Should monetary policy care about redistribution? Should fiscal policy care about inflation? Optimal monetary policy with heterogeneous agents

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UCL, March 2020

Question

- The redistributive channels of monetary policy are now well-identified (Bilbiie, 2008; Algan, Challe Ragot, 2010; HANK, Kaplan, Moll, Violante 2018; Auclert 2018; Acharya and Dogra 2019...)
- How should optimal policy deal with redistributive effects? What about fiscal policy?
- Hard to think about optimal monetary policy without considering fiscal policy, which deals with redistribution.

What we do

- We solve for optimal monetary and fiscal policy (interest rate, capital tax, labor tax, public debt) in an heterogeneous-agent model with aggregate shocks (public spending and TFP shocks) : (HANK) model
- We derive analytical results depending on available fiscal tools
- Provide a quantification (employment risk)

How we do it? Projection theory and Lagrangian Approach

This is a very difficult problem : How to solve for Ramsey problem in Heterogeneous-agent model? Methodological contribution :

- "Lagrangian Approach" (Marcet and Marimon 2019) to solve for Ramsey problem in Heterogeneous-agent models with aggregate shocks.
  - works very well in HANK (deals with occasionally binding credit constraint + Phillips curve is an Euler equation)
- Going to the data : : Truncation to obtain a finite-dimensional state-space (Le Grand and Ragot, 2019). Can be simulated with simple perturbation methods (Dynare), different from Reiter 2009; Preston, Roca 2007; Kim, Kolman, Kom 2010.

What we find

- Irrelevance result. With simple tools : linear capital and labor taxes, public debt monetary policy should not care about redistribution
- If either labor or capital taxes are missing, monetary policy must consider redistribution : price stability
- With imperfect fiscal policy : three new channels
  - Capital-tax channel
  - Real-wage channel
  - Public finance channel
- Quantitatively : Small deviation from price stability, except when labor and capital taxes are not optimally time-varying.

Selected literature Review

1. Redistributive effect Bilbiie 2008; Algan and Ragot, 2010; Kaplan, Moll Violante (2018), Auclert (2019), Acharya and Dogra (2019); Bayer, Luetticke, Pham-Dao, Tjaden (2019)
2. Fiscal, monetary interactions Chari, Kehoe (1999); Correia, Nicolini, Teles (2008) (consumption taxes).
3. Optimal monetary policy heterogeneous agents ;Nuno Thomas (2017); Bilbiie Ragot, (2017); Challe (2019); Acharya, Challe and Dogra (2020)
4. Bhandari, Evans, Golosov, and Sargent (2018) : lump-sum tax and income tax, primal approach : difficult to characterize the steady-state distribution. Acikgoz et al. (2019) optimal fiscal policy at the steady state.

Outline of the presentation

1. Environment
2. Optimal policy
3. Bringing the model to the data : The truncation theory
4. Numerical application

1 - Environment

- Unit mass of agents, discount factor β, (GHH)

$$U(c,l) = u\left(c - \frac{l^{1+1/\phi}}{1+1/\phi}\right).$$

- Y idiosyncratic states, $y_t \in \mathcal{E} \equiv \{1,\ldots,Y\}$, productivity y_t.
- Markovian transition matrix for idiosyncratic risk T.

Households

The program (obvious notations)

$$\max_{\{c_t^i, l_t^i, a_t^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u \left(c_t^i - \chi^{-1} \frac{l_{t+1}^{i,1+1/\varphi}}{1+1/\varphi} \right)$$

$$c_t^i + a_t^i = R_t a_{t-1}^i + w_t y_t^i l_t^i$$

$$a_t^i \geq -\bar{a},\ c_t^i > 0,\ l_t^i > 0.$$

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First-order conditions

$$U_c(c_t^i, l_t) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) U_c(c_{t+1}^i, l_{t+1})\right] + \nu_t^i$$

$$l_t^{i,1/\varphi} = \chi w_t y_t^i$$

Production

Standard assumption to get the Philipps curve

- Monopolistic competition, demand elasticity ε
- Rotemberg price adjustment cost

$$\frac{\kappa}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 Y_t$$

- Steady-state subsidy (no steady-state distortion)
- Marginal utility of firm owner M_t

Production

Firm produce with capital and labor a differentiated product.

$$Z_t \tilde{k}_t(j)^\alpha \tilde{l}_t(j)^{1-\alpha}$$

Cost minimization gives the marginal cost is $\zeta_t(j)$

$$\zeta_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t}{\alpha}\right)^\alpha \left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}$$

Production

Phillips curve

$$\Pi_t(\Pi_t - 1) = \frac{\varepsilon - 1}{\kappa}(\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}$$

Pricing kernel : Formally, the expression of M_t is:

$$M_t = \beta^t \int_i a_{t-1}^i U_c(c_t^i, l_t^i)\ell(di)$$

Monoploy profits Ω_t taxed by the government

$$\Omega_t = \left(1 - \zeta_t - \frac{\kappa}{2}\pi_t^2\right) Y_t.$$

Government

- One-period debt B_t, all assets have the same return r_t in period t.
- (Exogenous) Public spending G_t financed by taxes on capital τ_t^K and labor τ_t^L.

$$w_t = (1-\tau_t^L)\tilde{w}_t, 1 + r_t = R_t = (1-\tau_t^K)\frac{1+i_{t-1}}{\Pi_t}$$

- Budget constraint (post tax, as in Chamley)

$$G_t + B_{t-1} + r_t\left(B_{t-1} + K_{t-1}\right) + w_t L_t = B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t.$$

Market equilibria

$$\int_i a_t^i \ell(di) = B_t + K_t$$

$$\int_i c_t^i \ell(di) + K_t = Y_t + (1-\delta)K_{t-1}$$

$$\int_i y_t^i l_t^i \ell\,(di) = L_t$$

Question

How should the government choose time-varying monetary and fiscal policy i_{t-1}, τ_t^K, τ_t^L,B_t, after shocks on G_t or z_t ?

2 - Optimal policy

2 - Optimal policy

$$\max_{\left(R_t, w_t, i_t, \tilde{R}_t, \tilde{w}_t, \tau_t^K, \tau_t^L, B_t, \Pi_t, (a_t^i, c_t^i, l_t^i)_i\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega_t^i U(c_t^i, l_t^i) \ell(di)\right],$$

$$G_t + B_{t-1} + r_t\left(B_{t-1} + K_{t-1}\right) + w_t L_t = B_t + \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t.$$

$$\begin{aligned}
a_t^i + c_t^i &= y_i l_t^i w_t + (1 + r_t) a_{t-1}^i \\
U_c(c_t^i, l_t^i) &= \beta \mathbb{E}_t \left[U_c(c_{t+1}^i, l_{t+1}^i)(1 + r_{t+1})\right] + \nu_t^i, \\
l_t^{i,1/\varphi} &= \chi w_t y_t^i, \\
\nu_t^i (a_t^i + \overline{a}) &= 0 \\
\Pi_t(\Pi_t - 1) &= \frac{\varepsilon - 1}{\kappa}\left(\zeta\left(\tilde{R}_t, \tilde{w}_t\right) - 1\right) \\
&\quad + \beta \mathbb{E}_t \Pi_{t+1}(\Pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \frac{M_{t+1}}{M_t} \\
K_t + B_t &= \int_i a_t^i \ell(di), L_t = \int_i y_t^i l_t^i \ell(di),
\end{aligned}$$

Reformulation

Key idea : Using Marcet and Marimon (2019) methodology :

- Factorization of the Lagrangian to get rid of expectations.
- α_t Lagrange multiplier on the Phillips cuve
- μ_t Lagrange multiplier on the budget of the State
- λ_t^i Lagrange multiplier on the Euler equation in period t
- KEY : $\lambda_t^i \times \nu_t^i = 0$ (Euler equation or not!)

Reformulation

$$
\begin{aligned}
J =&\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \\
&\int_i \omega_t^i U(c_t^i, l_t^i)\ell(di) - \beta^t \int_i \left(\omega_t^i \lambda_t^i - (1+r_t)\lambda_{t-1}^i \omega_{t-1}^i\right) U_c(c_t^i, l_t^i)\ell(di) \\
&- (\alpha_t - \alpha_{t-1}) \left(\Pi_t \left(\Pi_t - 1\right) Y_t M_t\right) + \frac{\varepsilon - 1}{\kappa} \alpha_t \left(\zeta_t - 1\right) Y_t M_t \\
&- \mu_t \left(G_t + B_{t-1} + r_t \left(B_{t-1} + K_{t-1}\right) + w_t L_t - B_t - \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t\right)
\end{aligned}
$$

subject to

$$
\begin{aligned}
a_t^i + c_t^i &= \theta_i l_t^i w_t + (1+r_t) a_{t-1}^i \\
l_t^{i,1/\varphi} &= \chi w_t y_t^i, \\
K_t + B_t &= \int_i a_t^i \ell(di)\,, L_t = \int_i y_t^i l_t^i \ell\left(di\right),
\end{aligned}
$$

Analytical Results : 1

Define the real-economy as the economy without monetary friction $(\kappa = 0)$

Proposition

When both labor and capital taxes are available, $\Pi_t = 1$*.*
The government reproduces the (time-varying) real-economy allocation after shocks on G_t *or* z_t

The real economy is the constrained-efficient economy : Useful benchmark

Intuition

- $\Pi_t = 1$ can be obtained by several combinations of instruments. Simple policy :

$$r_t \;\; = \;\; (1 - \tau_t^K) i_{t-1}.$$

- and the policy rule ($\phi^\Pi > 1$ for determinacy)

$$i_{t-1} = \mathbb{E}_t R_{t+1} + \phi^\Pi \left(\Pi_t - 1 \right),$$

- $\mathbb{E}_t \tau_{t+1}^K = 0$: The capital tax in the period only adjusts to the new information
- Not enough : The labor tax τ_{t+1}^K is time-varying to adjust to obtain the right labor share. set.

Analytical Results

In the paper, analytical characterization of :

- Optimal labor tax and monetary policy , with constant capital tax
- Optimal capital tax and monetary policy, with constant labor tax
- Optimal monetary policy, with exogenous fiscal system

Involved algebra, simulation instead

3 - Bringing the model to the data : The truncation theory

- For agents, the state space is the joint distribution $\Lambda(\lambda, a)$ over Lagrange multiplier and wealth
- No algorithm to find the steady state (see Acikgoz et al. 2019 for an attempt)
- Main Idea : construct a consistent finite dimensional model, that can be simulated.
- We go back in the sequential representation (history representation) to provide a theory based on truncation.
- Instead of keeping track of the whole history of each agent, assume an insurance system implies that only the last N periods are relevant : Finite number of different agents

Consistent Truncated model

- Consider an integer $N \geq 1$.
- Agents have history $y^{i,t} = \{y_{1,\ldots}, y_t\}$ at period $t \geq 1$.
- Number of histories for the last N periods
  $y^k = \{y_{-N,\ldots}, y_0\}$,$k = 1...Y^N$
- Knowing your history at period t, y^k, can easily deduce the probability $\Pi_{k,k'}$ of $y^{k'}$ next period.
- Easy to compute the measure of agents having each history S_{y^k}

Approximated model : Asumptions

- Agents face a fiscal system, which depends on the history over the last $N+1$ periods.
- Agents face preference shocks which depend on their current N-period history: ξ_{y^k} (Important, see below)

Recursive representation ($k = 1 ... Y^{N+1}$)

$$V\left(a, y^{k,N+1}, X\right) = \max_{c,a',l} \xi_{y^{k,N}} U\left(c,l\right) + \beta \mathbb{E} V\left(a', y^{k',N+1}, X'\right)$$

subject to

$$\begin{aligned} c + a' &= Ra + wy_i l + T\left(y^{k,N+1}\right) \\ a' &\geq 0 \end{aligned}$$

Proposition

There exists a balanced fiscal system such that all agents with the same history $y^{k,N}$ have the same beginning of period wealth.

The economy is, for each $k = 1...Y^N$

$$\begin{aligned} \xi_k U'(c_k, l_k) &= \beta \mathbb{E} R' \xi_{k'} U'(c_{k'}, l_{k'}) + \nu_k \\ c_k + a'_k &= R\tilde{a}_k + w y_k l_k \\ l_k &= (\chi y_k w)^{\phi} \end{aligned}$$

with

$$\tilde{a}_k = \sum_{l=1...Y^N} \frac{S_{l,-1}}{S_k} \Pi_{l,k} a'_{l,-1}.$$

Total savings $A = \sum_{k=1}^{Y^N} S_k a'_k$.

Consistent model : What is ξ?

Consider the (true) Bewley model without aggregate shock.

Denote as $\Lambda\left(a, y^k\right)$, $k = 1...Y^N$, the distribution of wealth of agents having history y^k.

Average consumption

$$\begin{aligned}
\hat{a}'_k &= \int a'\left(a, y^k\right)\Lambda\left(a, y^k\right) \\
\hat{c}_k &= \int c\left(a, y^k\right)\Lambda\left(a, y^k\right) \\
\hat{l}_k &= \int l\left(a, y^k\right)\Lambda\left(a, y^k\right)
\end{aligned}$$

Proposition

For any $N \geq 1$, there are preferences parameters ξ_k, $k = 1...Y^N$, such that the allocations are the same in the truncated in "true" Bewley model :

$$
\begin{aligned}
\hat{a}'_k &= a_k \\
\hat{c}_k &= c_k \\
\hat{l}_k &= l_k
\end{aligned}
$$

In addition, when $N \to \infty$, $\xi_k \to 1$.

Intuition

Average marginal utility by history in the "true" Bewley model:

$$u'_k \;=\; \int U'_c\left(c\left(a,y^k\right),l_k\left(a,y^k\right)\right)\Lambda\left(a,y^k\right)$$

Then

$$\xi_k \simeq \frac{u'_k}{U'_c\left(c_k,l_k\right)}$$

We solve

$$\max_{\left(R_t,w_t,i_t,\tilde{R}_t,\tilde{w}_t,\tau_t^K,\tau_t^L,B_t,\Pi_t,(a'_{k,t},c_{k,t},l_{k,t})_{k=1..Y^N}\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\sum_{k=1}^{Y^N}S_{k,t}\xi_k U(c_{k,t},l_{k,t})\right]$$

$$\begin{aligned}
\xi_k U'\left(c_{k,t},l_{k,t}\right) &= \beta\mathbb{E}R_{t+1}\xi_{k'}U'\left(c_{k',t+1},l_{k'_{t+1}}\right)+\nu_{k,t}\\
c_{k,t}+a_{k,t} &= R_t\tilde{a}_{k,t}+w_t y_k l_{k,t}\\
l_{k,t} &= \left(\chi y_{k,t}w_t\right)^{\phi}\\
\tilde{a}_{k,t} &= \sum_{l=1\dots Y^N}\frac{S_{l,t-1}}{S_{k,t}}\Pi_{l,k,t}a_{l,t-1}\\
A_t &= \sum_{k=1}^{Y^N}S_{k,t}a_{k,t}
\end{aligned}$$

+ production side and government budget constraint.

Algorithm to find the steady state and simulatation

1. Choose N
2. Choose instruments $\left(i, \tau^K, \tau^L, B\right)$
3. Solve for the true Bewley model, compute ξ_k
4. Find the equilibrium values of Lagrange multipliers on the Truncated model λ_k (easy).
5. Check the first-order conditions of the planner on the Truncated model
6. Iterate on the instruments $\left(i, \tau^K, \tau^L, B\right)$
7. Check that doesn't depend on N
8. Use perturbation methods to solve the model with aggregate shocks (DYNARE)

4 - Numerical application

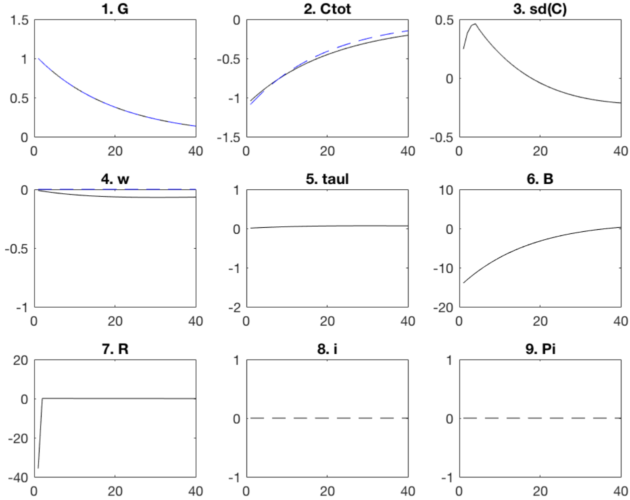
Preliminary - Economy without capital, emplyoment risk

	Values	Description
β	0.99	Discount factor
σ	1	Log utility
ϕ	0.5	Chetty et al. (2011)
δ/wl	50%	Replacement rate
κ	100	Rotemberg coeff.
ϵ	6	Elasticity of sub.
f	80%	Job finding rate
s	5%	Job separation rate
G	0	Steady-state gov. spend.
ρ_g	97%	Persistence shock
σ_g	7%	St. dev
N	4	Thus $2 * 2^4 = 32$ agents

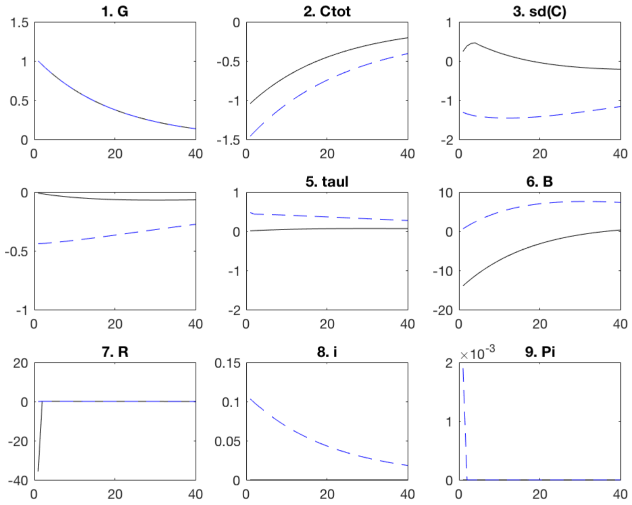
Steady-state results

- Optimal inflation rate 0%, as expected
- Public debt 42% of quarterly GDP, consistent with (Le Grand and Ragot 2017) in an economy with capital
- Capital tax 0.001% very small
- Labor tax 4% to pay interest on public debt.

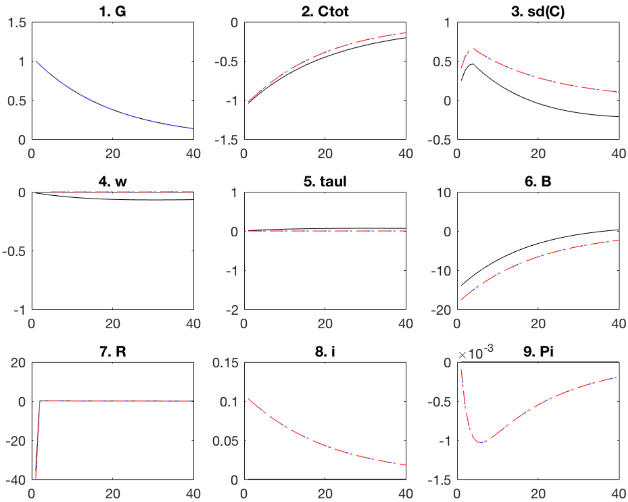
Comparison between first-best and constrained-efficient economy (CE)



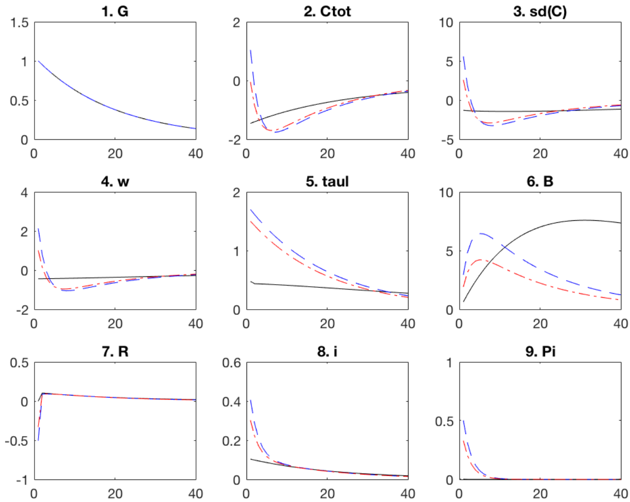
Comparison CE and labor tax only



Comparison CE and capital tax only



Comparison monetary policy only, no optimal fiscal policy



Conclusion

- We derive optimal monetary policy with heterogeneous agents, with rich set of fiscal tools.
- Projection theory and Lagrange approach : very efficient tool for monetary policy with heterogenous agents
- Strong deviation from price stability with exogenous fiscal dynamics (fiscal dominance)
- Next : More quantitative...

Reformulation

The Ramsey program on the projected model

$$\max_{\left((a_{h,t},c_{h,t},l_{h,t})_{h\in\mathcal{H}},\phi_t,\tau_t\right)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\sum_{h\in\mathcal{H}} S_{h,t}\xi^U_{h,t}U(c_{h,t},l_{h,t})\right],$$

with

$$\forall h\in\mathcal{H}\setminus\mathcal{C},\ \xi^u_{h,t}U_c(c_{h,t},l_{h,t})-\beta\mathbb{E}(1+r_{t+1})\sum_{\tilde{h}\in\mathcal{H}}\Pi^u_{h\tilde{h},t+1}\xi^u_{\tilde{h},t+1}U_c(c_{\tilde{h},t+1},l_{\tilde{h},t+1})=0,$$

$$\forall h\in\mathcal{C},\ a_{h,t}=-\bar{a},$$

$$\forall h\in\mathcal{H},\ l_{h,t}=\chi^{\varphi}w_t^{\varphi}y_h^{\varphi},$$

$$\forall h\in\mathcal{H},\ c_{h,t}+a_{h,t}\leq(1+r_t)\sum_{\tilde{h}\in\mathcal{H}}\Pi^a_{\tilde{h}h,t}\frac{S_{\tilde{h},t}}{S_{h,t}}a_{\tilde{h},t-1}+l_{h,t}y_hw_t,$$

Reformulation with the projected model(Go back)

Ramsey problem can be simplified into:

$$\max_{\left((a_{h,t},c_{h,t},l_{h,t})_{h\in\mathcal{H}}\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{h\in\mathcal{H}} S_{h,t} \left(\xi^U_{h,t} U(c_{h,t}, l_{h,t}) \right.$$
$$\left. - \left(\lambda_{h,t} - (1+r_t)\Lambda_{h,t}\right) \xi^u_{h,t} U_c(c_{h,t}, l_{h,t})\right) \tag{1}$$

$$\text{s.t. } \lambda_{h,t} = 0 \text{ if } a_{h,t} = -\overline{a}, \tag{2}$$

with

$$\Lambda_{h,t} \equiv \frac{\sum_{\tilde{h}\in\mathcal{H}} S_{\tilde{h},t} \Pi^S_{\tilde{h}h,t} \lambda_{\tilde{h},t-1}}{S_{h,t}} \tag{3}$$