Gradual Bargaining in Decentralized Asset Markets

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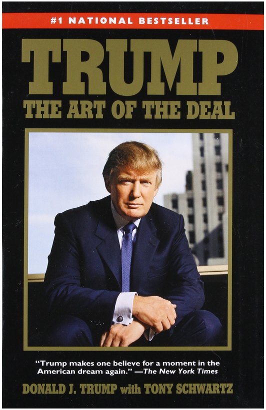
The art of the deal (in asset markets)

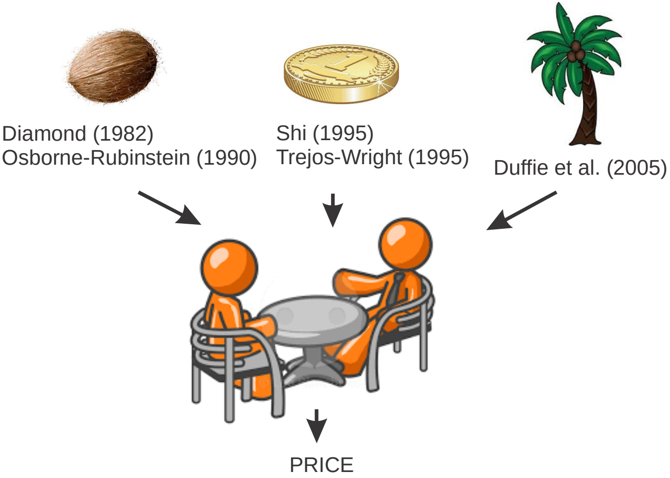


Background

- Models of decentralized asset markets
  - to explain asset/market liquidity
- Two approaches
  - New Monetarist approach: Assets as media of exchange
  - Finance approach: Illiquid assets traded over the counter
- Based on search paradigm with two core components:
  1. search frictions and pairwise meetings
  2. bargaining
- This paper is about **bargaining**.

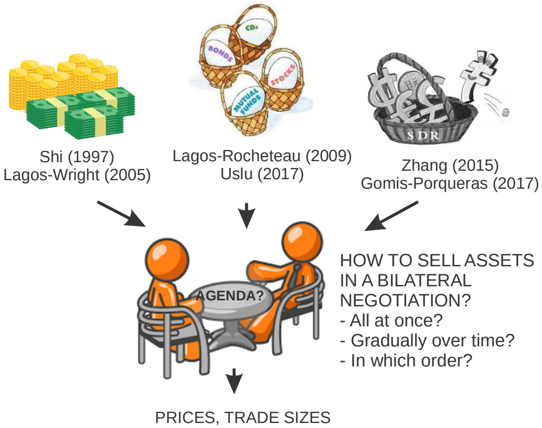
Background: 2nd generation of models

- Restricted asset holdings: $a \in \{0, 1\}$



Background: 3rd generation of models

- Portfolio of divisible assets: $\mathbf{a} \in \mathbf{R}_+^J$



Background: How is bargaining handled?

- Bargaining with $\mathbf{a} \in \mathbf{R}_{+}^{J}$ like with $a \in \{0, 1\}$
  - Generalized Nash or Kalai solution.
  - Agents negotiate their portfolio all at once.
- Is this agenda (all-at-once bargaining) restrictive?
- Is it the agenda that agents/society would choose?
- Does the agenda matter for allocations and prices?

What we do

1. A new approach to bargaining over asset portfolios
   - Assets are sold gradually over time
   - Both axiomatic and strategic foundations
   - A new asset characteristic: negotiability
2. Incorporate into models of decentralized asset markets
   - New Monetarist models (Lagos-Wright)
   - Models of OTC markets (Duffie et al.)
3. Two applications
   - Money and bonds and OMOs
   - Multiple currencies and exchange rates

Insights

1. **Bargaining theory**
   Extensive-form bargaining games, endogenous agenda
2. **Asset prices**
   Negotiability premia, distributions of asset returns and velocities
3. **Monetary theory**
   rate-of-return dominance, exchange rate determination, OMOs

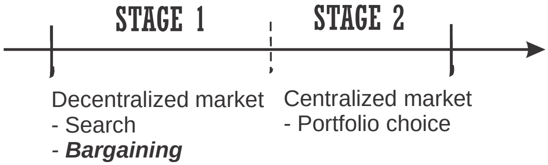
Literature

1. **Gradual bargaining with an agenda**: O'Neill et al. (2004)
   - Application to money in Rocheteau and Waller (2005)
2. **Strategic bargaining games:** Rubinstein (1982)
   - Applications to money: Shi (1995), Trejos and Wright (1995)
   - Non-stationary environment: Coles and Wright (1998)
   - Delays under asymmetric information: Tsoy (2016)
3. **Models of decentralized asset markets**
   1. New Monetarist: Geromichalos et al. (2007); Lagos (2010)
   2. Finance: Duffie et al. (2005); Lagos and Rocheteau (2009)
   3. Money and finance: Herrenbrueck and Geromichalos (2016), Lagos and Zhang (2017), Wright, Xiao, and Zhu (2017)

ENVIRONMENT

Time, goods

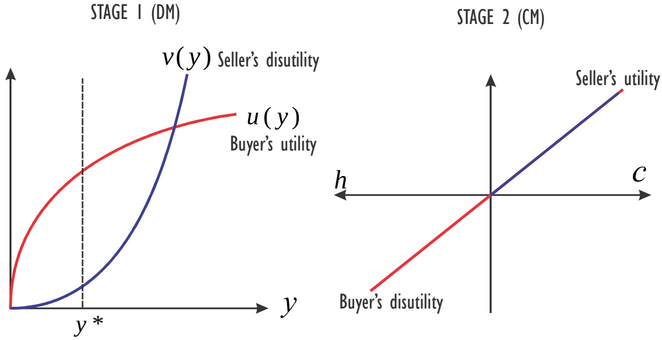
- Time: $t = 0, 1, 2..., \infty$
- Each period has two stages:
  1. Decentralized market (DM): Pairwise meetings / bargaining
  2. Centralized market (CM)
- DM good is perishable
- CM good taken as numeraire



Agents

- Agents divided into two types
  1. Consumers: consume DM good and produce numeraire
  2. Producers: produce DM good and consume numeraire
- A unit measure of each type
- In the DM, $\alpha \in (0, 1]$ pairwise meetings between consumers and producers

Preferences



- Discount factor: $\beta = 1/(1+\rho)$
- Efficient DM output: $u'(y^*) = v'(y^*)$

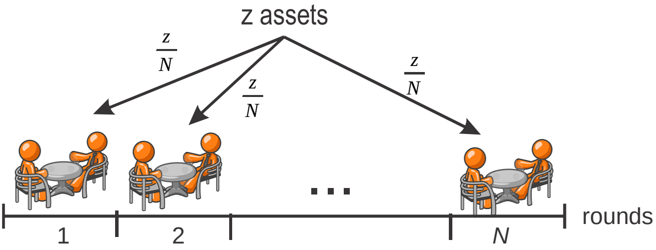
Assets

- Lucas trees: pay off $d \geq 0$ in the CM
  - Fiat money: $d = 0$
- Exogenous supply: $A_{t+1} = (1 + \pi)A_t$
  - if $d > 0$, $\pi = 0$
- Asset price in terms of the numeraire: ϕ_t
- No private IOUs: Agents cannot commit

GRADUAL BARGAINING

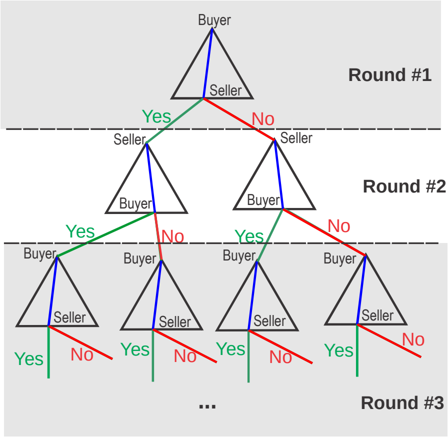
Bargaining game

- Asset owner has z units of assets (in terms of numeraire)
- Divided into N equal sizes: z/N
- Game has N rounds
- In each round, agents negotiate the sale of z/N assets for some output y



Alternative ultimatum offer game

- N two-stage rounds
  1. Stage 1: One player makes an offer
  2. Stage 2: Other player accepts/rejects
- Identity of the proposer alternates



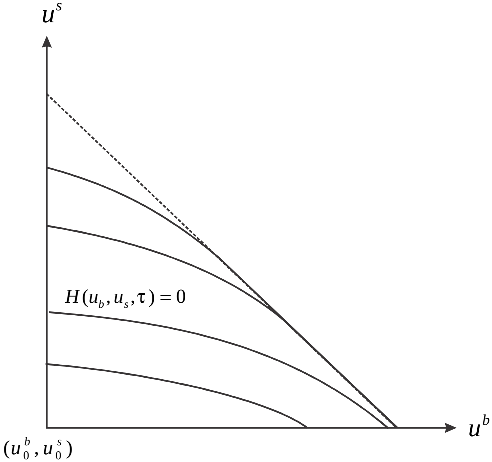
Intermediate Pareto frontiers

- Denote $\tau \equiv nz/N$ where $n = 1, ..., N$
- For each τ, feasibility constraint on asset sales: $p(\tau) \leq \tau$
- For each τ, a Pareto frontier:

$$\max u_b(\tau) \quad \text{s.t.} \quad u_s(\tau) \geq u_s \text{ and } p(\tau) \leq \tau$$

$$\Downarrow$$

$$H(\overset{-}{u^b}, \overset{-}{u^s}, \overset{+}{\tau}) = 0.$$



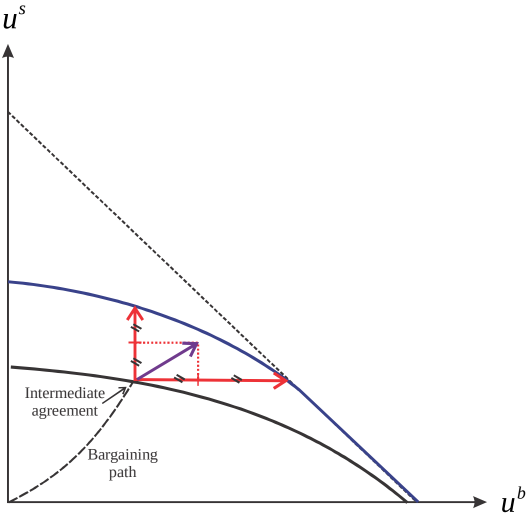
u^s
$H(u_b, u_s, \tau) = 0$
u^b
(u_0^b, u_0^s)

Subgame perfect equilibrium

There exists a unique SPE. As N approaches ∞, payoffs solve:

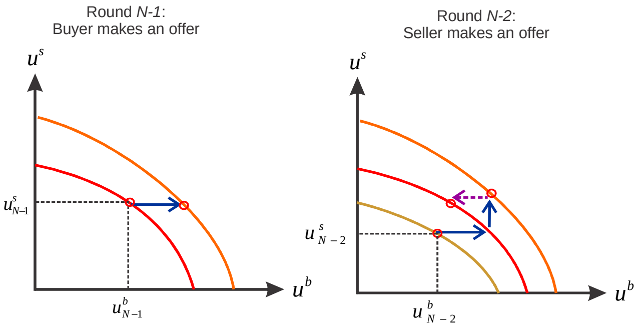
$$u^{\chi\prime}(\tau) = -\frac{1}{2}\frac{\overbrace{\partial H(u^b, u^s, \tau)/\partial\tau}^{\text{shift of Pareto frontier}}}{\underbrace{\partial H(u^b, u^s, \tau)/\partial u^\chi}_{\text{expressed in utils of player } \chi}}, \quad \chi \in \{b, s\},$$

(i.e., z/N approaches to 0)



u^s
u^b
Intermediate agreement
Bargaining path

Sketch of proof



Robustness: Axiomatic approach

- From O'Neill et al. (2004), $\langle u^b(\tau), u^s(\tau) \rangle$ is also the unique solution satisfying

1. Pareto optimality
2. Scale invariant
3. Symmetry
4. Directional continuity
5. Time consistency

- The solution is **ordinal**, i.e., invariant to order-preserving transformations.

Solution in terms of allocations/prices

- Asset price (in terms of DM goods) solves:

$$y'(\tau) = \frac{1}{2}\left(\overbrace{\frac{1}{\upsilon'(y)}}^{\text{bid price}} + \overbrace{\frac{1}{u'(y)}}^{\text{ask price}}\right) \quad \text{for all } y < y^*.$$

- Suppose $\upsilon'(y) = 1$. Asset price is:

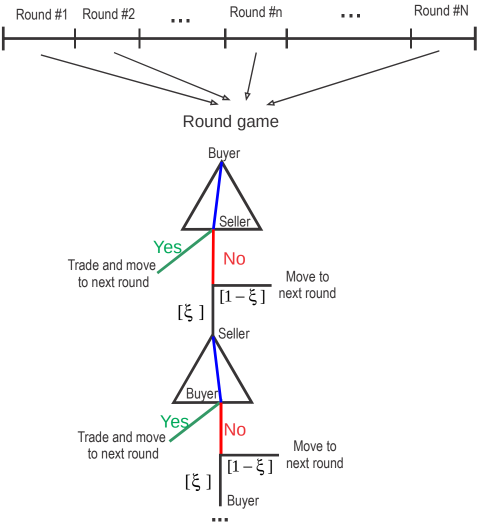
$$\frac{1}{2}\left(1 + \frac{1}{u'(y)}\right).$$

It increases with the size of the trade.

AGENDA OF THE NEGOTIATION

Part 1: Optimal number of rounds

Repeated Rubinstein game

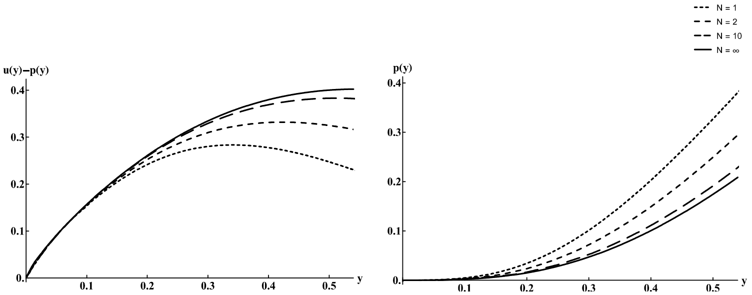


Bundled vs gradual sales

- Intermediate output levels, $\{y_n\}_{n=1}^{N}$, solve:

$$\int_{y_{n-1}}^{y_n} \overbrace{\frac{v'(y_n)}{u'(y_n)+v'(y_n)}}^{\text{producer's share}} u'(x) + \overbrace{\frac{u'(y_n)}{u'(y_n)+v'(y_n)}}^{\text{consumer's share}} v'(x)\,dx = \frac{z}{N}$$

Proposition: *Consumers (asset owners) prefer $N = +\infty$ to any $N < +\infty$.*

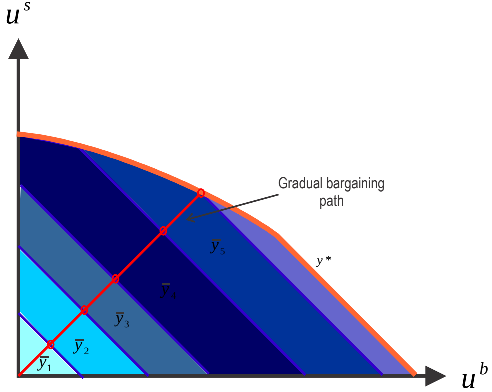


AGENDA OF THE NEGOTIATION

Part 2: gradual bargaining over DM goods

Gradual bargaining over DM goods

- Suppose agents bargain gradually over y in exchange for money



Gradual bargaining over DM goods (cont'ed)

- The payment for y units of DM goods is:

$$p(y) = \frac{1}{2}\left[u(y) + v(y)\right].$$

- Implications:
  1. New strategic and axiomatic foundations for the egalitarian solution
  2. Egalitarian is not scale invariant while gradual solution is ordinal!

Endogenous agenda

- Different agendas lead to different outcomes
- Suppose we pick one player to choose the agenda.
    - The buyer chooses to bargain gradually over z
    - The seller chooses to bargain gradually over y.

FROM PARTIAL TO GENERAL EQUILIBRIUM

Asset negotiability

- Agenda indexed by time, τ
  - An implicit mapping between τ and z
- New asset characteristic: Negotiability
  - $\delta > 0$ units of assets can be sold per unit of time
- What is negotiability in practice:
  - time to authenticate assets
  - time to value complex assets
  - time to execute trade and transfer ownership (e.g., blockchain technologies)

Making time relevant

- Random time to negotiate asset sales: $\bar{\tau} \sim \text{Exp}(\lambda)$
  - negotiation breakdown, proxy for discounting
- Formally:

$$\overbrace{p(y)}^{\text{Asset sales}} \leq \overbrace{\delta}^{\text{Negotiability}} \times \overbrace{\bar{\tau}}^{\text{Time to negotiate}}$$

Pricing of Lucas trees

- Interest rate spread (liquid vs non-liquid):

$$\overbrace{s}^{\text{spread}} = \overbrace{\alpha}^{\text{search}} \times \overbrace{\theta}^{\text{bargaining}} \times \overbrace{e^{-\frac{\lambda}{\delta}p(y)}}^{\text{negotiability}} \times \overbrace{\ell(y)}^{\text{liquidity needs}} .$$

  where $\ell(y) \equiv u'(y)/v'(y) - 1$
- $e^{-\frac{\lambda}{\delta}p(y)}$ akin to a pledgeability coefficient
  - endogenous with $\neq$ comparative statics
- s decreases with Ad but increases with δ and $1/\lambda$

Endogenous negotiability

- Consumers choose δ when a match is formed but before $\bar{\tau}$ is realized
- Cost to enhance negotiability: $\psi(\delta)$

Proposition

1. *If A is not too large, an increase in A reduces s, but raises δ.*
2. *If A is not too large, asset negotiability is too low for all bargaining powers*
   - A pecuniary externality

Multiple assets

- J one-period lived Lucas trees in fixed supply A_j
- Each tree pays off one unit of numeraire
- Fiat money: $j = 0$
- Negotiability of asset j is δ_j with

$$\delta_0 \geq \delta_1 \geq \delta_2 \geq ... \geq \delta_J$$

- Pecking order: sell assets with high negotiability first

Asset prices

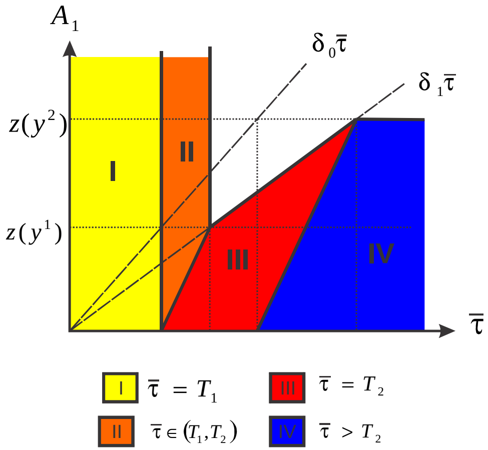
$$\underbrace{s_j}^{\text{spread}} = \overbrace{\alpha\theta}^{\text{search\&bargaining}} \overbrace{\lambda \sum_{k=j+1}^{J} \int_{T_k}^{T_{k+1}} \frac{(\delta_j - \delta_k)}{\delta_j} e^{-\lambda\tau} \ell[y(\tau)] d\tau}^{\text{negotiability premium}}$$

$$+\alpha\theta \overbrace{e^{-\lambda T_{J+1}} \ell[y(T_{J+1})]}^{\text{liquidity premium}}$$

- If $\delta_j > \delta_{j+1}$, then $s_j \geq s_{j+1}$.
- Negotiability premium is asset specific
  - depends on asset supplies, negotiability differentials

APPLICATION #1: MONEY AND BONDS

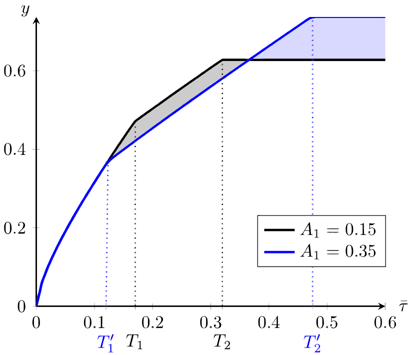
Money and bonds



Regime 1: CIA
Regime 2: Marginally illiquid bonds
Regime 3: Effective OMOS
Regime 4: Rate of return equality

OMOs: negotiability vs liquidity

- In Regime 3, an increase in A_1 (bond supply) leads to a reduction in output
  - The most negotiable assets are replaced with less negotiable ones
- Suppose $\bar{\tau}$ is stochastic



APPLICATION #2: MULTIPLE CURRENCIES

Multiple fiat monies

- Multiple cryptocurrencies: Bitcoins, Litecoin, Ethereum ...
- Confirmation times vary across currencies
  - Different currencies have different δ
- 2 currencies: 0 and 1
- $\delta_0 > \delta_1$ but $\pi_0 > \pi_1$

Dual currency equilibrium

- Suppose $i_0 > i_1$.
- For all $\tau \in (\bar{\tau}_0, \bar{\tau}_1)$ there exists a unique SSE where both currencies are valued.
- $\partial y / \partial \pi_0 < 0$ and $\partial y / \partial \pi_1 > 0$
- Currency 0 appreciates vis-a-vis currency 1 as α or θ increases or as $\bar{\tau}$ decreases
  - because agents put more weight on negotiability

OTC MARKET WITH MONEY

Re-interpretation: OTC markets

- Each agent is endowed with Ω units of short-lived assets
- Asset payoff: $\varepsilon f(\omega)$ where $\varepsilon \in \{\varepsilon_\ell, \varepsilon_h\}$ is idiosyncratic
- Assets reallocated in pairwise meetings
- quantity of assets sold by ℓ to h: y
- Utility of buyer: $u(y) \equiv \varepsilon_h [f(\Omega + y) - f(\Omega)]$
- Disutility of seller: $v(y) \equiv \varepsilon_\ell [f(\Omega) - f(\Omega - y)]$
- A competitive interdealer market with price q

Bid and ask prices

- In meeting with dealers, the investor chooses the agenda
- Ask price: gradual bargaining over investors' money

$$p^a(y) = q\int_0^y \frac{2\varepsilon_h f'(\Omega + x)}{\varepsilon_h f'(\Omega + x) + q}dx$$

- Bid price: gradual bargaining over investor's illiquid assets

$$p^b(y) = \int_0^y \frac{q + \varepsilon_\ell f'(\Omega - x)}{2}dx$$

- Efficient trade sizes at the Friedman rule

Conclusion

- A new approach to bargaining over asset portfolios in decentralized asset markets
  - Axiomatic and strategic foundations
  - Tractable
  - More general: encompasses Nash and Kalai solutions for specific agendas
- Insights
  - normative: gradual bargaining desirable individually and socially
  - positive: negotiability premia, distribution of asset returns, determinacy of exchange rate, OMOs