# FIRST DEGREE PRICE DISCRIMINATION AND QUALITY CUSTOMISATION UNDER DATA PROTECTION REGULATIONS: A SPATIAL MODEL

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**Abstract**

In response to privacy and ethical concerns, data protection laws such as General Data Protection Regulations (GDPR) are now in place and ought to have an impact on the industries that are closely associated with price and quality customisation. Consumers now have a say on personal data and can legally opt out of the data-oriented personalisation scheme. In this paper, I develop a Hotelling-styled spatial model to explore the interaction between the regulations and the industry in a duopolistic setting. I show when both firms use personalised price and quality (PPQ) for consumers who give consent and non-targeted non-linear pricing (NNP) or uniform pricing for consumers who opt out of the scheme, consumers always give consent and choose the personalised offer in equilibrium.

## Introduction

Tailored price and quality is becoming a new norm amid rapid growth in so-called 'database relationship marketing' thanks to the booming digital market and ever connected internet. General public keeps refreshing the realisation of the profound impact by mass processing and analysing of personal data on the society. Shiller (2014) shows that online service provider such as Netflix could earn more profit when using web-browsing variables comparing to just demographics to price their products. This is one of the examples[2] that show how those data could reveal the types and willingness to pay of the consumers for the benefits of the firms. News like data breach or unethical use of data often trembles the society and inflates the concern over data privacy, which led to the enforcement of data protection laws in many parts of the world. This attracts enquires regarding the economic impacts on the market and social welfare. Hence, this paper aims to provide an economic prospective to the debates over the legislation and contributes to the studies of price and product personalisation.

The discussion of personalising technology and price discrimination has never been restricted to the field of economics and both of them are ones of the most controversial topics. In a survey by Kahneman et. al. (1986), 91% of respondents viewed personalised pricing as unfair. How firms utilise the marketing technology without triggering backlash has been a real challenge. (Tanner, 2014). Now being viewed as a classic case study, Amazon once experimented personalised pricing but had to retract after the move went backfired. Without going further into case studies and surveys, it's obvious and intuitive that people are generally uncomfortable with personalised pricing. Besides those suggested in behavioural economics such as regret aversion (Loomes and Sugden, 1982), consumers feel unfair because they are paying different prices for products that are exactly the same as in the Amazon case. However, the psychology changes when firms conduct versioning: the extra premium on high quality product could be easily justified and accepted. This means personalisation marketing might not necessarily get backfired when product quality is customised as well.

In the wake of 'Cambridge Analytica' series in 2018, European legislators quickly respond to upgrade data regulation regulations in place. The regulations on the usage of personal data is at the most scrutinised level in the history. Under General Data Protection Regulation (GDPR)[3] the binding law across European Union and European Economic Area, consumers' consents are due for firms to process personal data and consumers can 'opt out' of the agreement anytime by discretion. The scholars in legal studies commonly agree that GDPR applies to personalised pricing in general and cookies with unique identifiers should be regarded as personal data (Borgesius and Poort, 2017; Steppe, 2017). Therefore, consumers are legally granted more power for them to play a more important role in the game of personalisation. Some studies incorporate the strategic behaviours of consumers for example by delaying consumption in the early period to gain an advantage in the future (Chen and

[2] For a detailed survey on price personalisation in real world practice, refer to Borgesius and Poort, 2017

[3] eugdpr.org

Zhang, 2009). The ability to opt out of personalisation programme should have more robust implications, which is the prime motivation of this research.

**Related Literature**

Stigler (1978) defined price discrimination (PD) to be '*the sale of two or more similar goods at prices that are in different ratios to marginal cost'*. Theoretical literature of industrial organisation in the past decades saw a variety of interpretations and approaches taken when modelling price discrimination. The rise of PD is commonly considered as a product of imperfect competition and asymmetric information and is conducted for the purpose of surplus extraction. However, the first-degree PD was lesser discussed for its nature of impracticality and infeasibility until this millennium. Particularly because the rapid development of e-commerce sector gives the prevalence of `personalisation' and allows personal data collecting with ease (e.g., online cookies usually record the web browsing data and feed back to the retailers and advertisers). Together with the purchasing mode of e-commerce that transaction happen in a relatively isolated manner instead of posted pricing seen in a traditional market, one of the most important conditions for the existence of PD is sufficed: consumers can be segmented either or indirectly. In the case of first-degree PD, consumers must be segmented to individual level, which is technically possible in the current state of technology.

The majority of literature focus on oligopoly as it's thought that firms must have some market power to adopt the generally known costly personalised technology and have access to sufficiently rich data, of which the cost is normally standardised to zero by symmetry. Hotelling's linear city model and Salop's circle model often serve as the base of the analytical literature should competition need to be introduced. The framework also provides opportunity to incorporate consumers' heterogeneity in types which could be interpreted as brand/location preference or willingness to pay, subject to the research question.

Horizontal product differentiation stands for the difference in brand preference of the consumer represented through dimensional location advantage while in vertical product differentiation, firms rank the consumer types identically hence consumer's taste of quality independent of brand preferences. Single-dimensional models incur inadequacy to capture both brand preferences and the marginal value of consumption. The multi-dimensional model is approached either through simulation (Borenstein and Rose, 1994) or discrete-choice approach (Mussa and Rosen, 1978; Rochet and Stole, 2002). Ghose et. al. (2009) manage to capture both quality and brand preference by making a stylised assumption when specifying their model: the location of the consumer simultaneously denote both preferences. In other words, it's assumed that larger the brand preference of a given consumer for one firm, the larger is the quality preference of that consumer for that firm's product, which is well backed by empirical studies and intuitions.

In the setup of Ghose et. al., 2009, it's convenient to investigate both personalised pricing and quality customisation, contrasting the majority of the other models. This is an important feature as it's closely associated with one of the most common practice in real world marketing known as versioning. Recall the definition of price discrimination by Stigler: the exact word he used 'two similar goods' instead of two identical goods. Firms usually have a line-up of products that fall into the same cluster on the spectrum of the product differentiation in the whole market but still differentiate in feature and performance for different target groups. For instance, Apple annually release two or three new generation Iphones that are overall similar but differentiate in subtle features. The premier version is typically sold in a much higher margin. It has also seen a prevailing trend that consumers are in favour of tailored, personalised product and service, which provides room for firm to achieve product customisation at its extreme form with data analytic technology. This means personal data processing is not only concerning the price but also the quality and neither should be omitted from analysis. Therefore, the modelling of this work is based on the framework of Ghose et. al. which is also benchmarked. Hence details of the model are deferred to later sections where I introduce the set-up of the model.

## The model

As a standard Hotelling-style setup, two firms, call them firm L and firm R, are located on the two ends of the 'linear city' where consumers are uniformly distributed and ranked by type. Quasi-linear utility function is assumed: the gross utility to a consumer with type θ buying from the firm located at 0, firm L is

$$u^L(q,\theta) = q \times (1-\theta)$$

his gross utility derived from buying from the firm located at 1, firm R, is

$$u^R(q,\theta) = q \times \theta$$

the firm sets a price according to consumer types (targeted or not) $p^L(\theta)$, $p^R(\theta)$ the indirect utility/consumer surplus function of the consumers are

$$s^L(q,\theta) = q \times (1-\theta) - p^L(\theta)$$

$$s^R(q,\theta) = q \times (\theta) - p^R(\theta)$$

Identical to G09, assume the marginal cost of production is invariant with quantity but depends on the quality of the product and depending on the quality schedule, both firms have the identical cost function

$$c(q) = \frac{q^\alpha}{\alpha}, \alpha > 1$$

The sequence of the game is as following: In the first period, the firms simultaneously choose the pricing strategy they will use for two pools of consumers: those who opt in and opt out[4] of the personalisation programme. Firms are neither able to process the data of out-consumers nor use against them by perfect targeting. In other words, once consumers opt out, their location becomes private information and targeting is disabled, which means they wouldn't get the product with price and quality the firms personalise just for them. Therefore, personalised price and quality (PPQ) is ruled out for the out-consumers by law. In this model, we consider three pricing strategies: personalised price and quality (PPQ); non-targeting non-linear pricing (NNP); uniform pricing. PPQ is the firm's offering of a pair of price and quality designed specifically for some consumer type, the consumer either accepts the offer or does not purchase. NNP is a classic form of second-degree PD that relies on the consumer's self-selection: firms set a price and quality schedules for each types of the consumers and consumers self-select them into buying the quality-price pair that maximises their utility from the menu. NNP is subject to consumer incentive compatibility and individual rationality constraints such that he purchases the product with price and quality on his type. Uniform pricing is when firm doesn't discriminate at all and offer same quality and price for every consumer. Therefore, there are those cases of combination of firms' choice to consider:

- Both firms use PPQ for in-consumers and NNP for out-consumers

[4] henceforth in-consumers and out-consumers

- Both firms use PPQ for in-consumers and uniform pricing for out-consumers
- Both firm use PPQ for in-consumers; one firm uses NNP for out-consumers, one firm uses uniform pricing for out-consumers
- One firm uses PPQ for in-consumers, one firm uses NNP for in-consumers; both firms use NNP for out-consumers
- One firm uses PPQ for in-consumers, one firm uses uniform pricing for in-consumers; both firms use uniform pricing for out-consumers

Note the latter two cases consider the scenarios where only one firm has the access to personalised technology or forego the technology. Because the firms are symmetric hence the analysis can depart from one firm and the conclusion still holds for the other firm.

As the data protection regulations allow consumers to pull out any time and firms are obliged to comply, we can assume that consumers make the decision after observing the pricing strategy by the firms. This also assumes that consumers have the price and quality information in both segments hence prefers the segment in which they get higher surplus.

The game precedes as follows. At the beginning of a round, the firms face two segments of consumers where different pricing strategies decided in the initial stage are used. Based on the ex-post information about the size of two segments, firm will respond with updated price and quality schedule to maximise profits. Subsequently, consumers observe the updated price-quality schedules and make strategic switch of the segments, whichever offers higher surplus. Every time after consumers decide to opt in/out, we call it end of a round. Those rounds will keep happening until an equilibrium is reached. Note it's assumed that firms do not switch pricing strategies as it complicates analysis. From G09, we learn that the consumers at the middle always gain the highest surplus from PPQ while the most loyal consumers (those locate close to 1 or 0) get the lowest with surplus fully extracted at the extreme case. Hence, the in-consumer segment will be continuous in the middle thanks to the quasi-linearity of the utility function. To help analysis, we can preliminary categorise that the firm faces an out-consumer segment loyal to it, the in-consumer segment in the middle and then an out-consumer segment loyal to the other firm.

Due to the limit of space, the analysis and associated proof of first two subgames have been placed in the appendix. But preliminarily, we may conclude that when both firms use PPQ for in-consumers and NNP for out-consumers (subgame 1); and when both firms use PPQ for in-consumers, uniform pricing for out-consumers (subgame 2), all consumers opt in and receive PPQ offer in equilibrium. Social welfare is at the highest and total consumer surplus is at lowest.

**Preliminary conclusions**

So far I've examined the first two subgames out of five set out: both firms use PPQ for in-consumers and NNP for out-consumers; both firms use PPQ for in-consumers, uniform pricing for out-consumers. We reach the similar equilibrium for both cases: all consumers opt in at equilibrium. The result is somewhat surprising as one should naturally expect consumers surplus increasing when consumers are given the discretionary choice about pricing. However, competition under PPQ has given the rise of unevenly distributed consumer surplus: those in the middle receive the highest surplus while consumer at the ends get fully extracted. This leads to some consumers better off staying in while some consumer better off staying out. And when consumers make corresponding strategic switch, the firms acquire the information about the size of the two segments from the consumer type at the margin. It is this screening process that reveals the types of out consumers as firms discover how loyal the segments are and update their price-quality accordingly. Note that it is convenient for firms to exercises monopoly power because the in-segment blocks the competition from the rival firm. Hence in the end, it's as if consumers do not have a choice when they are 'forced in'. Nevertheless, the social welfare is maximised in equilibrium as socially optimal quality is offered under PPQ. The total consumer surplus is lowest under all pricing. More on welfare distribution regarding those equilibria will be discussed and compared in the remainder of this dissertation in addition to finishing the analysis on first stage.

The next step is to finish the analysis of the remaining subgame such that analysis on first stage can be conducted. This is to better understand the firms' motive in choosing their pricing strategies under the context of data protection regulations.

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# Appendix

## 1 Subgame I: both firms use PPQ for in-consumer, NNP pricing for out-consumer

Suppose for consumer type $\theta \in [\hat{\theta}, 1-\hat{\theta}]$ choose to opt in and otherwise opt out. Denote $A = \hat{\theta}; \quad C = 1 - \hat{\theta}$ such that $A + C = 1$ with A and C being the marginal consumers on the edge of opting out. Denote the marginal consumer who feels indifferent between the personalised offer by both firms by $\theta = B$. In this subgame, each firm offers a menu i.e. quality-price schedule for out consumers $\theta \in [0, A); \quad \theta \in (C, 1]$ and a pair of personalised price and quality for in-consumers $\theta \in [A, C]$. The decision variables of the firms are $q(\theta)$ and $p(\theta)$ for all segment. Focusing the analysis on firm R, the objective function is given by

$$\max_{p^R(\theta), q^R(\theta)} \pi^R, \quad \text{where} \quad \pi^R = \int_B^1 \left[ p^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} \right] d\theta \tag{1}$$

Starting from the out-consumer segment, standard in self-selection literature, we need an incentive compatibility (IC) constraint to ensure that consumer purchases the product designed for his type instead of other types such that his utility is maximised from the purchase. Additionally, the consumer should derive non-negative indirect utility for him to accept any price-quality bundle at all, which forms an individual rationality (IR) constraint. Therefore the optimisation problem is subject to the following:

- **IC:** $\theta = \arg\max_t \quad \theta \times q^R(t) - p^R(t), \forall \theta \in [C, 1]$ where t denotes the type of the chosen product.

- **IR:** $s^R(\theta) \geq 0, \forall \theta \in [C, 1]$

Note the reason why the domain of the constraints are restricted to $[c, 1]$ is not the constraints only apply for the consumers of the segment: all consumers are still free to choose between two firms in this model. Because of the specification of the utility function, it is easy to show that in equilibrium compared with marginal consumer $C$, the consumers to the left of $C$, those in the out-consumer segment loyal to firm L strictly prefer buying from firm L, vice versa. Therefore, firm R serves $[C, 1]$ and firm L serves $[0, A]$. This is inherited from Mussa and Rosen (1987[1]) and G09.

Transform the decision variable the price schedule $p^R(\theta)$ to a function of $s^R(\theta)$. We have $p^R(\theta) = \theta q^R(\theta) - s^R(\theta)$ and rewrite the objective function of firm R as:

$$\max_{s^R(\theta), q^R(\theta)} \pi^R, \quad \text{where} \quad \pi^R = \int_C^1 \left[\theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] d\theta \tag{2}$$

The objective function for firm L by symmetry can be written as following:

$$\max_{s^L(\theta), q^L(\theta)} \pi^L, \quad \text{where} \quad \pi^L = \int_0^A \left[(1-\theta) q^L(\theta) - s^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] d\theta \tag{3}$$

As per IC constraint, consumer gains the indirect utility:

$$S^R(\theta) = \max_t \quad \theta \times q^R(t) - p^R(t) \tag{4}$$

The first order condition is:

$$\theta \times \frac{\partial q^R(t)}{\partial t} - \frac{\partial p^R(t)}{\partial t} = 0 \tag{5}$$

Using the envelop theorem, the equation holds at $t = \theta$ because the consumer self-select the the price and quality pair designed for his type. By differentiating 4:

$$\frac{ds^R(\theta)}{d\theta} = q^R(\theta) + \theta \frac{\partial q^R(\theta)}{\partial \theta} - \frac{\partial p^R(\theta)}{\partial \theta} \tag{6}$$

Hence we have:

**Lemma 1.**

$$\frac{ds^R(\theta)}{d\theta} = q^R(\theta) \tag{7}$$

$$\frac{ds^L(\theta)}{d\theta} = q^L(\theta) \tag{8}$$

This implies that the quality schedule $q^R(\theta)$ equals to the slope of the consumer surplus $s^R(\theta)$ and we obtain that

$$s^R(\theta) = s^R(C) + \int_C^\theta q^R(t) dt \tag{9}$$

$$s^L(\theta) = s^L(A) + \int_\theta^A q^L(t) dt \tag{10}$$

In G09, firms compete on the marginal consumer who is indifferent from buying from two firms. Hence there is an additional IR constraint to ensure that $s^R(B) = s^L(B)$ and the firms thus compete by lowering the pricing schedule by a constant, $s^R(B)$. As a result, loyal consumers receive higher surplus termed as information rent. However, in the setting of this model, the existence of the in-consumer segment in the middle ground forms a barrier to facilitate two local monopoly: it's straightforward that for the marginal consumers of two segments $A \neq C$, they strictly prefer the offer by the closer firm as it is not profitable for the further firm to poach the marginal consumer of the loyal out-consumer segment of the rivalry due to location disadvantage. Therefore, similar to a monopoly, firms drive the surplus of the marginal consumer to zero such that $s^L(A) = s^R(C) = 0$. The optimisation problem can be rewritten as:

$$\max_{\{s^R(\theta), q^R(\theta)\}} \pi^R, \quad \text{where} \quad \pi^R = \int_C^1 \left[\theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] d\theta \tag{11}$$

$$s.t. s^R(\theta) \geq 0, s^R(C) = s^L(A) = 0 \tag{12}$$

substituting for $s^R(\theta)$, we have

$$\max_{s^R(\theta)} \pi^R, \quad \text{where} \quad \pi^R = \int_C^1 \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - \int_C^\theta q^R(t)dt\right] d\theta \tag{13}$$

Changing the integration order of last term in the bracket:

$$\int_C^1 [\int_C^\theta q^R(t)dt]d\theta = \int_c^1 [\int_C^1 q^R(t)d\theta]dt = \int_C^1 q^R(t)(1-t)dt = \int_C^1 q^R(\theta)(1-\theta)d\theta \tag{14}$$

then the objective function becomes

$$\begin{aligned} \max_{q^R} \pi^R \text{where} \pi^R &= \int_C^1 \left[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha} - q^R(\theta)(1-\theta)\right] d\theta \\ &= \int_C^1 \left[(2\theta - 1)q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\right] d\theta \end{aligned} \tag{15}$$

Similarly ,the optimisation problem for firm L follows:

$$\max_{q^L(\theta)} \pi^L \text{where} \quad \pi^L = \int_0^A \left[(1-2\theta)q^L(\theta) - \frac{(q^L)^\alpha(\theta)}{\alpha}\right] d\theta \tag{16}$$

To obtain the quality schedule, differentiate the terms in (5) with respect to $q^R(\theta)$:

$\theta - (q^R)^{\alpha-1}(\theta) - (1-\theta) = 0$

$\Rightarrow$

**Lemma 2.** *The quality schedules are* $q^R(\theta) = (2\theta - 1)^{\frac{1}{\alpha-1}}$ *and* $q^L(\theta) = (1 - 2\theta)^{\frac{1}{\alpha-1}}$.

Now we can obtain the consumer surplus function by Lemma 1 and Lemma 2:

$$s^L(\theta) = 0 + \int_{\theta}^{A} q^L(t)dt \tag{17}$$

$$s^L(\theta) = 0 + \int_{C}^{\theta} q^R(t)dt \tag{18}$$

We can obtain the price schedules by $p^L(\theta) = (1 - \theta)q^L(\theta) - s^L(\theta)$ and $p^R(\theta) = \theta q^R(\theta) - s^R(\theta)$. Therefore we have the optimal price schedule:

$$p^L(\theta) = (1 - \theta)(1 - 2\theta)^{\frac{1}{\alpha-1}} - \frac{\alpha - 1}{2\alpha}(1 - 2\theta)^{\frac{\alpha}{\alpha-1}} + \frac{\alpha - 1}{2\alpha}(1 - 2A)^{\frac{\alpha}{\alpha-1}} \tag{19}$$

$$p^R(\theta) = \theta(2\theta - 1)^{\frac{1}{\alpha-1}} - \frac{\alpha}{\alpha - 1}(2\theta - 1)^{\frac{\alpha}{\alpha-1}} + \frac{\alpha - 1}{2\alpha}(2B - 1)^{\frac{\alpha}{\alpha-1}} \tag{20}$$

The first two terms are identical to G09 in the case of both firms conducting NNP. The NNP price is strictly higher for the in the presence of the in-consumer segment since the last term is strictly positive. The increase in price should fully extract the consumer surplus of the marginal consumer at the edge of out-segment.

Now consider the in-consumer segment, the analysis will be less different from the G09 since the segment is just a truncated market as in their original model. Unlike the out-consumer segments, the in-consumer segment will be a continuum of consumers at the middle and firms face direct competition from each other: there may be incentive to poach consumers for market share. Recall the objective functions of two firms:

$$\max_{\{s^R(\theta), q^R(\theta)\}} \pi^R, \quad \text{where} \quad \pi^R = \int_{C}^{1} \left[\theta q^R(\theta) - s^R(\theta) - \frac{(q^R)^{\alpha}(\theta)}{\alpha}\right] d\theta \tag{21}$$

$$\max_{\{s^L(\theta), q^L(\theta)\}} \pi^L, \quad \text{where} \quad \pi^L = \int_{0}^{A} \left[(1 - \theta)q^L(\theta) - s^L(\theta) - \frac{(q^L)^{\alpha}(\theta)}{\alpha}\right] d\theta \tag{22}$$

Due to perfect targeting by PPQ, there is no self-selection concern here hence we can make IC constraint redundant. Therefore $s^L(\theta)$ and $s^R(\theta)$ are equal to the socially optimal surplus. This contrasts the quality degradation due to the fear of product cannibalisation seen in NNP case. First order condition gives the quality schedule:

$$\begin{aligned} \frac{\partial \pi^L(\theta)}{\partial q^L(\theta)} &= (1 - \theta) - (q^R)^{\alpha-1}(\theta) = 0 \\ \Leftrightarrow q^L(\theta) &= (1 - \theta)^{\frac{1}{\alpha-1}} \end{aligned} \tag{23}$$

$$\frac{\partial \pi^R(\theta)}{\partial q^R(\theta)} = \theta - (q^R)^{\alpha-1}(\theta) = 0$$
$$\Leftrightarrow q^R(\theta) = \theta^{\frac{1}{\alpha-1}} \tag{24}$$

As firms have the perfect information about the in-consumers, they will compete in a Bertrand manner at the individual level, which means the closer firm will offered exactly the highest possible surplus offer by the other firm to keep the consumer. Each consumers will get offers from both firms and should feel indifferent from accepting each one of them. The closer firm appropriates the remaining surplus. Thus the consumer surplus $s^L(\theta)$ and $s^R(\theta)$ should equal to the socially optimal surplus by the rival firm. The rival firm offers socially optimal quality that maximise its profit and marginal cost (due to Bertrand price competition) to the consumers located closer to the other firm. We obtain

$$s^L(\theta) = \max_{q^R(\theta)} \Big[\theta q^R(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\Big] = \Big(1 - \frac{1}{\alpha}\Big)\theta^{\frac{\alpha}{\alpha-1}}, \theta \in [A, 1/2] \tag{25}$$

$$s^R(\theta) = \max_{q^L(\theta)} \Big[(1-\theta) q^L(\theta) - \frac{(q^R)^\alpha(\theta)}{\alpha}\Big] = \Big(1 - \frac{1}{\alpha}\Big)(1-\theta)^{\frac{\alpha}{\alpha-1}}, \theta \in [1/2, C] \tag{26}$$

In equilibrium, all consumers in $[A, 1/2]$ buy from firm L and all consumers in $[1/2, C]$ buy from firm R. Substituting in optimal quality schedule, we have the consumer surplus

$$S^L(\theta) = (1 - \frac{1}{\alpha})\theta^{\frac{\alpha}{\alpha-1}} \tag{27}$$

$$S^R(\theta) = (1 - \frac{1}{\alpha})(1-\theta)^{\frac{\alpha}{\alpha-1}} \tag{28}$$

By $p(\theta) = u(q(\theta), \theta) - s(\theta)$ we have the price schedule for in-consumer segment:

$$p^L(\theta) = (1-\theta)^{\frac{\alpha}{\alpha-1}} - (1 - \frac{1}{\alpha})\theta^{\frac{\alpha}{\alpha-1}}, \theta \in [A, 1/2] \tag{29}$$

$$p^R(\theta) = \theta^{\frac{\alpha}{\alpha-1}} - (1 - \frac{1}{\alpha})(1-\theta)^{\frac{\alpha}{\alpha-1}}, \theta \in [1/2, C] \tag{30}$$

Now we have the quality and price schedule for both segments as we arrive at

**Lemma 3.** *Given the size of the segments, when both firms use PPQ for in-consumers and NNP for out-consumers, the best response prices, quality schedules and surplus function are the following:*

$$
q^L(\theta) = \begin{cases} (1-2\theta)^{1/(\alpha-1)} \text{if} & \theta \in [0, A] \\ (1-\theta)^{1/(\alpha-1)} \text{if} & \theta \in [A, C] \\ 0 \quad \text{if} & \theta \in [C, 0] \end{cases}
$$

$$
q^R(\theta) = \begin{cases} (2\theta-1)^{1/(\alpha-1)} \text{if} & \theta \in [C, 1] \\ \theta^{1/(\alpha-1)} \text{if} & \theta \in [A, C] \\ 0 \quad \text{if} & \theta \in [0, A] \end{cases}
$$

$$
s^L(\theta) = \begin{cases} \frac{\alpha-1}{2\alpha}(1-2\theta)^{\alpha/(\alpha-1)} - \frac{\alpha-1}{2\alpha}(1-2A)^{\alpha/(\alpha-1)} \text{if} & \theta \in [0, A] \\ (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)} \quad \text{if} & \theta \in [A, 1/2] \end{cases}
$$

$$
s^R(\theta) = \begin{cases} \frac{\alpha-1}{2\alpha}(2\theta-1)^{\alpha/(\alpha-1)} - \frac{\alpha-1}{2\alpha}(2C-1)^{\alpha/(\alpha-1)} \text{if} & \theta \in [C, 1] \\ (1-\frac{1}{\alpha})(1-\theta)^{\alpha/(\alpha-1)} \quad \text{if} & \theta \in [1/2, C] \end{cases}
$$

$$
p^L(\theta) = \begin{cases} (1-\theta)(1-2\theta)^{\frac{1}{\alpha-1}} - \frac{\alpha-1}{2\alpha}(1-2\theta)^{\frac{\alpha}{\alpha-1}} + \frac{\alpha-1}{2\alpha}(1-2A)^{\frac{\alpha}{\alpha-1}} \text{if} & \theta \in [0, A] \\ (1-\theta)^{\alpha/(\alpha-1)} - (1-\frac{1}{\alpha})\theta^{\alpha/(\alpha-1)} \quad \text{if} & \theta \in [A, 1/2] \\ 0 \quad \text{if} & \theta \in [1/2, 1] \end{cases}
$$

$$
p^R(\theta) = \begin{cases} \theta(2\theta-1)^{\frac{1}{\alpha-1}} - \frac{\alpha}{\alpha-1}(2\theta-1)^{\frac{\alpha}{\alpha-1}} + \frac{\alpha-1}{2\alpha}(2B-1)^{\frac{\alpha}{\alpha-1}} \text{if} & \theta \in [C, 1] \\ p^R(\theta) = \theta^{\frac{\alpha}{\alpha-1}} - (1-\frac{1}{\alpha})(1-\theta)^{\frac{\alpha}{\alpha-1}} \quad \text{if} & \theta \in [1/2, C] \\ 0 \quad \text{if} & \theta \in [0, 1/2] \end{cases}
$$

The original model of G09 considers the scenario for all consumers where both firms engage in PPQ and both firms engage in NNP, which can be considered as a special case with all consumers opt in and all consumers opt out: $A = C = 1/2$. We can also consider G09 as a state of world where data protection regulation have not been introduced yet as this can serve as the first round of the game. Hence, after firms decide on the pricing strategies, in round 0, they set price-quality for in-consumers as if all consumers are in and price-quality schedules for out-consumers as if all consumers are out. We know that the consumer in the middle enjoys the highest surplus while the most loyal consumer getting surplus fully extracted when both firms use PPQ. The reverse is true when both firms use NNP. This is because in PPQ the least loyal consumers (the ones at the middle) benefit from the fierce competition between firms drive price down to prevent theirs customers from being poached. Under NNP, the most loyal consumers enjoy 'information rent' (Mussa and Rosen, 1978[1]) due to firm's fear of product cannibalisation. To find the marginal consumer $\hat{\theta}$ that is indifferent between opting in/out, equate the consumer surplus (purchasing from firm L) for both pricing and solve for $\hat{\theta}$:

$$\begin{aligned} \frac{\alpha-1}{2\alpha}(1-2\hat{\theta})^{\alpha/(\alpha-1)} =& (1-\frac{1}{\alpha})\hat{\theta}^{\alpha/(\alpha-1)} \\ \Rightarrow \hat{\theta} =& \frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}} \end{aligned} \tag{1}$$

By symmetry,

$$1-\hat{\theta} = 1-\frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}} \tag{2}$$

Therefore in round 0, the consumers with type $\theta \in \left[0, \frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}}\right]$ and $\left[1-\frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}}, 1\right]$ the loyal types, want to opt out to receive higher surplus while the less loyal types prefer to opt in. Therefore this forms a first round segmentation: consumer type $\theta \in \left[\frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}}, 1-\frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}}\right]$ choose to opt in and otherwise opt out. Subsequently, firms update their price-quality menu for the out-consumer segment. Since the competitive nature hasn't changed, firms continue to offer the PPQ price and quality same as before when there is no out-segment. From the derivation before, we know that the quality schedule follows the IC constraint and is not determined by the size of the segment. Hence price schedule effects all the rent extraction at this stage. Recall the price schedule for out-consumers:

$$p^L(\theta) = (1-\theta)(1-2\theta)^{\frac{1}{\alpha-1}} - \frac{\alpha-1}{2\alpha}(1-2\theta)^{\frac{\alpha}{\alpha-1}} + \frac{\alpha-1}{2\alpha}(1-2A)^{\frac{\alpha}{\alpha-1}} \tag{3}$$

$$p^R(\theta) = \theta(2\theta-1)^{\frac{1}{\alpha-1}} - \frac{\alpha}{\alpha-1}(2\theta-1)^{\frac{\alpha}{\alpha-1}} + \frac{\alpha-1}{2\alpha}(2B-1)^{\frac{\alpha}{\alpha-1}} \tag{4}$$

The size of increase in price, $\frac{\alpha-1}{2\alpha}(1-2A)^{\frac{\alpha}{\alpha-1}}$ $\frac{\alpha-1}{2\alpha}(2B-1)^{\frac{\alpha}{\alpha-1}}$ depends on the size of the out-segment indicated by $A, C$ and should appropriate all the consumer surplus for those at the margin. Plugging in $A = \hat{\theta} = \frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}}$ and $C = 1 - \hat{\theta} = 1 - \frac{2^{(1-\alpha)/\alpha}}{1+2^{1/\alpha}}$ we can obtain the updated price schedule for out-segment. In the next round, it's straightforward to show that some consumers who have opted out in the last round is worse off due to the new price schedule and would like to opt in. The in-segment is hence larger and firms update the price schedule accordingly. As shown in Figure 1[1], this iteration process keeps going until only the most loyal consumer, $\theta = 0$ or $\theta = 1$ is left in the out-segment. At that time, they should be indifferent to opting out and opting out as their surplus are fully extracted either way. Therefore we arrive at:

**Proposition 1.** *When both firms use PPQ for in-consumers and NNP for out-consumers, all consumers choose to opt in and receive PPQ offer in equilibrium.*

Intuitively, because firms hold the location information of the in-consumers and hence the size of the segment, firms do not directly need the consumer to opt in for them to know the type of the consumers at the margin of out-segment. The information is crucial as firms can accordingly extract maximum surplus by updating price schedule and there is no fear of rival firm poaching customers.

In this subgame, even though consumers are given the discretionary choice of staying in and out, the typical consumer is weakly worse off when opting out, that is, the utility-maximising product from the quality-price menu for the out-segment delivers no greater surplus comparing to the offer that is customised for the consumer. From the perspective social welfare, it's straightforward to show that the social welfare is highest in equilibrium and it's socially optimal. Firms offer sub-optimal quality schedule, a.k.a. product degradation under NNP to prevent product cannibalisation under NNP but offer socially optimal quality with PPQ. More on welfare discussion shall be discussed in more detail in the later section.

[1] the simulation takes value $\alpha = 2$

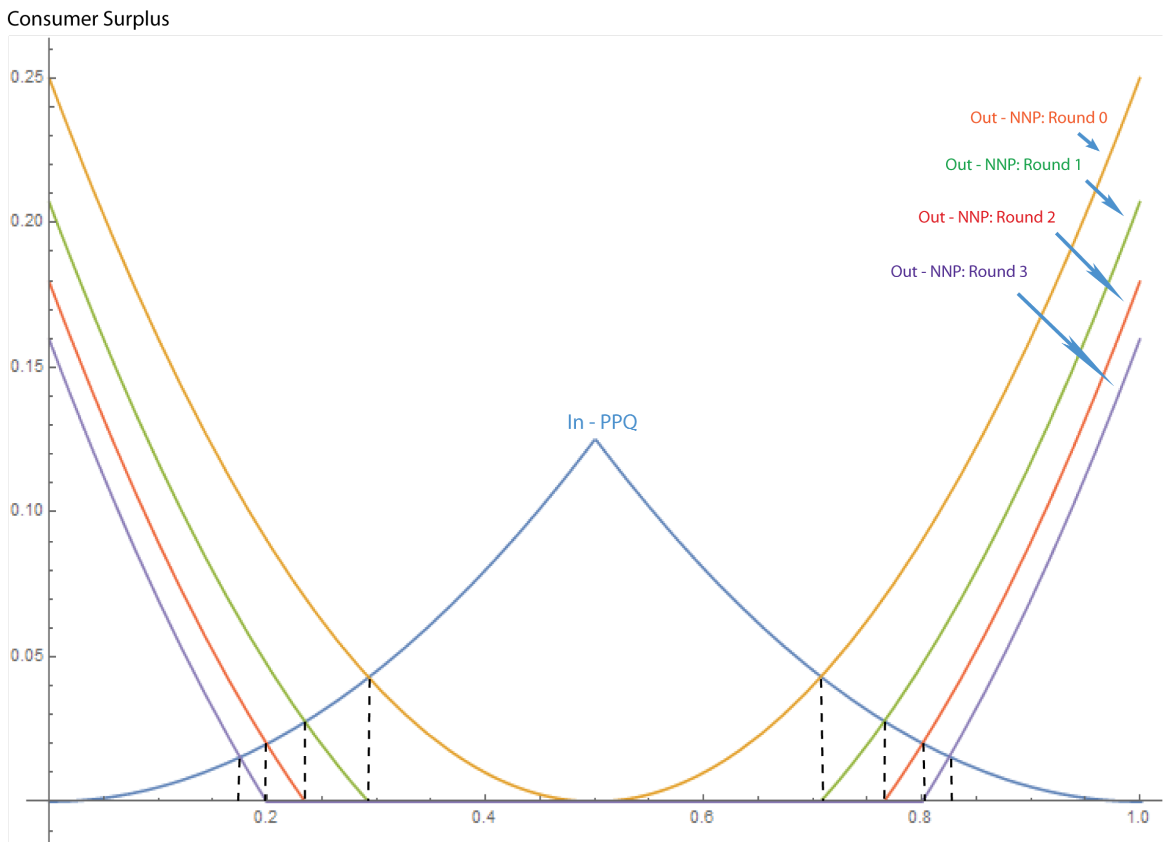


Figure 1: Consumer surplus in case I subgame

# 2 Subgame II: both firms use PPQ for in-consumers, uniform pricing for out-consumers

In this subsection, I analyse the subgame when both firms use PPQ for in-consumers and uniform pricing for out-consumers. The analysis for in-consumer segment will remain the same as in previous section for same pricing being used. For out-consume segment, firms will offer a flat quality and price for every consumers with no regards to their types in this case. Firms choose a price and quality that maximises profits. Consumers choose to purchase from the firm when they receive surplus equal or higher than 0. This implies that the consumer who is indifferent between buying and not buying, i.e. $s(\theta) = 0$ determines the optimal market share for firms: consumers will not purchase from the firm unless he is closer (or equally close) to the firm than the indifferent consumer. Hence instead of considering firms set price and quality to maximise profit, we can consider firms choose the optimal market share to serve with quality that maximise profits. Without loss of generality, we again depart from the perspective of firm R and the analysis for firm L should follow by symmetry. We also start with the stage when no one has opted in. As the indifferent consumer (denoted by $\theta_R^*$) derives zero surplus, from $s(\theta) = \theta q - p$, we have $p = \theta^* q$. We have the optimisation problem for firm R:

$$\max_{\theta_R^*, q_R} \pi^R \quad \text{where} \quad \pi^R = (1 - \theta_R^*)(\theta_R^* q_R - \frac{q_R^\alpha}{\alpha}) \tag{5}$$

the optimisation problem for firm L:

$$\max_{\theta_L^*, q_L} \pi^L \quad \text{where} \quad \pi^L = \theta_L^*((1 - \theta_L^*) q_L - \frac{q_L^\alpha}{\alpha}) \tag{6}$$

The first order conditions for firm R are:

$$\frac{\partial \pi^R}{\partial \theta_R^*} = q_R - 2\theta_R^* q_R + \frac{q_R^\alpha}{\alpha} = 0 \tag{7}$$

and that for firm L are

$$\frac{\partial \pi^L}{\partial q_R} = \theta_R^* - q^{\alpha-1} - \theta^{*2} + \theta_R^* q^{\alpha-1} = 0 \tag{8}$$

Solving for $\theta_R^*$ and $q_R$, we have

$$\theta_R^* = \frac{\alpha}{2\alpha - 1} \tag{9}$$

$$q_R = \Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{1}{1-\alpha}} \tag{10}$$

Then we obtain

**Lemma 4.** *When all consumers opt out, the best response uniform price, quality, consumer surplus set by the firms are the following:*

$$p_L = p_R = \Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{2-\alpha}{1-\alpha}} \tag{11}$$

$$q_L = q_R = \Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{1}{1-\alpha}} \tag{12}$$

$$s^R(\theta) = \begin{cases} \theta\Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{1}{1-\alpha}} - \Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{2-\alpha}{1-\alpha}} & \text{if} \quad \theta \in \Big[\frac{\alpha}{2\alpha - 1}, 1\Big] \\ 0 & \text{Otherwise} \end{cases}$$

$$s^L(\theta) = \begin{cases} (1-\theta)\Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{1}{1-\alpha}} - \Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{2-\alpha}{1-\alpha}} & \text{if} \quad \theta \in \Big[0, \frac{\alpha - 1}{2\alpha - 1}\Big] \\ 0 & \text{Otherwise} \end{cases}$$

We can see that as $\alpha \to \infty$, $\frac{\alpha}{2\alpha-1} \to \frac{1}{2}$ and $\frac{\alpha}{2\alpha-1} < \frac{1}{2}$ for $\alpha > 1$ Therefore, it's never optimal for the firm to cover more than half of the market and there will always be consumers not purchasing i.e. the market is not covered. Equilibrium test is easily passed as there isn't a profitable deviation for firms to seek larger market share from rival's territory.

To find the marginal consumer who is indifferent between opting in/out, i.e. receives same surplus from uniform price and quality and PPQ, equate the consumer surplus (purchasing from firm R) function under two pricing:

$$(1 - \frac{1}{\alpha})(1-\theta)^{\frac{\alpha}{\alpha-1}} = \theta\Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{1}{1-\alpha}} - \Big(\frac{\alpha}{2\alpha - 1}\Big)^{\frac{2-\alpha}{1-\alpha}} \tag{1}$$

The consumer type of $\theta$ that satisfies the above equation is the marginal consumer of the in-consumer segment. The consumers to the left of the marginal consumer either receive a lower surplus comparing to the surplus they would have received under PPQ or doesn't purchase under uniform pricing. Hence it's optimal for them to opt in and receive higher surplus. Note that after the first round of opting in, it's straightforward to show that the market size of the out-consumer segment shrinks such that the optimal market share is no longer attainable because some consumers who purchase under uniform pricing will opt in once given the option. Hence corner solution arises: firms cover the whole out-segment and sets price that exactly extract all the consumer surplus at the margin to maximise profits. Then the iteration process as shown in figure 2[2] follows the same manner as in case I and reaches an equilibrium when consumers locating at the end, $\theta = 0$, $\theta = 1$ are indifferent between the uniform pricing offer and PPQ offer. We arrive at

**Proposition 2.** *When both firms use PPQ for in-consumers, uniform pricing for out-consumers, all consumers opt in and receive PPQ offer in equilibrium.*

This case is more intuitive than the last one where both firms use NNP for out-segment. Because of the nature of uniform pricing, the firms will leave out some consumer such that the market is not covered. Then there is nothing stopping the consumers in the middle from opting into the personalisation scheme to achieve higher surplus comparing to none. Given firms always maximise profits subject to the size of the out-segment, recall the first order condition:

$$\frac{\partial \pi^L}{\partial q_R} = \theta_R^* - q^{\alpha-1} - \theta^{*2} + \theta_R^* q^{\alpha-1} = 0 \tag{2}$$

Solve for $q_R$, given the constraint $C = \theta$

$$q_R = \left(\frac{C^2 - C}{C - 1}\right)^{\frac{1}{\alpha-1}} \tag{3}$$

We can see for $\alpha > 1$, it's easy to show that $q_R$ strictly increases in C. Hence as the out-segment get smaller, the quality provided by the firms will increase along price until they reach the maximum that the most loyal consumers are willing to pay.

[2]the simulation take value $\alpha = 2$

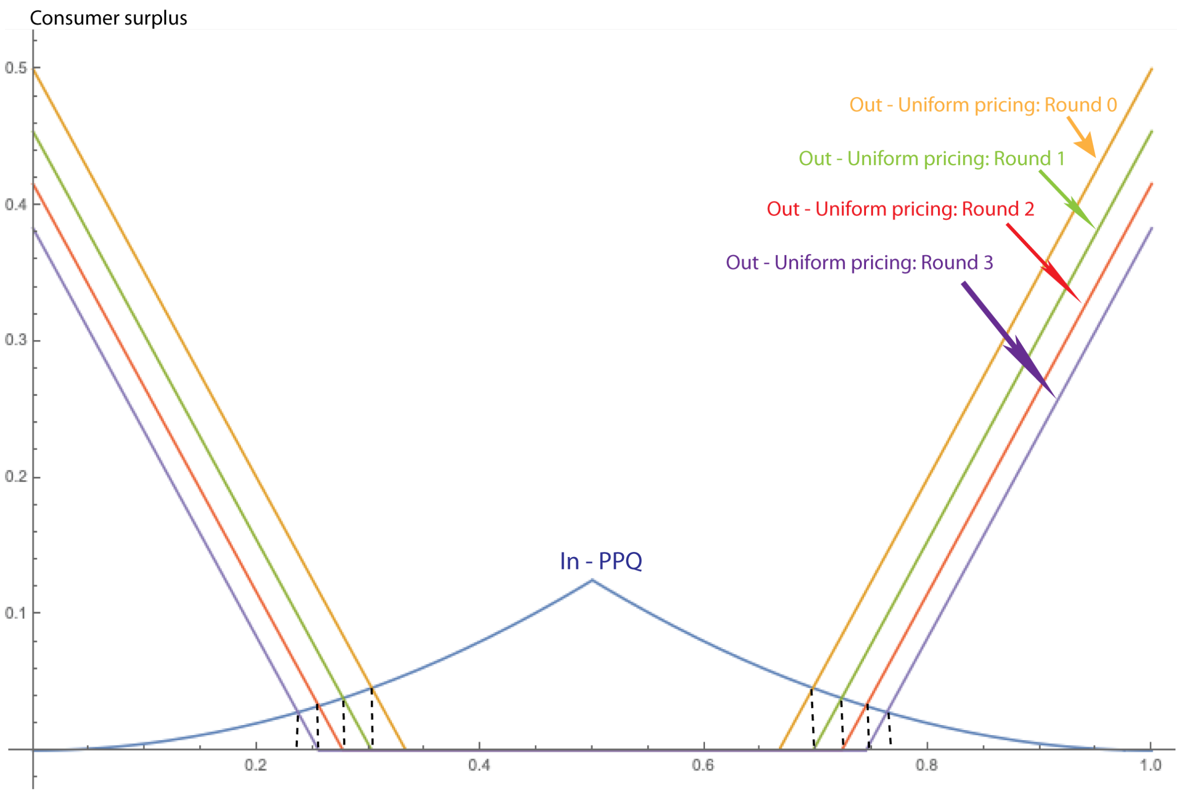


Figure 2: Consumer surplus in case II subgame