard incomplete markets model with heterogeneous households that make consumption-savings and extensive-margin labor supply decisions in the presence of both idiosyncratic productivity risk and aggregate risk (Chang and Kim, 2007; 2014).[4] Our model also incorporates progressive government transfers, captured by a parsimonious, yet flexible, nonlinear function. We calibrate our model economy to match salient features in the micro-level data including the degree of progressivity in the government transfer programs obtained from the Survey of Income and Program Participation (SIPP) data as well as the persistence of idiosyncratic wage risk obtained from the Panel Study of Income Dynamics (PSID) data.

We find that our baseline model features the aggregate labor market dynamics that differ con-

[2] Our model features a binary employment choice, which makes the curvature of utility along the leisure (or hours worked) irrelevant.

[3] See also Kim (2017).

[4] This class of models in turn builds on a standard incomplete markets model without aggregate risk, pioneered by Imrohoroğlu (1988), Huggett (1993) and Aiyagari (1994).

siderably from its nested versions, abstracting from either government transfers (a model similar to Chang and Kim, 2007; 2014) or household heterogeneity (a model similar to Hansen, 1985). Specifically, we find significant improvements in the business cycle statistics regarding aggregate labor market fluctuations. First, our baseline model with the nonlinear government transfer schedule generates considerably lower correlations of average labor productivity with output (0.56 vs. 0.35 in the data) than its nested versions (0.81 in the absence of government transfers and 0.78 in the absence of household heterogeneity). At the same time, in our baseline model, the cyclical volatility of aggregate hours relative to output is 0.86, which is quite close to 0.96 in the data. It is striking that this performance is comparable to or is even slightly better than 0.85, the value obtained from its representative-agent counterpart, which is often considered to be the upper bound due to the representative agent's utility function having the lowest curvature in labor supply.[5]

To illustrate the key mechanism underlying our quantitative success, we conduct impulse response exercises. We find that, in our baseline model, total hours fall considerably more and average labor productivity changes nontrivially following a negative shock in total factor productivity (TFP). We also compute the impulse responses of total hours at the disaggregated level. We find that labor supply is generally less elastic among households with high productivity (or wage), consistent with our analytical finding in a simple static model. This pattern of heterogeneity in labor supply (i.e., disproportionately more elastic labor supply among low-wage workers) and the resulting compositional changes following the aggregate TFP shock underlie the quantitative success of the baseline model with government transfers. However, in the heterogeneous-agent model without government transfers, we find that the labor supply of low-wage households is remarkably inelastic. Therefore, we argue that, for our main quantitative results, the presence of household heterogeneity *per se* is not sufficient in the presence of incomplete markets, thereby explaining why the existing heterogeneous-agent model (e.g., Chang and Kim, 2007; 2014) is unable to deliver our main quantitative results.

Finally, we use micro data from the PSID to empirically explore heterogeneity in labor supply

[5] This finding is also noteworthy given that the presence of household heterogeneity seems to make it more challenging to generate a large volatility of aggregate hours, according to the recent findings in the business cycle literature. For example, Chang and Kim (2014) reports that the volatility of aggregate hours relative to output is 0.58 in their model with indivisible labor.

responses. In particular, we use the panel structure of the PSID, which allows us to keep track of the same individuals over time. We document two key empirical findings using two different approaches. First, we find that the individual-level probability of adjusting labor supply along the extensive margin is significantly higher among low-wage workers. In particular, we show that this probability tends to decrease with wage. Second, we document that during the last six recessions, the employment rate has fallen most sharply in the first and second wage quintiles whereas it was relatively stable among the top wage quintile. Although the above two approaches capture different aspects of labor supply adjustments over different time horizons, these two findings are remarkably similar. This demonstrates the robustness of our empirical result that lower wage workers adjust labor supply along the extensive margin more elastically. More importantly, both of these empirical findings are consistent with the pattern of heterogeneity in labor supply responses in our model economy, thereby supporting our key mechanism of the model.

Our underlying mechanism regarding the interaction of heterogeneous households and progressive government transfers builds upon Yum (2018). In a stationary equilibrium environment, Yum (2018) finds that providing government transfers to the wealth-poor households, who lack savings for self-insurance in the incomplete markets environment, reduces their precautionary motives of labor supply, and makes their labor supply more elastic with respect to permanent tax changes. Our results herein suggest that the presence of progressive government transfers in this class of incomplete markets environments not only matters for the long run employment effects of labor taxes, as studied in Yum (2018), but it also has important implications in hours volatility over the business cycle. More importantly, our paper provides a tractable simple theory with closed-form solutions illustrating the key mechanisms of the fully-fledged heterogeneous-agent models and empirical evidence supporting the key mechanism of the model. Finally, the analysis of our paper covers not only aggregate labor supply elasticity but also average labor productivity and other macroeconomic variables.

Our quantitative business cycle model is based on Chang and Kim (2007; 2014) yet it differs from theirs in at least two important respects. First, as highlighted above, we bring the institutional feature of progressive government transfers, as observed in the micro-level data (SIPP), into the model. Chang, Kim, and Schorfheide (2013) consider a version of the model in Chang and Kim (2007) with flat lump-sum transfers. However, given the different focus of their paper, they report limited number of standard business cycle statistics, which are rather the main focus of our paper. Second, note that the estimation of the idiosyncratic productivity process is not trivial since wages are only available for those who choose to work in the data. Chang and Kim (2007; 2014) deal with this selection problem outside the model by applying the Heckman (1979) correction. In contrast, we deal with the selection problem and potential temporal aggregation bias (quarterly model vs. annual micro-level data) using the model simulation directly. Specifically, we use the simulated data where selection is endogenously taken care of within the model and then perform temporal aggregation using the simulated quarterly data to obtain the simulated annual data. Moreover, our calibration targets the persistence of idiosyncratic wage risk estimated following Heathcote, Storesletten, and Violante (2010). As a result, our calibration strategy leads to a fairly high estimate of the persistence of idiosyncratic productivity shocks, which interacts with the presence of progressive government transfers in improving the performance of the incomplete-markets business cycle model.[6]

The paper is organized as follows. Section 2 presents some analytic results on the key mechanism of this paper. Section 3 describes the model environment of the equilibrium business cycle models, defines equilibrium, and discusses the numerical solution methods. In Section 4, we describe how parameters are calibrated and show the steady-state properties of the quantitative models. Section 5 presents the main quantitative results from the calibrated models. Section 6 presents empirical evidence on heterogeneity in labor supply using the panel structure of the PSID. Section 7 concludes.

2 Interplay of household heterogeneity and progressive transfers

In this section, we present a simple model to illustrate the key mechanism through which the interplay of household heterogeneity and progressive transfers influences aggregate labor market fluctuations. For analytical tractability and a clear illustration, we consider a static economic

[6] Specifically, the persistence estimate of the idiosyncratic productivity shocks at the annual frequency ranges from 0.89 to 0.93 in our paper whereas it is around 0.75 in Chang and Kim (2007). In Section 2, we discuss how the persistence matters for our results.

environment.

There is a continuum of agents in the unit interval. The productivity level of the each type i is defined as $x_i \in \{x_l, x_h\}$. The mass of each type is denoted by π_l and π_h satisfying $\pi_l + \pi_h = 1$. A subscript $i \in \{l, h\}$ denotes the type of the agent throughout this section. Since our focus is on the extensive margin, the agent can choose to either work full time or not at all: $n_i \in \{0, 1\}$.[7]

The decision problem of each type is given by

$$\max_{c_i \geq 0, n_i \in \{0,1\}} \{\log c_i - bn_i\}$$

subject to

$$c_i \leq zx_i n_i + a + T_i, \quad i = l, h$$

where c denotes consumption, n is labor supply, a is the level of assets, and $b > 0$ is disutility constant. We use z to denote aggregate productivity state (or wage level) capturing aggregate economic conditions. Finally, note that we allow for a productivity-dependent public insurance scheme $T_i \geq 0$. We assume that T_l is greater than T_h, implying that it is progressive.

The above maximization problem describes the optimal decisions of an individual. Specifically, comparing the utility conditional on working to not working, the agent chooses to work if

$$\log(zx_i + T_i + a) - b \geq \log(T_i + a).$$

Note that this can be equivalently written as

$$b \leq \log\left(\frac{zx_i + T_i + a}{T_i + a}\right) = \log\left(1 + \frac{zx_i}{T_i + a}\right),$$

or

$$a \leq zx_i - T_i$$

[7] Our analytical framework in this section builds on the theoretical framework presented in Doepke and Tertilt (2016). Since the focus of the analysis is different, their model is based on two gender types and continuous preference heterogeneity whereas our model is based on two productivity types and continuous asset heterogeneity. Moreover, our results cover not only the labor supply elasticity but also the average labor productivity.

where we assume that the constant b is equal to $\log(2) > 0$ without loss of generality. This decision rule shows that the agent is more likely to work if the aggregate condition z or individual productivity x is higher. Also, note that the agent is less likely to work if the size of transfers is higher.

In our model of the extensive margin labor supply, aggregate employment is shaped by both the decision rule and the distribution. Let $F_i(a)$ be the conditional (differentiable) distribution function of assets with its (continuous) marginal density being $f_i(a) = F_i'(a)$. Specifically, we use exponential function which is often used to approximate wealth distribution. Specifically, for $a \geq 0$,

$$F_i(a) = 1 - \exp(-a),$$
$$f_i(a) = F_i'(a) = \exp(-a).$$

This density function has the mode at 0 and is strictly decreasing in a. In addition, its density decreases slower with a, which generates a long right tail of asset distribution as in the data. Given the density function, the fraction of agents working (i.e., the employment rate) for each type is given by

$$N_i = F(\tilde{a}_i) = 1 - \exp(-\tilde{a}_i)$$

where

$$\tilde{a}_i = zx_i - T_i.$$

In other words, the employment rate N_i is the integral of those whose asset level is lower than the threshold level $\tilde{a}_i$. We now present some theoretical results based on this model. All proofs are provided in Appendix.

Proposition 1 *Let ε_i be the labor supply elasticity of the type i.*

$$\varepsilon_i \equiv \frac{\partial N_i}{\partial z}\frac{z}{N_i}.$$

Assume $T_i = 0$. The labor supply elasticity of the low type is greater than that of the high type.

That is, $\varepsilon_l > \varepsilon_h$.

This shows that our model of the extensive margin delivers the heterogeneity in the labor supply elasticity. Note that the threshold asset level of employment for the low-type agents is lower than that for the high-type agents: $\tilde{a}_l < \tilde{a}_h$. Since there are more marginal households around $\tilde{a}_l$, the same change in z, which in turn shifts the employment thresholds to the same degree, has a stronger impact on the employment rate of the low type.

We now consider the role of government transfers and how they interact with heterogeneity. To simplify the algebra, we impose symmetry. Specifically, we assume that $\pi_l = \pi_h = 0.5$. In addition, $x_h = 1 + \lambda$ and $x_l = 1 - \lambda$ where $\lambda \in [0, 1]$ measures the degree of inequality.

To show the effect of progressivity in the transfer schedule, T_i is assumed to be determined by

$$
\begin{aligned}
T_l &= T\left(1 + \omega\lambda\right) \\
T_h &= T\left(1 - \omega\lambda\right)
\end{aligned}
$$

where $T \in [0, z]$ is the scale of transfers, and $\omega \in [0, \frac{1}{\lambda}]$ captures progressivity. Note that a changes in progressivity ω does not affect the aggregate amount of transfers since

$$
\begin{aligned}
\sum \pi_i T_i &= \pi_l T\left(1 + \omega\lambda\right) + \pi_h T\left(1 - \omega\lambda\right) \\
&= T + (\pi_l - \pi_h)\omega\lambda = T.
\end{aligned}
$$

Given the above assumptions, we can derive the type-specific employment rate:

$$
\begin{aligned}
N_l &= 1 - \exp(-\tilde{a}_l) \\
N_h &= 1 - \exp(-\tilde{a}_h)
\end{aligned}
$$

where $\tilde{a}_l = zx_l - T_l = \max\left\{0, z(1-\lambda) - T - T\omega\lambda\right\}$ and $\tilde{a}_h = zx_h - T_h = \max\left\{0, z(1+\lambda) - T + T\omega\lambda\right\}$.

Proposition 2 *Greater progressivity of transfers increases the labor supply elasticity of the low-type agents, yet it decreases the labor supply elasticity of the high-type agents.*

Intuitively, greater progressivity (or a higher ω) shifts $\tilde{a}_l$ to the left where the distribution of assets is denser. In contrast, greater progressivity shifts $\tilde{a}_h$ to the right around which the distribution of assets is scarcer.

Proposition 3 *Let N denote the aggregate employment rate: $N = \pi_l N_l + \pi_h N_h$. Let ε be the aggregate labor supply elasticity:*

$$\varepsilon \equiv \frac{\partial N}{\partial z}\frac{z}{N}.$$

The aggregate labor supply elasticity is higher with greater progressivity. That is, $\frac{\partial \varepsilon}{\partial \omega} > 0$.

Although the proof is tedious, this result is rather straightforward given Propositions 1 and 2. This result underlies one of our main findings in Section 5 showing that incorporating progressive government transfers allows the quantitative business cycle model with rich heterogeneity to generate a large volatility of aggregate hours over the business cycle.

Next, we consider the implications for the cyclicality of average labor productivity. Define average labor productivity as

$$\chi \equiv \frac{\sum_{j\in\{l,h\}} \pi_i \left(z x_i N_i\right)}{\sum_{j\in\{l,h\}} \pi_i N_i} = z\frac{\sum_{j\in\{l,h\}} \pi_i \left(x_i N_i\right)}{\sum_{j\in\{l,h\}} \pi_i N_i}.$$

Letting $\chi = z\chi_0$, we can see that the first term z directly makes average labor productivity to be procyclical as in real business cycle models. On the other hand, The second term captures the effects of worker composition on the ALP. The following two propositions focus on the second part.

Proposition 4 *The effect of z on average labor productivity through worker composition effects is negative. That is, $\frac{\partial \chi_0}{\partial z} < 0$.*

Proposition 5 *The average labor productivity becomes less procyclical with greater progressivity. That is, $\frac{\partial}{\partial \omega}\left(\frac{\partial \chi_0}{\partial z}\right) < 0$.*

Proposition 5 tells us that progressivity in the transfer schedule shapes the cyclicality of average labor productivity through worker composition effects. Specifically, it implies that greater progressivity would make the average labor productivity to be *less* procyclical, as illustrated in a numerical example in the right panel of Figure 1. The intuition behind this result is in fact related to Propositions 1 and 2. Note that the positive impact of progressivity on aggregate labor supply responsiveness in Proposition 3 (as depicted in the left panel of Figure 1) is driven by the low type having a large labor supply response to a change in z. In other words, if a fall in z (i.e., a recession) generates a large fall in the labor supply of the low type, especially relative to the high type, it would then tend to raise average labor productivity when output falls. This force dampens the tight positive link between z and average labor productivity.

It is worth discussing our assumption that $F_l(a) = F_h(a) = 1 - \exp(-a)$. In other words, the conditional distribution of assets for each type is identical. In fact, it should be noted that this assumption is the most conservative one. In other words, as we assume that $F_l(a)$ has a lower mean than $F_h(a)$ does (as is consistent with the data), the distribution of assets for the low productivity type would become more packed around the threshold asset level, which in turn would strengthen the above results (due to a greater labor supply elasticity of the low-type).[8]

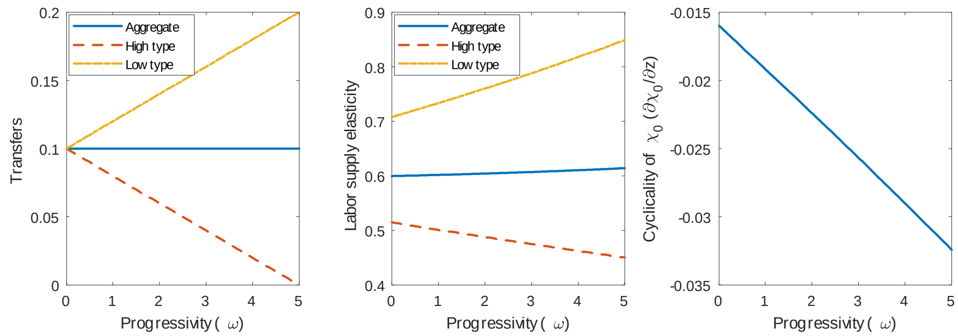
In the next sections, we imbed these key insights into standard equilibrium business cycle models with more realistic household heterogeneity to explore how this mechanism can alter the model-implied dynamics of macroeconomic aggregates.

3 Quantitative business cycle models

In this section, we describe the economic environment of the quantitative business cycle models studied in this paper. We consider four model specifications. The first is the baseline model with household heterogeneity and progressive government transfers. The other alternative specifications are considered to illustrate the importance of the interplay of government transfers and household heterogeneity.

[8] Moreover, note that this highlights the importance of the persistence of idiosyncratic shocks in the full dynamic model. This is because a higher persistence would enlarge the difference of the mean of assets across individual productivity types in equilibrium. For example, when the idiosyncratic shock has zero persistence (i.i.d.), the equilibrium asset distribution for each type would become identical.

Figure 1: Impact of progressivity on aggregate labor market



Note: A numerical example of Propositions 1-5, based on $b = 2, \lambda = 0.2, T = 0.1$, and $z = 1.19$ to match the aggregate employment rate of 65% when $\omega = 0$.

3.1 Heterogeneous-agent models

In this subsection, we introduce the first two model specifications with heterogeneous households. The first is the baseline model with government transfers, denoted as Model (HA-T). The second, denoted as Model (HA-N), is simply a nested specification of the baseline model by shutting down government transfers. This model roughly corresponds to a standard incomplete-markets real business cycle model with household heterogeneity and endogenous labor supply at the extensive margin (Chang and Kim, 2007; 2014). In other words, the baseline model economy extends Chang and Kim (2007, 2014) by incorporating labor taxes and progressive government transfers.

Households:

The model economy is populated by a continuum of infinitely-lived households. It is convenient to describe the infinitely-lived household's decision problem recursively. At the beginning of each period, households are distinguished by their asset holdings a and productivity x_i. We assume that x_i takes a finite number of values N_x and follows a Markov chain with transition probabilities π^x_{ij} from the state i to the state j. In addition to the individual state variables, a and x_i, there are aggregate state variables including the distribution of households $\mu(a, x_i)$ over a and x_i and aggregate total factor productivity shocks z_k. We also assume that z_k takes a finite number of values N_z following a Markov chain with transition probabilities π^z_{kl} from the state k to the state

l. We assume that the Markov processes for individual productivity and aggregate productivity (or total factor productivity) capture the following continuous AR(1) processes in logs.

$$\log x' = \rho_x \log x + \varepsilon_x' \tag{1}$$

$$\log z' = \rho_z \log z + \varepsilon_z' \tag{2}$$

where $\varepsilon_x \sim N(0, \sigma_x^2)$ and $\varepsilon_z \sim N(0, \sigma_z^2)$. Finally, we assume competitive markets; in other words, households take as given the wage rate per efficiency unit of labor $w(\mu, z_k)$ and the real interest rate $r(\mu, z_k)$, both of which depend on the aggregate state variables. Households take as given government policies.

The dynamic decision problem of households can be written as the following functional equation:

$$V(a, x_i, \mu, z_k) = \max \left\{ V^E(a, x_i, \mu, z_k), V^N(a, x_i, \mu, z_k) \right\}$$

where

$$V^E(a, x_i, \mu, z_k) = \max_{\substack{a' > \underline{a}, \\ c \geq 0}} \left\{ \log c - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \mu', z_l') \right\} \tag{3}$$

$$\text{subject to} \quad c + a' \leq (1-\tau) w(\mu, z_k) x_i \bar{n} + (1 + r(\mu, z_k)) a + T(x_i)$$

$$\mu' = \Gamma(\mu, z_k).$$

and

$$V^N(a, x_i, \mu, z_k) = \max_{\substack{a' > \underline{a}, \\ c \geq 0}} \left\{ \log c + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \mu', z_l') \right\} \tag{4}$$

$$\text{subject to} \quad c + a' \leq (1 + r(\mu, z_k)) a + T(x_i) \tag{5}$$

$$\mu' = \Gamma(\mu, z_k). \tag{6}$$

Households maximize utility by choosing optimal consumption c, asset holdings in the next period

a', and labor supply n.[9] The labor supply decision is assumed to be discrete $n \in \{0, \bar{n}\}$. The total disutility of work is captured by $B\bar{n} > 0$. Households understand that the expected future value, discounted by a discount factor β, is affected by stochastic processes for individual productivity x' and aggregate productivity z' as well as the whole distribution μ'. The evolution of μ is governed by the law of motion in (6). The budget constraint states that the sum of current consumption c and asset demands for the next period a' should be less than or equal to the sum of net-of-tax earnings $(1-\tau)w(\mu, z_k)x_i n$, current asset holdings and capital income $(1 + r(\mu, z_k))a$, and government transfers $T(x_i)$. Household face a borrowing limit $\underline{a} = 0$.[10]

Government:

There is a government that taxes labor earnings at a fixed rate of τ. The government uses the collected tax revenue to finance transfers T to households. The remaining tax revenue is spent as government spending, which is not valued by households.

Following Krusell and Rios-Rull (1999), we assume that transfers T consist of two components (i.e., $T = T_1 + T_2$). The first component T_1 is given to all households equally whereas the second component T_2 captures the income security aspect of transfers. In the U.S., there are various means-tested programs such as food stamps, the Supplemental Nutrition Assistance Program and the Temporary Assistance for Needy Families (formerly the Aid to Families with Dependent Children). As shown in Section 4, these programs lead to the observation that the amount of transfers is negatively associated with wealth. To replicate the progressivity observed in the U.S. data, we assume that T_2 depends on productivity and decreases with it.[11]

[9] A variable with a prime denotes its value in the next period.

[10] Allowing a moderate non-zero borrowing limit does not affect our main results, given that we target the same other moments.

[11] Oh and Reis (2012) also use the transfer schedule which depends on productivity. Alternatively, one can consider an income-dependent transfer schedule (e.g., Yum, 2018). Since aggregate income fluctuates over the business cycle, it inevitably introduces the countercyclicality of the aggregate transfer, which in turn can amplify the labor supply responses with respect to aggregate shocks. In contrast, since the marginal distribution of productivity is invariant over the business cycle, our results using the productivity dependent transfer schedule are not affected by the cyclicality of aggregate transfers. Although this amplification channel could be interesting, it complicates the comparison exercise relative to the representative agent models which assumes the fixed amount of transfers over the business cycle. Therefore, we leave this investigation for future work.

Specifically, T_2 for each productivity type is determined by the following functional form

$$T_2(x) = \omega_s(1+x)^{-\omega_p}. \tag{7}$$

This parametric assumption adds two parameters. First, $\omega_s \geq 0$ is a scale parameter in the sense that it determines the size of transfers when the argument is zero ($T(0) = \omega_s$). The next parameter $\omega_p \geq 0$ governs the shape of progressivity. Specifically, a higher ω_p would make T_2 decrease faster with x. Note that $\omega_p = 0$ would imply that the transfer schedule is independent of income, which is commonly used in the literature.

Firm:

Aggregate output Y is produced by a representative firm. The firm maximizes its profit

$$\max_{K,L} \{z_k F(K,L) - (r(\mu, z_k) + \delta)K - w(\mu, z_k)L\} \tag{8}$$

where $F(K,L)$ captures a standard neoclassical production technology in which K denotes aggregate capital, L denotes aggregate efficiency units of labor inputs, and δ is the capital depreciation rate. As is standard in the literature, we assume that the aggregate production function follows a Cobb-Douglas function with constant returns to scale:

$$F(K,L) = K^\alpha L^{1-\alpha}. \tag{9}$$

The first-order conditions for K and L give

$$r(\mu, z_k) = z_k F_1(K,L) - \delta, \tag{10}$$

$$w(\mu, z_k) = z_k F_2(K,L). \tag{11}$$

Equilibrium:

A recursive competitive equilibrium is a collection of factor prices $r(\mu, z_k), w(\mu, z_k)$, the household's decision rules $g_a(a, x_i, \mu, z_k)$, $g_n(a, x_i, \mu, z_k)$, government policy variables $\tau, G, T(\cdot)$, a value function $V(a, x_i, \mu, z_k)$, a measure of households $\mu(a, x_i)$ over the state space, the aggregate capital and labor $K(\mu, z_k), L(\mu, z_k)$, and the aggregate law of motion $\Gamma(\mu, z_k)$ such that

1. Given factor prices $r(\mu, z_k), w(\mu, z_k)$ and government policy $\tau, G, T(\cdot)$, the value function $V(a, x_i, \mu, z_k)$ solves the household's decision problems defined above, and the associated household decision rules are

$$a'^* = g_a(a, x_i, \mu, z_k) \tag{12}$$

$$n^* = g_n(a, x_i, \mu, z_k). \tag{13}$$

2. Given factor prices $r(\mu, z_k), w(\mu, z_k)$, the firm optimally chooses $K(\mu, z_k)$ and $L(\mu, z_k)$ following (10) and (11).

3. Markets clear

$$K(\mu, z_k) = \sum_{i=1}^{N_x} \int_a a d\mu \tag{14}$$

$$L(\mu, z_k) = \sum_{i=1}^{N_x} \int_a x_i g_n(a, x_i, \mu, z_k) d\mu. \tag{15}$$

4. Government balances its budget

$$G + \sum_{i=1}^{N_x} \int_a T d\mu = \tau w L(\mu, z_k).$$

5. The law of motion for the measure of households over the state space $\mu' = \Gamma(\mu, z_k)$ is consistent with individual decision rules and the stochastic processes governing x_i and z_k.

3.2 Representative-agent models

In addition to the heterogeneous-agent model specifications, i.e., Models (HA-T) and (HA-N), we consider two additional specifications of the representative-agent model. First, Model (RA-T) shuts down only household heterogeneity while maintaining the fiscal environment including taxes and transfers, as in Model (HA-T). Given the indivisible labor assumption, our representative agent version of the model is essentially the business cycle model studied in Hansen (1985) augmented with tax and transfers. Next, Model (RA-N) shuts down both household heterogeneity and government transfers. We consider the decentralized competitive equilibrium given distortionary labor taxation.

Representative-agent model environment:

At the beginning of each period, the stand-in household has the current period's asset k. The aggregate state variables are the aggregate capital K and the aggregate productivity z_k. The aggregate productivity follows the same stochastic process as in the baseline model. Taking the wage rate $w(K, z_k)$ and the real interest rate $r(K, z_k)$, as well as the aggregate law of motion $\Gamma(K, z_k)$ as given, the dynamic decision problem of the representative household can be written as the following functional equation:

$$V(k, K, z_k) = \max_{\substack{k' \geq 0, c \geq 0 \\ n \in [0,1]}} \left\{ \log c - Bn + \beta \sum_{l=1}^{N_z} \pi^z_{kl} V(k', K', z'_l) \right\}$$

$$\begin{aligned} \text{subject to} \quad & c + k' \leq (1-\tau) w(K, z_k) n + (1 + r(K, z_k)) k + T \\ & K' = \Gamma(K, z_k) \end{aligned}$$

The household maximize utility by choosing optimal consumption c, the next period's capital k' and labor supply n. Our stand-in household has a linear disutility of work B due to the aggregation in Rogerson (1988). The budget constraint states that the sum of consumption c and the next period's capital k' should be less than or equal to the sum of net-of-tax labor income $(1-\tau)w(K, z_k)n$, current capital k, capital income $r(K, z_k)k$ and government transfers T.

As in the baseline model, government collects taxes on labor earnings τwn to finance transfers T and government expenditure G. We assume the amount of government transfers is constant, implying that the ratio of transfers to output is countercyclical, as in the data. We maintain the firm side as in the baseline model. The resulting first-order conditions for K and L are the same as those in (10) and (11).

Equilibrium:

A recursive competitive equilibrium is a collection of factor prices $r(K, z_k)$, $w(K, z_k)$, the household's decision rules $g_k(k, K, z_k)$, $g_n(k, K, z_k)$, government policy variables τ, G, T, the household's value function $V(k, K, z_k)$, the aggregate labor $L(K, z_k)$ and the aggregate law of motion for aggregate capital $\Gamma(K, z_k)$ such that

1. Given factor prices $r(K, z_k)$, $w(K, z_k)$ and government policy τ, G, T, the value function $V(k, K, z)$ solves the household's decision problem, and the associated decision rules are

$$k'^* = g_k(k, K, z_k)$$
$$n^* = g_n(k, K, z_k).$$

2. Prices $r(K, z_k)$, $w(K, z_k)$ are competitively determined following (10) and (11).

3. Government balances its budget

$$G + T = \tau w(K, z_k) L(K, z_k).$$

4. Consistency is satisfied:

$$K' = \Gamma(K, z_k) = g_k(K, K, z_k)$$
$$L(K, z_k) = g_n(K, K, z_k).$$

3.3 Solution method

Our heterogeneous-agent models (i.e., Model (HA-T) and Model (HA-N)) cannot be solved analytically, are thus solved numerically. Several key features make the numerical solution method nontrivial. First, a key decision in our model economy is a discrete employment choice. Therefore, our solution method is based on the nonlinear method (i.e., the value function iteration) applied to the recursive representation of the problem described above. Second, the aggregate law of motion and the state variable involve an infinite-dimensional object: the distribution μ. Therefore, we solve the model by approximating the distribution of wealth by the mean of the distribution following Krusell and Smith (1998). In addition, since market-clearing is nontrivial in our model with endogenous labor, our solution method incorporates a step to find market-clearing prices in each period when simulating the model.

We describe the solution method briefly.[12] Following Krusell and Smith (1998), we assume that households use a smaller object that approximates the distribution when they forecast the future state variables to make current decisions. More precisely, we approximate $\mu(a, x_i)$ by its mean of the asset distribution $K = \int_a \sum_{i=1}^{N_x} a d\mu$. Also, the next period's aggregate capital K', real wage rate w and real interest rate r are assumed to be functions of (K, z) instead of (μ, z). We impose the parametric assumptions to approximate the aggregate law of motion $K' = \Gamma(K, z)$ and $w = w(K, z)$ following

$$\hat{K}' = \hat{\Gamma}(K, z) = \exp\left(a_0 + a_1 \log K + a_2 \log z\right) \tag{16}$$

$$\hat{w} = \hat{w}(K, z) = \exp\left(b_0 + b_1 \log K + b_2 \log z\right). \tag{17}$$

as in Chang and Kim (2007, 2014) and Takahashi (2014, 2017). Based on these forecasting rules, households obtain the forecasted $\hat{r}$ through the first-order conditions of firm's profit maximization.

Given the above forecasting rules, the model is solved in the two steps. First, we solve for the individual policy functions given the forecasting rules using the value function iterations (*the inner loop*). Then, we update the forecasting rules by simulating the economy using the individual

[12] See Appendix for more details.

policy functions (*the outer loop*). As noted above, it is important to note that, since our model environment with endogenous labor supply involves non-trivial factor market clearing unlike the benchmark Krusell-Smith (1998), we find market-clearing factor prices based on the forecasting rules (Chang and Kim, 2014; Takahashi, 2014).[13] The consistency between the law of motion and individual decision rules is obtained as we repeat this procedure until the coefficients in the forecasting rules converge. We provide more details in Appendix.

It is more straightforward to solve the representative-agent version of the model. Due to the distortionary tax, we solve the decentralized competitive equilibrium. For the purpose of comparison, we keep the same assumption on the discretization of the aggregate productivity process as in the heterogeneous-agent model. The steady-state can be obtained analytically, which is used in calibration (see Appendix). For the solutions with aggregate uncertainty, we use the policy function iteration method.

4 Calibration and model properties in steady state

The model is calibrated to U.S. data. A period in the model is a quarter, as is standard in the business cycle literature. There are two sets of parameters. The first set of parameters is calibrated externally in line with the business cycle literature. These parameter values are commonly set in all model specifications. The second set of parameters is calibrated to match the same number of relevant target statistics. The jointly calibrated parameter values differ across model specifications.

We begin with the first set of parameters that is calibrated externally. Most of these parameters are commonly used in the real business cycle literature. The capital share, α, is chosen to be consistent with the capital share of 0.36. The quarterly depreciate rate, δ, is 2.5 percent. In our model specifications with a binary labor supply choice, the level of hours worked, $\bar{n}$, can be arbitrarily set since it simply determines the scale of the calibrated disutility parameter B. We set it to 1/3, implying that working individuals spend a third of their time endowment on working.

[13] We have also checked the results when market-clearing is ignored. We find that although R^2 and Den-Haan statistics look reasonably good, the key business cycle statistics are considerably different from those obtained with market-clearing prices. In particular, we find that the difference is even larger than those reported in Takahashi (2014) in our model with more persistent idiosyncratic shocks.

Table 1: Parameter values chosen internally

	Model				
	(HA-T)	(HA-N)	(RA-T)	(RA-N)	Description
$B =$	.8760	1.092	1.056	1.254	Disutility of work
$\beta =$	.9863	.9841	.9901	.9901	Subject discount factor
$\rho_x =$	.9816	.9726	-	-	Persistence of $\ln x$
$\sigma_x =$	.0931	.1310	-	-	S.D. of innovations to $\ln x$
$T_1 =$	.0705	-	.2564	-	Transfer scale
$\omega_s =$	.498	-	-	-	Transfer schedule shape
$\omega_p =$	4.08	-	-	-	Transfer schedule shape

Note: Model (HA-T) is the baseline specification: a heterogeneous-agent incomplete markets model with government transfers. Model (HA-N) shuts down government transfers but keeps household heterogeneity. Model (RA-T) abstracts from household heterogeneity but keeps government transfers. Model (RA-N) shuts down both heterogeneity and government transfers. All model specifications have the same labor taxation.

The labor income tax, τ, is set to 27.9 percent as in Yum (2018) who follows Mendoza et al. (1994) and Trabandt and Uhlig (2011). We do not allow borrowing (i.e., $\underline{a} = 0$) in our baseline model. Finally, we should note that the goal of this paper is not to investigate the relative importance of different sources of aggregate fluctuations. Rather, our focus is on the transmission mechanism of aggregate shocks while taking a stochastic process for aggregate productivity shocks as given exogenously (Kydland and Prescott, 1982). Hence, we employ standard values for the aggregate productivity shocks (i.e., $\rho_z = 0.95$ and $\sigma_z = 0.007$) (Cooley and Prescott, 1995), which are also used by recent related papers such as Chang and Kim (2007, 2014) and Takahashi (2017).[14]

The second set of parameters is jointly calibrated for each specification of the model. As shown in Table 1, there are seven parameters for Model (HA-T), and four parameters for Model (HA-N) which shuts down government transfers with the restriction of $\omega_s = 0$. Unlike the heterogeneous agent models which require simulation to calibrate these parameters, the representative-agent models are easy to calibrate using the analytical optimality conditions (see Appendix). Our discussion herein focuses on the heterogeneous-agent model specifications. The parameter values reported in Table 1 are the calibrated values via matching the same number of target statistics in Table 2.

[14] The estimation of aggregate risk within a model with household heterogeneity is an important yet difficult task partly due to the computational burden. This important task is out of scope of this paper.

We now explain how each parameter is linked to the target statistics. The first parameter is B, which captures the disutility of work. The most relevant target moment is the employment rate of 65.2 percent in our SIPP samples. The next parameter β captures the discount factor of households. As is standard in the literature, it is targeted to match the quarterly interest rate of 1 percent.

The next two parameters, ρ_x and σ_x, govern the dynamics of idiosyncratic labor productivity, which is closely linked to wages if a household chooses to work. Note that there are two issues we would like to highlight when it comes to these parameters. First, there is a discrepancy in the data frequency. Specifically, the model period is a quarter while the wage data that are frequently used for estimating wage or earnings processes in the literature are at the annual frequency. This may lead to a non-straightforward temporal aggregation bias since labor supply is endogenous in the model. Moreover, and relatedly, in the data, there is a selection issue. That is, we only observe wages if households choose to work. To deal with both issues, we first estimate the persistence of annual wage using the PSID following a standard method in the literature (e.g., Heathcote et al. 2010), as described in detail in Appendix. The estimation result shows that the persistence of wages at the annual frequency is 0.953, which is in line with the estimates in the literature. Then, we calibrate the model so that the persistence of annual wages, which are constructed by the simulated quarterly data from the model, matches 0.953 as well.[15] This approach allows us to deal with the selection problem and temporal aggregation problem. Next, the standard deviation of innovations to the AR(1) process, σ_x, in (1) is calibrated to match the overall dispersion of annual earnings, which are also constructed by simulating the model. This allows the two heterogeneous agent models, which may have different persistence of idiosyncratic productivity risk, to have the same degree of the observed earnings inequality in U.S. data.

The last three parameters, T_1, ω_s and ω_p, govern the two components of transfers. Note that the two components can be distinguished in Model (HA-T): the parameter T_1 determines the size of

[15] More precisely, we simulate the model and construct annual wages by dividing annual earnings by annual hours worked through explicit temporal aggregation. Note that, although labor supply is a binary choice at the quarterly frequency, there are richer variations in the *annual* hours worked driven by the number of quarters worked, which is in turn affected by both idiosyncratic risk and aggregate risk. Erosa, Fuster and Kambourov (2016) highlight a similar point in a stationary environment in the absence of business cycles.

Table 2: Target statistics in the data and in the model

Target	Data	Model (HA-T)	(HA-N)	(RA-T)	(RA-N)
Employment rate	.652	.652	.652	.652	.652
Real interest rate	.010	.010	.010	.010	.010
Persistence of annual worker wages	.953	.953	.953	-	-
S.D. of log annual worker earnings	.623	.624	.624	-	-
Ratio of T_1 to output	.068	.068	-	-	-
Ratio of $T_1 + T_2$ to output	.106	.106	-	.106	-
$E(T_2\|\text{1st income quintile})/E(T_2)$	1.95	1.95	-	-	-

Note: See Table 1 for the description of the model specifications.

flat government transfers and ω_s determines the size of progressive transfers. Their relevant target statistics are set to be 6.8% and 3.8% of output, respectively (Krusell and Rios-Rull, 1999). Since Model (RA-T) lacks heterogeneity, T_2 is irrelevant. Hence, T_1 is directly calibrated to match an aggregate transfers-output ratio of 10.6%. Next, note that ω_p shapes the degree of progressivity in government transfers. Each parameter affects progressivity in different ways. Our calibration strategy is to let the model have an empirical reasonable degree of progressivity. For this purpose, we measure the degree of progressivity in the U.S. transfer programs using the SIPP data. We construct a broad measure of government transfers including means-tested programs and social insurance (as defined in Appendix) and use the relative amount of transfers in each wealth quintile. Since income security is highly relevant for the poor households, we choose the ratio of the average means-tested transfers received by the first income quintile to its unconditional mean (1.95) as a target statistic.

Table 2 shows that all model specifications do a good job of matching the target statistics. This does not necessarily mean that the model does a good job of accounting for other relevant statistics. We thus present some distributional aspects of the model economy in steady state as non-targeted moments. Table 3 summarizes the share of wealth, employment rates by wealth quintile. Overall, both heterogeneous-agent model specifications do a good job of accounting for the share of wealth by wealth quintile. A closer look reveals that Model (HA-T) does a noticeably better

Table 3: Characteristics of wealth distribution

Unit: %	Wealth quintile				
	1st	2nd	3rd	4th	5th
Share of wealth					
U.S. Data	-.017	.015	.077	.189	.742
Model (HA-T)	.002	.016	.084	.226	.672
Model (HA-N)	.014	.027	.104	.241	.626
Employment rate					
U.S. Data	.636	.708	.659	.639	.624
Model (HA-T)	.635	.782	.720	.627	.496
Model (HA-N)	.955	.725	.619	.532	.428

Note: The source of U.S. data is the Survey of Income and Program Participation 2001. See Table 1 for the description of the model specifications.

job of accounting for the wealth concentration at the top of the wealth distribution. Specifically, the relative shares of the fourth and fifth quintiles are noticeably closer to the data (18.9% and 74.2%, respectively) in Model (HA-T) (22.6% and 67.2%, respectively) compared to Model (HA-N) (24.1% and 62.6%, respectively). Note that, in the presence of government transfers, households' incentive to save declines (Hubbard, Skinner and Zeldes, 1995). This force is especially stronger for the low-income households since our nonlinear government transfer schedule implies that transfers decline with productivity. Therefore, this force tends to raise the relative share of wealth by the richer households in the baseline model.

When we look at the employment rate by wealth quintile, it is clear that Model (HA-T) does a significantly better job of accounting for the cross-sectional employment-wealth relationship. In the U.S., the employment rate of the first wealth quintile is relatively low (63.6%), compared to that of the second quintile (70.8%), and then it declines with wealth.[16] This weakly inverse-U shape of the employment rates across wealth quintiles in the data are well captured in Model (HA-T). On the other hand, Model (HA-N) predicts that employment falls sharply with wealth, consistent with the findings in Chang and Kim (2007). The sharp difference in the cross-sectional wealth-employment relationship between Model (HA-T) and Model (HA-N) is due to the presence of

[16] See Yum (2018) for the evidence in the Survey of Consumer Finances.

Table 4: Progressivity of income-security transfers

Unit: %	Income quintile				
	1st	2nd	3rd	4th	5th
Conditional mean/unconditional mean					
U.S. Data	1.95	1.70	0.79	0.37	0.19
Model (HA-T)	1.95	1.37	1.00	0.51	0.17

Note: The source of U.S. data is the Survey of Income and Program Participation 2001.

government transfers, which mitigates the excessively strong precautionary motive of labor supply among the poor households in this class of the incomplete markets framework (Yum, 2018).

Lastly, Table 4 also show the joint relationship between income and transfers. More precisely, the reported numbers are the ratio of average transfers in each income quintile to the unconditional mean transfers. In the U.S., there is a clear negative relationship between the amount of government transfers and income. Note that, in the model, this is a complicated equilibrium object, which is shaped not only by the parametric assumption on the nonlinear transfer schedule (7) but also by the endogenous household heterogeneity (which is in turn shaped by consumption-saving and labor supply decisions). Despite the relatively simple functional form in (7), we can see that the model does a fairly good job of replicating the degree of progressivity in equilibrium.

5 Quantitative results

In this section, we report the main business cycle results and illustrate the mechanism underlying the main quantitative results.

5.1 Business cycle statistics

We first compare business cycle statistics of key macroeconomic variables from simulations of the models to those from the data. We filter all the series using the Hodrick-Prescott filter with a smoothing parameter of 1600. The U.S. data statistics are computed using the aggregate data from 1955Q1 to 2011Q4 (see Appendix for more details). Table 4 summarizes the cyclical volatility of the

Table 5: Volatility of aggregate variables

		Model			
	U.S. data	(HA-T)	(HA-N)	(RA-T)	(RA-N)
σ_Y	1.56	1.63	1.51	1.88	1.79
σ_C/σ_Y	0.57	0.22	0.27	0.20	0.23
σ_I/σ_Y	2.82	2.78	2.74	2.85	2.83
σ_L/σ_Y	-	0.70	0.64	-	-
σ_H/σ_Y	0.96	0.86	0.70	0.85	0.82
$\sigma_{Y/H}/\sigma_Y$	0.55	0.34	0.43	0.20	0.23

Note: See Table 1 for the description of the model specifications. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Volatility is measured by the percentage standard deviation of each variable. The U.S. statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

key aggregate variables: Y is output, C is consumption, I is investment, L is aggregate efficiency unit of labor, H is aggregate hours, and Y/H is average labor productivity. The volatility is measured by the percentage standard deviation, as is standard in the literature. Except for the output volatility, we report the relative volatility, computed as the absolute volatility of each variable divided by that of output.

The most notable finding in Table 5 is that a very high volatility of aggregate hours observed in U.S. data ($\sigma_H/\sigma_Y = 0.96$) is best accounted for by Model (HA-T) among all the model specifications. This finding is remarkable for several reasons. First, note that standard real business cycle models are known to have difficulties in generating a large relative volatility of hours without relying on a low curvature of the utility function (or a high Frisch elasticity). In fact, Models (RA-T) and (RA-N) have the stand-in household whose disutility is linear in aggregate hours. When the utility function features zero curvature in labor supply, we can see that these models do generate a quite substantial relative volatility of hours (0.85 and 0.82, respectively), as shown in Hansen (1985). What is striking is that our baseline model, Model (HA-T), delivers a comparably high volatility of hours (0.86).

In fact, Chang and Kim (2007; 2014)'s results have demonstrated that a large relative volatility of hours obtained through indivisible labor (Rogerson, 1988) in Hansen (1985) may not be robust to

Table 6: Cyclicality of aggregate variables

		Model			
	U.S. data	(HA-T)	(HA-N)	(RA-T)	(RA-N)
$Cor(Y,C)$	0.82	0.79	0.84	0.78	0.82
$Cor(Y,I)$	0.91	0.99	0.99	0.99	0.99
$Cor(Y,L)$	-	0.98	0.97	-	-
$Cor(Y,H)$	0.85	0.95	0.93	0.99	0.99
$Cor(Y,Y/H)$	0.35	0.56	0.81	0.78	0.82
$Cor(H,Y/H)$	-0.21	0.26	0.54	0.69	0.72

Note: See Table 1 for the description of the model specifications. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Cyclicality is measured by the correlation of each variable with output. The statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

incomplete markets economies with heterogeneous households.[17] We can see this point also in Table 5 showing that Model (HA-N) is considerably less successful in accounting for the large volatility of hours among the four model specifications using indivisible labor. However, our result from Model (HA-T) suggests that, once progressive government transfers are incorporated, the heterogeneous agent incomplete-markets model may even perform better than the Hansen-Rogerson (stand-in household) economy in terms of a large fluctuation of hours over the business cycle.

Having highlighted the most notable difference across the four models, we also note that there are also interesting differences in the volatility of macroeconomic aggregates. For instance, the volatility of consumption and average labor productivity over the business cycle tends to be more consistent with the data in the heterogeneous-agent models than in the representative-agent models.[18] In both the heterogeneous-agent and representative-agent models, the introduction of transfers tends reduce the volatility of consumption and average labor productivity. This is not surprising given the nature of government transfers, effectively providing insurance against aggregate shocks.

We now move on to the focus of this paper: the cyclicality of macroeconomic variables. The first five rows of Table 6 show the correlations of output with other aggregate variables considered in Table 5.[19] The last row shows the correlation between total hours and labor productivity. As

[17] The relative volatility of hours is less than 0.6 (Chang and Kim, 2014; Takahashi, 2014), given a lower persistence of idiosyncratic shocks.
[18] Note that in the representative-agent models, average labor productivity is proportional to consumption.
[19] Again, all aggregate variables are detrended using the Hodrick-Prescott filter, as above.

is well known in the literature (e.g., King and Rebelo, 1999), most macroeconomic variables such as consumption, investment, and total hours are highly procyclical in the U.S. Table 6 show that the strongly positive correlations with output are fairly well replicated in all model specifications regardless of heterogeneity. Therefore, one may conclude that heterogeneity seems irrelevant at least when it comes to explaining highly procyclical macroeconomic variables over the business cycle.

However, we emphasize that there is a key difference in average labor productivity. In the U.S., average labor productivity does not feature strong procyclicality (i.e., $Cor(Y, Y/H) = 0.35$). A related observation is that the correlation between total hours and average labor productivity is even weakly negative (-0.21). In contrast, it has been well-known that canonical real business cycle models generate highly procyclical average labor productivity, and thus fail to replicate the cyclicality of average labor productivity. High correlations between output and average labor productivity in Models (RA-T) and (RA-N) (0.78 and 0.82, respectively) manifest this weakness as well.

A notable finding in Table 6 is that the cyclicality of average labor productivity is much smaller (0.56) in Model (HA-T), being closer to the data (0.35). The literature has suggested various possibilities to dampen strongly positive correlations of average labor productivity with output and hours (Benhabib, Rogerson, and Wright, 1991; Christiano and Eichenbaum, 1992; Braun, 1994; and Takahashi, 2017). In contrast to the existing literature which relies on additional exogenous shocks, the key to our result is the interaction between household heterogeneity and the presence of government transfers, both of which generates heterogeneous labor supply behavior across households. In fact, Model (HA-N) which features household heterogeneity still generates a very high correlation of 0.83, implying that heterogeneity per se cannot dampen highly procyclical average labor productivity in the real business cycle models. In the next subsection, we investigate the mechanism underlying our quantitative success.

Before moving on to the impulse responses, we briefly discuss the performance of the model economies in terms of the persistence of the macroeconomic variables. On the whole, all model specifications tend to produce similar persistence of output, consumption, and investment in Table

Table 7: Persistence of aggregate variables

		Model			
	U.S. data	(HA-T)	(HA-N)	(RA-T)	(RA-N)
$\rho(Y)$	0.85	0.70	0.69	0.68	0.68
$\rho(C)$	0.84	0.85	0.82	0.84	0.81
$\rho(I)$	0.89	0.68	0.67	0.67	0.67
$\rho(L)$	-	0.68	0.65	-	-
$\rho(H)$	0.84	0.68	0.67	0.67	0.67
$\rho(Y/H)$	0.52	0.32	0.52	0.84	0.81

Note: See Table 1 for the description of the model specifications. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Persistence is measured by the first-order autocorrelation of each variable. The statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

7. Although Model (HA-T) performs slightly better than the other models in replicating the highly persistent total hours ($\rho(H) = 0.84$), the difference is not substantial. Heterogeneity helps dampen the highly persistent average labor productivity in the representative-agent models (around 0.8).[20]

5.2 Impulse responses

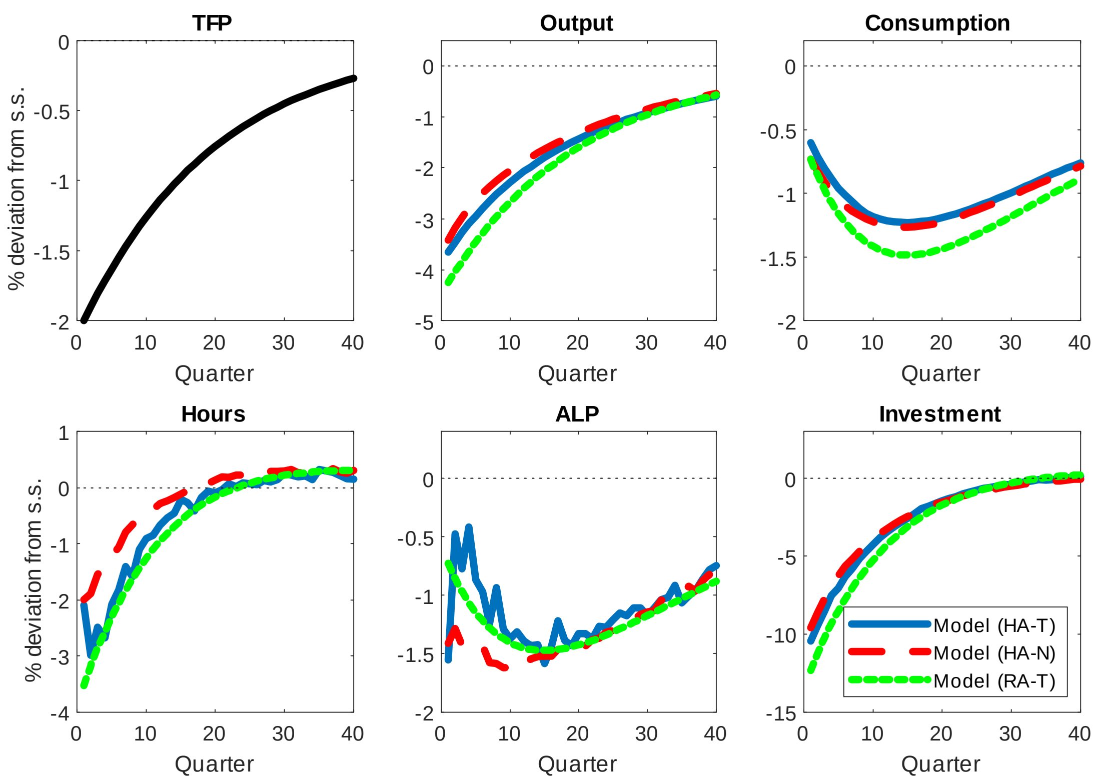
We now present impulse response functions to better understand the main findings in the previous subsection. Figure 2 shows the impulse responses of the key aggregate variables such as output, consumption, total hours, average labor productivity, and investment following a persistent negative 2% shock to z (or TFP) in the first three model specifications.[21] The impulse response of total hours clearly confirms that Model (HA-T) (solid line) delivers a substantially larger fall in total hours than Model (HA-N). This reinforces the main finding on a large relative volatility of hours in Model (HA-T) in Table 4. Interestingly, the fall in aggregate hours in Model (HA-T) immediately following a negative TFP shock is not as strong.

Another important difference to note is the impulse responses of average labor productivity. In the representative-agent models, the impulse response of average labor productivity follows that of consumption, exhibiting an inverse hump-shape. This is the case also for our heterogeneous-agent

[20] In particular, Model (HA-N) matches the persistence of average labor productivity very well.

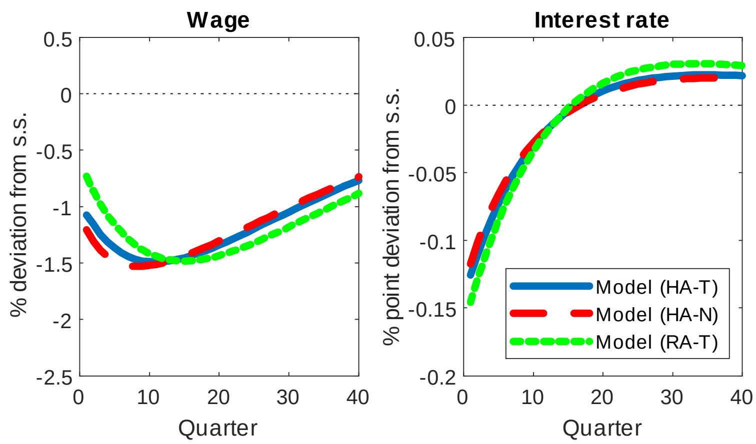
[21] Since the impulse responses from Model (RA-T) and Model (RA-N) are very similar, figures in this subsection does not report the results from Model (RA-N).

Figure 2: Impulse responses of macroeconomic aggregates



Note: TFP denotes the total factor productivity (or aggregate risk). ALP refers to the average labor productivity (or output divided by aggregate hours). The figures display the responses of macroeconomic aggregates to a negative 2 percent TFP shock with persistence ρ_z.

Figure 3: Impulse responses of equilibrium prices



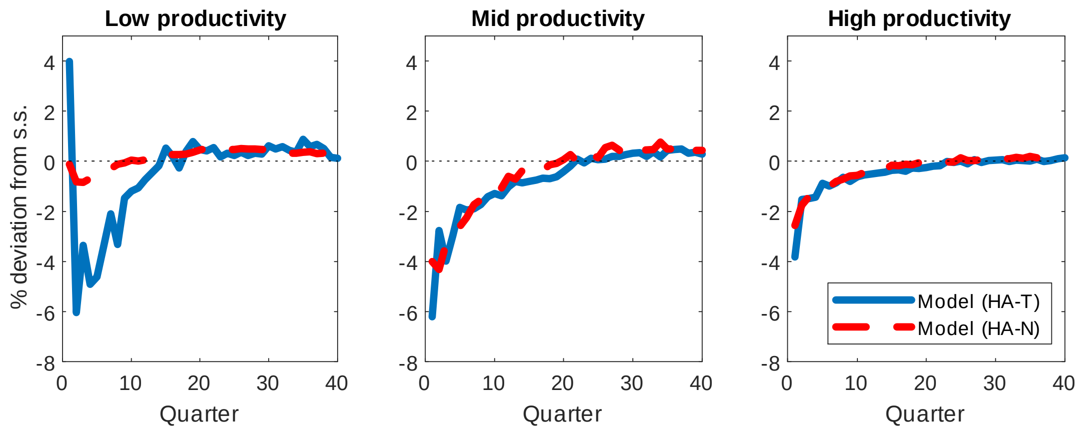
Note: The figures display equilibrium market-clearing price responses to a negative 2 percent TFP shock with persistence ρ_z.

model without transfers (i.e., Model (HA-N)). In contrast, Model (HA-T) delivers a nontrivial average labor productivity response following a negative TFP shock; it falls sharply, and then it reverts close to zero and the falls again. This nontrivial response of average labor productivity clearly illustrates why the cyclicality of average labor productivity in Model (HA-T) is different from those from the other model specifications.

It is interesting to note that the responses in aggregate output, consumption and investment from Model (HA-T) resemble those from Model (HA-N) very closely and even those from the representative model fairly closely. We now attempt to show how our baseline full model (i.e., Model (HA-T)) delivers the impulse responses of total hours and average labor productivity that are markedly different from the other models. In particular, we focus on illustrating why heterogeneity per se is not sufficient to generate a large response of total hours and non-trivial average labor productivity dynamics.

An obvious candidate is the dynamics of equilibrium prices. Figure 3 displays the changes in market-clearing wage and interest rates following the same negative TFP shock for the first three model specifications. It appears that the difference across the model specifications is not substantial. This suggests that our main results are not mainly driven by the difference in equilibrium price

Figure 4: Impulse responses of hours by productivity



Note: Households are grouped into low productivity (below median), mid productivity (median), and high productivity (above median). The figures display employment responses in each group to a negative 2 percent TFP shock with persistence ρ_z.

dynamics.

Next, it is useful to look at the impulse responses at more disaggregated level. Figure 4 plots the impulse responses of hours worked by productivity at the household level. Specifically, we categorize households into three groups: (i) low productivity $\{x_i\}_{i=1}^{8}$; (ii) mid productivity x_9; and (iii) high productivity $\{x_i\}_{i=17}^{10}$.

We see that households with higher productivity tend to be less elastic in their labor supply. This result shows that the key intuition of Proposition 1 in a simple static model extends to our full heterogeneous-agent model framework. However, there is an exception of this pattern; in Model (HA-N), households with low productivity is highly inelastic in their labor supply. The reason for this exceptionally low labor supply elasticity is related to the finding in Yum (2018) who shows that the absence of public insurance in incomplete markets models raises the precautionary motive for labor supply among the poor households who lack self-insurance. When the precautionary motive is too high, this dominates the intertemporal substitution motive, thereby weakening the responses of hours with respect to a persistent fall in wages (Figure 4). Note also that this inelastic labor supply at the low productivity group provides a key reason for both a relative weaker volatility of total hours and a very procyclical average labor productivity in Model (HA-N) relative to Model (HA-T).

6 Microeconomic evidence on heterogeneity in extensive-margin labor supply responses

As shown in Section 5, the key element of our model is the existence of heterogeneous labor supply responses. In particular, low-wage workers are considerably more elastic in adjusting labor supply at the extensive margin, which weakens a highly procyclical average labor productivity and, at the same time, enlarges the volatility of aggregate hours worked over the business cycle. This section documents heterogeneity in labor supply responses using micro data to empirically investigate the key mechanism of our model.

Specifically, we exploit the panel structure of the PSID to explore whether extensive margin labor supply responses differ by their hourly wage. The panel structure is useful since we can keep track of the same people whose labor supply decisions are observed over time. Since labor supply adjustments can be measured in different ways, we consider two approaches. The first approach focuses on identifying the probability of extensive-margin labor supply adjustment for each individual, and illustrating how it differs by wage. On the other hand, the second approach focuses on differences in magnitude of the employment rate changes across wage groups over short time horizon. More precisely, we consider the periods before and in the middle of the last six recessions. We now present each empirical analysis in more detail.

As mentioned above, the key object of interest in the first approach is the probability of extensive-margin adjustment for each individual. This requires us to have a relatively long time-series observations for each individual to obtain a consistent estimate of the adjustment probability, based on the individual-level flow data.[22] Let us fix a year at j. Let i denote an individual index and t denote the year when the individual is observed. We define the extensive margin adjustment based on the full-time employment, E, consistent with the previous sections. In other words, an

[22] Since the frequency of the PSID survey has been annual until 1997 and became biannual since 1999, we use the samples observed annually from the 1969-1997 waves.

Table 8: Probability of extensive margin adjustment, by wage quintile

	Length of tracking each individual T		
	5 years	10 years	15 years
Wage quintile in base year			
1st	0.123	0.100	0.088
2nd	0.068	0.057	0.053
3rd	0.051	0.046	0.045
4th	0.046	0.041	0.039
5th	0.050	0.045	0.044
Base years	1969-1993 (25)	1969-1988 (20)	1969-1983 (15)
Avg. no. obs in base years	1,743	1,281	927
Total no. obs.	43,580	25,619	13,911
Avg. age in total sample	43.0	43.7	44.4

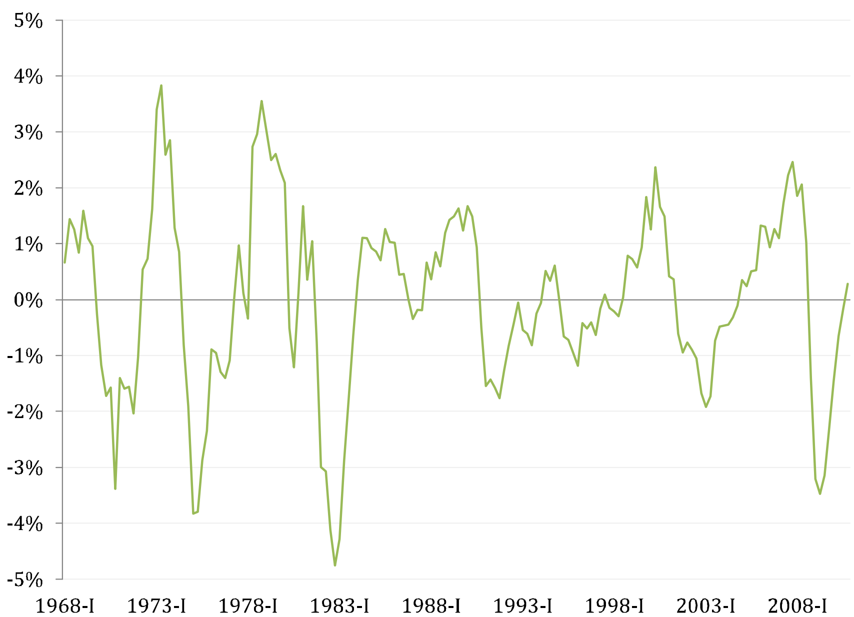
Note: Description here. obtained as average. Numbers in parentheses show the number of base years. PSID

individual i in year t is full-time employed (i.e., $E_{i,t} = 1$) if the annual hours worked is greater than 1,000 hours. Then, we define a binary variable of switching, $S_{i,t}$, such that $S_{i,t} = 1$ if $E_{i,t} \neq E_{i,t-1}$ and $S_{i,t} = 0$ otherwise. Note that, given the length of tracking each individual, T, there are $(T-1)$ number of $S_{i,t}$ for each individual i. Once we take the average over time, we obtain the individual-specific probability of extensive-margin adjustment at the annual frequency (i.e., $p_{i,j} \equiv \frac{1}{T-1}\sum_{t=j+1}^{j+T-1} S_{i,t}$). As we are interested in differences across wage, we compute p_j^q, the conditional mean of $p_{i,j}$, depending on each individual's wage quintile ($q \in \{1, 2, ..., 5\}$) determined in the base year j.

Table 8 reports the mean switching probabilities by wage quintile, averaged across the base years.[23] We consider three different values for the length of tracking each individual: $T \in \{5, 10, 15\}$. This is because a different value of T entails a trade-off. On the one hand, a larger number is beneficial since we are more likely to have a consistent estimate of the adjustment probability at the individual level. On the other hand, a longer time of tracking implies a stricter restriction on samples (since we only keep samples who are observed for T consecutive years). We compute

[23] For instance, when $T = 5$, we compute the conditional mean of p_i by wage quintile for $j = \{1969, 1970, ..., 1993\}$. Then, the reported values in the first column are the averages of the switching probability by wage quintiles over the base years. In fact, these statistics are quite robust across different base years.

Figure 5: Cyclical component of real GDP per capita



Note: A quarterly series of real GDP per capita is detrended using HP filter with a smoothing parameter of 1,600.

this by changing the base year. Then, the reported values are the averages over the base year, $p^q \equiv \frac{1}{J} \sum p_j^q$ where J is the number of base years.

It is clear to note the pattern that the individual-level probability of adjusting labor supply along the extensive margin is significantly higher among low-wage workers. For instance, when $T = 5$, the probability of switching to/from full-time employment among the first wage quintile is 12.3% at the annual frequency. In particular, we can see that this probability tends to decrease with wage. For the third to fifth quintiles, this probability is around 5%. When T increases, we also find that the key pattern of extensive-margin adjustment probabilities across wage quintile is still present. As the samples become slightly older and T becomes longer, however, we also see that the switching probabilities become generally lower.

The above exercise is based on the long-run information on the labor market flow at the individual level. The next empirical exercise, on the other hand, uses the differences in magnitude of the employment level changes across wage groups. In addition, we now focus on the changes

Table 9: Employment changes in recessions, by wage quintile

	Recession					
	1969-70	1973-75	1980-82	1990-91	2000-02	2006-10
Wage quintile in peak year						
1st	-6.3	-10.2	-11.4	-3.9	-9.3	-16.5
2nd	-4.0	-8.0	-3.4	-2.8	-5.0	-14.4
3rd	-2.8	-7.8	-1.4	0.1	-3.8	-11.0
4th	-3.0	-2.2	-6.9	-0.9	-4.1	-9.5
5th	0.1	-3.0	-3.4	-0.8	-1.3	-5.9
No. obs in peak year	1,928	2,135	2,496	2,533	3,404	3,628

Note: The year range denotes the peak and trough years of each recession, chosen based on the deviations of GDP in Figure 5 and the availability of annual PSID data. Reported values are the percentage point changes in employment rate by wage quintile (in the peak year of each recession), constructed by following the individuals who are present in the peak year.

in short time horizons, investigating the changes in employment over the last six recessions. More specifically, we choose six recessions, as evident from Figure 5 which plots the cyclical component of quarterly real GDP per capita.[24] For each recession, we choose a peak year and a trough year, based on Figure 5 and the aggregate employment changes in our PSID samples. Note that our definition of peak and trough years should be regarded only roughly since the PSID data set is available annually (until 1997) or biannually (since 1999). The resulting year combinations for each recession are shown in Table 9.

For each recession, we compute the conditional mean of full-time employment by wage quintile in the peak year, $\frac{1}{N_{peak}} \sum_i E^q_{i,peak}$ where N_{peak} is the number of observations in the peak year of a recession and q is the wage quintile in the peak year. Then, we measure the percentage point changes in the employment rate by wage quintile in the corresponding trough year: that is, $\frac{1}{N_{peak}} \sum_i \left(E^q_{i,trough} - E^q_{i,peak} \right)$. It is important to note that we keep the set of households in each wage group fixed by assigning a wage quintile to each household in the peak year. That way, our measured changes in employment by wage quintile are not affected by compositional changes, but are rather based on labor supply decisions of the same households.

[24] A quarterly series of real GDP per capita is detrended using HP filter with a smoothing parameter of 1,600.

Table 9 clearly shows that the employment rate fell most sharply in the first wage quintile in all of the recessions. Furthermore, the magnitude of the fall in employment tends to be smaller as the wage quintile increases. For example, in the first recession we considered, the employment rate among the first wage quintile fell by 6.3 percentage point whereas the employment rate among the fifth wage quintile barely changed. This pattern of the employment changes by wage quintiles is quite robust across different recessions despite the variations in the overall changes in employment rate. Note that the overall magnitude of fall in employment is relatively stronger in the recessions of 1973-75, 1980-82 and 2006-10. This is in fact consistent with relatively larger amplitudes of these recessions, as shown in Figure 5.

Overall, the above two empirical exercises support our key mechanism of the heterogeneous-agent model with progressive government transfers. It is remarkable that, although the two approaches are designed to capture different aspects of labor supply adjustments (i.e., individual-level flows over the long-run vs. short-run aggregate changes in magnitude, respectively), they yield the consistent result. This demonstrates the robustness of our empirical result that lower wage workers adjust labor supply along the extensive margin more elastically. More importantly, both of these empirical findings are consistent with the pattern of heterogeneity in labor supply responses in our model economy, thereby supporting our key mechanism of the model.

7 Conclusion

In this paper, we have explored the interplay of household heterogeneity and progressive government transfers in shaping the dynamics of macroeconomic aggregates over the business cycle. We first presented the key insights using analytical results obtained from a stylized static model with two types. We then constructed a full general equilibrium business cycle model with household heterogeneity. We have shown that in our heterogeneous-agent model with progressive government transfers, calibrated to match the salient facts in the micro-level moments, micro-level heterogeneity shapes the dynamics of aggregate labor market variables substantially when household heterogeneity interacts with progressive government transfers. In particular, our baseline real business cycle model delivers moderately positive correlations of average labor productivity with output and total hours while generating a large relative volatility of total hours over the business cycle.

Using the panel structure of the PSID, we have documented that the individual-level probability of adjusting labor supply along the extensive margin is significantly higher among low-wage workers. In addition, we have also found that the magnitude of a decline in employment rate is considerably larger among low-wage workers in all of the last six recessions. The microeconomic evidence supports the key mechanism of our heterogeneous-agent model with progressive government transfers.

References

Ahn, SeHyoun, Greg Kaplan, Benjamin Moll, Thomas Winberry, and Christian Wolf. 2017. "When Inequality Matters for Macro and Macro Matters for Inequality." In *NBER Macroeconomics Annual* 2017, Volume 32: University of Chicago Press.

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *The Quarterly Journal of Economics* 109 (3): 659-684.

Altonji, Joseph G. and Lewis M. Segal. 1996. "Small-Sample Bias in GMM Estimation of Covariance Structures." *Journal of Business & Economic Statistics* 14 (3): 353-366.

Benhabib, Jess, Richard Rogerson, and Randall Wright. 1991. "Homework in Macroeconomics: Household Production and Aggregate Fluctuations." *Journal of Political Economy* 6: 1166-1187.

Braun, R. Anton. 1994. "Tax Disturbances and Real Economic Activity in the Postwar United States." *Journal of Monetary Economics* 33 (3): 441-462.

Chang, Yongsung and Sun-Bin Kim. 2014. "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations: Reply." *The American Economic Review* 104 (4): 1461-1466.

———. 2007. "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations." *American Economic Review* 97 (5): 1939-1956.

———. 2006. "From Individual to Aggregate Labor Supply: A Quantitative Analysis Based on a Heterogenous Agent Macroeconomy." *International Economic Review* 47 (1): 1-27.

Chang, Yongsung, Sun-Bin Kim, Kyooho Kwon, and Richard Rogerson. 2014. "Individual and Aggregate Labor Supply in a Heterogenous Agent Economy with Intensive and Extensive Margins." Working paper.

Chang, Yongsung, Sun-Bin Kim, and Frank Schorfheide. 2013. "Labor-Market Heterogeneity, Aggregation, and Policy (in) Variance of DSGE Model Parameters." *Journal of the European Economic Association* 11: 193-220.

Christiano, Lawrence J. and Martin Eichenbaum. 1992. "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations." *American Economic Review* 82 (3): 430-450.

Cociuba, Simona E., Edward C. Prescott, and Alexander Ueberfeldt. 2012. "US Hours and Productivity Behavior using CPS Hours Worked Data: 1947-III to 2011-IV." Working paper.

Coeurdacier, Nicolas, Hélène Rey, and Pablo Winant. 2015. "Financial integration and growth in a risky world." National Bureau of Economic Research No. w21817.

Den Haan, Wouter J. 2010. "Assessing the Accuracy of the Aggregate Law of Motion in Models with Heterogeneous Agents." *Journal of Economic Dynamics and Control* 34 (1): 79-99.

Doepke, Matthias and Michele Tertilt. 2016. "Families in Macroeconomics." In *Handbook of Macroeconomics*. Vol. 2, 1789-1891: Elsevier.

Erosa, Andrés, Luisa Fuster, and Gueorgui Kambourov. 2016. "Towards a Micro-Founded Theory of Aggregate Labour Supply." *The Review of Economic Studies* 83 (3): 1001-1039.

Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics* 16 (3): 309-327.

Heathcote, Jonathan. 2005. "Fiscal Policy with Heterogeneous Agents and Incomplete Markets." *The Review of Economic Studies* 72 (1): 161-188.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante. 2010. "The Macroeconomic Implications of Rising Wage Inequality in the United States." *Journal of Political Economy* 118 (4): 681-722.

———. 2009. "Quantitative Macroeconomics with Heterogeneous Households." *Annual Review of Economics* 1 (1): 319-354.

Heckman, James J. 1979. "Sample Selection Bias as a Specification Error." *Econometrica* 47 (1): 153-161.

Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes. 1995. "Precautionary Saving and Social Insurance." *Journal of Political Economy*: 360-399.

Huggett, Mark. 1993. "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies." *Journal of Economic Dynamics and Control* 17 (5): 953-969.

Imrohoroğlu, Ayşe. 1989. "Cost of Business Cycles with Indivisibilities and Liquidity Constraints." *Journal of Political Economy* 97 (6): 1364-1383.

Keane, Michael and Richard Rogerson. 2015. "Reconciling Micro and Macro Labor Supply Elasticities: A Structural Perspective." *Annual Review of Economics* 7 (1): 89-117.

Khan, Aubhik and Julia K. Thomas. 2008. "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics." *Econometrica* 76 (2): 395-436.

Kim, Heejeong. 2017. "Inequality, Portfolio Choice, and the Business Cycle." Working paper.

King, Robert G. and Sergio T. Rebelo. 1999. "Resuscitating Real Business Cycles." *Handbook of Macroeconomics* 1: 927-1007.

Kopecky, Karen A. and Richard MH Suen. 2010. "Finite State Markov-Chain Approximations to Highly Persistent Processes." *Review of Economic Dynamics* 13 (3): 701-714.

Krueger, Dirk, Kurt Mitman, and Fabrizio Perri. 2016. "Macroeconomics and Household Heterogeneity." *Handbook of Macroeconomics* 2: 843-921.

Krusell, Per and Jr Smith Anthony A. 2006. "Quantitative Macroeconomic Models With Heterogeneous Agents," in *Advances in Economics and Econometrics: Theory and Applications, Ninth*

World Congress. Econometric Society Monographs, Vol. 41, ed. by R. Blundell, W. Newey, and T. Persson. Skatteverket: Cambridge University Press, 298-340.

———. 1998. "Income and Wealth Heterogeneity in the Macroeconomy." *Journal of Political Economy* 106 (5): 867-896.

Kydland, Finn E. and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica* 50 (6): 1345-1370.

Mendoza, Enrique G., Assaf Razin, and Linda L. Tesar. 1994. "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption." *Journal of Monetary Economics* 34 (3): 297-323.

Rios-Rull, Victor. 1999. "Computation of Equilibria in Heterogenous Agent Models." In *Computational Methods for the Study of Dynamic Economies*: Oxford University Press.

Rogerson, Richard. 1988. "Indivisible Labor, Lotteries and Equilibrium." *Journal of Monetary Economics* 21 (1): 3-16.

Rouwenhorst, K. Geert. 1995. "Asset pricing implications of equilibrium business cycle models." *In: Cooley, T.F.(Ed.), Frontiers of Business Cycle Research.* Princeton University Press, Princeton, NJ, pp. 294–330.

Takahashi, Shuhei. 2017. "Time-Varying Wage Risk, Incomplete Markets, and Business Cycles." Working paper.

———. 2014. "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations: Comment." *American Economic Review* 104 (4): 1446-1460.

Thomas, Julia K. 2002. "Is Lumpy Investment Relevant for the Business Cycle?" *Journal of Political Economy* 110 (3): 508-534.

Trabandt, Mathias and Harald Uhlig. 2011. "The Laffer Curve Revisited." *Journal of Monetary Economics* 58 (4): 305-327.

Yum, Minchul. 2018. "On the Distribution of Wealth and Employment." *Review of Economic Dynamics* 30: 86-105.

Appendix

A Proofs in Section 2

Proof of Proposition 1 Assume $T_i = 0$. Then, we can rewrite

$$\tilde{a}_i = zx_i.$$

Therefore,

$$N_i = 1 - \exp(-zx_i)$$

Given this, note that

$$\begin{aligned} \varepsilon_i \equiv \frac{\partial N_i}{\partial z}\frac{z}{N_i} &= x_i \exp(-zx_i)\frac{z}{1-\exp(-zx_i)} \\ &= \frac{zx_i \exp(-zx_i)}{1-\exp(-zx_i)} \end{aligned}$$

For expositional convenience, assume that x is continuous for now.

$$\varepsilon(x) = \frac{zx\exp(-zx)}{1-\exp(-zx)}$$

$$\begin{aligned} \frac{\partial \varepsilon(x)}{\partial x} &= \frac{\left[z\exp(-zx) - z^2 x\exp(-zx)\right][1-\exp(-zx)] - zx\exp(-zx)\left[z\exp(-zx)\right]}{[1-\exp(-zx)]^2} \\ &= \frac{\exp(-zx)z\left[1-zx\right][1-\exp(-zx)] - z^2 x\exp(-zx)\left[\exp(-zx)\right]}{[1-\exp(-zx)]^2} \\ &= \frac{z\exp(-zx)\left\{1-zx-\exp(-zx)\right\}}{[1-\exp(-zx)]^2} \end{aligned}$$

Since $\exp(-zx) < 1$ for all $z, x > 0$,

$$\frac{\partial \varepsilon(x)}{\partial x} = \frac{z\exp(-zx)\,(1 - zx - \exp(-zx))}{[1-\exp(-zx)]^2} < \frac{z\exp(-zx)\,(1-zx-1)}{[1-\exp(-zx)]^2}$$
$$= \frac{z\exp(-zx)\,(-zx)}{[1-\exp(-zx)]^2} < 0.$$

Proof of Proposition 2 Since

$$\frac{\partial N_l}{\partial z} = \exp(-\tilde{a}_l)(1-\lambda),$$
$$\frac{\partial N_h}{\partial z} = \exp(-\tilde{a}_h)(1+\lambda).$$

we have

$$\frac{\partial}{\partial \omega}\left(\frac{\partial N_l}{\partial z}\right) = \exp(-\tilde{a}_l)(1-\lambda)T\lambda > 0,$$
$$\frac{\partial}{\partial \omega}\left(\frac{\partial N_h}{\partial z}\right) = -\exp(-\tilde{a}_h)(1+\lambda)T\lambda < 0.$$

Also, note that

$$\frac{\partial N_l}{\partial \omega} = -\exp(-\tilde{a}_l)T\lambda < 0$$
$$\frac{\partial N_h}{\partial \omega} = \exp(-\tilde{a}_l)T\lambda > 0.$$

Proof of Proposition 3 Since

$$\varepsilon \equiv \frac{\partial N}{\partial z}\frac{z}{N}$$
$$= \left(\pi_l \frac{\partial N_l}{\partial z} + \pi_h \frac{\partial N_h}{\partial z}\right)\frac{z}{\pi_l N_l + \pi_h N_h}$$

the aggregate labor supply elasticity is given by

$$\varepsilon = z\frac{\exp(-\tilde{a}_l)(1-\lambda) + \exp(-\tilde{a}_h)(1+\lambda)}{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)}$$

where

$$\tilde{a}_l = z(1-\lambda) - T - T\omega\lambda$$
$$\tilde{a}_h = z(1+\lambda) - T + T\omega\lambda.$$

Then, we have

$$\frac{\partial\varepsilon}{\partial\omega} = z\frac{\begin{array}{c}[\exp(-\tilde{a}_l)(1-\lambda)(-1)(-T\lambda) + \exp(-\tilde{a}_h)(1+\lambda)(-1)T\lambda]\left[2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right] \\ -\left[\exp(-\tilde{a}_l)(1-\lambda) + \exp(-\tilde{a}_h)(1+\lambda)\right]\left[-\exp(-\tilde{a}_l)(-1)(-T\lambda) - \exp(-\tilde{a}_h)(-1)T\lambda\right]\end{array}}{\left[2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right]^2}$$
$$= zT\lambda\frac{\begin{array}{c}[\exp(-\tilde{a}_l)(1-\lambda) - \exp(-\tilde{a}_h)(1+\lambda)]\left[2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right] \\ +\left[\exp(-\tilde{a}_l)(1-\lambda) + \exp(-\tilde{a}_h)(1+\lambda)\right]\left[\exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right]\end{array}}{\left[2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right]^2}$$

The sign of $\frac{\partial\varepsilon}{\partial\omega}$ is equal to that of the numerator, which can be rewritten as

$$\begin{aligned}\text{Numerator} &= 2(1-\lambda)\exp(-\tilde{a}_l) - (1-\lambda)\exp(-2\tilde{a}_l) - (1-\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) \\ &- 2(1+\lambda)\exp(-\tilde{a}_h) + (1+\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) + (1+\lambda)\exp(-2\tilde{a}_h) \\ &+ (1-\lambda)\exp(-2\tilde{a}_l) - (1-\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) \\ &+ (1+\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) - (1+\lambda)\exp(-2\tilde{a}_h) \\ &= 2\left[(1-\lambda)\exp(-\tilde{a}_l) - (1+\lambda)\exp(-\tilde{a}_h) + 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)\right].\end{aligned}$$

Letting $\theta = \frac{(1-\lambda)}{(1+\lambda)}$, we can rewrite

$$\begin{aligned}&2(1+\lambda)\left[\frac{(1-\lambda)}{(1+\lambda)}\exp(-\tilde{a}_l) - \exp(-\tilde{a}_h) + \frac{2\lambda}{(1+\lambda)}\exp(-\tilde{a}_h - \tilde{a}_l)\right] \\ &= 2(1+\lambda)\left[\theta\exp(-\tilde{a}_l) + (1-\theta)\exp(-\tilde{a}_h - \tilde{a}_l) - \exp(-\tilde{a}_h)\right].\end{aligned}$$

Since $\exp(-x)$ is convex, we know

$$\begin{aligned}\theta \exp(-\tilde{a}_l) + (1-\theta)\exp(-(\tilde{a}_h + \tilde{a}_l)) &> \exp\left(-\left\{\theta \tilde{a}_l + (1-\theta)(\tilde{a}_h + \tilde{a}_l)\right\}\right) \\ &= \exp\left(-\left\{(1-\theta)\tilde{a}_h + \tilde{a}_l\right\}\right).\end{aligned}$$

Applying this inequality, we have

$$\begin{aligned}\text{Numerator} &= 2(1+\lambda)\left[\theta \exp(-\tilde{a}_l) + (1-\theta)\exp(-\tilde{a}_h - \tilde{a}_l) - \exp(-\tilde{a}_h)\right] \\ &> 2(1+\lambda)\left[\exp\left(-\left\{(1-\theta)\tilde{a}_h + \tilde{a}_l\right\}\right) - \exp(-\tilde{a}_h)\right] \geq 0\end{aligned}$$

if and only if

$$\begin{aligned}(1-\theta)\tilde{a}_h + \tilde{a}_l &\leq \tilde{a}_h \\ \tilde{a}_l &\leq \theta \tilde{a}_h \\ (1+\lambda)\left[z(1-\lambda) - T - T\omega\lambda\right] &\leq (1-\lambda)\left[z(1+\lambda) - T + T\omega\lambda\right] \\ z(1+\lambda)(1-\lambda) - (1+\lambda)T - (1+\lambda)T\omega\lambda &\leq z(1+\lambda)(1-\lambda) - (1-\lambda)T + (1-\lambda)T\omega\lambda \\ -(1+\lambda) - (1+\lambda)\omega\lambda &\leq -(1-\lambda) + (1-\lambda)\omega\lambda \\ -1 &\leq \omega\end{aligned}$$

which is always satisfied.

Proof of Proposition 4 Note that

$$\begin{aligned}\chi_0 &= \frac{(1-\lambda)(1-\exp(-\tilde{a}_l)) + (1+\lambda)(1-\exp(-\tilde{a}_h))}{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)} \\ &= \frac{1-\lambda-\exp(-\tilde{a}_l) + \lambda\exp(-\tilde{a}_l) + 1 + \lambda - \exp(-\tilde{a}_h) - \lambda\exp(-\tilde{a}_h)}{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)} \\ &= \frac{2 - (1-\lambda)\exp(-\tilde{a}_l) - (1+\lambda)\exp(-\tilde{a}_h)}{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\frac{\partial \chi_0}{\partial z} &= \frac{\left[(1-\lambda)^2 \exp(-\tilde{a}_l) + (1+\lambda)^2 \exp(-\tilde{a}_h)\right] \left[2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right]}{\left(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right)^2} \\
&- \frac{\left[2 - (1-\lambda)\exp(-\tilde{a}_l) - (1+\lambda)\exp(-\tilde{a}_h)\right] \left[\exp(-\tilde{a}_l)(1-\lambda) + \exp(-\tilde{a}_h)(1+\lambda)\right]}{\left(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right)^2} \\
&= \frac{1}{\left(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right)^2} \left\{ \begin{array}{c}
2(1-\lambda)^2 \exp(-\tilde{a}_l) + 2(1+\lambda)^2 \exp(-\tilde{a}_h) \\
-(1-\lambda)^2 \exp(-2\tilde{a}_l) - (1+\lambda)^2 \exp(-\tilde{a}_h - \tilde{a}_l) \\
-(1-\lambda)^2 \exp(-\tilde{a}_h - \tilde{a}_l) - (1+\lambda)^2 \exp(-2\tilde{a}_h) \\
-2(1-\lambda)\exp(-\tilde{a}_l) - 2(1+\lambda)\exp(-\tilde{a}_h) \\
+(1-\lambda)^2 \exp(-2\tilde{a}_l) + (1+\lambda)(1-\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) \\
+(1+\lambda)(1-\lambda)\exp(-\tilde{a}_h - \tilde{a}_l) + (1+\lambda)^2 \exp(-2\tilde{a}_h)
\end{array} \right\} \\
&= \frac{2\lambda(\lambda-1)\exp(-\tilde{a}_l) + 2\lambda(\lambda+1)\exp(-\tilde{a}_h) - 4\lambda^2 \exp(-\tilde{a}_h - \tilde{a}_l)}{\left(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right)^2} \\
&= \frac{2\lambda\left\{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)\right\}}{\left(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right)^2} < 0.
\end{aligned}$$

Proof of Proposition 5 Define

$$\Phi(\omega) \equiv \log\left(\frac{\partial \chi_0}{\partial z}\right).$$

Since the log transformation preserves monotonicity, it suffices to show that $\Phi'(\omega) < 0$. As

$$\begin{aligned}
\Phi(\omega) &= \log 2\lambda + \log\left\{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)\right\} \\
&\quad - 2\log\left(2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right)
\end{aligned}$$

we have

$$
\begin{aligned}
\Phi'(\omega) &= \frac{-T\lambda(\lambda-1)\exp(-\tilde{a}_l) + T\lambda(\lambda+1)\exp(-\tilde{a}_h)}{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)} \\
&\quad - 2\frac{T\lambda\exp(-\tilde{a}_l) - T\lambda\exp(-\tilde{a}_h)}{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)} \\
&= \frac{\underbrace{T\lambda(1-\lambda)\exp(-\tilde{a}_l) + T\lambda(\lambda+1)\exp(-\tilde{a}_h)}_{\text{positive}}}{\underbrace{(\lambda-1)\exp(-\tilde{a}_l) + (\lambda+1)\exp(-\tilde{a}_h) - 2\lambda\exp(-\tilde{a}_h - \tilde{a}_l)}_{\text{negative}}} \\
&\quad - 2\frac{\underbrace{T\lambda\left[\exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)\right]}_{\text{positive}}}{\underbrace{2 - \exp(-\tilde{a}_l) - \exp(-\tilde{a}_h)}_{\text{positive}}} \\
&< 0.
\end{aligned}
$$

B Aggregate data

The business cycle statistics are based on the aggregate time-series data covering from 1955Q1 to 2011Q4. For output, we use “Real Gross Domestic Product (millions of chained 2009 dollars)” in Table 1.1.6 of the Bureau of Economic Analysis (BEA). For consumption, we use “Personal Consumption Expenditure” after subtracting durable goods in Table 2.3.5 of the BEA. Investment is constructed as the sum of durable goods in Table 2.3.5 and private fixed investment in Table 5.3.5. The real values of consumption and investment are calculated using the price index for Gross Domestic Product in Table 1.1.4. Data on total hours worked are obtained from Cociuba et al. (2012). We modified all of the raw time series into those per capita by dividing the raw data by quarterly population in Cociuba et al.(2012).

C Micro data

For the statistics obtained at the micro level, we use data from the Survey of Income and Program Participation (SIPP). This data set is representative of the non-institutionalized U.S. population.

The survey period is in a monthly basis. The SIPP covers a wide range of information on income, wealth, and participation in various transfer programs. We choose the samples from the first wave to the ninth wave of the SIPP in 2001, covering from 2001 to 2003.

The period of variables used in our analysis is quarterly given a structural feature in the SIPP. The data set is composed of a main module and several topical modules. While the main module contains monthly information on income and transfers, variables for medical expenses and wealth are quarterly reported in the topical modules.[25] To do this, we adjust the time frequency of variables in the main module to a quarterly basis so that information from both modules is consistent in terms of the time frequency.

We construct variables at household level. Data sets in the SIPP contain not only household variables but also individual variables. To generate a household variable from its corresponding individual variable, we take the following steps. First, we identify households with sample unit identifier (SSUID) and household address id in sample unit (SHHADID). Second, we add up the values of a variable for all members in a household. The government transfers that is used to infer the degree of progressivity is based on a broad range of transfer programs including Supplemental Security Income (SSI), Temporary Assistant for Needy Family (TANF), Supplemental Nutrition Assistance Program (SNAP), Supplemental Nutrition Program for Women, Infants, and Children (WIC), childcare subsidy and Medicaid. We do not include Social Security and Medicare since these programs are targeted at old populations only. We construct a variable of *household income* broadly since the payment of transfer programs depends on total income in the U.S. Therefore, it consists of labor income, income from financial investments, and property income. We exclude households in which the age of head is less than or equal to 20. We convert all of their nominal values to the values in 2001 US dollar using the CPI-U.

D Estimation of the persistence of full-time worker wage

We estimate the persistence of wage in the United States using data from the Panel Study of Income Dynamics (PSID). We choose samples for the period of 1969-2010. Our measure of labor produc-

[25] Specifically, the 3rd, 6th, 9th topical modules contain the information.

tivity is defined as a worker's relative hourly wage to other individuals. This labor productivity is measured by a worker's earnings divided by hours worked. To avoid the oversampling of low income household heads, we exclude households from the Survey of Economic Opportunity. We consider household heads whose age is between 23 and 60. We drop the samples whose wage is below a half of the minimum wage. The nominal values are converted into the value of US dollar in 2000 with the CPI-U.

We run the ordinary least square regression of the log of the productivity (hourly wages) on a dummy for male, a cubic polynomial in potential experience (age minus years of education minus five), a time dummy, and a time dummy interacted with a college education dummy. We take its residual, $x_{i,j}$, as an idiosyncratic productivity that contains a wide range of individual abilities in the labor market. This stochastic process is composed of the summation of a persistent, $\eta_{i,j}$ and a transitory process, $\nu_{i,j}$:

$$x_{i,j} = \eta_{i,j} + \nu_{i,j}, \nu_{i,j} \sim N(0, \sigma_\nu) \tag{18}$$
$$\eta^{'}_{i,j} = \rho_\eta \eta_{i,j-1} + \epsilon^{'}_{i,j}, \epsilon^{'}_{i,j} \sim N(0, \sigma_\epsilon)$$

We use a Minimum Distance Estimator to estimate the parameters of the process. The mechanism is to find parameters that minimizing the distance between empirical and theoretical moments. We take the covariance matrix of the residual $x_{i,j}$ as our moments. Let's denote θ as a vector of $(\rho_\eta, \sigma_v, \sigma_\epsilon)$. Let $m_{j,j+n}(\theta)$ be the covariance of the labor productivity between age j and $j+n$ individuals. $\hat{m}_{j,j+n}$ is defined as the empirical counterpart of $m_{j,j+n}(\theta)$. Then,

$$E\left[\hat{m}_{j,j+n} - m_{j,j+n}(\theta)\right] = 0 \tag{19}$$

where

$$\hat{m}_{j,j+n} = \frac{1}{N_{j,j+n}} \sum_{i=1}^{N_{j,j+n}} x_{i,j} \cdot x_{i,j+n}$$

The moments can be represented by as an upper triangle matrix:

$$\bar{m}(\theta) = \begin{bmatrix} m_{0,0}(\theta) & m_{0,1}(\theta) & \cdots & \cdots & m_{0,J-1}(\theta) & m_{0,J}(\theta) \\ 0 & m_{1,1}(\theta) & \cdots & \cdots & m_{1,J-1}(\theta) & m_{1,J}(\theta) \\ 0 & 0 & m_{2,2}(\theta) & \cdots & m_{2,J-1}(\theta) & m_{2,J}(\theta) \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & m_{J-1,J-1}(\theta) & m_{J-1,J}(\theta) \\ 0 & 0 & 0 & \cdots & 0 & m_{J,J}(\theta) \end{bmatrix}$$

We denote a vector of $\bar{M}(\theta)$ by vectorizing $\bar{m}(\theta)$ with length $(J+1)(J+2)/2$. To estimate parameters θ, we solve

$$\min_{\theta} \left[\hat{\bar{M}} - \bar{M}(\theta)\right]' W \left[\hat{\bar{M}} - \bar{M}(\theta)\right]$$

where the weighting matrix W is set to be an identity matrix.[26]

E More on calibration for the representative-agent models

It is straightforward to calibrate the parameters of the representative-agent models using the steady state equilibrium equations. First, β is directly obtained by

$$\beta = (1+r)^{-1}$$

Then, given the target of $T/Y = 0.106$ or 0 and $L = 0.652$, B is obtained by

$$B = \frac{(1-\tau)(1-\alpha)}{\left(1-\delta\frac{K}{Y}-\frac{G}{Y}\right)L}$$

[26] Using the identity matrix has been common in the literature since Altonji and Segal (1996) show that the optimal weighting matrix generate severe small sample biases.

where

$$\frac{K}{Y} = \frac{\alpha}{r+\delta}$$
$$\frac{G}{Y} = \tau(1-\alpha) - \frac{T}{Y}.$$

Finally, since $\frac{Y}{K} = \left(\frac{K}{L}\right)^{\alpha-1}$, we can obtain $\frac{K}{L}$, which in turn gives K and thus Y. Then, T is obtained using $T/Y = .106$.

F Heterogeneous-agent model with divisible labor

The assumption of indivisible labor is important for the aggregate labor supply to depend on the joint distribution of wealth and productivity. We illustrate this point by considering a heterogeneous-agent model with divisible labor. The economic environment is identical to Model (HA-T) except that the household makes a continuous labor supply choice. More precisely, the household's decision problem becomes:

$$W(a, x_i, \mu, z_k) = \max_{\substack{a' > \underline{a}, \\ c \geq 0, \\ h \in [0,1]}} \left\{ \log c - B \frac{h^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z W(a', x_j', \mu', z_l') \right\}$$

$$\text{subject to} \quad c + a' \leq (1-\tau) w(\mu, z_k) x_i h + (1 + r(\mu, z_k)) a + T(x_i)$$
$$\mu' = \Gamma(\mu, z_k).$$

We calibrate the model using the same target statistics used for Model (HA-T). The model fit in terms of the target statistics is as good as Model (HA-T). The following three tables (Tables A1-A3) report the main business cycle statistics. We consider three values of $\gamma : \gamma \in \{0.5, 1, 2\}$.

Table A1 shows the tight link between γ and the implied volatility of macroeconomic aggregates. In particular, a lower curvature (i.e., higher γ) leads to a greater volatility of aggregate hours. However, even with a relatively large value of γ (e.g., 2), the volatility of aggregate hours is

Table A1: Volatility of aggregate variables

		Divisible labor model					
		In the presence of T			In the absence of T		
	U.S. data	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
σ_Y	1.56	1.08	1.20	1.35	1.07	1.17	1.31
σ_C/σ_Y	0.57	0.31	0.30	0.29	0.30	0.30	0.29
σ_I/σ_Y	2.82	2.61	2.64	2.67	2.62	2.64	2.67
σ_L/σ_Y	-	0.25	0.39	0.52	0.24	0.36	0.49
σ_H/σ_Y	0.96	0.24	0.35	0.47	0.22	0.32	0.43
$\sigma_{Y/H}/\sigma_Y$	0.55	0.77	0.65	0.54	0.79	0.69	0.59

Note: Reported values are obtained from a model economy with divisible labor and progressive government transfers. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Volatility is measured by the percentage standard deviation of each variable. The U.S. statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

considerably smaller than the data.

More importantly, Table A2 clearly shows that the model even with the progressive government transfers is not able to deliver mildly procyclical average labor productivity. In fact, the correlation between output and average labor productivity is nearly one. This highlights the importance of indivisible labor in our main result.

G More on numerical methods for the heterogeneous-agent models

G.1 Solving for the equilibrium with aggregate risk

The models with aggregate risk are solved in the following two steps. First, we solve for the individual policy functions given the forecasting rules (*the inner loop*). Then, we update the forecasting rules by simulating the economy using the individual policy functions (*the outer loop*). We iterate the two steps until the forecasting rules converge. That is, the difference between the old forecasting rule used in the inner loop and the new forecasting rule generated in the outer loop is small enough.

Table A2: Cyclicality of aggregate variables

		Divisible labor model					
		In the presence of T			In the absence of T		
	U.S. data	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
$Cor(Y, C)$	0.82	0.88	0.87	0.86	0.89	0.88	0.87
$Cor(Y, I)$	0.91	0.99	0.99	0.99	0.99	0.99	0.99
$Cor(Y, L)$	-	0.99	0.99	0.99	0.98	0.98	0.98
$Cor(Y, H)$	0.85	0.99	0.99	0.99	0.98	0.98	0.98
$Cor(Y, Y/H)$	0.35	1.00	1.00	0.99	1.00	1.00	0.99
$Cor(H, Y/H)$	-0.21	0.98	0.97	0.96	0.97	0.96	0.94

Note: Reported values are obtained from a model economy with divisible labor and progressive government transfers. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Cyclicality is measured by the correlation of each variable with output. The statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

Table A3: Persistence of aggregate variables

		Divisible labor model					
		In the presence of T			In the absence of T		
	U.S. data	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 0.5$	$\gamma = 1$	$\gamma = 2$
$\rho(Y)$	0.85	0.71	0.71	0.71	0.71	0.71	0.71
$\rho(C)$	0.84	0.81	0.82	0.82	0.80	0.81	0.81
$\rho(I)$	0.89	0.70	0.70	0.70	0.70	0.70	0.70
$\rho(L)$	-	0.70	0.70	0.70	0.70	0.70	0.69
$\rho(H)$	0.84	0.70	0.70	0.70	0.70	0.70	0.70
$\rho(Y/H)$	0.52	0.72	0.72	0.73	0.72	0.72	0.73

Note: Reported values are obtained from a model economy with divisible labor and progressive government transfers. Each quarterly variable is logged and detrended using the Hodrick-Prescott filter with a smoothing parameter of 1600. Persistence is measured by the first-order autocorrelation of each variable. The statistics are based on aggregate time-series from 1955Q1 to 2011Q4.

Inner loop In the inner loop, we solve for the valued function $V(a, x_i, K, z_k) = \max\left\{V^E(a, x_i, K, z_k), V^N(a, x_i, K\right.$ These value functions are approximated by the non-evenly spaced grid for a and the evenly spaced grid for K with the number of grid points $n_a = 200$ and $n_K = 5$. Unlike Chang and Kim (2007; 2014) and Takahashi (2014), we discretize stochastic processes x_i and z_k by using Rouwenhorst (1995) method with $n_x = 17$ and $n_z = 5$. We find that the approximation of the continuous processes for our highly persistent shocks is substantially better with the Rouwenhorst method.[27] To obtain $V(a, x_i, K, z_k)$ at each grid point, we solve the following problems

$$V^E(a, x_i, K, z_k) = \max_{\substack{a' > \underline{a}, \\ c > 0}} \left\{ \log c - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \hat{K}', z_l') \right\} \tag{20}$$

$$\text{subject to} \quad c + a' \leq (1 - \tau)\hat{w}(K, z_k)x_i z_k \bar{n} + (1 + \hat{r}(K, z_k))a + T(x_i)$$

and

$$V^N(a, x_i, K, z_k) = \max_{\substack{a' > \underline{a}, \\ c > 0}} \left\{ \log c + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \hat{K}', z_l') \right\} \tag{21}$$

$$\text{subject to} \quad c + a' \leq (1 + \hat{r}(K, z_k))a + T(x_i).$$

To evaluate the functional value of the expected value function on $(a', \hat{K}')$ which are not on the grid points, we use the bivariate cubic spline interpolation. By solving these problems, we also obtain the individual policy functions for savings conditional on working status:

$$\begin{aligned} g^E(a, x_i, K, z_k) = \underset{a' > \underline{a}}{\operatorname{argmax}} \Big\{ & \log\left((1 - \tau)\hat{w}(K, z_k)x_i z_k \bar{n} + (1 + \hat{r}(K, z_k))a + T(x_i) - a'\right) \\ & - B\bar{n} + \beta \sum_{j=1}^{N_x} \pi_{ij}^x \sum_{l=1}^{N_z} \pi_{kl}^z V(a', x_j', \hat{K}', z_l') \Big\} \end{aligned}$$

[27] Specifically, we use the simulated data from Rouwenhorst and Tauchen's methods and estimate the persistence and the standard deviation of error terms in the AR(1) processes for both aggregate productivity shocks and idiosyncratic shocks (available upon request). See also Kopecky and Suen (2010).

and

$$g^N(a,x_i,K,z_k) = \underset{a'>\underline{a}}{\operatorname{argmax}} \Big\{ \log\big((1+\hat{r}(K,z_k))a + T(x_i) - a'\big) + \beta \sum_{j=1}^{N_x} \pi^x_{ij} \sum_{l=1}^{N_z} \pi^z_{kl} V(a',x'_j,\hat{K}',z'_l) \Big\}.$$

Outer loop In the outer loop, we simulate the model economy using the individual policy functions. Here, two things are important and worth noting. First, we need to make sure that $V^E(a,x_i,K,z_k)$ and $V^N(a,x_i,K,z_k)$ satisfy a single-crossing property for a so that there is a threshold asset for each individual productivity level $a^*(x_i,K,z_k)$ (conditional on the aggregate state) below which $V^E(a,x_i,K,z_k) > V^N(a,x_i,K,z_k)$ holds and households choose to work. Second, we need to find the equilibrium factor prices and associated total employment in each period of the simulation (Takahashi, 2014).

The measure of households $\mu(a,x_i)$ is approximated by a finer (non-evenly spaced) grid on a than that in the inner loop with the number of grid points equal to 2000 (Rios-Rull, 1999). K is constructed based on the measure of households following $K = \int_a \sum_{i=1}^{N_x} a\mu(da,x_i)$. In each simulation period, we use a bisection method to obtain the equilibrium factor prices as follows:

1. Set an initial range of (w_L, w_H) and calculate the aggregate labor demand $L^d = (1-\alpha)^{\frac{1}{\alpha}}(z_k/w)^{\frac{1}{\alpha}}K$ for each w implied by the firm's FOC. Note that r is obtained by using the relationship $r = z_k^{\frac{1}{\alpha}}\alpha\left(\frac{w}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} - \delta$, which is implied by (10) and (11).
2. Calculate the aggregate labor supply L^s at each w and make sure that the excess labor demand $L^d - L^s > 0$ at w_L and $L^d - L^s < 0$ at w_H.
3. (Bisection) Let $\tilde{w} = (w_L + w_H)/2$ and obtain $L^d - L^s$ at $\tilde{w}$. If $L^d - L^s > 0$, set $w_L = \tilde{w}$; otherwise, set $w_H = \tilde{w}$.
4. Continue updating (w_L, w_H) until $\|w_L - w_H\|$ is small enough.

Taking the measure of households $\mu(a,x_i)$, the aggregate state (K,z_k), and factor prices w and r as given, we compute the aggregate labor supply $L^s(K,z_k)$ by using the threshold asset

$a^*(x_i, K, z_k)$ for each individual productivity. Specifically, we solve (20) and (21) given the expected value function in the next period using interpolation. Note that we use the valued function obtained in the inner loop and the forecasting rule (16) for $\hat{K}' = \Gamma(K, z_k)$ which is not on the grid points of K. Then, the individual household decision rules are given by

$$n = g_n(a, x_i, K, z_k) = \begin{cases} \bar{n} & \text{if } a < a^*(x_i, K, z_k), \\ 0 & \text{otherwise}, \end{cases}$$

where $a^*(x_i; K, z)$ is the level of asset holding with which $V^E(a^*(x_i, K, z), x_i, K, z_k) = V^N(a^*(x_i, K, z), x_i, K, z_k)$ holds. Having $n = g_n(a, x_i, K, z_k)$ for each grid point (a, x_i) on μ at hand, the aggregate labor supply is obtained by $L^s(K, z_k) = \int_a \sum_{i=1}^{N_x} x_i g_n(a, x_i, K, z_k)\mu(da, x_i)$. After finding the market-clearing prices, we update the measure of households in the next period by using

$$a' = g_a(a, x_i, K, z_k) = \begin{cases} g^E(a, x_i, K, z_k) & \text{if } a < a^*(x_i, K, z_k), \\ g^N(a, x_i, K, z_k) & \text{otherwise}, \end{cases}$$

and the stochastic process for x_i. We simulate 3,500 periods and the first 500 periods of which are discarded when computing statistics. We have experimented with a greater number of periods, and found that the results are robust.

Finally, the coefficients $(a_0, a_1, a_2, b_0, b_1, b_2)$ in the forecasting rules

$$\log K' = a_0 + a_1 \log K + a_2 \log z, \tag{22}$$

$$\log w = b_0 + b_1 \log K + a_2 \log z, \tag{23}$$

are updated by ordinary least squares with the simulated sequence of $\{K', w, K\}$. Our parametric assumption on the forecasting rules are the same as those in Chang and Kim (2007; 2014) and Takahashi (2014; 2017). We repeat the whole procedure of the inner and outer loops until the coefficients in the forecasting rules converge.

As is clear in the forecasting rules (22) and (23), households predict prices and the future distributions of capital only with the mean capital stock. Therefore, it is important to check whether the equilibrium forecast rules are precise or not. We summarize results for the accuracy of the forecasting rules for the future mean capital stock K' in Table $A1$ and for the wage w in Table $A2$. It is clear that all R^2 are very high in both specifications of the model. We also present accuracy statistics suggested by Den Haan (2010). Since our dependent variables are in logs, we multiply the statistics by 100 to interpret them as percentage errors. We note that the mean errors are sufficiently small (considerably less than 0.1 percent) and the maximum errors are also reasonably small ranging from 0.3 to 0.4 percent for both models.

Table A4: Estimates and accuracy of forecasting rules

Model	Dependent variable	Coefficient Const.	$\log K$	$\log z$	R^2	Den-Haan (2010) Max (%)	Mean (%)
Model (HA-T)	$\log K'$	0.1314	0.9445	0.1199	.99999	0.4041	0.0865
	$\log w$	-0.3558	0.5158	0.5778	.99953	0.3141	0.0551
Model (HA-N)	$\log K'$	0.1269	0.9467	0.1090	.99999	0.3213	0.0621
	$\log w$	-0.3825	0.5240	0.6563	.99954	0.3296	0.0517

G.2 Impulse response functions

To compute impulse response functions, we first simulate the economy for a sufficiently long time so that the economy reaches the stochastic steady state (Coeurdacier et al., 2015). Then, we hit the economy with an exogenous disturbance to z, which follows the AR(1) process, and let the economy evolve according to the shock realizations. The economy is simulated long enough so that it goes back to the original stochastic steady state.

As we solve the model using a Markov chain discretizing the aggregate TFP shocks, it is a nontrivial task to obtain impulse response functions according to its original continuous TFP shocks. We construct the impulse responses based on the weighted averages using the linear interpolation for z. Specifically, given a value of z which follows the original AR(1) process, we compute the weight for z by $\omega = (z_{i+1} - z)/(z_{i+1} - z_i)$ where $z \in [z_i, z_{i+1}]$ and z_i and z_{i+1} are the two nearest grid points of the Markov chain. Taking K as given, we calculate the individual decision rules $g_a(a, x_i; K, z_k)$ and $g_n(a, x_i; K, z_k)$ for each $k = i$ and $i + 1$. Note that the market-clearing factor prices are obtained for each k. The individual decision rules and the equilibrium factor prices are obtained as the weighted averages such as

$$g_a(a, x_i; K, z) = \omega g_a(a, x_i; K, z_i) + (1 - \omega) g_a(a, x_i; K, z_{i+1}),$$

$$g_n(a, x_i; K, z) = \omega g_n(a, x_i; K, z_i) + (1 - \omega) g_n(a, x_i; K, z_{i+1}),$$

$$w(K, z) = \omega w(K, z_i) + (1 - \omega) w(K, z_{i+1}),$$

$$r(K, z) = \omega r(K, z_i) + (1 - \omega) r(K, z_{i+1}).$$