Low Real Interest Rates and the Zero Lower Bound

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Abstract

How do low real interest rates constrain monetary policy? Is the zero lower bound optimal if the real interest rate is sufficiently low? What is the role of forward guidance? A model is constructed that incorporates sticky price frictions, collateral constraints, and conventional monetary distortions. The model has neo-Fisherian properties. If the zero lower bound is a problem, then a symptom is inflation above the central bank's inflation target. Extended periods of low nominal interest rates are useful in bringing inflation down and relaxing financial constraints, not for forward guidance reasons. The ZLB may be suboptimal under tight collateral constraints.

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1 Introduction

Non-monetary factors that cause the short-term real rate of interest to be low can contribute to circumstances where central banks will lower their nominal interest rate targets to zero (the zero lower bound or ZLB), or to some lower effective lower bound (ELB). What are the symptoms associated with a low real interest rate? Does a sufficiently low real interest rate imply that the ZLB (or something lower) is optimal? If so, how should the central bank manage the transition from the ZLB state? In this paper, we construct a tractable analytical model of low real interest rates and monetary policy. The goal is to study optimal monetary policy using a model which is explicit about the reason for low real interest rates – a scarcity of safe collateral – and which can incorporate alternative sources of inefficiency so as to understand how those interact with collateral scarcity. The model can incorporate sticky prices, financial frictions, monetary exchange, and open market operations, in a straightforward way.

Interest in the ZLB in macroeconomics is at least as old as Keynes's *General Theory* (Keynes 1936). Keynes emphasized the importance of the liquidity trap – the neutrality of open market operations at the ZLB. However, Keynes's arguments seem to have been viewed as of little practical importance, except perhaps for explaining what was going on in the Great Depression. Indeed, in the period after the 1951 Treasury/Fed Accord until 2002, monetary policy in the United States rarely got within shouting distance of the ZLB. For example, between 1951 and 2002, the 3-month Treasury bill rate dipped below 1% only twice, and only for short periods of time, in 1954 and 1958.

The ZLB did not become a widely discussed monetary policy issue until the late 1990s and early 2000s. Interest at that time appears to have been sparked by experience in Japan, as well as in the United States, where the federal funds rate target stood at 1% from mid-2002 to mid-2003. This experience led to macroeconomic research using modern macroeconomic theory to understand the importance of the ZLB for macroeconomic activity and for monetary policy. Krugman (1998) used a Lucas-type dynamic cash-in-advance model to show how temporarily-sticky prices could lead to a situation in which current monetary policy would be thwarted by the ZLB, but a promise to raise future prices through future monetary expansion would permit escape from a liquidity trap. This idea was later fleshed out in a more complete New Keynesian framework by Eggertsson and Woodford (2003), who argued that a binding ZLB constraint could cause deflation and low output. This was then characterized as a policy problem that could be mitigated by commitment to future policy actions. Once the underlying factor that had been causing the ZLB to bind – a low "natural real rate of interest" – dissipates, the future inflation-targeting central banker will revert to a policy that the present-day policymaker would view as suboptimal. According to Eggertsson and Woodford, a forward guidance policy by which the policymaker could commit to a future policy of low nominal interest rates and higher inflation - after the natural real rate rises - would mitigate the ZLB problem.

An influential policy view from this period (see Bernanke et al. 2004, page

1) is neatly summarized as follows:

> Should the nominal rate hit zero, the real short-term interest rate—at that point equal to the negative of prevailing inflation expectations—may be higher than the rate needed to ensure stable prices and the full utilization of resources. Indeed, an unstable dynamic may result if the excessively high real rate leads to downward pressure on costs and prices that, in turn, raises the real short-term interest rate, which depresses activity and prices further, and so on.

This policy view is in some ways consistent with Eggertsson and Woodford (2003), but differs in two ways. First, there is no emphasis on forward guidance. Second, Bernanke thinks that a ZLB monetary policy leads to unstable dynamics and the potential for a deflationary black hole. This idea is hard to trace in the academic literature, but seems to have been part of the public policy discussion (see for example Krugman 2002).

Since the early 2000s, macroeconomic shocks and the policy responses to those shocks have provided macroeconomists interested in ZLB issues with more information. The global financial crisis and the resulting worldwide recession in 2008-09 ultimately resulted in ZLB (or negative interest rate) monetary policy in many countries, including the US, the UK, Canada, the Euro area, and Sweden. In Japan, the Bank of Japan has now pursued ZLB policies, or negative interest rate policies, for about 23 years. As well, there has been extensive use of the unconventional monetary policies specified in Bernanke et al. (2004) for example, including large central bank balance sheets, increases in the average maturity of central bank asset portfolios, and forward guidance.

More recent research by Werning (2012) reinforces the New Keynesian analysis of Eggertsson and Woodford (2003). Werning's contribution is to analyze optimal monetary policy in the context of a temporarily low natural real rate of interest. As in Eggertsson and Woodford (2012), the ZLB period is characterized by low inflation and low output. Forward guidance, in the form of commitment to low nominal interest rates and high inflation in the future, mitigates the problem. Werning also argues that the ZLB problem worsens as price flexibility increases.

Williams (2014) summarizes mainstream monetary policymakers' views of the state of the art in ZLB monetary policy. What appears to have disappeared from the policy discussion (relative to 2003) is concern with the deflationary-black-hole potential formerly thought to be inherent in ZLB episodes. This change in the consensus policy view has occurred for good reasons. In particular, it would be hard to characterize the 23-year low-nominal-interest-rate regime in Japan as unstable, and Japan has not experienced a persistent deflation. On average, the inflation rate in Japan since 1995 has been close to zero. As well, during the seven-year period (2008-2015) in the United States when the fed funds rate target range was 0-0.25%, the inflation rate (headline personal consumption deflator) in the US averaged 1.3%. This rate was lower than the Fed's target of 2%, but hardly a deflationary black hole.

From Williams (2014) and the New Keynesian literature on the ZLB discussed above, a current consensus policy view emerges:

1. The real rate of return on safe assets, including government debt, has been persistently low, and this low real interest rate is expected to persist into the indefinite future.

2. The low real interest rate implies that the ZLB problem will arise more frequently in the future than it has in the past.

3. This frequently-occurring binding ZLB will be reflected in more frequent undershooting of central bank inflation targets, if such targets remain at 2%.

4. For the long run, higher inflation targets should be considered.

There are good reasons to be skeptical of this consensus policy view. First, under the consensus view, the ZLB is sometimes discussed as an inevitable response to a set of economic shocks. But the ZLB binds in practice as the result of policy choices. Benhabib et al. (2001) tells us that adherence to the Taylor rule by a central bank, following the Taylor principle (increase the nominal interest rate more than one-for-one in response to an increase in the inflation rate) can lead the central bank into a policy trap. At low nominal interest rates, the Fisher effect sets in, inflation is low, and the central banker lowers the nominal interest rate to zero, expecting that inflation will go up. But this just leads to persistently low inflation, and the central bank is stuck at the ZLB.

Second, some recent research on the properties of New Keynesian models raises doubts about how the results in Eggertsson and Woodford (2002) and Werning (2012), for example, should be interpreted. Work by Cochrane (2016, 2017), Rupert and Sustek (2016), and Williamson (2018c) comes to *Neo-Fisherian* conclusions concerning the properties of New Keynesian models, and of the broader set of mainstream macroeconomic monetary models. That is, low (high) inflation tends to be caused by low (high) nominal interest rate settings by the central bank. While there may be nonneutralities of money that result in liquidity effects (an increase in the central bank's nominal interest rate target raises the real interest rate in the short run), such effects are dominated by Fisher effects, even in the short run. As a result, higher (lower) nominal interest rates are associated with higher (lower) inflation. This alternative view of inflation dynamics might make us question the consensus policy view of the ZLB problem. Maybe inflation is low when the ZLB binds because of monetary policy, not because of what monetary policy is responding to.

Third, as Werning (2012) notes as a caveat in his work:

> ...the analysis ...omits ... financial constraints and other frictions which may be relevant in these situations.

A good case can be made that such constraints and frictions are indeed relevant in such situations – i.e. situations in which the ZLB may bind, or should bind, according to policymakers. That is, macroeconomists seem to agree that real interest rates are currently low for reasons independent of monetary policy. In particular, low productivity growth and demographic factors may cause the real interest rate to be low. And theory and evidence suggest that a key factor leading to low real rates of interest on safe assets is the high demand and low supply of safe and liquid assets. For theoretical support, see Caballero et al. (2016) and Andolfatto and Williamson (2015), and for empirical evidence, see Vissing-Jorgensen (2012), and Del Negro et al. (2017). Thus, it seems that it would be useful in analyzing the implications of low real interest rates to model a safe asset shortage explicitly.

To explore these issues, we need a model, and many monetary policymakers think it important that policy modeling include price and/or wage rigidity. But following the conventional New Keynesian (NK) route – Dixit-Stiglitz monopolistic competition, Calvo pricing, etc., leads to complicated reduced-form linearized derivations, which can obscure what is going on. And a key simplification in such models is to eliminate central bank balance sheets and the details of retail transactions from the analysis, as in Woodfordian "cashless" models (see Woodford 2003). But once we enter the realm of financial frictions, looking out for the details of central bank asset swaps and retail transactions can be critical (Andolfatto and Williamson 2015, Williamson 2016, 2018a, 2018b). If we want to introduce other frictions than sticky prices, and include details of central bank assets and liabilities and their roles in the economy, it seems unwise to try to build on a conventional NK framework, which was designed with something else in mind.

The tractable model constructed here will permit us to put in and take out particular frictions, so that we can understand where the results come from. In general, the model can include sticky prices, a safe asset shortage reflected in binding collateral constraints, monetary exchange, and explicit open market operations. To keep things simple, production and consumption are carried on at the household level, and when prices change they are determined competitively, not be price-setting firms. Every period, some prices are flexible, while sticky prices remain at their previous-period levels. In markets in which prices are sticky, output is demand-determined, just as in mainstream NK setups.

We start with a cashless model, in the spirit of Woodford (2003) (though without monopolistic competition and Calvo pricing). All goods are purchased with secured credit, and the available collateral in the model is government debt. In this version of the model, there is a natural inflation target, which is zero whether the collateral constraint binds or not. If the collateral constraint binds, the real interest rate is low, and there is a liquidity premium associated with government debt. In the cashless model, the ZLB may bind when the central bank is conducting policy optimally. But, a binding ZLB is always reflected in inflation that is above the natural inflation target – i.e. above the inflation rate that eliminates the relative price distortion caused by sticky prices. Further, future monetary policy is irrelevant for current inflation, so there is no role for forward guidance. That is, all of the effects of policy flow from present to future. Current monetary policy can matter for the future liquidity premium on government debt, which can indirectly constrain future monetary policy at the ZLB and affect future inflation.

In the cashless model, a collateral constraint that is tight on average, and tight enough so that the ZLB binds on a regular basis, implies that, under optimal monetary policy, the inflation rate is above the natural inflation target when the ZLB binds, and below the target when it does not bind. This is the opposite of the consensus policy view about how to manage a binding ZLB constraint.

In the cashless model, a scenario similar to Werning (2012) is considered, where the economy experiences a temporary period with a low real interest rate, followed by an indefinite period with a "normal" real interest rate. That is, the collateral constraint is tight for a specified period of time, and then the supply of safe assets increases permanently, relaxing the collateral constraint. Under this scenario, while the collateral constraint is tight the ZLB constraint binds at the optimum for the central bank, output is low, and inflation is high. Once the quantity of safe assets falls permanently, there is a temporary period when it is optimal for the central bank to keep the nominal interest rate low. But that is because this policy provides the fastest disinflation path to the natural target inflation rate, at the same time relaxing the collateral constraint in the quickest fashion. In the Eggertsson and Woodford (2002) and Werning (2012) analyses, a prolonged period of low nominal interest rates, after the low "natural rate" period has passed, acts to increase welfare during the low-natural-rate period.

The model is then extended to permit retail transactions using currency, and open market operations by the central bank to support its interest rate policy. So that we can understand how all the frictions in this setup fit together, we first consider the case with flexible prices. In this case there are two potential sources of inefficiency: a traditional Friedman rule inefficiency according to which high inflation and a positive nominal interest rate make exchange in the cash market inefficient; and a binding collateral constraint, whereby scarce collateral makes exchange inefficient in the cash-and-credit market. If the collateral constraint does not bind, then a Friedman rule is optimal and the nominal interest rate should be zero. However, if the collateral constraint binds under any conditions, it binds when the nominal interest rate is low, and at the optimum the nominal interest rate should be greater than zero. That is, an open market sale of government bonds by the central bank at the ZLB raises the nominal interest rate and relaxes the collateral constraint. This raises welfare. The results for the cashless model are just the opposite. The nominal interest rate should be greater than zero when the collateral constraint does not bind, and the ZLB is optimal when the collateral constraint is sufficiently tight.

But what happens in the case in which we include sticky prices, retail payments using currency, and secured credit? Then, there are potentially three inefficiencies at work: a sticky price inefficiency that distorts the relative price of flexible-price and sticky-price goods, a Friedman-rule inefficiency which causes inefficiency in the market for goods purchased with currency, and a safe asset scarcity, which causes inefficiency in the market for goods purchased with secured credit, and generally constrains the demand for goods.

In this full-blown model, if the collateral constraint does not bind, then the nominal interest rate should be positive, and optimal policy trades off the costs of Friedman rule inefficiency and sticky price inefficiency. Optimal inflation falls somewhere between Friedman-rule deflation and zero inflation. In a region of the parameter space, a tighter collateral constraint and a low real interest rate implies that the optimal nominal interest rate increases as the collateral constraint gets tighter. That is, sticky prices are insufficient to induce an optimal ZLB policy when the collateral constraint is tight. For the ZLB to be optimal, it is necessary, but not sufficient, that the demand for sticky-price goods purchased with credit be highly interest-elastic. Like the cashless version of our model, this version has the implication that forward guidance in monetary policy is irrelevant.

So, the conclusions here differ markedly from the consensus view of the ZLB "problem," and the results have a Fisherian flavor. A safe asset shortage that causes the real interest rate to be low implies that inflation tends to be high – perhaps higher than desirable. If monetary factors are deemed to be irrelevant for the problem, this implies an extended period with low nominal interest rates, and this serves to bring inflation down quickly. Once we take monetary factors into account, the ZLB may be suboptimal when the collateral constraint is tight. In contrast to arguments in Woodford (2003), the cashless economy behaves differently - in important ways - from the economy with currency transactions and open market operations.

In terms of the theoretical approach to monetary exchange and monetary policy, this paper is closest to Andolfatto and Williamson (2015), and shares an approach to safe asset scarcity with Williamson (2016, 2018a, 2018b). The modeling of sticky prices is new here, and certainly different from standard New Keynesian frameworks, e.g. Woodford (2003), Gali (2015).

The remainder of the paper is organized as follows. The baseline cashless model is constructed and analyzed in the second section. Then, in the third section, an extended model that includes retail exchange with currency, along with open market operations, is developed and analyzed. The final section is a conclusion.

2 Cashless Model

To start, we will construct a cashless model, in the spirit of baseline New Keynesian models (e.g. Woodford 2003, Gali 2015), except that we take a different approach to price stickiness. A key element in the model will be secured credit, and the possibility that a binding credit constraint induces a low real interest rate. To focus on the issues of interest here, we assume no aggregate uncertainty.

There is a continuum of households with unit mass, with each maximizing

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t^f) + u(c_t^s) - (n_t^f + n_t^s) \right] \tag{1}$$

Here, $0 < \beta < 1$, c_t^f is consumption by the household of the flexible-price good, c_t^s is consumption of the sticky-price good, and n_t^f and n_t^s denote, respectively, household labor supplied to produce the flexible-price and sticky-price goods, respectively. The fact that the utility function is linear in labor supply is an important restriction, which will lend tractability to the problem by eliminating wealth effects. In standard New Keynesian constructs (see e.g. Gali 2015 for the details), quasilinear utility would be immaterial, as output is effectively demand-determined in those frameworks. In contrast, though there is price stickiness in our setup, supply considerations will come into play in markets in which prices are flexible, so labor supply elasticities and wealth effects will matter, in general.

As in mainstream New Keynesian cashless models, goods are denominated in terms of money, and money does not serve as a medium of exchange, only as a unit of account. Let P_t denote the price of flexible-price goods in units of money. The spot market in flexible-price goods clears every period, but households are technologically constrained (in the spirit of the technological constraints in any sticky-price or sticky-wage setting) to sell sticky-price goods at the price P_{t-1}, and must satisfy whatever demand arises for sticky-price goods at that price. Demand is assumed to be distributed uniformly among households in the sticky-price goods market.

This setup is equivalent to a world in which there are two physically distinct goods, which for convenience can be denoted *even* and *odd* goods. The even (odd) good is the flexible price good in even (odd) periods . Then in an even (odd) period, the price is determined competitively for the even good, and the odd (even) good must be sold at its price in the previous period. Thus, for a particular good, the price stays the same for two periods running, and there is staggered price determination. This yields the setup we have specified, with this period's flexible-price good being next period's sticky-price good.

The approach to sticky prices taken here is different from, for example, either Calvo pricing (e.g. Gali 2015) or state-dependent pricing (Golosov and Lucas 2007), under which individual monopolistically competitive firms make forward-looking pricing decisions. Our approach avoids some of the technical modeling complications of these alternative approaches but, as we show, the model captures the fundamentals of New Keynesian sticky price economics – relative price distortions and Phillips curve effects. We will show, though, that a binding collateral constraint will alter Phillips curve relationships.

The household produces goods using a linear technology, identical for the two goods. In particular, normalize so that one unit of labor supplied produces one unit of either good, and assume goods are perishable. Further, a household cannot consume its own output, but must purchase goods from other households using secured credit. There is only one asset in this economy, which is a one-period nominal government bond. A bond issued in period t sells for one unit of money in period t and pays off R_t units of money in period $t+1$, so R_t is the gross nominal interest rate. Let the flexible price good be the numeraire, and let $\frac{1}{\pi_t}$ denote the price of the sticky price good in terms of the flexible price good, that is $\pi_t = \frac{P_t}{P_{t-1}}$, so π_t is both an intratemporal relative price (the price of the flexible price good in terms of the sticky price good) and an intertemporal relative price (the price of this period's flexible price good relative to last period's flexible price good).

In the model, π_t will reflect the effects of monetary policy and will give us a measure of the inefficiency wedge resulting from sticky prices. As shorthand we will refer to π_t as the "gross inflation rate," with some abuse of language. The actual measured gross inflation rate in period t in this economy is

$$i_t = \frac{P_{t-1}c_t^s + P_t c_t^f}{P_{t-2}c_{t-1}^s + P_{t-1}c_{t-1}^f} = \frac{c_t^s + \pi_t c_t^f}{\frac{c_{t-1}^s}{\pi_{t-1}} + c_{t-1}^f}.$$

Under some special conditions, $i_t = \pi_t$, but in general $i_t \neq \pi_t$.

Secured credit is necessary in this economy as there is limited commitment. The household consists of a seller, who supplies labor to produce flexible-price and sticky-price goods in exchange for IOUs, and a buyer, who purchases goods with IOUs. The IOUs are then settled at the end of the period, after production, goods market exchange, and consumption take place. The household could default on its debts at the end of the period, but in equilibrium its does not, because the household has secured its debts with sufficient government debt.

To be more specific, the household's period t budget constraint is

$$\bar{b}_t + c_t^f + \frac{c_t^s}{\pi_t} = n_t^f + \frac{n_t^s}{\pi_t} + \frac{R_{t-1}}{\pi_t}\bar{b}_{t-1} + \tau_t. \tag{2}$$

In equation (2), the household enters the period with $\bar{b}_{t-1}$ government bonds acquired in period $t-1$, in units of the period $t-1$ flexible-price good, receives the gross real rate of return $\frac{R_{t-1}}{\pi_t}$ on each bond, and receives a lump-sum transfer τ_t from the government. The household seller works during the period, acquiring $n_t^f + \frac{n_t^s}{\pi_t}$ claims on end-of-period consumption, which it uses along with its beginning-of-period wealth to extinguish $c_t^f + \frac{c_t^s}{\pi_t}$ claims on consumption issued to purchase goods and to purchase $\bar{b}_t$ government bonds.

To prevent the household from running away on its within-period IOUs, the household must hold enough government bonds to secure its debt. So, the following collateral constraint must hold:

$$c_t^f + \frac{c_t^s}{\pi_t} \leq \hat{q}_t \bar{b}_t, \tag{3}$$

where $\hat{q}_t$ is the price of bonds at the end of the period. That is, if the household were to run away on its debts at the end of the period, its creditors confiscate the bonds posted as collateral by the household, each of which is worth $\hat{q}_t$ to household at the end of the period. Therefore, the collateral constraint (3) states that the household prefers paying its debts each period to running away on its debts and giving up its posted collateral. Collateral constraints such as (3) are familiar from Kiyotaki and Moore (1997), or Venkateswaran and Wright (2013), and Williamson (2016, 2018a, 2018b).

The government issues one-period debt backed by lump sum taxes and transfers, and the government's budget constraint is

$$b_t = \frac{R_{t-1}b_{t-1}}{\pi_t} + \tau_t, \tag{4}$$

for $t = 1, 2, ...$, where b_t denotes the value of government bonds issued in period t, in units of the period t flexible price good. In period 0, there is no outstanding debt, so

$$b_0 = \tau_0. \tag{5}$$

2.1 Equilibrium

In period t, the household chooses c_t^f, c_t^s, n_t^f, and $\bar{b}_t$. The labor input for production of the sticky-price good, n_t^s, is determined by the household's share of demand for the sticky-price good at market prices. From the first-order conditions for an optimum from the household's problem,

$$u'(c_t^f) = 1 + \lambda_t, \tag{6}$$

$$u'(c_t^f) = \pi_t u'(c_t^s), \tag{7}$$

$$-1 + \frac{\beta u'(c_t^f)R_t}{\pi_{t+1}} = 0, \tag{8}$$

where λ_t denotes the multiplier associated with the collateral constraint (3). Note that quasilinear utility implies that the marginal utility of wealth (the multiplier associated with the budget constraint (2)) is a constant equal to unity, – there are no wealth effects.

The price of a government bond at the end of the period, after production and consumption takes place, is

$$\hat{q}_t = \frac{\beta R_t}{\pi_{t+1}} \leq \frac{\beta u'(c_t^f)R_t}{\pi_{t+1}} = 1. \tag{9}$$

That is, $\hat{q}_t$ is the fundamental price of the government bond, as it does not reflect the payoff from using the bond as collateral. Note that the weak inequality in (9) is a strong inequality when $\lambda_t > 0$ and the collateral constraint (3) binds, from (6). That is, when $\lambda_t > 0$, there is an inefficiency in the market for flexible price goods, which is reflected in a liquidity premium on government debt in equation (8). Efficient output in each market is c^*, where $u'(c^*) = 1$, i.e. the marginal utility of consumption equals the marginal utility of wealth

In equilibrium, each household optimizes, markets clear, and the government budget constraints hold. That is, households optimize, i.e. (2), (3) and (6)-(8) hold, for $t = 0, 1, 2, ...$, the bond market clears, i.e.

$$\bar{b}_t = b_t,$$

for $t = 0, 1, 2, ...$, demand for goods is satisfied at market prices, i.e.

$$c_t^f = n_t^f, c_t^s = n_t^s,$$

for $t = 0, 1, 2, ...$, and government budget constraints are satisfied, i.e. (4) holds for $t = 1, 2, 3, ...$, and (5) holds.

Then, in equilibrium, in period t we can have one of two cases. In the first case, the collateral constraint (3) does not bind, so

$$u'(c_t^f) = 1, \tag{10}$$

from (6), equation (7) holds,

$$-1 + \frac{\beta R_t}{\pi_{t+1}} = 0,$$

from (8), and (3) holds, which from (7) and (9) gives

$$c^* + c_t^s u'(c_t^s) \leq b_t. \tag{11}$$

In the second case, the collateral constraint (3) binds, so (7) and (8) hold, and (3) holds with equality, so from (7) and (9),

$$c_t^f u'(c_t^f) + c_t^s u'(c_t^s) = b_t. \tag{12}$$

We will assume that fiscal policy sets a path for real government debt, $\{b_t\}_{t=0}^{\infty}$, and that the central bank chooses a sequence of gross nominal interest rates $\{R_t\}_{t=0}^{\infty}$ or, alternatively, chooses a policy rule for the nominal interest rate. Thus, we are assuming that the fiscal authority sets the path for real government debt exogenously, and then manipulates lump sum taxes in response to fiscal policy so as to achieve this path for government debt. Potentially, $\{b_t\}_{t=0}^{\infty}$ is suboptimal in that, under optimal fiscal policy, the collateral constraint would not bind. Our interest is in analyzing optimal monetary policy in response to potentially-suboptimal fiscal policy, not in analyzing optimal fiscal policy. That is, we want to give advice to central bankers. The particular policy rule we assume is convenient, as it is the path for the real government debt that matters in a straightforward way for the determination of equilibrium quantities and prices in the model.

2.2 Characterizing an Equilibrium

In order to understand the properties of the equilibrium, including the effects of monetary policy, it helps to make some further regularity assumptions. To that end, first define

$$f(c) \equiv cu'(c)$$

Assume that

$$f'(c) > 0, \tag{13}$$

i.e. the coefficient of relative risk aversion is less than one for all c. As well, for any (c_1, c_2) with $0 < c_1 < c_2 < c^*$, assume

$$f'(c_1) - f'(c_2) > f'(c_1)u'(c_2) - f'(c_2)u'(c_1). \tag{14}$$

Also, assume that

$$f(0) = 0. \tag{15}$$

and

$$f'(0) = \infty. \tag{16}$$

Assumptions (13)-(16) are satisfied, for example, if the coefficient of relative risk aversion of $u(c)$ is constant and less than one for all c. Assumption (13) assures that the demand for collateral is strictly increasing in consumption for both the flexible-price good and the sticky-price good, while assumptions (14) and (16) are sufficient for the optimal monetary policy problem to be well-behaved.

Before analyzing policy, we will provide a general characterization of the equilibrium. Given the equilibrium conditions, the initial gross inflation rate π_0 is indeterminate – there is nothing to tie down the initial relative price of the sticky price good. But, given π_0, either the collateral constraint (3) binds, in which case (c_0^f, c_0^s) is determined by (7) and (12), or it does not bind, so (c_0^f, c_0^s) is determined by (7) and (10). Then, from equation (8), the current gross inflation rate in the next period, π_1 is determined by R_0 and c_0^f. Then π_1 determines the period 1 consumption allocation, and R_1 and c_1^f determine π_2, etc. Thus, given monetary policy and fiscal policy, the model solves forward, given the initial gross inflation rate π_0.

We want to show that the collateral constraint binds in period t for high π_t, and does not bind for low π_t, given fiscal policy.

Proposition 1 *(i) If $b_t > c^*$, then the collateral constraint (3) binds in equilibrium in period t if and only if $\pi_t > \hat{\pi}$, and does not bind for $\pi_t \leq \hat{\pi}$, where*

$$\hat{\pi} = \frac{1}{u'(\hat{c})}, \tag{17}$$

and $\hat{c}$ is uniquely determined by

$$c^* + \hat{c}u'(\hat{c}) = b_t. \tag{18}$$

(ii) If $b_t \leq c^*$, then the collateral constraint binds in equilibrium for all $\pi_t > 0$.

Proof. If the incentive constraint (3) binds, then (12) and (7) hold with equality. Assumption (13) implies that $g'(c) > 0$, so that right-hand side of (12) is strictly increasing in c_t^f and in c_t^s. Therefore, the locus defined by (12) is strictly decreasing in (c_t^s, c_t^f) space, given π_t, and given the properties of $u(\cdot)$, the locus defined by (7) is strictly increasing in (c_t^s, c_t^f) space, given π_t. Therefore, given π_t, (12) and (7) solve uniquely for (c_t^s, c_t^f), given π_t. Further, higher (lower) π_t implies higher (lower) c_t^s and lower (higher) c_t^f. So, as $\pi_t \to 0$, the solution to (12) and (7) has $c_t^s \to 0$ and $c_t^f \to \tilde{c}$, where $\tilde{c}$ solves $\tilde{c}u'(\tilde{c}) = b_t$. If $b_t > c^*$, then $\tilde{c} > c^*$, which implies, from (6), that $\lambda_t \geq 0$ is violated, so the collateral constraint (3) cannot bind as $\pi_t \to 0$. However, if $b_t \leq c^*$, then $\tilde{c} \leq c^*$, so the collateral constraint binds as $\pi_t \to 0$. As $\pi_t \to \infty$, the solution to (12) and (7) has $c_t^f \to 0$ and $c_t^s \to \tilde{c}$, so $u'(c_t^f) \to \infty$ as $\pi_t \to \infty$, and from (6) the collateral constraint (3) binds. Therefore, by continuity, (i) and (ii) hold. ■

Proposition 1 states that, if the current government debt is sufficiently low, then the collateral constraint will bind for any inflation rate. Further, part of the proof of the proposition shows that, when the collateral constraint binds, then higher inflation causes substitution from flexible-price goods to sticky-price goods. That is, higher inflation implies a lower relative price of sticky-price goods to flexible-price goods, and our assumptions on preferences guarantee that substitution effects dominate income effects, in terms of the demand for goods. Further, higher inflation tightens the collateral constraint, i.e. the multiplier associated with the collateral constraint increases. Figure 1 shows the determination of equilibrium (c_t^s, c_t^f) given inflation, in the case where $b_t \leq c^*$. If inflation increases from π^A to π^B then the equilibrium shifts from A to B, with consumption of the sticky price good rising, and consumption of the flexible price good falling.

When the quantity of government debt is sufficiently high, i.e. $b_t > c^*$ then, as in the low-government-debt case, higher inflation causes substitution from flexible-price goods to sticky-price goods. However, there will be a critical gross inflation rate $\hat{\pi}$, below which the collateral constraint does not bind, and above which it binds. The high-government-debt case is illustrated in Figure 2. In the Figure, we have shown a case where a high inflation rate π^A implies a binding collateral constraint, and a low inflation rate π^B implies a nonbinding collateral constraint.

2.2.1 The Phillips Curve

When the collateral constraint (3) does not bind, then from (6), $c_t^f = c^*$ and, from (7), $u'(c_t^s) = \frac{1}{\pi_t}$, so higher inflation necessarily increases total output because it does not affect output of flexible price goods (production is efficient in the flexible price sector), but increases output of sticky price goods. Thus, there is a Phillips curve effect, as sticky-price producers with low relative prices produce to satisfy demand, and this demand is higher the larger is π_t, i.e. the lower is the relative price of sticky-price goods.

However, if the collateral constraint binds, then things are not so clear-cut. Dropping t subscripts, and totally differentiating (12) and (7), we can determine the derivative of total output with respect to π, when the collateral constraint

(3) binds:

$$\frac{dc^f}{d\pi} + \frac{dc^s}{d\pi} = \frac{u'(c^s)\left[u'(c^f) - u'(c^s) - c^s u''(c^s) + c^f u''(c^f)\right]}{-\pi u''(c^s)\left[u'(c^f) + c^f u''(c^f)\right] - u''(c^f)[u'(c^s) + c^f u''(c^s)]} \tag{19}$$

Then, (13) and (14) imply that the expression on the right-hand side of (19) is strictly positive if and only if $c^f < c^s$, and is strictly negative if and only if $c^f > c^s$. Therefore, from equation (7), when the collateral constraint binds total output is strictly decreasing in π for $\pi < 1$, and strictly increasing in π for $\pi > 1$.

There are therefore three cases. In Figures 3-5, y denotes total output, while π denotes the gross inflation rate. In the first case, depicted in Figure 3, $b_t > c^*$ and $\hat{\pi} > 1$, so there is a sufficient quantity of government debt that the collateral constraint binds for only very high inflation rates. Then, there is a Phillips curve for $\pi < \hat{\pi}$, but even if output and consumption has increased enough that the collateral constraint binds, for $\pi > \hat{\pi}$ there is still a Phillips curve effect. In Figure 4, $b_t > c^*$ and $\hat{\pi} < 1$, in which case we obtain a non-monotonic relationship between output and inflation. In the region where $\hat{\pi} < \pi < 1$, increases in output are associated with decreases in inflation. Finally, in Figure 5, $b_t \leq c^*$, in which case the collateral constraint always binds, and output first decreases as inflation increases, and then output increases as inflation increases.

Thus, there is a conventional Keynesian mechanism at work, which will tend to produce a conventional Keynesian Phillips curve. But when the collateral constraint binds the real quantity of government debt constrains output. In the region where the collateral constraint binds, the relationship between inflation and output is determined by the demand for collateral, and how that relates to the substitution between flexible-price and fixed price goods as inflation changes.

2.3 Optimal Monetary Policy

In this model, the central bank has the power to set the gross nominal interest rate R_t, and is assumed to treat fiscal policy, $\{b_t\}_{t=0}^{\infty}$, as given. We showed above that, given how the model solves, R_t is irrelevant for the determination of inflation and consumption allocations in periods $0, 1, 2, ..., t-1$. Thus, there is no role for forward guidance, whereby future monetary policy matters for current economic activity and inflation. From equation (8),

$$\pi_{t+1} = \beta u'(c_t^f) R_t. \tag{20}$$

which gives us the effect of monetary policy in period t on inflation in period $t+1$, given period t consumption of the flexible price good.

If the collateral constraint were not binding in period $t+1$, then from (10), (7), and (20), if the central bank maximizes period utility of the household in period $t+1$, it solves

$$\max_{\pi_{t+1}, c_{t+1}^s} \left[u(c^*) + u(c_{t+1}^s) - c^* - c_{t+1}^s\right]$$

subject to

$$u'(c^s_{t+1}) = \frac{1}{\pi_{t+1}},$$

$$\pi_{t+1} \geq \beta u'(c^f_t),$$

where the last constraint is the zero lower bound constraint, from (20). The solution to this problem is

$$\pi_{t+1} = \max\left[1, \beta u'(c^f_t)\right]$$

and from (20) the monetary policy that maximizes period $t+1$ utility is

$$R_t = \max\left[1, \frac{1}{\beta u'(c^f_t)}\right]. \tag{21}$$

But (21) is also the optimal monetary policy in period t as, so long as the collateral constraint does not bind in period $t+1$, $u'(c^f_{t+1}) = 1$ and c^f_{t+1} is maximized, so the central bank's choices of interest rates in periods $t+1$, $t+2$, ..., are not restricted by the policy choice in period t.

Things get more complicated however, if a binding collateral constraint comes into play in how the central bank sets the nominal interest rate. Suppose we consider the same problem as above, except suppose that for the range of policy choices of interest the collateral constraint binds. Then, if the central bank were to choose R_t so as to maximize period utility for the household in period $t+1$, from (7), (12), and (20) it would solve

$$\max_{\pi_{t+1}, c^f_{t+1}, c^s_{t+1}} \left[u(c^f_{t+1}) + u(c^s_{t+1}) - c^f_{t+1} - c^s_{t+1}\right] \tag{22}$$

subject to

$$u'(c^f_{t+1}) - \pi_{t+1} u'(c^s_{t+1}) = 0, \tag{23}$$

$$c^f_{t+1} u'(c^f_{t+1}) + c^s_{t+1} u'(c^s_{t+1}) = b_{t+1}, \tag{24}$$

and

$$\pi_{t+1} \geq \beta u'(c^f_t), \tag{25}$$

Then, (13) and (14) imply that, on the locus defined by (24), the value of the objective function in (22) increases (decreases) if $c^f_{t+1} > c^s_{t+1}$ ($c^f_{t+1} < c^s_{t+1}$) when c^f_{t+1} decreases and c^s_{t+1} increases. Therefore, in the case of a binding collateral constraint, the period-utility-maximizing monetary policy is given by (21). This is the same as in the case when the collateral constraint does not bind.

What is different in the case with a binding collateral constraint, is that (25) will necessarily affect monetary policy in periods leading up to the one in which the zero lower bound constraint binds. That is, suppose that the collateral constraint binds, and (25) binds in some period, i.e. $R_t = 1$ maximizes period $t+1$ utility. Also, suppose that $R_{t-1} > 1$ maximizes period t utility and the collateral constraint binds in period $t-1$. Then, a marginal reduction in R_{t-1} reduces π_t and reduces $u'(c_t^f)$. With $R_t = 1$, this reduces π_{t+1}, from (20). Therefore, utility increases in period $t+1$, but utility does not change at the margin in period t, so utility goes up for periods t and $t+1$. We can work backward and show that, if the ZLB constraint binds in any period T, so $R_T = 1$, and if the ZLB constraint does not bind and the collateral constraint binds in the previous s periods, i.e. $R_t > 1$ for $t = T-s-1, T-s, ...T-1$, then $\pi_T > 1$ and $\pi_t < 1$ for $t = T-s-1, T-s, ...T-1$.

So, if the collateral constraint is sufficiently tight that the ZLB constraint binds in any period, then the central bank sets interest rates lower in the periods leading up to this ZLB episode in which the collateral constraint binds, in such a way that inflation is lower than what would maximize period utility in those pre-ZLB periods. This contrasts with a forward guidance policy, under which the central bank commits to a future policy that would otherwise be suboptimal. In this model, if the central bank sets the current nominal interest rate lower than would be the case if the central bank were ignoring the future ZLB event, this results in lower inflation. Inflation is too high given the ZLB constraint, and reducing inflation in advance of the ZLB event will lower inflation when the ZLB binds.

2.4 Equilibria Under Optimal Monetary Policy

To understand the implications of binding collateral constraints and low real interest rates for optimal policy, it helps to consider a set of examples. What happens if the quantity of safe collateral fluctuates, so that there are recurrent ZLB episodes? Could inefficiencies persist even if safe collateral is plentiful? What happens if the ZLB constraint binds temporarily? What happens if it binds permanently? What are the inflation dynamics when the ZLB constraint binds?

2.4.1 Example 1: Alternating ZLB Episodes

In this case, suppose that $b_t = b^e$ in even periods, and $b_t = b^o$ in odd periods, and confine attention to the equilibrium in which all variables depend only on whether the period is even or odd. Further, assume that $u(c) = 2c^{\frac{1}{2}}$, which permits a closed-form equilibrium solution. Assume that $b^e < 2$ and $b^o < 2$, which is sufficient for collateral constraints to bind in every period given an optimal monetary policy. From (7), (8), and (12), letting the superscripts e and o denote variables in even and odd periods, respectively, the equilibrium consumption allocation, in terms of gross inflation rates, is

$$c^{fi} = \left(\frac{b^i}{1+\pi^i} \right)^2,$$

$$c^{si} = \left(\frac{b^i \pi^i}{1+\pi^i} \right)^2,$$

for $i = e, o$, where π^e and π^o solve

$$\pi^o = \frac{\beta R^e(1+\pi^e)}{b^e}, \tag{26}$$

and

$$\pi^e = \frac{\beta R^o(1+\pi^o)}{b^o}. \tag{27}$$

First, if $b^e \geq 2\beta$ and $b^o \geq 2\beta$, i.e. if collateral constraints are not too tight, then $\pi^o = \pi^e = 1$ is optimal, and this is supported by a monetary policy

$$R^i = \frac{b^i}{2\beta}, \tag{28}$$

for $i = e, o$, and given our assumptions the ZLB constraint does not bind at the optimum. Note, in (28), that the nominal interest rate is low (high) in periods when the quantity of safe collateral is low, i.e. when the collateral constraint is tighter. That is, a tighter collateral constraint reduces the real interest rate, implying that inflation is higher, given the nominal interest rate, so the nominal interest rate must fall to keep the inflation rate at zero.

Next, suppose that we start with $b^e \geq 2\beta$ and $b^o \geq 2\beta$, so that the policy given by (28) is optimal, with gross inflation rates determined by (26) and (27), as depicted in Figure 6 by the intersection of lines $A1$ and $B1$. So, initially, the inflation rate is zero in all periods. Then, suppose that b^e falls, holding nominal interest rates constant. This results in a shift in $B1$ to $B2$, resulting in increases in inflation rates in both even and odd periods. So now, given the initial settings for nominal interest rates, inflation is too high in both even and odd periods. This is because a lower supply of safe collateral in even periods acts to tighten the collateral constraint and reduce consumption in even periods. The real interest rate falls in even periods and, given the nominal interest rate, inflation rises in odd periods, reducing consumption of flexible price goods in odd periods, and reducing the odd period real interest rate, which increases even period inflation.

To increase welfare, then, the central bank reduces nominal interest rates in both even and odd periods. However, b^e is sufficiently low that the ZLB constraint binds in even periods. A feasible allocation is the one that achieves $\pi^e = 1$, but $\pi^o > 1$, as depicted by the intersection of lines $A2$ and $B3$. However this allocation is not optimal, as the central bank can increase welfare by reducing R^o further, shifting $A2$ to $A3$. At the optimum, $\pi^e < 1$, and $\pi^o > 1$, as monetary policy trades off distortions in the allocation of flexible-price and sticky-price goods in even and odd periods.

It is sometimes asserted (e.g. Williams 2014) that, if the real interest rate is on average low, then the ZLB will bind more frequently. Some policymakers claim that it would then be preferable to raise the inflation target, as then the ZLB would bind less frequently. In our setting, a low supply of safe collateral will indeed lower the real interest rate, and there are conditions, as illustrated in this example, under which the ZLB will bind more frequently. However, the inflation target in the example is determined optimally in this model: the optimal inflation rate is always zero. A binding ZLB constraint will result in an inflation rate that is too high when the ZLB constraint binds. The optimal response to this problem is to tolerate inflation below the inflation target in periods when the constraint does not bind.

2.4.2 Example 2: With Sufficiently Plentiful Collateral, Eliminating Both Inefficiencies Can Be Feasible in the Long Run

In this case, we will assume that $b_t = b$, a constant, for all t, and that $b \geq 2c^*$, so that $\hat{\pi} \geq 1$. Recall that $\hat{\pi}$ is defined by (17) and (18), as the gross inflation rate above which the collateral constraint binds. In this case there is sufficient government debt that the collateral constraint binds only for inflation rates greater than zero.

Given our assumptions, there exists an equilibrium in which monetary policy supports an optimal allocation from the first date, indefinitely. That is, if the central bank sets the nominal interest rate at $R_t = \frac{1}{\beta}$ for all t, then from (7), (8), and (10), one equilibrium is $\pi_t = 1$, and $c_t^f = c_t^s = c^*$ for all t. But what about all the other equilibria? We need to determine inflation dynamics for all $\pi_0 > 0$, as the initial inflation rate is indeterminate.

In any period t, given the current gross inflation rate, π_t, which is determined from previous monetary policy decisions (except in period 0, when the gross inflation rate is arbitrary), if it is feasible for the central bank to set R_t so that $\pi_{t+1} = 1$, then that is optimal. That is, from (20), if $\pi_t \leq \tilde{\pi}$ where $\tilde{\pi}$, $\tilde{c}$, and $\tilde{c}^s$ satisfy

$$u'(\tilde{c}) = \frac{1}{\beta} \tag{29}$$

$$1 = \beta \tilde{\pi}_0 u'(\tilde{c}^s) \tag{30}$$

$$f(\tilde{c}) + f(\tilde{c}^s) = b, \tag{31}$$

then the optimal monetary policy for period t, from (20), is

$$R_t = \frac{1}{\beta u'(c_t^f)},$$

Then, $\pi_{t+1} = 1 < \tilde{\pi}$, and so by induction optimal policy achieves $\pi_s = 1$, for $s = t+2, t+3, \ldots$.

But what if $\pi_t > \tilde{\pi}$? Then, the optimal policy in period t is $R_t = 1$, as that maximizes period $t+1$ utility, and relaxes constraints on future monetary policy as much as possible. This implies, from (7) and (20), that

$$\pi_{t+1} = \beta u'(c_t^s) \pi_t. \tag{32}$$

But given that $b > c^*$, from (12), $\beta u'(c_t^s) < 1$, so $\pi_{t+1} < \pi_t$. Therefore, in finite time, the gross inflation rate will pass the threshold for which the ZLB constraint binds. That is, there exists some $s \geq t+1$ such that $\pi_s \leq \tilde{\pi}$. As a result, $\pi_t = 1$ for $t = s+1, s+2, \ldots$.

We can then conclude that, given a sufficiently large quantity of safe collateral, any equilibrium path under which monetary policy is conducted optimally converges in finite time to the equilibrium in which period utility is maximized and the inflation rate is zero.

2.4.3 Example 3: Even With a Permanently Binding Collateral Constraint, Elimination of the ZLB Inefficiency Can Be Feasible in the Long Run

In this case, suppose that $b_t = b$ for all t, with $b < 2c^*$, so that the collateral constraint binds when $\pi_t = 1$, and assume that $\tilde{c} < \bar{c}$, where $\bar{c}$ solves

$$f(\bar{c}) = \frac{b}{2}.$$

As in the previous example, if $\pi_t \leq \tilde{\pi}$ for any t, then $\pi_s = 1$, for $s = t+1, t+2, ...,$ under optimal monetary policy. If $\pi_t > \tilde{\pi}$, then $R_t = 1$ is optimal, by similar arguments to Example 2, and inflation evolves according to (32). But, since $\tilde{c} < \bar{c}$, therefore if $\pi_t > \tilde{\pi}$, $\beta u'(c_t^s) < 1$, so $\pi_{t+1} < \pi_t$ under the optimal policy.

Therefore, just as in the previous case, all equilibria under optimal policy converge to an equilibrium with $\pi_t = 1$ in finite time. The only difference here from Example 2 is that the collateral constraint binds in the long run.

2.4.4 Example 4: Collateral Constraint and ZLB Can Both Bind in the Long Run

In this case, we will make the same assumptions as in the case in the previous subsection – Example 3 – except that $\tilde{c} \geq \bar{c}$ and $f(\tilde{c}) < b$. In this case, there is a steady state in which $c_t^s = \tilde{c}$, and $c_t^f = \tilde{c}^f < \tilde{c}$, where, from (12), $\tilde{c}^f$ solves

$$f(\tilde{c}^f) + f(\tilde{c}) = b. \tag{33}$$

Since $\tilde{c}^f < \tilde{c}$, therefore in the steady state $\pi_t > \tilde{\pi}_0$, so the ZLB constraint binds. This is a steady state as, under the ZLB, inflation evolves according to (32). The steady state inflation rate, from (20) is

$$\pi^{ss} = \beta u'(\tilde{c}^f) > 1. \tag{34}$$

We can then show, by totally differentiating (7), (12), and (20), given $R_t = 1$, that

$$\frac{d\pi_{t+1}}{d\pi_t} = \frac{\beta u''(c_t^f)u'(c_t^s)f'(c_t^s)}{f'(c_t^f)\pi_t u''(c_t^s) + u''(c_t^f)f'(c_t^s)}.$$

So,

$$0 < \frac{d\pi_{t+1}}{d\pi_t} < 1.$$

Therefore, for any initial inflation rate π_0, inflation converges monotonically to the steady state.

So, in this case, the collateral constraint is tight enough that the ZLB constraint must bind in the long run.

2.4.5 Example 5: With A Very Tight Collateral Constraint, Inflation Increases Without Bound

In this case, we make the same assumptions as in Example 4, except that $b < f(\tilde{c})$, so now the steady state in Example 4 is not attainable. This implies that $\pi_t > \tilde{\pi}_0$, $c_t^f < \tilde{c}$, and $c_t^s < \tilde{c}$ for all t. Therefore,

$$\pi_{t+1} = \beta u'(c_t^s) R_t \pi_t,$$

so since $\beta u'(c_t^s) R_t > \beta u'(\tilde{c}) > 1$, therefore $\pi_{t+1} > \pi_t$ for $t = 0, 1, 2, ...$. Therefore, in all equilibria inflation increases without bound.

So, if the economy is extremely short of safe collateral, implying an extremely low real interest rate, then inflation increases indefinitely. In this economy, deflationary black holes are not a possibility, but hyperinflations are, under extremely tight collateral constraints.

2.5 A Period of Tight Collateral Constraints Followed By Relaxed Constraints

This experiment is in the spirit of the scenario Werning (2011) considers, according to which the real interest rate is low for an extended period, and optimal monetary policy will imply a zero nominal interest rate for that extended period. After this extended period at the ZLB, the real interest rate rises permanently. How should monetary policy be conducted after the low real interest rate period ends?

Here, we will assume that $b_t = b^l$ for $t = 0, 1, 2, ..., T$, and $b_t = b^h$ for $t = T+1, T+2, T+3, ...$. First, assume that the period when $b_t = b^l$ looks like example 4 above, so $\tilde{c} \geq \bar{c}$ and $f(\tilde{c}) < b^l$. Therefore, for the period $t = 0, 1, 2, ..., T$, the collateral constraint must bind, and if $b_t = b^l$ were permanent, under optimal monetary policy the economy would converge to a steady state with the nominal interest rate at zero and inflation above zero.

Also, assume that $b^h \geq 2c^*$, so once the quantity of safe collateral rises in period $T+1$, we have conditions as in Example 2, so that optimal monetary policy from date $T+1$ forward will imply convergence in finite time to a long run steady state in which $\pi_t = 1$ and neither the ZLB constraint nor the collateral constraint binds.

It is straightforward to put Example 2 together with Example 4, to derive results for this scenario. First, suppose we focus on the steady state equilibrium under the tight collateral constraint as the equilibrium up to period T. This implies that, for periods $t = 0, 1, 2, 3, ..., T$, given optimal monetary policy, $R_t = 1$ (the ZLB binds),

$$c_t^s = \tilde{c},$$

and, from (34) and (33),

$$c_t^f = \tilde{c}^f < \tilde{c},$$

where $\tilde{c}^f$ solves

$$f(\tilde{c}^f) + f(\tilde{c}) = b^l, \tag{35}$$

The gross inflation rate for periods $t = 0, 1, 2, 3, ..., T$ is then

$$\pi^l = \beta u'(\tilde{c}^f) > 1. \tag{36}$$

In period $T+1$, the supply of safe collateral increases, though the optimal policy in period T is the ZLB policy $R_T = 1$, and in period $T+1$ inflation remains high, i.e. $\pi_{T+1} = \beta u'(\tilde{c}^f) = \pi^l$. Because of the increase in government debt to b^h, consumption of both flexible-price and fixed-price goods must increase in period $T+1$, so total output increases. Then, we simply follow Example 2 to determine monetary policy from period $T+1$ onward.

Over the period $t = T+1, T+2, ...$, the goal of monetary policy is to reduce inflation as quickly as possible to $\pi_t = 1$. From Example 2, we know that the optimal policy is determined by the current inflation rate, which is given by history. Specifically, if $\pi_t \geq \tilde{\pi}$, then $R_t = 1$ is optimal, and $\pi_{t+1} = \beta u'(c_t^f)$. However, once $\pi_t \leq \tilde{\pi}$, then $R_t = \frac{1}{\beta u'(c_t^f)} < \frac{1}{\beta}$, and $\pi_{t+1} = 1$. From (29), (30), and (31), since $\pi_{T+1} = \pi^l > \tilde{\pi}$, therefore $R_T = 1$ at the optimum. That is, there is at least one period in the high-safe-collateral period when the ZLB binds, and one and only one period after that when the ZLB does not bind, but $R_t < \frac{1}{\beta}$, so the nominal interest rate is lower than its steady state value.

Here, a low supply of safe collateral produces an episode during which the collateral constraint binds, the real interest rate is low, and monetary policy is constrained by the zero lower bound. In contrast to conventional analysis of ZLB episodes, e.g. Werning (2012), in our model the ZLB episode is reflected in inflation above the central bank's target. Just as in Werning's model, output is low during the ZLB episode, and optimal monetary policy is to extend the period of low interest rates beyond the period when the cause of the tight collateral constraint goes away. In our model, however, the ZLB problem does not get worse the longer is the ZLB episode – under the tight collateral constraint the economy will converge to a unique steady state. Also, the reason the nominal interest rate is low after the low-safe-collateral period is that this brings inflation down quickly to the inflation target, relaxes the collateral constraint, and corrects the sticky-price inefficiency.

3 Money, Collateral, and Credit

The next step is to expand the model to include a richer set of assets. This will allow for retail payments using currency, and for a more explicit treatment of monetary policy, in that open market operations are required to support the central bank's interest rate policy. Our interest is in showing how these model elements make a difference for how monetary policy should deal with low-real-interest rate episodes.

As a result of the changes to the model, there will potentially be three distortions to be concerned with: (i) a standard Friedman-rule distortion under which there is a suboptimally low quantity of currency, in real terms; (ii) a shortage of interest-bearing debt, reflected in a low real rate of interest; (iii) a sticky price friction. To understand how this version of the model works, it will help to first consider a setup with flexible prices, which includes only the first two distortions. After we have done that analysis, we will consider the sticky price case, which includes all three distortions.

3.1 Flexible Prices

This case will work in a manner similar to Andolfatto and Williamson (2015), though a key difference from that work is in the role that government debt plays in the model. In particular, in this model government debt serves as collateral rather than being traded directly, as in Andolfatto and Williamson (2015).

We need to be explicit about how exchange works. Assume that a household consists of a continuum of consumers with unit mass, and a producer. Each consumer in the household has a period utility function $u(c_t)$, and there are two markets on which goods are sold. In the *cash-only market*, sellers of goods accept only money, as there is no technology available to verify collateral if the consumer attempts to make a credit transaction. In the *cash-and-credit market*, sellers are able to verify the ownership of government debt posted as collateral in a credit transaction, and sellers will also accept money. Unsecured credit is not a possibility in purchasing goods, as the memory (recordkeeping) needed to support this does not exist. The household would always default on unsecured credit, so none is extended. Each consumer in a household receives a shock which determines the market he or she participates in. With probability θ the consumer goes to the cash-only market, and with probability $1-\theta$, he or she goes to the cash-and-credit market. The household allocates assets to each consumer in the household – money and any government debt to be posted as collateral – and consumers consume on the spot in the markets where they arrive. That is, consumption cannot be shared within the household.

The producer in the household supplies labor n_t and, as in the cashless model, can produce one unit of output for each unit of labor input. Output is perfectly divisible and can be sold on either the cash-only market or the cash-and-credit market, or both.

A household maximizes

$$\sum_{t=0}^{\infty} \beta^t \left[\theta u(c_t^m) + (1-\theta)u(c_t^b) - n_t,\right]$$

where c_t^m denotes the consumption of each consumer who goes to the cash-only market, while c_t^b is consumption of each consumer in the cash-and-credit market.

At the beginning of the period, the household trades on the asset market and faces the constraint

$$b_t + \theta c_t^m + m_t' \leq \frac{m_{t-1} + R_{t-1}b_{t-1}}{\pi_t} + \tau_t. \tag{37}$$

On the right-hand side of inequality (37), the household has wealth at the beginning of the period consisting of the payoffs on currency and bonds held over from the previous period and the lump-sum transfer from the fiscal authority. Here, m_{t-1} denotes beginning-of-period currency balances in units of the period $t-1$ cash market consumption good. The left-hand side of (37) includes purchases of one-period nominal government bonds, currency (in units of the period t cash market good) the household requires for cash market goods purchases, and currency, m_t', that is sent with consumers to the cash-and-credit market.

In the cash-and-credit market, consumers from the household can purchase goods with currency m_t', or with credit secured by government debt, so the following constraint must hold:

$$(1-\theta)c_t^b \leq R_t b_t + m_t'. \tag{38}$$

In inequality (38), note that the IOUs issued by the household (by way of consumers in the household) are settled at the end of the period, at which time the bonds the household acquired at the beginning of the period are worth $R_t b_t$. That is, at the end of the period, the forthcoming payoffs on government bonds are equivalent to cash. Inequality (38) states that, for cash-and-credit purchases in excess of what is paid for with cash, the household will prefer to pay its debt at the end of the period rather than face seizure of the bonds posted as collateral.

Finally, the household must satisfy its budget constraint

$$\theta c_t^m + (1-\theta)c_t^b + b_t + m_t \leq n_t + \frac{m_{t-1} + R_{t-1}b_{t-1}}{\pi_t} + \tau_t. \tag{39}$$

We can summarize the first order conditions from the household's problem with the following three equations:

$$u'(c_t^b)R_t = u'(c_t^m), \tag{40}$$

$$1 = \underbrace{\beta\left[\frac{u'(c_{t+1}^m)}{u'(c_t^m)\pi_{t+1}}\right]}_{\text{fundamental}} + \underbrace{\frac{u'(c_t^m)-1}{u'(c_t^m)}}_{\text{liquidity premium}}, \tag{41}$$

$$\frac{1}{R_t} = \underbrace{\beta\left[\frac{u'(c_{t+1}^m)}{u'(c_t^m)\pi_{t+1}}\right]}_{\text{fundamental}} + \underbrace{\frac{u'(c_t^b)-1}{u'(c_t^m)}}_{\text{liquidity premium}}. \tag{42}$$

First, (40) reflects intratemporal optimization. Note that the price of goods purchased on the cash market relative to the price of goods on the cash-and-credit market is R_t, which is also the gross nominal interest rate. Second, equations (41) and (42) have been written so as to show the similarities in asset pricing between money and government debt, respectively. In equation (41), the left-hand side is the current price of money, normalized, while the right hand side consists of the fundamental and a liquidity premium. The fundamental is the expected payoff on money in the next period, appropriately discounted, while the liquidity premium is related to the inefficiency in the market for goods purchased with cash. That is, $u'(c_t^m)-1$ is an inefficiency wedge in this market. The liquidity premium on money is something we observe in most mainstream monetary models, and it typically disappears if the central bank runs a Friedman rule. Similarly, in equation (42), the price of government debt, appropriately normalized, on the left-hand side, is equal to the sum of a fundamental plus a liquidity premium, on the right-hand side. The fundamental is identical to the one in equation (41), since the explicit payoff on the asset is the same as for money (appropriately normalized). But government debt has a different liquidity premium, which is related to the inefficiency wedge, $u'(c_t^b) - 1$, in the cash-and-credit market.

The consolidated government budget constraints are:

$$m_0 + b_0 = \tau_0, \tag{43}$$

$$m_t + b_t - \frac{m_{t-1} + R_{t-1}b_{t-1}}{\pi_t} = \tau_t, \tag{44}$$

where m_{t-1} denotes the real quantity of currency outstanding at the beginning of period t, before government intervention occurs, in units of the period $t-1$ cash market good. Here, we will assume that the fiscal authority fixes exogenously the path for the real value of the consolidated government debt, i.e.

$$v_t = m_t + b_t, \tag{45}$$

where v_t is exogenous. This follows the approach of Andolfatto and Williamson (2015) and Williamson (2016, 2018a, 2018b), and is an extension of our treatment of fiscal policy in the cashless version of the model. Here, the fiscal policy rule whereby v_t is exogenous provides a nice demarcation between fiscal and monetary policy: fiscal policy determines the total real quantity of consolidated government debt, and monetary policy determines its composition. As in the cashless model, the fiscal authority is assumed to set an endogenous path for lump sum taxes to achieve a particular path for consolidated government debt, given monetary policy. Further, the path for v_t may be suboptimal, just as in the cashless model.

Solving for an equilibrium, in any period t, (40) and (41) hold and either

$$u'(c_t^b) = 1$$

and

$$\theta c_t^m + \frac{(1-\theta)c_t^b}{R_t} \leq v_t$$

or

$$u'(c_t^b) > 1$$

and

$$\theta c_t^m + \frac{(1-\theta)c_t^b}{R_t} = v_t$$

Thus, in period t, either exchange is efficient in the cash-and-credit market and the collateral constraint (38) does not bind, or exchange is inefficient in the cash-and-credit market and the collateral constraint binds.

3.1.1 Optimality

Note that the model solves period-by-period for c_t^m and c_t^b, and thus for labor supply, with output equal to consumption in equilibrium,

$$n_t = \theta c_t^m + (1-\theta)c_t^b.$$

Letting c^* denote the solution to $u'(c^*) = 1$, as in the cashless model, if

$$v_t \geq c^*,$$

then $R_t = 1$ at the optimum – the nominal interest rate is zero – and $c_t^m = c_t^b = c^*$. This is essentially a Friedman rule result. If the collateral constraint does not bind, then exchange is efficient in the cash-and-credit market. Therefore, if $R_t = 1$, and the collateral constraint does not bind, exchange is efficient in both markets in period t.

However, if

$$v_t < c^*, \tag{46}$$

then the household's collateral constraint binds for $R_t = 1$. When the collateral constraint binds, then

$$\theta c_t^m + \frac{(1-\theta)c_t^b}{R_t} = v_t, \tag{47}$$

$$u'(c_t^b)R_t = u'(c_t^m), \tag{48}$$

and equations (47) (48) solve for (c_t^m, c_t^b) given policy (v_t, R_t). It will prove convenient, for purposes of comparison with the previous model, and when we include sticky prices in this model, to rewrite (47) using (48), obtaining

$$\theta c_t^m u'(c_t^m) + (1-\theta)c_t^b u'(c_t^b) = v_t u'(c_t^m) \tag{49}$$

Then, totally differentiate (49) and (48), and drop t subscripts for convenience, to get

$$\frac{dc^m}{dR} = \frac{(1-\theta)u'(c^b)\left[u'(c^b) + c^b u''(c^b)\right]}{\begin{array}{c}\left\{\theta\left[u'(c^m) + c^m u''(c^m)\right] - vu''(c^m)\right\} Ru''(c^b) \\ +(1-\theta)\left[u'(c^b) + c^b u''(c^b)\right] u''(c^m)\end{array}}, \tag{50}$$

$$\frac{dc^b}{dR} = \frac{-u'(c^b)\left\{\theta\left[u'(c^m) + c^m u''(c^m)\right] - vu''(c^m)\right\}}{\begin{array}{c}\left\{\theta\left[u'(c^m) + c^m u''(c^m)\right] - vu''(c^m)\right\} Ru''(c^b) \\ +(1-\theta)\left[u'(c^b) + c^b u''(c^b)\right] u''(c^m)\end{array}}, \tag{51}$$

Recall that we have assumed that $f'(c) > 0$, where $f(c) = cu'(c)$, so we can sign both derivatives. That is, $\frac{dc^m}{dR} < 0$ and $\frac{dc^b}{dR} > 0$, so that an increase in the nominal interest rate target of the central bank, given fiscal policy v, causes substitution from cash market transactions to cash-and-credit market transactions.

It is important to recognize that in this economy, in contrast to the one in the previous section, the central bank's interest rate policy must be supported by appropriate open market operations. That is, the fiscal authority sets v, the total quantity of consolidated government debt, and the central bank swaps outside money for government debt so as to supply the private sector with the appropriate mix of consolidated-government liabilities. This appropriate mix will support desired transactions by the private sector at the nominal interest rate target chosen by the central bank.

So, our results tell us that activity in the cash market declines, and activity in the cash-and-credit market increases as the nominal interest rate increases. Therefore, the collateral constraint is relaxed, i.e. the degree of inefficiency in the cash-and-credit market, $u'(c^b) - 1$, falls, as R increases. We can then conclude that the collateral constraint binds if and only if it binds for $R = 1$, i.e. if and only if (46) holds.

But, suppose (46) holds. Do conditions exist under which the collateral constraint will not bind? That is, does there exist a critical nominal interest rate $\tilde{R}$ such that, if (46) holds and $R < \tilde{R}$, then the collateral constraint binds, and if $R \geq \tilde{R}$ it does not bind? If $\tilde{R}$ exists, then from (48) and (49), $\tilde{R}$ and $\tilde{c}$ solve the following two equations:

$$u'(\tilde{c}) = \tilde{R} \tag{52}$$

$$\theta \tilde{c} u'(\tilde{c}) + (1-\theta)c^* = vu'(\tilde{c}) \tag{53}$$

where $\tilde{c}$ is the quantity of cash-market consumption when the collateral constraint just binds. It is straightforward to show that, given (46), there always exists a unique solution to (52) and (53) with $1 < \tilde{R} < \infty$, and $0 < \tilde{c} < c^*$. We can then conclude that the binding collateral constraint – and the low real interest rate that goes with it – is a low-*nominal*-interest-rate phenomenon. Given v, the central bank can relax the collateral constraint and eliminate the liquidity premium on government debt if if raises the nominal interest rate sufficiently.

What about the welfare consequences of monetary policy when the collateral constraint binds at $R = 1$ and the real interest rate is low for low nominal interest rates? Do such conditions imply that the central bank should adopt a zero lower bound policy, as in the New Keynesian literature?

We can evaluate welfare period-by-period, with the period utility of the household equal to $W(R)$, where

$$W(R) = \theta\left[u(c^m) - c^m\right] + (1-\theta)\left[u(c^b) - c^b\right], \tag{54}$$

and c^m and c^b are determined by (48) and (49), dropping t subscripts. Then, using (50) and (51), we get

$$W'(R) = \frac{\begin{array}{c}\theta(1-\theta)u'(c^b)\left\{\begin{array}{c}\left[u'(c^m)-1\right]\left[u'(c^b)+c^b u''(c^b)\right] \\ -\left[u'(c^b)-1\right]\left[u'(c^m)+c^m u''(c^m)\right]\end{array}\right\} \\ +u'(c^b)(1-\theta)vu''(c^m)\left[u'(c^b)-1\right]\end{array}}{\begin{array}{c}\left\{\theta\left[u'(c^m)+c^m u''(c^m)\right] - vu''(c^m)\right\} Ru''(c^b) \\ +(1-\theta)\left[u'(c^b)+c^b u''(c^b)\right]u''(c^m)\end{array}}$$

When $R=1$, then from (47) and (48), we have $c^m = c^b = v$. Therefore, from (54), we get

$$W'(1) = (1-\theta)v\left[u'(v) - 1\right] > 0$$

so increasing the nominal interest rate above zero is optimal for the central bank if the collateral constraint binds when the nominal interest rate is zero. As well, from (54),

$$W'(\tilde{R}) = \frac{\theta(1-\theta)\left\{\left[\tilde{R}-1\right]\left[1+c^*u''(c^*\right]\right\}}{\begin{array}{c}\left\{\theta\left[u'(\tilde{c}) + \tilde{c}u''(\tilde{c})\right] - vu''(\tilde{c})\right\}\tilde{R}u''(c^*) \\ +(1-\theta)\left[1+c^*u''(c^*)\right]u''(\tilde{c})\end{array}} < 0.$$

Therefore, either v_t is sufficiently large that the collateral constraint does not bind for any $R \geq 1$, in which case $R=1$ is optimal, or v_t is low, the collateral constraint binds for $R_t \in [1, \tilde{R})$, and the optimal setting the nominal interest rate has the property $R_t \in (1, \tilde{R})$.

Therefore, so long as the supply of consolidated government debt is sufficiently low, the central bank should conduct open market operations so as to raise the nominal interest rate above zero, but it is not optimal for the central bank to entirely relieve the safe asset shortage, even though it can. When v_t and R_t are low enough, the collateral constraint binds, and the real interest rate is low. That is, if we price a real bond, which has the same characteristics as nominal bonds – in particular it can be posted as collateral in the cash-and-credit market – then the gross real interest rate is given by

$$r_t = \frac{u'(c_t^m)}{\beta u'(c_t^b)u'(c_{t+1}^m)},$$

so the factor $\frac{1}{u'(c_t^b)}$ captures the liquidity premium on the government debt. The greater the inefficiency in the cash-and-credit market, the higher is $u'(c_t^b)$ and the lower is the real interest rate. Then, our results above tell us that, for example in a stationary equilibrium in which $v_t = v$ and $R_t = R$ forever, a permanent increase in R lowers the liquidity premium, and raises the real interest rate – permanently.

Further, from (41), in equilibrium current inflation depends only on current consumption in the cash market, i.e.

$$\pi_t = \beta u'(c_t^m),$$

so from our results above, in a stationary equilibrium with $v_t = v$ and $R_t = R$ forever, a permanent increase in R will lower consumption in the cash market and raise inflation – a Fisher effect. But the Fisher effect is not one-for-one, as the real interest rate rises when the nominal interest rate rises, as long as the collateral constraint binds.

We obtain these results because the low real interest rate is caused by a shortage of safe collateral – a shortage of government debt. In the context of such a shortage, an increase in the nominal interest rate is accomplished if the central bank swaps government debt for money, thus relieving the collateral shortage. While this has the effect of making cash more scarce, so that the inefficiency wedge goes up in the cash-only market, the decrease in efficiency in this respect is more than offset by an increase in efficiency in the cash-and-credit market, evaluated in welfare terms.

It is useful to note the difference between the results in this section and the typical properties of New Keynesian models, for example Werning (2011). In Werning (2011), with a standard sticky price distortion, the nominal interest rate should be above zero when the "natural" real interest rate is high, and the nominal interest rate should go to zero when the natural real interest rate is sufficiently low. But, in this model with a standard monetary distortion, and a further friction caused by the safe asset friction, the nominal interest rate should be zero when the real interest rate is high, and the nominal interest rate should be above zero when the real interest rate is low.

There are also important differences between the policy conclusions in this model, and those from the cashless model we started with. Recall that the cashless model, in which the frictions are sticky prices and a potential scarcity of safe collateral, implies that the central bank's response to tight collateral constraints is to set the nominal interest rate to zero. As well, the nominal interest rate should be kept low for a period after the collateral scarcity goes away. In this model relative to the cashless model, we have added details of monetary exchange and taken out sticky prices, which essentially reverses the result – tight collateral constraints imply that the nominal interest rate should be off the ZLB. So, what explains the difference? Is it the details of exchange, or is it sticky prices? To answer that question, we need to put in sticky prices, as in the next subsection.

3.2 Sticky Prices

Next, we will extend our model of money and credit to include sticky prices, as in the baseline model. Assume, as in the previous subsection, that there exists a continuum of consumers in each household. Each period, an individual consumer in a household receives a shock that determines whether he or she receives utility from flexible-price or sticky-price goods. With probability $\frac{1}{2}$ the consumer gets utility only from the flexible price good, and with probability $\frac{1}{2}$ the consumer receives utility only from the sticky price good. As well, goods are sold in the cash-only market, and the cash-and-credit market. Each consumer in a household receives a shock each period determining their goods market participation. With probability θ the consumer goes to the cash-only market, and with probability $1-\theta$, he or she goes to the cash-and-credit market. Further, the preference shock and the shock determining market participation are independent for an individual consumer and are also independent across consumers.

On the production side, households can choose the quantities of flexible price goods to supply in each market. However, as in the cashless model with sticky prices, the demand for sticky price goods is distributed uniformly among households, and each household must supply the quantity of sticky price goods demanded at market prices. As above, one unit of labor supply produces one unit of any good sold in any market.

A household then maximizes

$$\sum_{t=0}^{\infty}\beta^t\left\{\frac{\theta}{2}\left[u(c_t^{mf})+u(c_t^{ms})\right]+\frac{(1-\theta)}{2}\left[u(c_t^{bf})+u(c_t^{bs})\right]-(n_t^f+n_t^s)\right\}. \quad (55)$$

Thus, there are now four different goods: c_t^{mf} (c_t^{ms}) denotes consumption of flexible-price (sticky-price) goods that are purchased in the cash-only market, while c_t^{bf} (c_t^{bs}) denotes consumption of flexible-price (sticky-price) goods purchased in the cash-and-credit market. At the beginning of the period, the household faces a financing constraint

$$b_t+\frac{\theta}{2}\left[c_t^{mf}+\frac{c_t^{ms}}{\pi_t}\right]+m_t'\leq\frac{m_{t-1}+R_{t-1}b_{t-1}}{\pi_t}+\tau_t. \quad (56)$$

The constraint (56) is a modification of (37) that includes both flexible-price and sticky-price goods purchases using cash on the left-hand side. Similarly, we can adapt (38) to include flexible-price and sticky-price goods so that the household's collateral constraint in the cash-and-credit market is

$$\frac{(1-\theta)}{2}\left[c_t^{bf}+\frac{c_t^{bs}}{\pi_t}\right]\leq m_t'+R_tb_t. \quad (57)$$

Finally, the household's budget constraint is

$$\begin{aligned}&b_t+\frac{\theta}{2}\left[c_t^{mf}+\frac{c_t^{ms}}{\pi_t}\right]+\frac{(1-\theta)}{2}\left[c_t^{bf}+\frac{c_t^{bs}}{\pi_t}\right]+m_t \quad (58)\\ \leq\ &\frac{m_{t-1}+R_{t-1}b_{t-1}}{\pi_t}+\tau_t+n_t^f+\frac{n_t^s}{\pi_t}\end{aligned}$$

The government's budget constraints are the same as in the flexible-price version of the model, i.e. (43) and (44) hold. As well, the fiscal authority follows the rule (45), i.e. the real value of the consolidated government debt is set exogenously at v_t in period t.

Given optimization and market clearing, we can characterize an equilibrium as follows. In each period, from the first-order conditions for the household's problem, the following hold:

$$1=\beta\left[\frac{u'(c_{t+1}^{mf})}{\pi_{t+1}}\right], \quad (59)$$

$$\pi_tu'(c_t^{ms})=u'(c_t^{mf}), \quad (60)$$

$$u'(c_t^{bf})R_t=u'(c_t^{mf}), \quad (61)$$

$$\pi_tR_tu'(c_t^{bs})=u'(c_t^{mf}), \quad (62)$$

Here, (59) is essentially an asset-pricing equation for currency, and (60)-(62) reflect intratemporal optimization with regard to the four goods in this economy. The relevant relative prices are π_t, the relative price of flexible price goods in terms of sticky price goods, and R_t, the relative price of flexible price cash goods in terms of flexible price cash-and-credit goods.

As well, either the collateral constraint does not bind, so

$$\frac{1}{R_t} = \beta \left[\frac{u'(c_{t+1}^{mf})}{u'(c_t^{mf})\pi_{t+1}} \right], \tag{63}$$

$$u'(c_t^{bf}) = 1, \tag{64}$$

(government bonds are priced at their fundamental price and the flexible-price cash-and-credit market is efficient) and

$$\theta \left[c_t^{mf} + \frac{c_t^{ms}}{\pi_t} \right] + \frac{(1-\theta)}{R_t} \left[c_t^{bf} + \frac{c_t^{bs}}{\pi_t} \right] \leq 2v_t \tag{65}$$

(the demand for collateral does not exceed the supply).

Alternatively,

$$\frac{1}{R_t} = \frac{u'(c_t^{bf}) - 1}{u'(c_t^{mf})} + \beta \left[\frac{u'(c_{t+1}^{mf})}{u'(c_t^{mf})\pi_{t+1}} \right], \tag{66}$$

$$u'(c_t^{bf}) > 1, \tag{67}$$

(government bonds are priced above their fundamental value and the flexible-price cash-and-credit market is inefficient) and

$$\theta \left[c_t^{mf} + \frac{c_t^{ms}}{\pi_t} \right] + \frac{(1-\theta)}{R_t} \left[c_t^{bf} + \frac{c_t^{bs}}{\pi_t} \right] = 2v_t. \tag{68}$$

(the demand for collateral is equal to the supply).

Thus, in period t, the collateral constraint (57) may not bind, in which case (63) holds – government debt sells at its fundamental price, the appropriately discounted value of the payoff stream on the asset – and (65) holds in equilibrium, i.e. the value of the consolidated government debt is large enough to finance all consumption purchases. Alternatively, (57) binds, so that there is a liquidity premium on government debt, reflected in a tight collateral constraint and a resulting inefficiency in the market for flexible price goods in the cash-and-credit market (inequality (67)). As well, in (68), the value of consolidated government debt is just sufficient to purchase all goods.

The initial inflation rate π_0 is indeterminate (the initial price, in units of money, of sticky price goods is arbitrary), but otherwise the model solves period-by-period. So, dropping t subscripts for convenience, and given fiscal and monetary policy (v, R), an unconstrained equilibrium in which the collateral constraint does not bind is determined, using (59)-(62), and (64), by

$$u'(c^{mf}) = R, \tag{69}$$

$$u'(c^{ms}) = \frac{1}{\beta}, \tag{70}$$

$$u'(c^{bf}) = 1, \tag{71}$$

$$u'(c^{bs}) = \frac{1}{R\beta}, \tag{72}$$

So, (69)-(72) determines the consumption allocation $(c^{mf}, c^{ms}, c^{bf}, c^{bs})$. The consumption allocation depends only on the current nominal interest rate, and not on v, as of course the collateral constraint does not bind. Recall that a particular market is efficient if $u'(c) = 1$ in that market, where c is consumption of that market's good. So, from (69), the distortion in the market for flexible price goods purchased with currency increases with the nominal interest rate, as is standard in monetary models. From (70), there is a distortion in the market for sticky-price goods purchased with currency, and this distortion is unaffected by monetary policy, because inflation distorts markets for all goods purchased with money in the same way, given the Fisher effect on the nominal interest rate. That is, from (59) and (69), in an unconstrained equilibrium,

$$R = \frac{\pi}{\beta}, \tag{73}$$

and the real interest rate is a constant, equal to the subjective rate of time preference. From (71), the market in flexible price goods purchased with credit is efficient, as the collateral constraint does not bind. Finally, (72) states that consumption in the market for sticky price goods consumption purchased with credit depends on the nominal interest rate. A higher nominal interest rate lowers the relative price of such goods, so that more are consumed – consumption in this market is demand-determined.

An unconstrained equilibrium exists if and only if the solution $(c^{mf}, c^{ms}, c^{bf}, c^{bs})$ to (69)-(72), which is unique, satisfies the collateral constraint, which we can write, using (59)-(62), (65), and (69)-(72) as

$$\theta c^{mf} R + \frac{\theta \tilde{c}}{\beta} + (1-\theta)c^* + \frac{(1-\theta)c^{bs}}{R\beta} \leq 2Rv, \tag{74}$$

where $\tilde{c}$ is defined as in the cashless model, by $u'(\tilde{c}) = \frac{1}{\beta}$.

Similarly, in a constrained equilibrium, $(c^{mf}, c^{ms}, c^{bf}, c^{bs})$ solves

$$u'(c^{bf})R = u'(c^{mf}), \tag{75}$$

$$\theta c^{mf} u'(c^{mf}) + \frac{\theta \tilde{c}}{\beta} + (1-\theta)c^{bf} u'(c^{bf}) + \frac{(1-\theta)c^{bs}}{R\beta} = 2u'(c^{mf})v, \tag{76}$$

and equations (70) and (72). It is straightforward to show that (75) and (76) solve uniquely for c^{mf} and c^{bf}, and of course (70) and (72) solve uniquely for c^{ms} and c^{bs}, respectively. The gross inflation rate, from (59), is given by

$$\pi = \beta u'(c^{mf}) = \beta R u'(c^{bf}),$$

so the tighter is the collateral constraint, the larger is $u'(c^{bf})$, which makes the real interest rate lower and the inflation rate higher. If this unique solution for the consumption allocation satisfies

$$u'(c^b) > 1, \tag{77}$$

then it is a constrained equilibrium.

It will be illuminating at this juncture to deal with a more restrictive specification, i.e. a constant relative risk aversion (CRRA) utility function. We want to answer two questions. First, for what policies (v, R) does the collateral constraint bind, and how is this affected by other parameters – risk aversion and θ, in particular? Second, under what conditions, if any, is a zero lower bound monetary policy, i.e. $R = 1$, optimal?

3.2.1 Constrained or Unconstrained Equilibrium?

Assume a constant CRRA utility function, that is $u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}$, with $0 < \alpha < 1$, in line with our maintained assumptions. This appears not to restrict the behavior of the model in any important ways, and helps us make the key points in a transparent fashion. Then, in an unconstrained equilibrium, from (69)-(72), the equilibrium consumption allocation is

$$(c^{mf}, c^{ms}, c^{bf}, c^{bs}) = (R^{-\frac{1}{\alpha}}, \beta^{\frac{1}{\alpha}}, 1, R^{\frac{1}{\alpha}}\beta^{\frac{1}{\alpha}}) \tag{78}$$

and the gross inflation rate is

$$\pi = \beta R.$$

The unconstrained equilibrium exists if and only if this solution satisfies (74), that is

$$\sigma(R) \leq 2v, \tag{79}$$

where

$$\sigma(R) = \theta \left[R^{-\frac{1}{\alpha}} + \beta^{\frac{1}{\alpha}-1}R^{-1}\right] + (1-\theta)\left[R^{-1} + \beta^{\frac{1}{\alpha}-1}R^{\frac{1}{\alpha}-2}\right]. \tag{80}$$

Similarly, in a constrained equilibrium, from (70), (72), (75), and (76),

$$(c^{ms}, c^{bs}) = (\beta^{\frac{1}{\alpha}}, R^{\frac{1}{\alpha}}\beta^{\frac{1}{\alpha}}) \tag{81}$$

and (c^{mf}, c^{bf}) is the recursive solution to

$$\left[\left(c^{mf}\right)^{1-\alpha} + \beta^{\frac{1}{\alpha}-1}\right]\left[\theta + (1-\theta)R^{\frac{1}{\alpha}-1}\right] = 2v\left(c^{mf}\right)^{-\alpha}, \tag{82}$$

and

$$c^{bf} = c^{mf}R^{\frac{1}{\alpha}}. \tag{83}$$

The consumption allocation determined by (81)-(83) is unique, and the solution is an equilibrium if and only if (77) is satisfied or, from (83), $c^{mf} < R^{-\frac{1}{\alpha}}$. From (82), this is equivalent to

$$\sigma(R) > 2v. \tag{84}$$

Therefore, from (80) and (84), an equilibrium always exists, and it is either a constrained or an unconstrained equilibrium. Further, from our analysis above, the equilibrium is unique. The key to determining the conditions under which the collateral constraint binds or does not, is uncovering the properties of the function $\sigma(R)$. Differentiating the left-hand side of (79), we get

$$\sigma'(R) = \theta\left[-\frac{1}{\alpha}R^{-\frac{1}{\alpha}-1} + -\beta^{\frac{1}{\alpha}-1}R^{-2}\right] + (1-\theta)\left[-R^{-2} - \left(2-\frac{1}{\alpha}\right)\beta^{\frac{1}{\alpha}-1}R^{\frac{1}{\alpha}-3}\right] \tag{85}$$

Proposition 2 *There are three cases of interest: (i)* $\frac{1}{2} \leq \alpha < 1$, *implying* $\sigma'(R) < 0$ *for* $R \geq 1$; *(ii)* $0 < \alpha < \frac{1}{2}$ *and* $\sigma'(R) < 0$ *for* $R \in [1, \tilde{R})$ *and* $\sigma'(R) > 0$ *for* $R > \tilde{R}$, *with* $\tilde{R} > 1$; *(iii)* $0 < \alpha < \frac{1}{2}$ *and* $\sigma'(R) \geq 0$ *for* $R \geq 1$.

Proof. Case (i) is clear from inspection of (85). For the other two cases, note that we can write (85) as

$$\sigma'(R) = R^{-2}\left\{-\left[\theta\frac{1}{\alpha}R^{-\frac{1}{\alpha}+1} + \theta\beta^{\frac{1}{\alpha}-1} + (1-\theta)\right] + (1-\theta)\left(\frac{1}{\alpha}-2\right)\beta^{\frac{1}{\alpha}-1}R^{\frac{1}{\alpha}-1}\right\}$$

Then, let

$$\phi(R) = \theta\frac{1}{\alpha}R^{-\frac{1}{\alpha}+1} + \theta\beta^{\frac{1}{\alpha}-1} + (1-\theta),$$

and

$$\omega(R) = (1-\theta)\left(\frac{1}{\alpha}-2\right)\beta^{\frac{1}{\alpha}-1}R^{\frac{1}{\alpha}-1}.$$

Then, if $0 < \alpha < \frac{1}{2}$, $\phi(0) = \infty$, $\phi(\infty) = \theta\beta^{\frac{1}{\alpha}-1} + (1-\theta)$, $\phi'(R) < 0$ for $R > 0$, $\omega(0) = 0$, $\omega(\infty) = \infty$, $\omega'(R) > 0$ for $R > 0$. So, by continuity, there exists $\tilde{R} > 0$ such that $\sigma'(\tilde{R}) = 0$, $\psi'(R) < 0$ for $0 < R < \tilde{R}$, and $\sigma'(R) > 0$ for $R > \tilde{R}$. This gives us cases (ii) and (iii), where in case (ii), $\tilde{R} > 1$, and in case (iii), $\tilde{R} \leq 1$. Whether we get case (ii) or (iii) depends on the sign of $\sigma'(1)$ when $0 < \alpha < \frac{1}{2}$. That is, for case (ii), $\sigma'(1) < 0$, and for case (iii), $\sigma'(1) \geq 0$. From (85),

$$\begin{aligned}
\sigma'(1) &= -\left[\theta\frac{1}{\alpha} + \theta\beta^{\frac{1}{\alpha}-1} + (1-\theta)\right] + (1-\theta)\left(\frac{1}{\alpha}-2\right)\beta^{\frac{1}{\alpha}-1} \\
&= \frac{1}{\alpha}\left[-\theta + (1-\theta)\beta^{\frac{1}{\alpha}-1}\right] - \beta^{\frac{1}{\alpha}-1}\left[\theta + 2(1-\theta)\right] - (1-\theta)
\end{aligned}$$

so, if θ is sufficiently large, we have case (ii), and if θ is sufficiently small and α is sufficiently small, we will have case (iii). ■

Figures 7 through 9 illustrate how equilibria depend on the policy (R, v). These figures depict the locus $v = \frac{\sigma(R)}{2}$, which separates the policy space into regions where the collateral constraint binds, and where it does not. In Figure 7, case (i), policy and parameters are such that the properties of the model are similar to the flexible price case. That is, given fiscal policy v, if the collateral constraint binds, then it will bind for low nominal interest rates. However, in

Figure 8, case (ii), if the collateral constraint binds, it could bind only for high nominal interest rates, it could bind for low nominal interest rates and high nominal interest rates, or it could bind for any nominal interest rate. Finally, in Figure 9, case (iii), the collateral constraint will bind only for high nominal interest rates, or it will bind for any nominal interest rate.

Why do we get different results here from the case with flexible prices? From (65), (68), (80), and (84), we can interpret v as the supply of safe collateral, and $\frac{\sigma(R)}{2}$ is the demand for safe collateral that would be forthcoming, given the nominal interest rate R, if the collateral constraint were not binding. In the event that $\frac{\sigma(R)}{2} > v$, then prices have to adjust, given R, so that the demand for collateral equals the supply. From the left-hand side of (79), if the collateral constraint does not bind in equilibrium, then the demand for safe collateral arising from transactions in the cash market, and transactions in the cash and credit market involving flexible price goods, is strictly decreasing in the nominal interest rate R. However, as R increases, note that the demand for fixed price goods in the cash and credit market is strictly increasing in R. If the demand for these goods is sufficiently price elastic (i.e. if α is sufficiently low), then demand could increase at a sufficiently high rate with R that total demand for safe collateral ultimately increases with R for R sufficiently high. So, there can be conditions under which the total demand for safe collateral is strictly decreasing in R, and conditions under which the demand for safe collateral will increase with R when R is sufficiently high.

3.2.2 Could the Zero Lower Bound be Optimal?

With flexible prices, we showed that, if the collateral constraint binds when $R = 1$, then it will bind when the nominal interest rate is low, but not when the nominal interest rate is high. Further, if the collateral constraint binds when $R = 1$, then $R = 1$ is a suboptimal setting for monetary policy. There is a welfare improvement if the nominal interest rate is positive. Thus, given conditions under which the zero lower bound would be optimal with sticky prices and no monetary exchange (the first model we considered) with flexible prices and monetary exchange the zero lower bound is not optimal. So, what makes the difference? Is it sticky prices or monetary exchange?

To explore this question, first consider the case in which the collateral constraint does not bind, so that the consumption allocation $(c^{mf}, c^{ms}, c^{bf}, c^{bs})$ is determined by (69)-(72). Let $W(R)$ denote the period utility of a household, and recall that, since the model solves period-by-period, the current nominal interest rate affects only the current consumption allocation. From (69)-(72), we get

$$W'(R) = \frac{(R-1)}{u''(c^{mf})} - \frac{\left(\frac{1}{\beta R} - 1\right)}{\beta R^2 u''(c^{bs})}.$$

So, $W'(1) > 0$, and $W'(R) < 0$ for $R \geq \frac{1}{\beta}$. Therefore, the optimal monetary policy is $R \in (1, \frac{1}{\beta})$, and this implies that the optimal gross inflation rate

$\pi \in (\beta, 1)$. Optimal inflation is then higher than Friedman rule deflation, but lower than zero inflation, as this trades off the distortions in two markets: the market for flexible price goods purchased with currency, and the market for sticky price goods purchased with cash and credit. A Friedman rule would correct the flexible-price monetary distortion, and zero inflation would correct the sticky price distortion in the sticky-price cash-and-credit market. So, with a nonbinding collateral constraint, the sticky price friction pushes optimal monetary policy off the zero lower bound. If the collateral constraint does not bind at the zero lower bound, the zero lower bound is suboptimal.

But what if the collateral constraint binds in the neighborhood of the zero lower bound? To make the analysis productive, consider the CRRA case $u(c) = \frac{c^{1-\alpha}-1}{1-\alpha}$, $0 < \alpha < 1$, as in the previous subsection. Then, from (81)-(83), we can derive

$$W'(1) = \frac{\begin{array}{c}(1-\theta)\left[\left(c^f\right)^{-\alpha} - 1\right]\left\{\left(c^f\right)^{1-\alpha} + \beta^{\frac{1}{\alpha}}\left[\beta^{-1} + 1 - \frac{1}{\alpha}\right]\right\} \\ +(1-\theta)\left(\beta^{-1} - 1\right)\beta^{\frac{1}{\alpha}}\left[\left(c^f\right)^{1-\alpha} + \beta^{\frac{1}{\alpha}-1} + \frac{1}{\alpha} - 1\right]\end{array}}{(1-\alpha)\left(c^f\right)^{-\alpha} + 2\alpha v\left(c^f\right)^{-\alpha-1}} \tag{86}$$

In (86), note that a sufficient condition for $W'(1) > 0$ is $g(\alpha) \geq 0$ where

$$g(\alpha) = \beta^{\frac{1}{\alpha}}\left[\beta^{-1} + 1 - \frac{1}{\alpha}\right]$$

So, if

$$\frac{\beta}{1+\beta} \leq \alpha < 1,$$

then $W'(1) > 0$. Further, we can show that $\lim_{\alpha \to 0} g(\alpha) = 0$, so $W'(1) > 0$ for α sufficiently small, by continuity.

Therefore, with sticky prices we know there exists a range of parameter values for which the zero lower bound is suboptimal, and a range of parameter values for which the zero lower bound may be optimal. However, if the zero lower bound is optimal, then such a policy can only be optimal when the collateral constraint binds. But, from (82), at the zero lower bound, a tighter collateral constraint (lower v) decreases c^{mf} at the zero lower bound, which from (59) increases inflation at the zero lower bound. Therefore, if the zero lower bound were optimal, such a state would arise when the inflation rate is high at the zero lower bound, just as in the cashless model we first analyzed. However, in typical analysis of baseline New Keynesian models (e.g. Werning 2012), optimal zero-nominal-interest-rate policy is associated with low inflation.

To show what can happen in cases for which the zero lower bound is never optimal, consider $u(c) = \ln c$. Then, if the collateral constraint does not bind, from (69)-(72) we get

$$\left(c^{mf}, c^{ms}, c^{bf}, c^{bs}\right) = \left(\frac{1}{R}, \beta, 1, R\beta\right),$$

and welfare as a function of R is

$$W(R) = \theta\left(-\ln R - \frac{1}{R} + \ln\beta - \beta\right) + (1-\theta)\left(-1 + \ln R + \ln\beta - \beta R\right)$$

Then, letting

$$\hat{R} = \frac{-2\theta + 1 + \left[(1-2\theta)^2 + 4(1-\theta)\theta\beta\right]^{\frac{1}{2}}}{2(1-\theta)\beta}$$

where $1 < \hat{R} < \frac{1}{\beta}$, we have $W'(R) > 0$ for $R \in [1, \hat{R})$, and $W'(R) < 0$ for $R > \hat{R}$.

Next, determining $\sigma(R)$ for the log utility case, we can determine that the equilibrium is unconstrained for $R \geq \max\left(\frac{1}{v}, 1\right)$, and is otherwise constrained.

In a constrained equilibrium,

$$\left(c^{mf}, c^{ms}, c^{bf}, c^{bs}\right) = \left(v, \beta, Rv, R\beta\right),$$

and welfare is given by

$$\begin{aligned} W(R) &= \theta\left[\ln v - v + \ln\beta - \beta\right] + (1-\theta)\left[\ln R + \ln R + \ln v + \ln\beta - Rv - R\beta\right] \\ &= \ln v + \ln\beta - \theta(v+\beta) + (1-\theta)\left[2\ln R - R(v+\beta)\right] \end{aligned}$$

For a constrained equilibrium, $v < 1$. So, $W'(1) > 0$ when equilibrium is constrained, and the optimal policy choice in a constrained equilibrium is given by

$$R = \min\left(\frac{2}{v+\beta}, \frac{1}{v}\right)$$

Therefore, if the equilibrium is constrained, and $\beta \leq v \leq 1$, then an optimal monetary policy is $R = \frac{1}{v}$, and if $v \leq \beta$, then the optimal policy is $R = \frac{2}{v+\beta}$. In the former case it is optimal to relax the collateral constraint at the optimum, and in the latter case it is not.

So, in conclusion, we have three cases:

1. If $v \geq \frac{1}{\hat{R}}$, then the optimal monetary policy is $R = \hat{R}$.

2. If $\beta \leq v < \frac{1}{\hat{R}}$, then the optimal policy is $R = \frac{1}{v}$.

3. If $0 < v < \beta$, then $R = \frac{2}{v+\beta}$.

Figure 10 depicts the optimal gross nominal interest rate policy as a function of fiscal policy v. So, in this case, if the collateral constraint never binds ($v \geq 1$), then as is more generally true, optimal monetary policy trades off Friedman-rule and sticky-price distortions, so there is deflation at the optimum, but not to the extent prescribed by the Friedman rule, i.e. $R = \hat{R} > 1$. If v is small enough that the collateral constraint binds for low nominal interest rates, but $R = \hat{R}$ implies the collateral constraint does not bind, then $R = \hat{R}$ is still optimal. But, once the collateral constraint becomes tight enough that it binds when $R = \hat{R}$, then it is optimal to raise the nominal interest rate to the point where the collateral constraint does not bind. If the collateral constraint is even tighter, then the constraint will bind at the optimum.

As shown in Figure 10, the optimal nominal interest rate is monotonically decreasing in v, so as the collateral constraint gets tighter, reducing the real interest rate, the nominal interest rate should increase. This result is driven by the same forces that arose in the flexible price case in that, with a tight collateral constraint, a higher nominal interest rate is brought about through a central bank open market sale of bonds, and this relaxes the collateral constraint. It may be possible that sticky price demand-driven effects could be enough to make a zero nominal interest rate optimal. But to get this, it is necessary that the demand for sticky price goods be sufficiently elastic with respect to inflation, so as to induce a large increase in the demand for collateral when the nominal interest rate goes up – an effect large enough to offset the increased supply of collateral.

4 Conclusion

In this paper, we developed a tractable model for the analysis of monetary policy in a low-real-interest-rate context. The model can incorporate various sticky-price, scarce-collateral, and standard monetary frictions, in alternative combinations. In several ways, the model contradicts consensus views concerning optimal monetary policy in low-real-interest-rate environments. In particular, the model has Neo-Fisherian properties.

In a cashless version of the model, in the spirit of baseline New Keynesian approaches, low real interest rates become a problem when the ZLB constraint binds, and inflation is greater than the natural inflation target. When there is a period of tight collateral constraints and low real interest rates, followed by a permanent relaxation in collateral constraints, then optimally-low nominal interest rates will extend beyond the period when collateral is low. This is not for forward guidance reasons, but because this lowers inflation quickly to the inflation target.

Extending the model to include retail currency transactions and open market operations, the ZLB is typically not optimal when collateral is scarce. That is, when the stock of safe assets and the real interest rate are low, an open market operation that increases the nominal interest rate also relaxes the collateral constraint, increasing welfare on net. This helps illustrate why the assumption of a cashless economy is not innocuous, in contrast to what is typically alleged in the New Keynesian literature (e.g. Woodford 2003). Lagos and Zhang (2018) come to a similar conclusion, for different reasons.

What do these results have to say about the post-financial-crisis period in the United States, 2009-2015, when the Fed kept the federal funds rate target in a range of 0-0.25%? By most accounts, measured real rates of return on government debt were historically low during this period, and a good case can be made that a scarcity of safe assets contributed in an important way. The financial crisis effectively destroyed part of the market in privately-supplied safe collateral, sovereign debt problems in some countries made US government debt more attractive, and new banking regulations increased the demand for safe collateral.

But, during the 2009-2015 period, the inflation rate in the US was on average lower than 2%. If we were looking at this experience through the lens of a New Keynesian model, we might say that the binding ZLB constraint was making inflation low, and the real interest rate was actually too high relative to the natural rate of interest. However, in the context of our model, we would say that it was monetary policy – low nominal interest rates – that was keeping inflation low during this period. That is, the ZLB constraint was not actually a problem for inflation control during this period. If it had been, than inflation would have been above 2%.

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Figure 1: Equilibrium with $b_t \leq c^*$

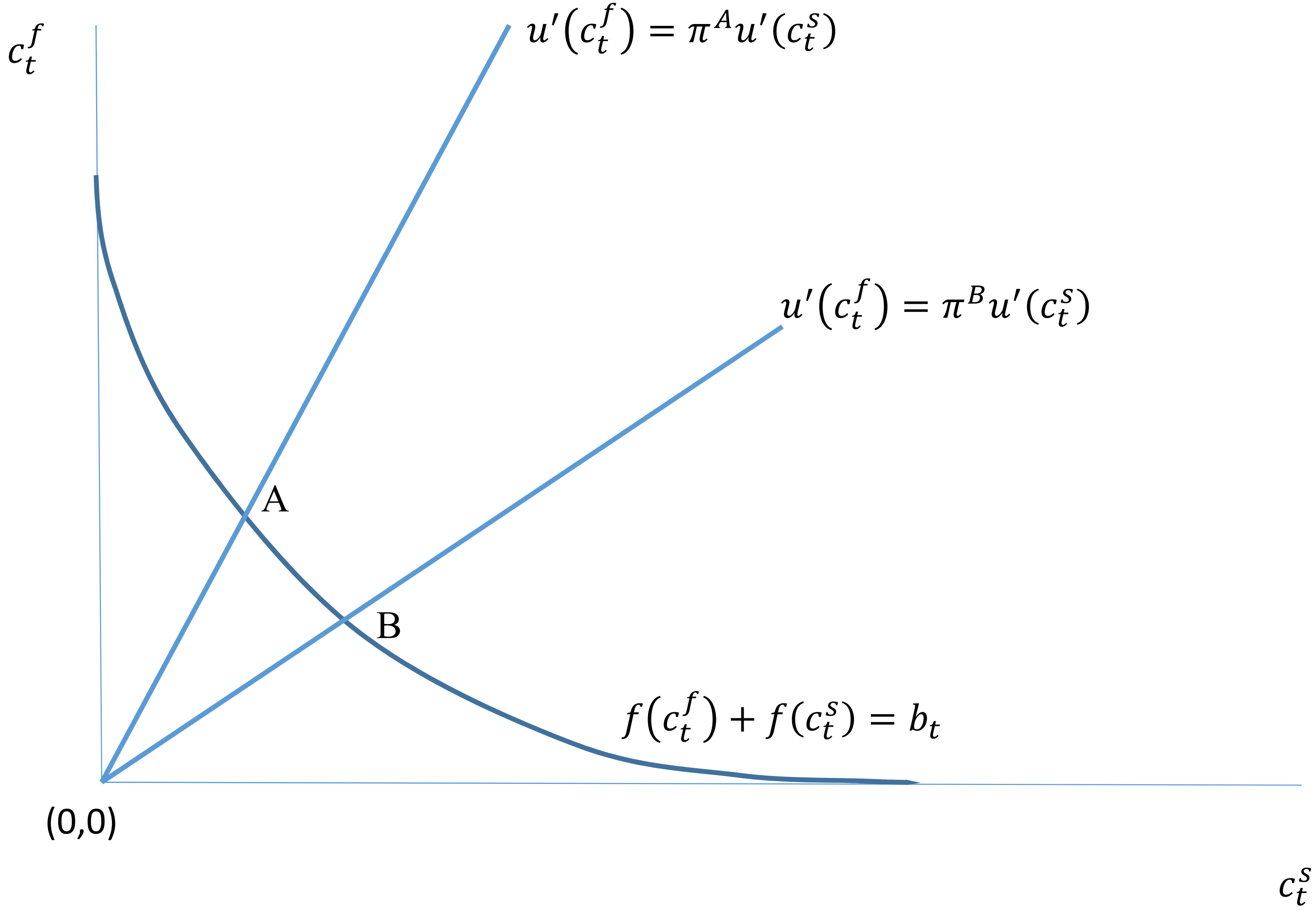

Figure 2: Equilibrium with $b_t > c^*$

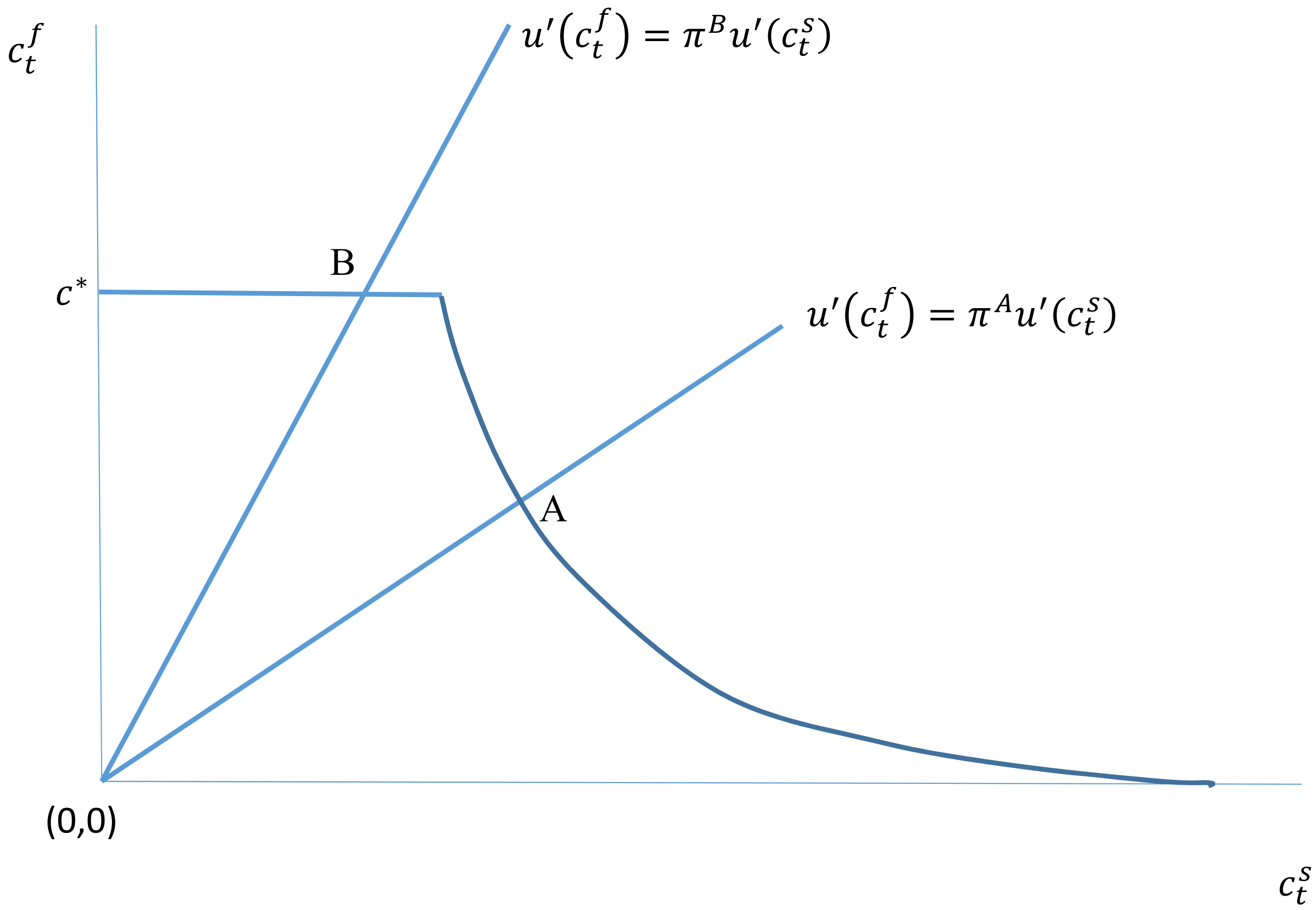

Figure 3: Phillips Curve, $b_t > c^*, \pi^\wedge > 1$

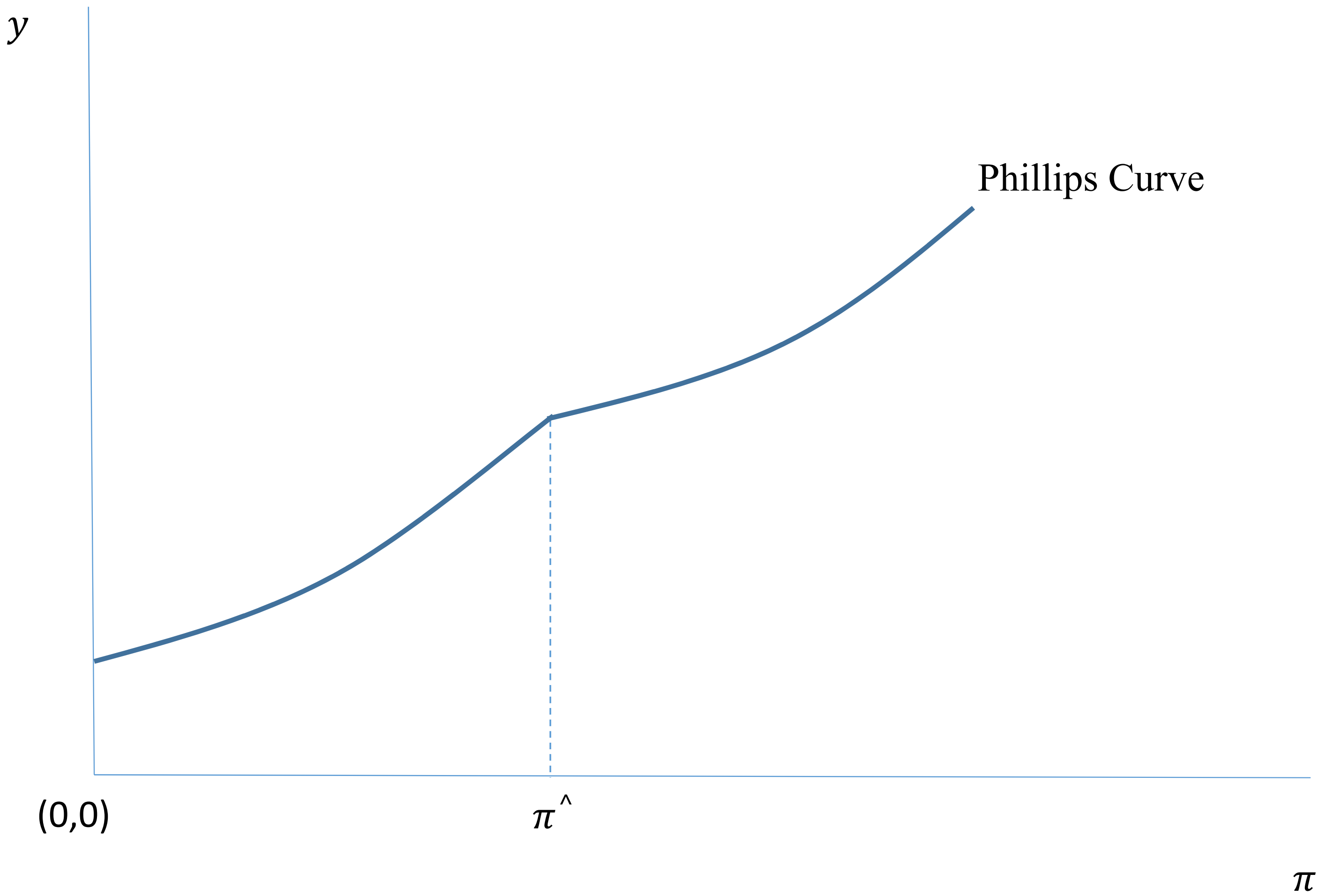

Figure 4: Phillips Curve, $b_t > c^*, \pi^\wedge < 1$

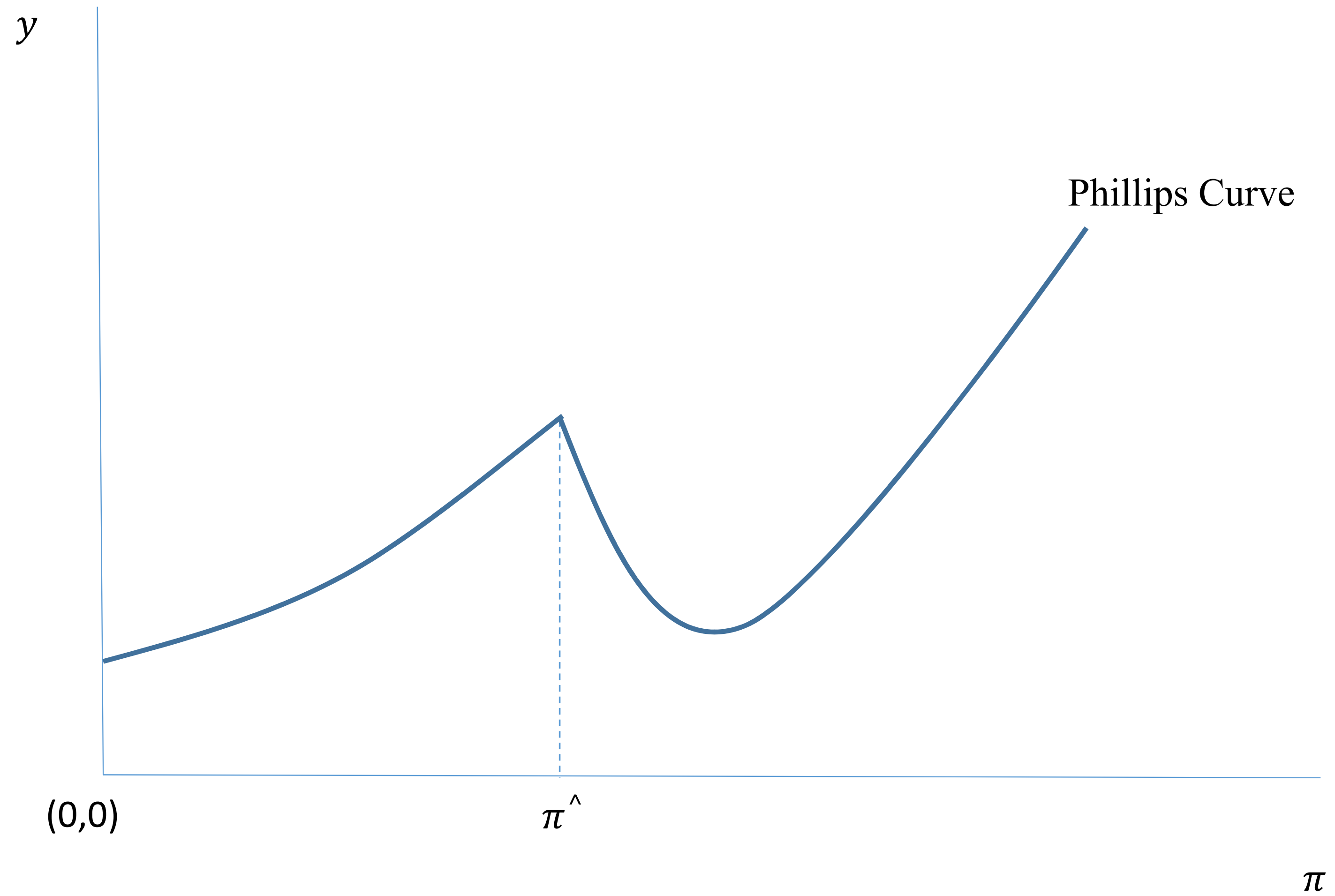

Figure 5: Phillips Curve, $b_t \leq c^*$

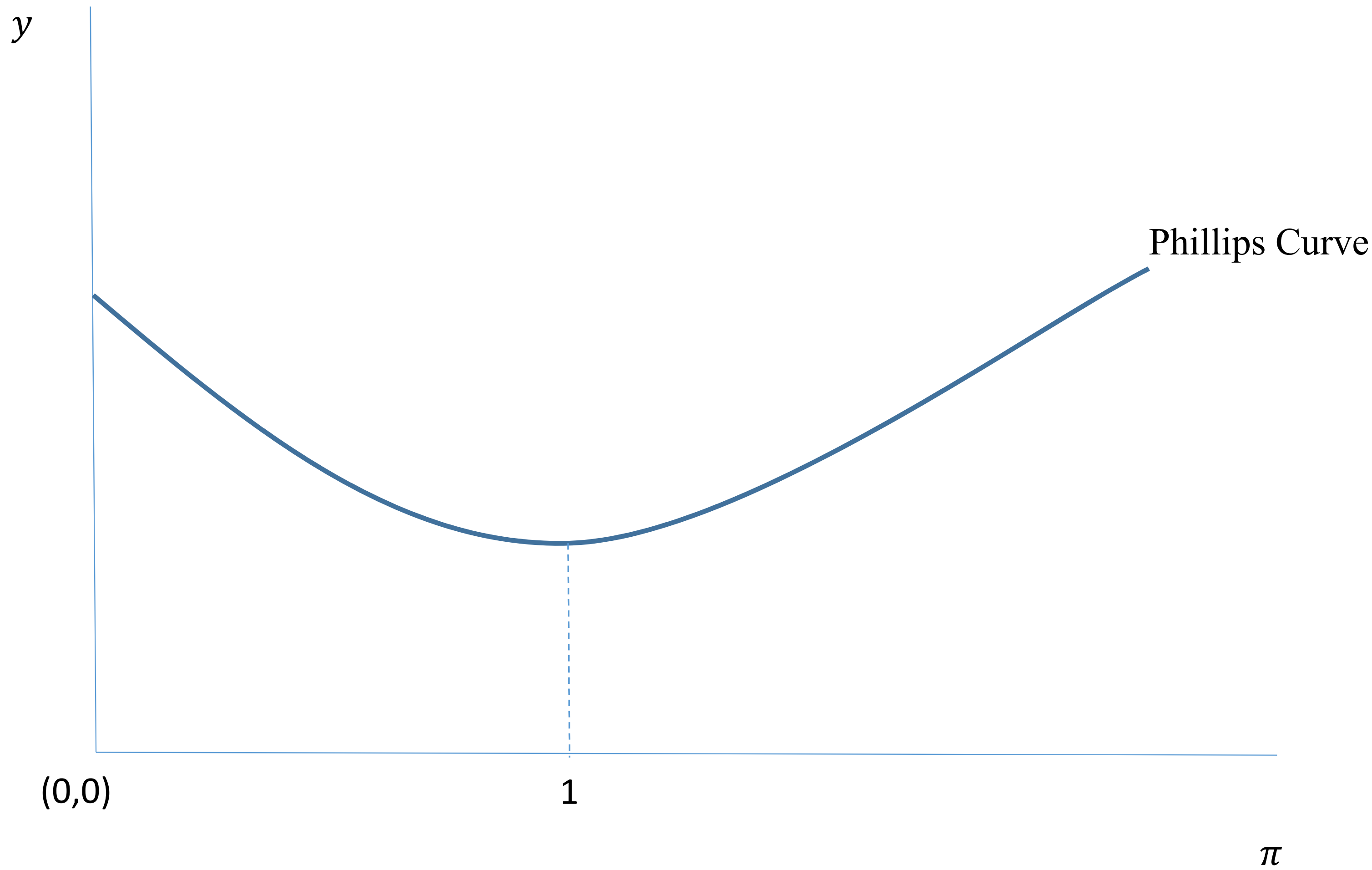

Figure 6: Equilibrium With Alternating ZLB Episodes

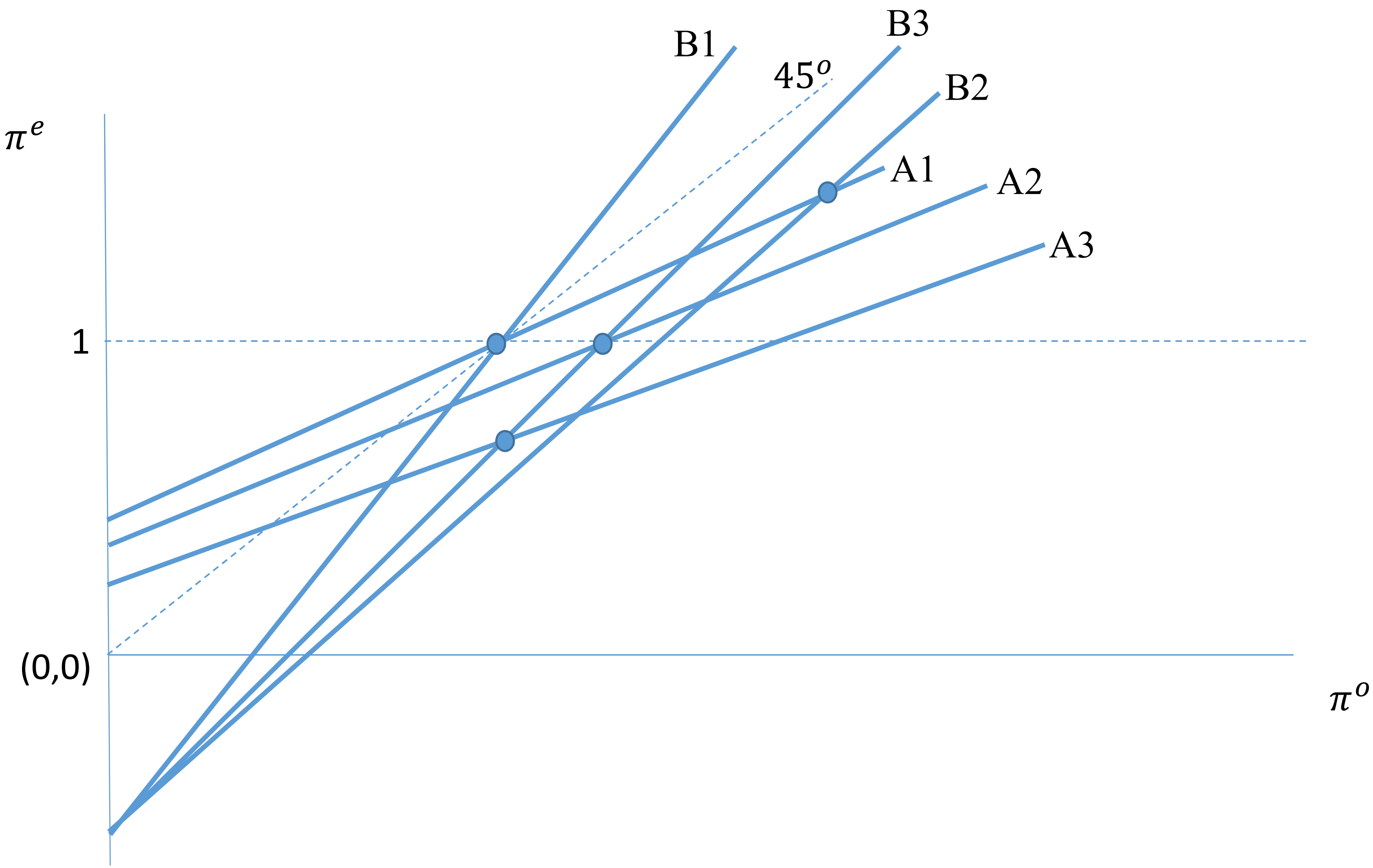

Figure 7: $\frac{1}{2} \leq \alpha < 1$

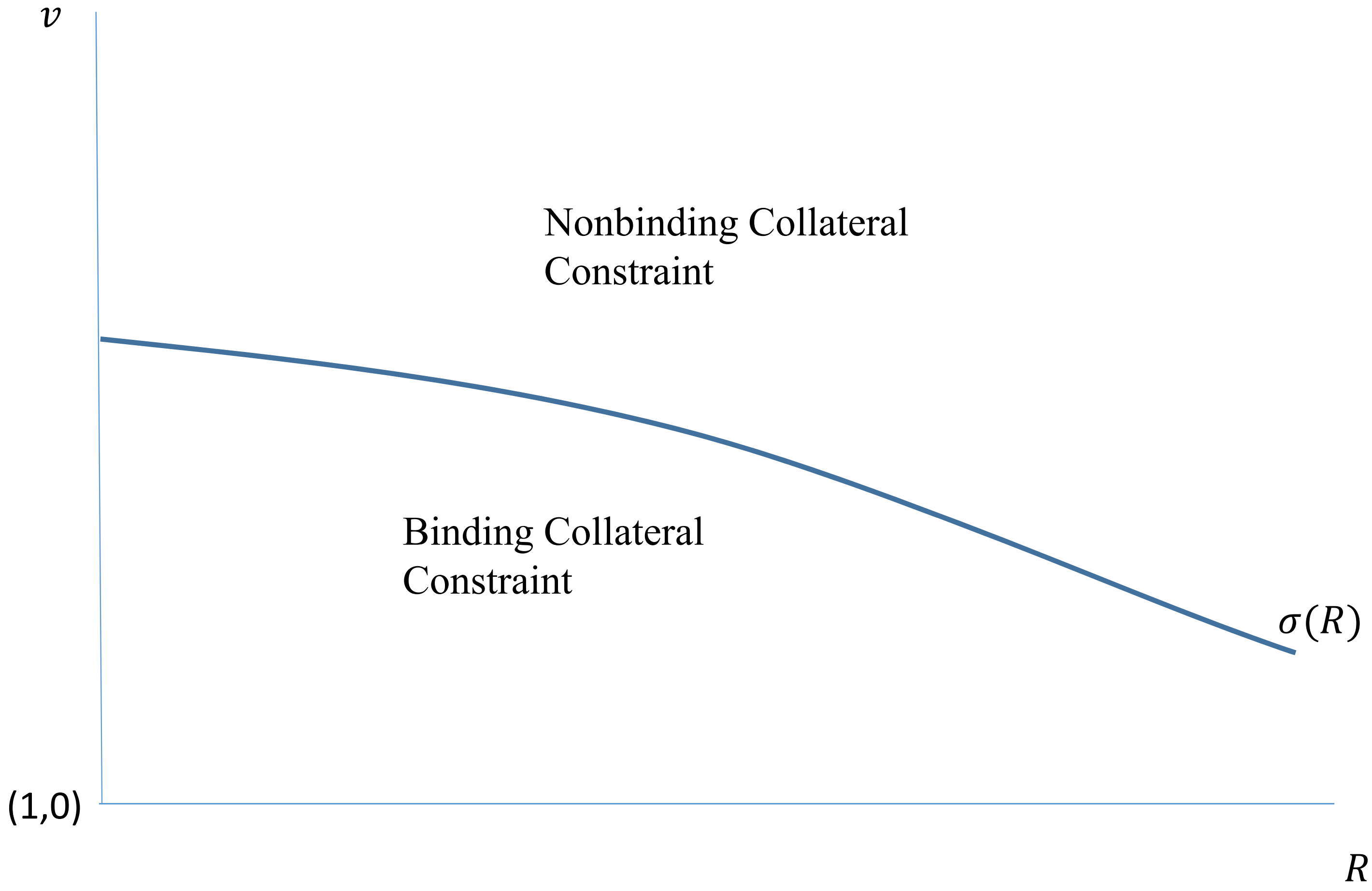

Figure 8: $0 < \alpha < \frac{1}{2}, Nonmonotonic\ \sigma(R)$

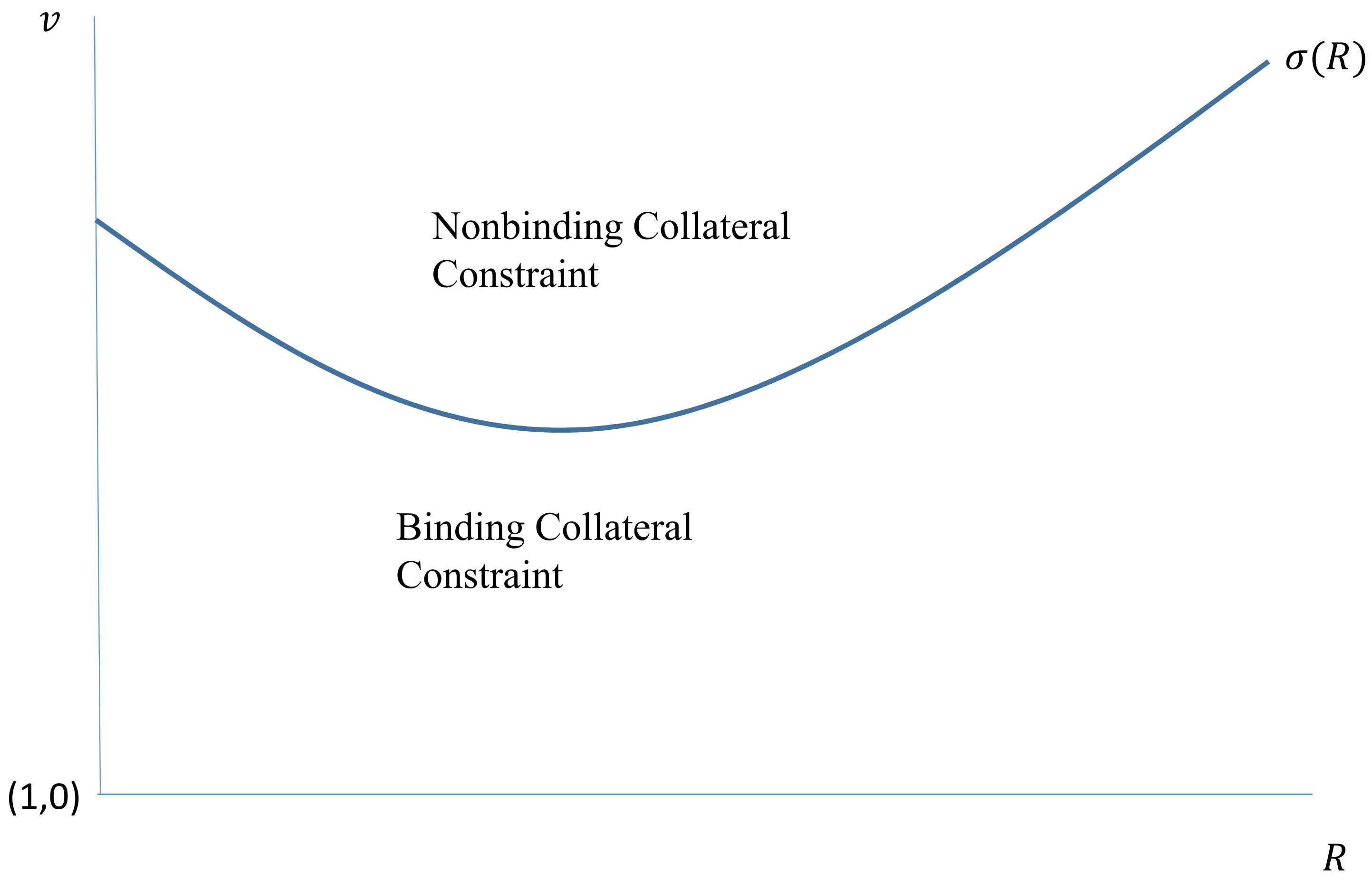

Figure 9: $0 < \alpha < \frac{1}{2}, Monotonic\ \sigma(R)$

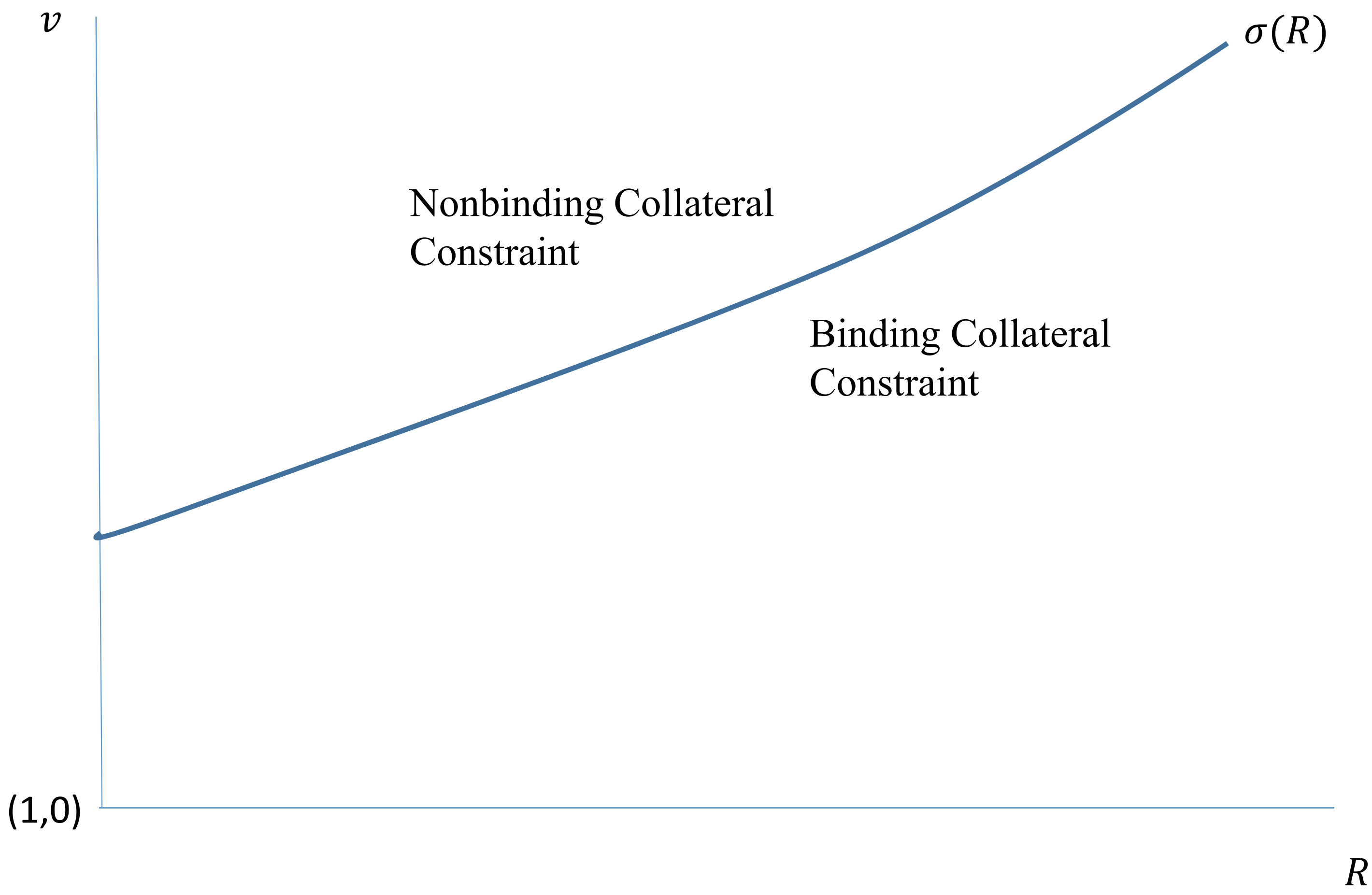

Figure 10: Log Example, Optimal Monetary Policy Given Fiscal Policy