

# Asset Price Booms and Macroeconomic Policy: a Risk-Shifting Approach\*

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## Abstract

This paper uses risk-shifting models to analyze particular policy responses to asset price booms and bubbles. We argue that the presence of risk shifting can generate many of the features of such booms and so is a reasonable framework to explore these issues. Our analysis offers several insights. First, we find that determining whether there is indeed a bubble in asset markets is unimportant, since risk-shifting leads to the same inefficiencies regardless of whether it gives rise to a bubble or not. Second, while risk shifting offers a reason for intervention, we find the leading proposals for interventions against booms have ambiguous welfare implications in our model. Specifically, we show tighter monetary policy may exacerbate some inefficiencies due to risk shifting even as it mitigates others, and that leverage restrictions may lead to higher asset prices and exacerbate excessive leverage rather than curb it.

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\*The views here do not represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

# Introduction

Policymakers have long debated how to respond to asset price booms and potential bubbles, i.e., when asset prices seem to surge above the value of the dividends assets are expected to yield. One view, summarized in Bernanke and Gertler (1999) and Gilchrist and Leahy (2002), argues that policymakers should not respond in these cases, but should instead wait to see what happens to asset prices. If asset prices collapse and drag down economic activity, policymakers should step in, just as they would whenever output is depressed. An alternative view, summarized in Borio and Lowe (2002), argues that policymakers should not stand idly by during these episodes. They argue that when asset booms coincide with credit booms, they predictably end in financial crises and recessions, a claim corroborated in subsequent work such as Jorda, Schularick, and Taylor (2015) and Mian, Sufi, and Verner (2017). The implication is that policymakers should raise interest rates during asset booms to dampen asset prices and mitigate the eventual crash.

The severity of the Global Financial Crisis in 2007 and the difficulty central banks faced in its wake to provide stimulus after lowering short-term nominal interest rates to zero shifted opinion in policy circles toward favoring a more proactive response to asset booms. But this led to a new debate over what type of intervention would be most appropriate. The two policies that attracted the most attention are monetary policy and macroprudential regulation. Svensson (2017) argues against monetary tightening during asset booms because its costs exceed its benefits. Stein (2013) argues against regulatory policy because it is likely to be circumvented in practice even if it could work in principle, advocating monetary tightening instead.

This paper revisits the question of how policymakers should respond to asset booms through the lens of risk-shifting models, i.e., models in which the agents who ultimately finance the purchase of assets cannot easily monitor the risk associated with any individual borrower they fund. We focus on risk-shifting because it seems to play an important role in actual booms. First, asset booms that end badly often feature extensive lending against assets. Second, booms are often associated with new and imperfectly understood technologies or with assets like housing that are valued idiosyncratically. In both cases, lenders will find it difficult to assess the risks on any given loan. With new technologies, lenders may find it hard to distinguish genuinely productive applications of new technologies (dot-com, blockchain, tranching securities) from speculative investments that may not pan out. With housing, lenders may not be able to distinguish illiquid borrowers who value home ownership from speculators who bet that house prices will rise and would walk away from their investments if house prices fell. In both cases, lenders will have a hard time discerning how likely the agent they lend to is to default if asset prices fall.<sup>1</sup>

The notion of risk-shifting has spawned a large literature since it was first introduced by Jensen and Meckling (1976). Much of this literature focuses on how leverage affects investment decisions. A smaller and more recent literature has examined how these investment decisions affect the prices of the assets that

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<sup>1</sup>While we describe situations where limited information is an exogenous feature of an asset, Asriyan, Laeven, and Martin (2018) argue asset booms can reduce the incentive to screen borrowers, so information about assets deteriorate endogenously.

leveraged agents buy. Allen and Gorton (1993) first showed that risk-shifting allows asset prices to exceed the fundamental value of the dividends assets can generate. This suggests these models may be relevant for analyzing asset booms. Subsequent work by Allen and Gale (2000), Barlevy (2014), Dow and Han (2015), Dubecq, Mojon, and Ragot (2015), and Bengui and Phan (2018) has built on this original result.<sup>2</sup>

This paper contributes to this literature in two ways. First, we explore risk-shifting in a general equilibrium setting that allows us to study the effects of monetary policy and macroprudential regulation, something the previous literature on risk-shifting has yet to do. Second, unlike previous work, we allow default to be costly. We model this as a cost of recovering resources from borrowers who default. While recovery costs are relatively small in practice, adding them allows our model to generate a decline in available resources at the end of an asset boom, in line with what empirical work suggests. Hoggarth, Reis, and Saporta (2002) and Reinhart and Rogoff (2009) estimate that an asset price crash is associated with a decline in GDP per capita of between 9 and 16%, while Atkinson, Luttrell, and Rosenblum (2013) estimate the cumulative loss in output in the US in the recent crisis was even larger. Such large costs are obviously not due to processing defaults but to how agents deal with losses from asset prices declines. For example, financial intermediaries who lent against assets during the boom may refuse to finance new investment when asset prices fall given the overhang of debt they face, while indebted households may be forced to delever and can drag down aggregate demand and output when prices are rigid.<sup>3</sup> Capturing these channels would require a substantially more complicated model. Our way of modelling default costs is simpler while still implying that larger asset booms translate into larger losses and more severe contractions.

At the heart of our model is an information asymmetry in which borrowers know the risks of their investments better than lenders. This encourages some agents to borrow and gamble on risky assets, knowing that lenders will bear the losses if their gamble fails. As speculators buy up assets, they drive the price of these assets up and the expected return on these assets down. This process leads to two distinct inefficiencies. The first involves misallocation. Since lenders cannot distinguish among the different types of investments, borrowers ultimately decide how funds are allocated. They will direct too many resources to risky investments that offer high private returns to speculation but low overall returns. This is consistent with evidence of misallocation during credit booms in Borio, Kharroubi, Upper, and Zampolli (2015) and Charles, Hurst, and Notowidigdo (2018). Second, even if resources are properly allocated, agents borrow too much because they fail to internalize the costs that others incur when they default. In our model these are the costs lenders incur in recovering assets, but more generally speculators would likely not internalize how their actions lead to lower output due to debt overhang or lower aggregate demand.

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<sup>2</sup>Shleifer and Vishny (1997) also study asset pricing with credit and limited information, but without involving risk-shifting. Asset prices in their model deviate from fundamentals because of noise traders and limits to arbitrage. They show how lender uncertainty can shape what creditors lend against assets and so how much asset prices deviate from fundamentals.

<sup>3</sup>See Phillippon (2010) on debt overhang and financial crises and Korinek and Simsek (2016) and Farhi and Werning (2016) on aggregate demand externalities and deleveraging. Rognlie, Shleifer, and Simsek (2018) suggest another channel involving investment overhang, whereby a glut of assets during the boom dampens the production of assets after the crash. Capturing this in our model is difficult given we assume assets are either endowed or created at date 0.

Our analysis yields new insights on when and how policymakers should address these inefficiencies. First, we find that even though risk-shifting can result in bubbles, these are a symptom of risk-shifting rather than the problem in and of itself. When default costs are large, assets will not exhibit bubbles even though risk-shifting remains a problem. Intuitively, large default costs discourage lending, helping keep asset prices in check and equal to fundamentals. But they don't discourage lending against risky assets altogether, and the lending that remains is distortionary. This provides a rigorous basis for the assertion in Borio and Lowe (2002) that policymakers should intervene during booms even if they cannot be sure if assets are overvalued, since whether assets are overvalued in our model is indeed irrelevant given evidence of risk-shifting.

The other key insight is that since risk-shifting models require productive uses of credit to cross-subsidize the lending that finances speculation, policy interventions that affect both activities can have ambiguous and even surprising effects. In particular, we find that tighter monetary policy helps alleviate excessive leverage associated with risk-shifting, but that it also exacerbates resource misallocation by reducing more productive investments that are already underfunded. This reveals a counterproductive aspect of tighter monetary policy. Nevertheless, tighter monetary policy can play a useful role if the first effect dominates, and there may be ways to mitigate some of the counterproductive aspects of tighter monetary policy if instead of tightening immediately, policymakers promise to tighten only if the boom continues into the future (and, by implication, to ease if the price of the asset collapses). We similarly find that macroprudential regulation can be counterproductive, increasing speculation rather than curbing it. This is because restricting leverage might especially curb productive investments, leaving more resources for speculation. Macroprudential regulation can thus make things worse, although in other circumstances it can play a useful role. The essence of our findings is that risk-shifting can offer a reason to intervene during an asset boom, but it also implies intervening will be difficult since policymakers will be hampered by the same informational frictions that give rise to risk-shifting in the first place. Indeed, one of the reasons a promise to act in the future is more useful in our framework than direct intervention is that it affects speculators and other borrowers differently, which means it can more effectively target speculation than a direct intervention would.

The paper is organized as follows. Section 1 introduces the basic setup, focusing on a simple case where assets are riskless. We then build on this framework in Section 2 and consider the case of risky assets. There, we argue that the equilibrium of our model can capture some of the key features of asset booms. Section 3 describes the ways in which the equilibrium of our model is inefficient and allow a possible role for policy intervention. Section 4 considers monetary policy, and Section 5 considers macroprudential regulation, specifically restrictions on leverage. Section 6 concludes.

## 1 Credit, Production, and Assets

We now set up our framework that includes credit, production, and assets. In this section we will consider the simple case where the asset involves no risk. In the next section we build on this setup and consider the more interesting case where assets are risky. It is in the latter case where credit and asset booms emerge.

Consider an overlapping generations economy where agents live for two periods and only value consumption when old. That is, agents born at date  $t$  value consumption  $c_t$  and  $c_{t+1}$  at dates  $t$  and  $t + 1$  at

$$u(c_t, c_{t+1}) = c_{t+1} \tag{1}$$

There is a fixed supply of identical assets normalized to one. For now, we assume these already exist at date 0, although later on we will consider the case where they must be produced. Each asset yields a constant real dividend  $d > 0$  per period. In the next section we will allow for a stochastic dividend.

There is a cohort of old agents at date 0 who start out owning all the assets. A new cohort is born at each  $t = 0, 1, 2, \dots$ . Each cohort consists of two types of agents. The first, whom we call savers, are endowed with an aggregate  $e$  units of the good when young. They cannot produce or store goods, and must either buy assets or trade intertemporally to convert their endowment into consumption when old. The second type, whom we call entrepreneurs, can convert a good at date  $t$  into  $1 + y$  goods at date  $t + 1$  where  $y > 0$ , but only up to a finite capacity of one unit of input. Each entrepreneur is endowed with  $w < 1$  goods while young. Since this is below their productive capacity, there is scope for savers and entrepreneurs to trade.

In principle,  $w$  and  $y$  can vary across entrepreneurs. For most of the analysis, though, we assume  $w = 0$  for all entrepreneurs, meaning they must borrow all of their inputs. We will allow  $w$  to vary across entrepreneurs in Section 5. We do assume  $y$  varies across entrepreneurs. Let  $n(y)$  denote the density of entrepreneurs with productivity  $y$ . We assume  $n(y) > 0$  for all  $y \in [0, \infty)$  and

$$e < \int_0^\infty n(y) dy < \infty \tag{2}$$

Condition (2) implies entrepreneurs could deploy more inputs than savers are endowed with.

We assume trade between savers and entrepreneurs is subject to the following frictions:

1. Savers cannot monitor whether those they finance produce or buy assets. They also cannot observe any of the agent's wealth beyond the particular project the lender finances.
2. Trade is restricted to debt contracts, i.e., for each unit of funding agents receive at date  $t$  they must promise to pay a fixed amount  $1 + R_t$  at date  $t + 1$
3. If borrowers fail to pay their obligation, lenders can get a court to transfer any proceeds from the project agents invested in, but the seizure wastes  $\Phi$  resources per unit invested in the project and incurs a utility cost  $\phi$  to the borrower who defaulted.

The first friction captures the idea that asset booms are often associated with uses that lenders cannot easily evaluate. For example, mortgage lenders may not be able to distinguish liquidity constrained agents who value homeownership and speculators who would default on their loans if prices fell. The return  $1 + y$

of entrepreneurs in our model would be analogous to the surplus to the former from homeownership.<sup>4</sup> Assuming wealth is unobservable implies borrowers face limited liability, since lenders can only go after any resources the borrowers own that they know about. Effectively, we are assuming agents can borrow via shell entities and hide their other resources, limiting their liability to the project they borrow for. The restriction to debt contracts accords with the empirical popularity of credit, which in practice is presumably due to the costs of enforcing contingent payments. Finally, although costly default is uncontroversial, the way we model these costs merits some discussion. We impose a utility cost  $\phi$  on borrowers only to ensure that they will not borrow if they expect to default with certainty. As such, we will focus on the limiting case where  $\phi \rightarrow 0$  to avoid explicitly accounting for  $\phi$ . Assuming seizure costs are proportional to the underlying investment captures the idea that auditing a borrower requires tracking what happened to all of the inputs the borrower invested. When we later allow for  $w > 0$  so that borrowers only need to finance a fraction of their investment, this will mean seizure is costly for large projects even if they involve little borrowing.

To recap, each period young savers allocate their endowment  $e$  to buying assets and making loans, young entrepreneurs choose whether to borrow to produce, and any young agent can choose to borrow to buy assets. When dividends are deterministic, as we have assumed so far, no agent who borrows will default. In that case, the inability to monitor borrowers, the restriction to debt contracts, and the cost of default are irrelevant. It is only when we assume stochastic dividends in the next section that these frictions matter.

An equilibrium for this economy consists of paths for asset prices  $\{p_t\}_{t=0}^{\infty}$  and interest rates on loans  $\{R_t\}_{t=0}^{\infty}$  that ensure both asset and credit markets clear when agents act optimally. To facilitate our exposition, we will proceed as if these paths are deterministic. In Appendix A we show there are no equilibria with stochastic prices. To solve for an equilibrium, we need to derive supply and demand for assets and credit. Agents in their last period of life do not participate in credit markets and so do not enter into either supply or demand for credit. They are the source of supply in asset markets, however, selling any assets they own if the asset price  $p_t > 0$ . Young savers are the only agents with resources to lend. They will compare the return to lending with the return to the asset and invest only in whatever offers the highest return. Young entrepreneurs are the only ones who can borrow to produce. Those with productivity  $y > R_t$  will find it profitable to do so. Finally, any young agent can borrow to buy assets, and would be willing to do so if the return to assets  $1 + r_t \equiv \frac{d+p_{t+1}}{p_t}$  is at least as high as the interest rate on loans  $1 + R_t$ .

Savers use their endowment to either buy assets or make loans. Their borrowers will then either produce or buy assets. Hence, the endowment is ultimately used to either finance production or buy assets, implying

$$\int_{R_t}^{\infty} n(y) dy + p_t = e \tag{3}$$

Since we assume  $n(y) > 0$  for all  $y$ , there is a unique interest rate  $R_t = \rho(p_t)$  that satisfies (3) for any asset price  $p_t$ . Moreover,  $\rho(p_t)$  is increasing in  $p_t$ . Intuitively, a higher  $p_t$  reduces the amount of goods available for productive investment, so the interest rate on loans  $R_t$  must rise to lower demand from entrepreneurs.

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<sup>4</sup>Of course, the surplus agents who value homeownership obtain are not a constant but depend on the price of housing. For an example of a proper risk-shifting model of housing, see Barlevy and Fisher (2018).

Next, we argue the interest rate on loans  $1 + R_t$  must equal the return to buying the asset  $1 + r_t \equiv \frac{d + p_{t+1}}{p_t}$ . For suppose  $R_t < r_t$ . Then agents could earn positive profits by borrowing and buying assets. Demand for borrowing would be infinite while the supply of credit can be at most  $e$ , so this cannot be an equilibrium. Next, suppose  $R_t > r_t$ . Then no one would buy the asset: Savers can earn a higher return from lending than buying the asset, and no agent would borrow to buy the asset knowing she would default. Since the old sell the asset whenever its price is positive, this would require  $p_t \leq 0$ . But if the price were nonpositive, demand for the asset would be infinite. For both the credit and asset market to clear, then, we must have

$$1 + R_t = \frac{d + p_{t+1}}{p_t} = 1 + r_t \quad (4)$$

Substituting (3) into (4) implies

$$\begin{aligned} p_{t+1} &= (1 + \rho(p_t)) p_t - d \\ &\equiv \psi(p_t) \end{aligned} \quad (5)$$

where  $\psi'(p_t) > 1$ ,  $\psi(0) = -d < 0$ , and  $\lim_{p \rightarrow \infty} \psi(p) > e$ . The graph of  $\psi(p)$  is illustrated in Figure 1 together with the 45° line. The two lines intersect at the unique value  $p^d$  at which  $p^d = \psi(p^d)$ . For any initial condition, the law of motion  $p_{t+1} = \psi(p_t)$  defines a unique path of asset prices. For any initial condition other than  $p_0 = p^d$ , the path will reach in finite time a value that is either negative or exceeds  $e$ , neither of which can be an equilibrium. Hence, the unique deterministic equilibrium is  $p_t = p^d$  and  $R_t = \rho(p^d)$  for all  $t$ . We make note that the steady state price  $p^d$  is increasing in the dividend  $d$ . Formally, setting  $p_t = p_{t+1} = p^d$  in the zero-profit condition (4), implies

$$d = \rho(p^d) p^d$$

The right hand side is increasing in  $p^d$ , so a higher  $d$  must imply a higher  $p^d$ . Graphically, a larger  $d$  will shift the curve  $p_{t+1} = \psi(p_t)$  in Figure 1 down, and so the steady state  $p^d$  will rise.

In Appendix A, we confirm that there are no stochastic equilibria, so  $p_t = p^d$  for all  $t$  is the unique equilibrium for this economy. We can summarize this result as follows.

**Proposition 1** *The unique equilibrium when  $d_t = d$  for all  $t$  features a constant asset price  $p_t = p^d$  and a constant interest rate  $R_t = \rho(p^d)$  for all  $t$ . Only entrepreneurs with productivity  $y > R_t$  produce.*

In equilibrium, then, savers both lend to entrepreneurs and invest in assets, either directly or by lending to others who buy assets and pass all the proceeds on to savers. Savers are indifferent between the two activities in equilibrium, and the returns to buying assets and making loans are equal. Denote this return by  $R^d = \rho(p^d)$ . Consider the present value of dividends discounted at this return. This is given by

$$f_t \equiv \sum_{j=1}^{\infty} \left( \frac{1}{1 + R^d} \right)^j d = d / R^d = p^d$$

The value of dividends discounted according to the return agents can earn thus coincides with the price of the asset. When  $d_t = d$  for all  $t$ , the asset will not be associated with a bubble.

**Remark 1:** We can easily allow for multiple riskless assets. Suppose there were  $J$  assets indexed  $j = 1, \dots, J$ , each with fixed supply of 1 but potentially different fixed dividends  $d_j$ . Let  $p_{jt}$  denote the price of the  $j$ -th asset at date  $t$ . Define  $d \equiv \sum_{j=1}^J d_j$  as the total dividends from all  $J$  assets and  $p_t \equiv \sum_{j=1}^J p_{jt}$  as the value of all  $J$  assets. Resources that don't finance production will be used to buy assets, so (3) continues to hold. In addition, the return on each asset  $1 + r_{jt} \equiv \frac{d_j + p_{j,t+1}}{p_{jt}}$  must equal the interest rate on loans  $1 + R_t$ . Combining these equalities implies (4). Hence, the equilibrium conditions for  $p_t$  and  $R_t$  are unchanged. We can reinterpret  $p_t$  from our model as the value of all assets, each offering return  $R_t$ . ■

**Remark 2:** With some modifications, we can also allow for a growing set of assets. This will be relevant in the next section, where the periodic arrival of new types of assets can serve as a trigger for asset booms. Suppose each period's old are endowed with a stock of new assets normalized to 1. Stocks pay dividends one period after arrival. For aggregate dividends to remain constant, dividends on any single asset must decay over time. Let  $d_{st}$  denote the dividend at date  $t$  on assets that arrived at date  $s$ , and set

$$d_{st} = \begin{cases} (1 - \theta)^{t-1} d & \text{if } s = 0 \\ (1 - \theta)^{t-(s+1)} \theta d & \text{if } s = 1, 2, 3, \dots \end{cases} \quad \text{for } t \geq s + 1$$

By design, total dividends  $\sum_{s=0}^{t-1} d_{st}$  in each period  $t$  sum to  $d$ . Let  $p_{st}$  denote the date- $t$  price of the asset that arrived at date  $s$ , and set  $p_t = \sum_{s=0}^t p_{st}$  as the total value of all assets around at date  $t$ . The market clearing condition (3) is unchanged. The return on each asset  $1 + r_{st} \equiv \frac{d_{s,t+1} + p_{s,t+1}}{p_{st}}$  will equal the interest rate on loans  $1 + R_t$ . Aggregating over all assets available at date  $t$  yields the following alternative to (4):

$$1 + R_t = \frac{d + (p_{t+1} - p_{t+1,t+1})}{p_t}$$

The equilibrium value of all assets  $p_t$  will be constant and equal to  $\frac{d}{R^d + \theta}$ , where  $R^d$  denotes the equilibrium interest rate on loans. The price of any individual asset equals  $p_{st} = \frac{d_{s,t+1}}{d} p_t = \frac{d_{s,t+1}}{R^d + \theta}$ . ■

We conclude our discussion with a brief comment on welfare. In equilibrium, the amount savers spend on assets equates the return on the asset to the productivity of the marginal entrepreneur. Is this efficient? At first, it might seem that any resources spent on the asset are wasted, since the asset will yield  $d$  regardless of how much is spent on it while lending to entrepreneurs yields additional output. However, shifting resources to production would rob the current owners of the asset from resources they earn selling their assets. Redirecting all resources towards production is therefore not Pareto improving. In fact, the equilibrium is efficient. Intuitively, suppose current asset owners could destroy their assets so that they stop yielding any dividends going forward. Paying the old for their assets can thus be viewed as an investment to preserve the asset. Efficiency dictates the returns to all investments should be equal at the margin, as they indeed are. In the next section, we show that returns will not be equated when assets are risky.



## 2 Risky Assets, Credit Booms, and Bubbles

We now consider the case where the asset pays stochastic dividends. We return to assuming only one asset. Let the dividend on this asset follow a regime-switching process such that the dividend  $d_t$  starts at  $D > d$  when  $t = 0$  and then switches to  $d$  with a constant probability  $\pi \in (0, 1)$  in each period if it has yet to switch. Once the dividend falls to  $d$ , it will remain equal to  $d$  forever.

An equilibrium still consists of paths for asset prices  $\{p_t\}_{t=0}^\infty$  and interest rates on loans  $\{R_t\}_{t=0}^\infty$ , but now also includes a path for the share of lending used to buy assets  $\{\alpha_t\}_{t=0}^\infty$ . These paths must be consistent with optimizing behavior by agents and must ensure asset and credit markets clear at all dates  $t$  for any  $d_t$ .

In what follows, it will prove convenient to distinguish for each date  $t$  whether the dividend  $d_t$  is  $D$  or  $d$ . If  $d_t = D$ , agents who buy the asset at date  $t$  will be unsure about the dividends  $d_{t+1}$  it will pay at  $t + 1$ . If  $d_t = d$ , agents who buy the asset at date  $t$  know it will pay a dividend of  $d$  at date  $t + 1$ . Let  $(p_t^D, R_t^D, \alpha_t^D)$  denote an equilibrium if  $d_t = D$  and  $(p_t^d, R_t^d, \alpha_t^d)$  denote an equilibrium if  $d_t = d$ . Once dividends fall, the equilibrium will be the same as in Section 1, with  $p_t^d = p^d$  and  $R_t^d = R^d$  for all  $t$ . Although Proposition 1 does not characterize  $\alpha_t^d$ , recall that when  $d_t = d$  those who borrow to buy assets know they will pass on all of the return from the asset to their lender. This implies  $\alpha_t^d$  is indeterminate and can assume any value in  $[0, \frac{p^d}{e}]$ . All that remains is to solve for  $\{(p_t^D, R_t^D, \alpha_t^D)\}_{t=0}^\infty$ .

We first show that we can solve for the equilibrium price  $p_t^D$  and interest rate on loans  $R_t^D$  independently of  $\alpha_t^D$ . As before, savers allocate all of their endowment  $e$  to either fund production or buy assets. The price  $p_t^D$  must thus continue to satisfy (3). Next, we argue that in equilibrium,

$$(1 + R_t^D) p_t^D = p_{t+1}^D + D \tag{6}$$

That is, the interest rate on loans  $1 + R_t^D$  is equal to the return on the asset if  $d_{t+1} = D$ . We first argue that  $p_{t+1}^D + D$  represents the maximum possible payoff to the asset, in the sense that

$$p_{t+1}^D + D > p^d + d$$

For suppose  $p_t^D + D \leq p^d + d$  for some  $t$ . Since  $D > d$ , this implies  $p_t^D < p^d$ . From (3), we know the equilibrium interest rate on loans  $R_t$  must equal  $\rho(p_t)$ . If  $p_t^D < p^d$ , then since  $\rho'(\cdot) > 0$ , we have

$$R_t^D = \rho(p_t^D) < \rho(p^d) = R^d$$

But then we would have

$$(1 + R_t^D) p_t^D < (1 + R^d) p^d = p^d + d.$$

This means that an agent who borrows to buy assets can make positive profits after paying their debt obligation at date  $t + 1$  if  $d_{t+1} = d$ . But then there would be infinite demand for borrowing to buy assets, which cannot be an equilibrium given supply of credit is finite. It follows that  $p_t^D + D > p^d + d$ .

To show that  $(1 + R_t^D) p_t^D$  must equal the highest return on the asset, suppose  $(1 + R_t^D) p_t^D < p_{t+1}^D + D$ . In this case, demand for borrowing would be infinite: Agents can earn positive profits if  $d_{t+1} = D$  but default and earn zero if  $d_{t+1} = d$ . Since the supply of credit is finite, this cannot be an equilibrium. Next, suppose  $(1 + R_t^D) p_t^D > p_{t+1}^D + D$ . In this case, no agent would borrow to buy the asset knowing they would surely default and incur the cost  $\phi$ . Nor would any agent buy the asset given they can earn more making loans. In particular, since no agent borrows to buy assets, the only agents who borrow are entrepreneurs with productivity  $y > R_t^D$ , and they will repay for sure. The return to lending  $R_t$  would then exceed the return to buying the asset. If no agent buys the asset, the price of the asset  $p_t^D$  would have to be nonpositive to ensure the old don't want to sell the asset. But this cannot be an equilibrium price, since if  $p_t^D \leq 0$  there would be infinite demand for the asset. The only remaining possibility is  $(1 + R_t^D) p_t^D = p_t^D + D$ .

Condition (6) is identical to the condition for an asset that offers a constant dividend  $d_t = D$  for all  $t$ . From the previous section, we know there is a unique path  $\{p_t^D, R_t^D\}_{t=0}^\infty$  that satisfies both this condition and (3). The equilibrium price  $p_t^D$  is constant and equal to  $p^D$ , where  $p^D$  solves

$$\rho(p^D) p^D = D$$

Likewise, the interest rate on loans  $R_t^D$  is constant and equal to  $R^D = \rho(p^D)$ . The fact that the asset trades as if it delivers  $d_t = D$  forever, even though the dividend can fall with a probability  $\pi$  that can be arbitrarily close to 1, suggests the asset is overvalued. We return to this point below.

The only part of the equilibrium we still need to solve for is  $\alpha_t^D$ . Let us first derive the expected returns to buying an asset  $\bar{r}_t^D$  and lending  $\bar{R}_t^D$ , respectively. The expected return to buying the asset satisfies

$$1 + \bar{r}_t^D = (1 - \pi) \left(1 + \frac{D}{p^D}\right) + \pi \left(\frac{d+p^d}{p^D}\right) \equiv 1 + \bar{r}^D \quad (7)$$

As for the expected return to lending, by definition a fraction  $\alpha_t^D$  of lending is used to buy assets. Since all of the proceeds from asset purchases accrue to the lender, the expected return to these loans is just the expected return to buying an asset net of default costs,  $1 + \bar{r}^D - \pi\Phi$ . All remaining loans are used to finance production and will be repaid in full, so the return on those loans is  $1 + R^D$ . This implies

$$\begin{aligned} 1 + \bar{R}_t^D &= (1 - \alpha_t^D) (1 + R^D) + \alpha_t^D (1 + \bar{r}^D - \pi\Phi) \\ &= (1 - \alpha_t^D) \left(1 + \frac{D}{p^D}\right) + \alpha_t^D (1 + \bar{r}^D - \pi\Phi) \end{aligned} \quad (8)$$

If  $\bar{R}_t^D > \bar{r}^D$ , savers would prefer lending over buying assets. The only agents who would buy assets would be those who borrow to do so, and so  $\alpha_t^D = \frac{p^D}{e}$ . If  $\bar{R}_t^D = \bar{r}^D$ , savers would be indifferent between buying assets and lending. This means  $\alpha_t^D$  can assume any value between 0 and  $\frac{p^D}{e}$ . Finally, if  $\bar{R}_t^D < \bar{r}^D$ , savers would prefer buying assets over lending. No agent would borrow to buy assets, implying  $\alpha_t^D = 0$ . Hence, the expected return to lending  $\bar{R}_t^D$  and the share of lending used to buy assets  $\alpha_t^D$  are jointly determined: We just described how  $\alpha_t^D$  depends on  $\bar{R}_t^D$  relative to  $\bar{r}^D$ , while (8) implies  $\bar{R}_t^D$  depends on  $\alpha_t^D$ .

To solve for  $\bar{R}_t^D$  and  $\alpha_t^D$ , consider first the case where  $\alpha^D = \frac{p^D}{e}$ . This can only be an equilibrium if

$\bar{R}_t^D \geq \bar{r}^D$  when  $\alpha_t^D = \frac{p^D}{e}$ , i.e., only if

$$\left(1 - \frac{p^D}{e}\right) \frac{D}{p^D} + \frac{p^D}{e} (\bar{r}^D - \pi\Phi) \geq \bar{r}^D$$

Rearranging this equation and substituting in for  $\bar{r}^D$  implies  $\alpha_t^D = \frac{p^D}{e}$  is an equilibrium only if

$$\Phi \leq \left(\frac{e}{p^D} - 1\right) \left(\frac{D+p^D-d-p^d}{p^D}\right) \equiv \Phi^* \quad (9)$$

Next, consider the case where  $\alpha_t^D \in \left(0, \frac{p^D}{e}\right)$ . This can only be an equilibrium if  $\bar{R}_t^D = r^D$  when we evaluate  $\bar{R}_t^D$  at the relevant  $\alpha_t^D$ . Since  $\bar{R}_t^D$  is decreasing in  $\alpha_t^D$ , this requires that  $\bar{R}_t^D < \bar{r}^D$  when  $\alpha_t^D = \frac{p^D}{e}$ , or

$$\Phi > \Phi^* \quad (10)$$

In this case, the equilibrium value of  $\alpha_t^D$  is the one that equates  $\bar{R}_t^D$  and  $\bar{r}^D$ , which implies

$$\alpha_t^D = \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} \quad (11)$$

Finally, there cannot be an equilibrium in which  $\alpha_t^D = 0$ . This would require  $\bar{R}_t^D \leq \bar{r}^D$  when  $\alpha_t^D = 0$ . But  $\alpha_t^D = 0$  implies  $\bar{R}_t^D = \frac{D}{p^D} > \bar{r}^D$ . Hence, the value of  $\alpha_t^D$  is unique and is either equal to  $\frac{p^D}{e}$  or some value between 0 and  $\frac{p^D}{e}$ , depending on the cost of default  $\Phi$ . We can summarize this result as follows:

**Proposition 2** *When the dividend process follows a regime-switching process, in the limit as  $\phi \rightarrow 0$ , the unique equilibrium is given by*

$$(p_t, R_t) = \begin{cases} (p^D, R^D) & \text{if } d_t = D \\ (p^d, R^d) & \text{if } d_t = d \end{cases}$$

The equilibrium share of loans  $\alpha_t$  is constant when  $d_t = D$  and given by

$$\alpha_t = \alpha^D = \begin{cases} \frac{p^D}{e} & \text{if } \Phi \leq \Phi^* \\ \frac{D+p^D-d-p^d}{D+p^D-d-p^d+\Phi p^D} & \text{if } \Phi > \Phi^* \end{cases}$$

When  $d_t = d$ , the equilibrium  $\alpha_t$  is indeterminate and can assume any value in  $\left[0, \frac{p^d}{e}\right]$ .

While dividends are high, some agents must borrow to bet on assets. Intuitively, if this were not the case, the interest rate on loans  $R_t$  would have had to equal the expected return on the asset for savers to be willing to both buy assets and make loans. But this would have made it profitable to borrow to buy assets, earn positive profits if  $d_{t+1} = D$  and default and shift losses to the lender if  $d_{t+1} = d$ . So some agents must speculate in equilibrium, and the price of the asset and the interest rate on loans must ensure zero expected profits from speculation to avoid the demand for borrowing for speculation to be infinite. At the same time, the model makes no predictions on which agents will borrow to buy assets. Less productive entrepreneurs who do not produce would certainly be willing to speculate. But since agents can hide any wealth not associated with the particular project they borrow for, entrepreneurs who borrow to produce and savers who make loans would be equally willing to borrow through a shell entity and speculate.

Now that we have characterized the equilibrium when assets are risky, we argue it can capture many features of the episodes documented by Borio and Lowe (2002), Jorda, Schularick, and Taylor (2015), and Mian, Sufi, and Verner (2017). Specifically, we show that our equilibrium can be associated with asset price booms and credit booms, asset bubbles, high realized returns but cheap borrowing during the boom, and an eventual crisis when asset prices collapse, speculators default, and consumption falls.

**Asset Price Booms:** We begin with asset price dynamics. The equilibrium price of the asset while  $d_t = D$  will be the same as in an economy in which the dividend is equal to  $D$  for all  $t$ . But as we noted earlier, the price of an asset with a fixed dividend is increasing in the value of the dividend. Hence,  $p^D > p^d$ . Our economy therefore starts with a high asset price that collapses when dividends fall.

Historically, asset price booms are characterized by both high prices and high price growth. But our environment implies a constant price within each regime. This is because we assume constant dividends within each regime, which simplifies the analysis by allowing us to solve for a single price within a regime rather than a price path. If we allowed dividend to rise within each regime, we could generate a rising price path during the boom. That is, suppose dividends in the high regime followed an increasing sequence  $\{D_t\}_{t=0}^{\infty}$  where  $D_0 \geq d$ , until a random date when dividends fall to  $d$  forever.<sup>5</sup> Intuitively, new technologies are often not immediately profitable but offer the possibility of an eventual stream of profits. Likewise, rents often don't rise quickly in hot housing markets, but there is an expectation that continuing housing demand could eventually lead to scarcity and high rents. The equilibrium interest rate on loans  $1 + R_t^D$  would still have to equal the maximum return on the asset at date  $t$ . Using the fact that  $R_t^D = \rho(p_t^D)$ , the price path  $p_t^D$  would have to satisfy the sequence of difference equations

$$p_{t+1}^D = (1 + \rho(p_t^D)) p_t^D - D_{t+1}$$

In the case where  $D_t$  is constant once  $t \geq T$  for some finite  $T$ , we can use backwards induction to solve for the unique equilibrium path  $p_t^D$  and show that it rises during the boom. Our model can thus accommodate price growth during the boom, but this offers little new insight at the cost of substantial complexity.

Our setup also abstracts from how an asset boom starts. Suppose  $d_t$  were equal to  $d$  at  $t = 0$  and then dividends switched with some probability to a high dividend regime that gives way with constant probability  $\pi$  to a permanently low dividend regime. Before dividends rise, agents would have an incentive to borrow and gamble on a high future dividend, and the asset would trade at  $p^D$ . The boom would then be present from inception, before dividends rose. This is reminiscent of the Diba and Grossman (1987) result that asset bubbles cannot suddenly appear but must be present from the inception of the asset. Martin and Ventura (2012) show we can get around the latter result and model the onset of bubbles by allowing for the constant arrival of new assets that cannot be traded before they come into existence. We can similarly capture the start of asset booms by allowing for new assets as per Remark 2. That is, we assume the economy keeps

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<sup>5</sup>Zeira (1999) previously analyzed asset pricing when dividends grow up to a stochastic date. He emphasized that asset prices can rise with dividends and then crash once dividends stop growing even if dividends themselves do not change. By contrast, we allow dividends to fall when growth stops. Zeira also did not consider risk-shifting or credit.

generating new assets, most of which pay a predictable but decaying return, but periodically new assets arrive which offer a temporarily high return that gives way to a predictable but decaying return. The arrival of these assets would trigger a boom on a new class of assets that did not previously trade.<sup>6</sup>

In short, although our model as specified only features a high initial asset price that eventually falls, it can be modified to incorporate periodic asset booms with rising prices that eventually crash.

**Credit Booms:** Next, we show that the asset boom is associated with a boom in lending against assets. Recall that lending by savers finances both production and asset purchases. When  $d_t = D$ , the amount agents borrow to buy assets is unique and given by

$$\frac{\alpha^D}{1 - \alpha^D} \int_{R^D}^{\infty} n(y) dy \quad (12)$$

By contrast, the amount of lending when  $d_t = d$  is indeterminate given agents are indifferent about borrowing to buy assets. However, this indeterminacy is not robust to the introduction of small transaction costs for borrowing; if it were even a little costly to borrow, agents would only buy riskless assets with their own wealth when  $d_t = d$ . By contrast, agents will continue to borrow both to buy risky assets and to produce when transaction costs are small. Thus, if we introduce small borrowing costs and take the limit as these costs go to zero, lending against assets will disappear when  $d_t = d$ . The boom in asset prices when  $d_t = D$  would then be associated with a boom in borrowing to buy assets.

If we cannot distinguish lending for speculation and lending for production, the relevant empirical measure of credit is total borrowing. Adding the amount borrowed to produce and to buy assets at date  $t$  yields

$$\frac{1}{1 - \alpha_t} \int_{R_t}^{\infty} n(y) dy$$

The term  $\frac{1}{1 - \alpha_t}$  will be higher when  $d_t = D$  and  $\alpha_t = \alpha^D \in (0, \frac{p^D}{e})$  than when  $d_t = d$  and  $\alpha_t = 0$ . But the interest rate on loans  $R_t$  will go from  $R^D$  to  $R^d$ , so the amount agents borrow to produce  $\int_{R_t}^{\infty} n(y) dy$  will be lower when  $d_t = D$ . Total lending can therefore either rise or fall when  $d_t$  falls to  $d$ . When  $\Phi \leq \Phi^*$ , savers lend out all of their endowment  $e$  when  $d_t = D$  but strictly less than  $e$  when  $d_t = d$ . Only when  $\Phi \gg \Phi^*$  is it possible for total borrowing to fall with dividends. The asset boom is therefore associated with a boom in lending for speculation, and with a boom in total lending when default costs are not too large.

**Asset Bubbles:** Our model also implies that during the boom, assets can trade at a price above their fundamental value, i.e., the present discounted value of the dividends assets are expected to generate. Although in practice it is difficult to measure the fundamental value of an asset, asset booms are often suspected to be associated with bubbles given how fast asset prices rise without any reasonable changes in the expected flow of dividends to justify this increase. In the model, of course, we can exactly compute the fundamental value of an asset and determine if asset booms coincide with bubbles.

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<sup>6</sup>To ensure the return on assets is riskless outside of booms may require one-off changes in the dividends of existing assets if the assets that arrive are risky to ensure the return on existing assets is the same as it would be if the new assets were riskless.

We begin by defining the fundamental value of the asset in our model. For this, it will help to distinguish among several rates of return when  $d_t = D$ . The first is the interest rate on loans  $R^D$  that borrowers are asked to repay. Recall this rate is the maximal possible return on the asset, i.e.,

$$1 + R^D = 1 + \frac{D}{p^D} \quad (13)$$

During the boom, lenders will not expect to always collect this interest in full, since a fraction  $\alpha^D > 0$  of lending is used to buy assets and may end in default. Lenders instead expect to earn  $\bar{R}^D$  given by

$$1 + \bar{R}^D = (1 - \alpha^D \pi) \left(1 + \frac{D}{p^D}\right) + \alpha^D \pi \left(\frac{d+p^d}{p^D} - \Phi\right) \quad (14)$$

Finally, the expected return to buying the asset in equilibrium is given by

$$1 + \bar{r}^D = \frac{(1-\pi)(D+p^D) + \pi(d+p^d)}{p^D} \quad (15)$$

These three returns can be ranked, with  $R^D > \bar{R}^D \geq \bar{r}^D$ . To derive the last inequality, note that if the expected return to buying the asset  $1 + \bar{r}^D$  exceeded  $1 + \bar{R}^D$ , no agent would agree to lend given they can buy the asset. But at any finite interest rate, demand for credit from entrepreneurs will be positive.

We now need to take a stand on the rate at which to discount dividends when we define the fundamental value of an asset. Given the information frictions in our economy, the relevant discount rate should arguably be the rate any agent with wealth would expect to earn in the absence of any information about who they might be lending to. The expected return such agents earn in our economy is the expected return savers anticipate. Since savers always lend in our economy, this expected return is equal to  $\bar{R}^D$ . Using the fact that the equilibrium is stationary, we can define the fundamental value of the asset  $f^D$  recursively as

$$f^D = \frac{\pi(d+p^d) + (1-\pi)(D+f^D)}{1 + \bar{R}^D} \quad (16)$$

Equation (16) incorporates  $1 + \bar{R}^D$  as the discount rate and uses the fact there  $p^d = f^d$ , since recall from the previous section that with constant dividends the price  $p^d$  coincides with the fundamental value of the asset  $d/R^d$ . Rearranging (16) implies

$$1 + \bar{R}^D = \frac{\pi(d+p^d) + (1-\pi)(D+f^D)}{f^D} \quad (17)$$

Comparing (17) with (15) shows that  $p^D > f^D$  whenever  $\bar{R}^D > \bar{r}^D$  and  $p^D = f^D$  whenever  $\bar{R}^D = \bar{r}^D$ . From Proposition 2 we can infer that when  $\Phi < \Phi^*$ , the expected return on loans  $\bar{R}^D$  exceeds the expected return on the asset  $\bar{r}^D$ . In this case the asset will exhibit a bubble. But when  $\Phi \geq \Phi^*$ , the expected return on loans  $\bar{R}^D$  will equal the expected return on the asset  $\bar{r}^D$ . In this case the price of the asset coincides with fundamentals. As we summarize in the next proposition, whether a bubble exists depends on  $\Phi$ :

**Proposition 3** *Let  $f^D$  denote the value of dividends discounted at the expected return on loans  $\bar{R}_t$ . Then the difference between the price of the asset and its fundamental value  $b^D = p^D - f^D$  is*

$$b^D = (\pi(d+p^d) + (1-\pi)D) \left[ \frac{1}{\pi + \bar{r}^D} - \frac{1}{\pi + \bar{R}^D} \right] \quad (18)$$

*Hence, there exists a bubble when  $\Phi < \Phi^*$  but not when  $\Phi \geq \Phi^*$ .*

Bubbles can arise because leveraged agents who have the option to default only care about the upside potential of the asset and are willing to pay above its expected value to buy it. When  $\Phi$  is large, savers will be reluctant to lend if there is too much speculation. This requires that savers buy some fraction of the assets. But since savers value an asset at its fundamentals, the asset cannot be a bubble in this case.

Although bubbles only arise when  $\Phi < \Phi^*$ , there is a sense in which agents spend too much on assets for all  $\Phi$ . To see this, note that since  $R^D > \bar{R}^D \geq \bar{r}^D$ , the marginal entrepreneur during the boom can earn a higher return  $R^D$  than the expected return  $\bar{r}^D$  an asset can generate regardless of whether  $\bar{R}^D > \bar{r}^D$  or  $\bar{R}^D = \bar{r}^D$ . Hence, it is not bubbles that lead to misallocation in our model, but risk shifting. This suggests that the relevant question for whether to intervene during an asset boom is not whether the asset is a bubble but whether there is evidence of risk shifting. We return to this point in the next section.

**Realized Returns and Interest Rates:** We next consider rates of return during the boom. Since  $R^D > R^d$ , the high dividend regime will be associated with a higher realized return on investment, both for those who buy assets and for those who lend.<sup>7</sup> A boom will appear to be a good time for savers.

However, there are two important caveats to this result. First, even if *realized* returns are higher during the boom, *expected* returns may be lower. The expected return to lending during the boom is  $\bar{R}^D$  and after the boom is  $R^d$ . The expected return  $\bar{R}^D$  defined in (14) is a weighted average of  $1 + \frac{D}{p^D}$  and  $\frac{d+p^d}{p^D}$  minus expected default costs. Since  $D/p^D = R^D > R^d$  and  $R^d = d/p^d > d/p^D$ , we have

$$1 + \frac{D}{p^D} > 1 + R^d > \frac{d+p^d}{p^D}$$

If the weighted average of  $1 + \frac{D}{p^D}$  and  $\frac{d+p^d}{p^D}$  gives enough weight to the latter, the expected return to lending will be below  $1 + R^d$  even before accounting for default costs. Asset booms can therefore be times of low expected returns even though realized returns while the boom continues are high.

Second, notwithstanding the high returns agents earn during the boom, there is a sense in which interest rates are too low during the boom. Consider an economy in which lenders could monitor borrowers. Lenders would charge borrowers different rates depending on whether they produce or buy assets. Those who borrow to buy assets would be charged an interest rate at least as high as the maximum return on the asset,  $1 + D/\hat{p}^D$ , where a hat denotes the asset price in the counterfactual economy with monitoring. If  $\Phi > 0$ , the only possible equilibrium with monitoring would involve no borrowing to buy the asset while  $d_t = D$ . Savers would instead have to buy the asset directly from the old. But they must also make loans to entrepreneurs. Hence, the expected return on the asset with monitoring,  $(1 - \pi)(1 + D/\hat{p}^D) + \pi(d + p^d)/\hat{p}^D$ , must be the same as the interest rate on loans to those who produce and repay in full. We can show that this implies  $\hat{p}^D < p^D$ . Intuitively, savers will lend more to entrepreneurs to produce when they can monitor borrowers, meaning fewer resources will be spent to buy the asset. This implies

$$R^D = \frac{D}{p^D} < \frac{D}{\hat{p}^D}$$

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<sup>7</sup>In our model, the high return on the asset is due to higher dividends. In practice, the high returns in boom are primarily due to asset price growth. As we noted above, we can introduce such growth by allowing for time-varying dividends.

The interest rate when lenders cannot monitor borrowers will be lower than the interest rate charged to those who buy assets with perfect monitoring. Lending against risky assets thus carries too low of an interest rate. Essentially, imperfect monitoring forces entrepreneurs to cross-subsidize speculators so the latter face interest rates that don't reflect the risk of the assets they buy.

**Fallout from the Crash:** Finally, we turn to how asset booms end in our model. When dividends fall, agents who previously borrowed to buy assets will be forced to default and will impose costs  $\Phi p^D$  on their lenders. The collapse in asset prices thus triggers a fall in the resources this cohort can consume above and beyond the decline in the dividend income they earn. By construction, the decline is proportional to the price of assets  $p^D$  during the boom. A larger boom thus implies a larger loss once the boom ends. In our model this is because recovery costs are larger the more resources were invested in assets. But, as we noted in the Introduction, we view this as a stand-in for other channels in which a fall in asset prices would lead to lower output, e.g. debt overhang and or deleveraging. These mechanisms also suggest the decline in output after a crash should depend on the magnitude of losses associated with the crash.

To recap, our model can capture several key features of asset booms. In the remainder of the paper, we examine whether there is a reason to intervene against a boom in our model and to analyze some of the interventions that have been proposed by policymakers.

### 3 Inefficiency of Equilibria

In this section, we argue that there are two distinct senses in which the equilibrium of our model is inefficient when  $d_t = D$ . The first concerns resource misallocation: The marginal return to production during the boom exceeds the expected return on assets, so there are potential gains to redirecting some of the resources spent on assets to production. The second concerns excessive leverage: Agents who borrow to buy assets ignore the default costs  $\Phi p^D$  they impose on others and take on too much debt. As we noted in the Introduction, there is suggestive evidence that these distortions indeed occur during booms.

We begin with misallocation. As we noted above, when  $d_t = D$ , the productivity of the marginal entrepreneur is  $R^D$ . The interest rate on loans  $R^D$  exceeds the expected return on loans  $\bar{R}^D$ , since agents who borrow to buy assets can default. At the same time,  $\bar{R}^D$  weakly exceeds the expected return on the asset  $\bar{r}^D$ . Otherwise, savers would refuse to lend and opt to buy assets instead, yet demand for credit from entrepreneurs is always positive. Hence,  $R^D > \bar{r}^D$ . This inequality implies that young agents could coordinate to earn a higher return by shifting some of what they spend on assets to the marginal entrepreneur. Intuitively, agents who borrow to buy the asset ignore the losses that their lenders incur. As a result, their private gain to buying the asset will exceed its social return, and they end up allocating too many resources to buying assets. This distortion only occurs during the boom. Once the boom ends, the return on the asset  $d/p^d$  will be the same as the productivity of the marginal entrepreneur  $R^d$ .



Although the young could earn a higher return by agreeing to shift some resources from buying assets to production, doing so would leave the old agents from whom they buy assets poorer. Redirecting resources to production would therefore not constitute a Pareto improvement. This point was already made in Grossman and Yanagawa (1993). They also studied an overlapping generations economy in which agents use resources to produce and buy assets. While their model did not feature risk shifting, it did feature a production externality that implied the return to production is higher than the return on the asset. One of their key results is that even though resources are misallocated, it will be impossible to make all agents better off by reallocating resources. We now argue that this impossibility result hinges on assuming an exogenously fixed endowment of assets. If agents must use resources to create assets, as is true for new technologies or housing, the resources buyers pay for assets are no longer pure rents to sellers. In this case, shifting resources from asset creation to entrepreneurs could potentially make all agents better off.

Formally, suppose the old at date 0 are endowed with neither goods nor assets, but they know how to convert goods into assets. For simplicity, suppose assets can only be created at date 0. The technology for producing assets is summarized by an increasing function  $c(q)$  which denotes the amount of goods needed to produce the  $q$ -th asset. Since goods are endowed to the young, the old must obtain goods from them. We will proceed as if the old collect the revenue from asset sales in advance. They will use some of the goods they receive as payment to produce assets and consume the rest. Optimality dictates they should create assets up to the point  $q^*$  at which the marginal cost of producing assets  $c(q^*)$  equals the price  $p_0$ . The old will then collect  $p_0 q^*$ , which exceeds the amount of goods  $C(q^*) \equiv \int_0^{q^*} c(q) dq$  they need to produce  $q^*$ .

With endogenous asset creation, the equilibrium condition (6) remains unchanged. However, we need to replace  $p_t$  in (3) with  $p_t q^* = p_t c^{-1}(p_0)$ . This expression is increasing in  $p_0$  for  $t = 0$ , and so we can show that the equilibrium remains unique and qualitatively similar to before. Suppose we intervene and marginally reduce the quantity of assets produced at date 0 from its equilibrium value  $q^*$ . Since  $c(q^*) = p_0$ , this intervention will leave the consumption of the old unchanged. Cohorts born at dates  $t \geq 0$  can redirect the  $p_t$  resources they would have spent on the last asset to entrepreneurs. If  $d_t = D$  at date  $t$ , the productivity of the marginal producer  $R^D$  exceeds the expected return on the asset  $\bar{r}^D$ . If  $d_t = d$  at date  $t$ , the return from the marginal producer  $R^d$  would equal the return on the asset  $r^d$ . Since  $\Pr(d_t = D) > 0$  for all  $t$ , a marginal reduction in  $q^*$  makes all cohorts better off ex ante. As long as the resources the young spend on the asset at date 0 are not pure rents to the old, there is room for a Pareto-improving intervention.

The inefficiency associated with misallocation we've described holds for all values of  $\Phi$ . But when  $\Phi > 0$ , our model admits an additional inefficiency associated with excessive leverage. Even if we hold the quantity of assets created at date 0 fixed, we can make all agents better off if we let lenders directly buy the assets their borrowers would have purchased and reimburse borrowers for their forgone income. This avoids the default costs  $\Phi p^D$  lenders incur when dividends fall. Essentially, there is no socially useful purpose for agents to borrow and buy risky assets. Yet in equilibrium they do so because they don't bear the costs of their default. The same would be true in models in which instead of recovery costs,  $\Phi$  represents the amount of forgone output when asset prices fall due to debt overhang or deleveraging.

These two sources of inefficiency suggest there may be scope for intervention to reduce the amount of assets created during a boom and to discourage borrowing against any assets that are created. But policymakers face the same difficulties as private agents in distinguishing between speculation and more productive uses of assets, and cannot design policies directed at those who create or purchase risky assets. However, policymakers might still rely on blunt tools, e.g. changing interest rates using monetary policy or restricting leverage for all borrowers. The remainder of the paper considers these two interventions.

To study these interventions, however, we need to relax some of the simplifying assumptions we have relied on so far. First, to capture the effects of monetary policy, we need to relax our assumption that the amount of resources  $e$  each cohort can allocate to production and assets is fixed. While this assumption is convenient, models of how monetary policy affects interest rates often assume that price rigidities allow economic activity to expand or contract when the monetary authority moves. In the next section, we drop the assumption that agents are endowed with a fixed amount of goods to incorporate monetary policy.

To capture the effects of leverage restrictions, we need to relax our assumption that entrepreneurs are endowed with nothing. When borrowers lack resources as we have assumed so far, there is no way to restrict leverage other than cutting off credit altogether. In Section 5 we return to assuming savers are endowed with a fixed amount of goods, but we assume entrepreneurs are also endowed with goods. While entrepreneurs without wealth must take on infinite leverage, those with wealth face a choice of how much leverage to take on. This requires us to move from a single credit market to many markets with different degrees of leverage. We can then study the effect of leverage restrictions that shut down markets with especially high leverage.

## 4 Monetary Policy

We begin with monetary policy. As we noted above, this requires us to abandon our assumption that savers are endowed with an exogenous amount of goods. We follow Galí (2014), who also considers monetary policy in an overlapping generations economy with assets. In particular, we introduce two modifications. First, we assume savers are endowed with labor rather than goods. Second, we introduce a monetary authority that moves after goods producers set their prices but before they hire labor. This allows the real wage – and consequently output – to respond to monetary policy.

We leave the formal details of the analysis to Appendix B and only sketch the results here. Our assumptions imply that labor supply by savers only depends on the real wage. This in turn implies that in the absence of money, the equilibrium real wage that clears the labor market will be constant over time and independent of  $d_t$ . Thus, in the absence of money, the reduced-form representation of our economy would be the same as the model we have assumed up to now: Each cohort of savers has a constant budget  $e$  which it must allocate between entrepreneurial activity and purchasing assets.

We then introduce money. As in Galí (2014), we consider an equilibrium in which money doesn't circulate.

In equilibrium, inflation adjusts to equate the real value of the nominal rate set by the monetary authority and the real return agents earn elsewhere. Agents are therefore indifferent to holding money. Producers set the prices of their goods each period. If they set prices after the monetary authority moves, or can perfectly anticipate what the monetary authority will do, the monetary authority will not be able to affect the real wage or any other real variable: Producers will set their prices in a way that leaves the real wage the same as without intervention. But if producers set their prices before the monetary authority moves and cannot perfectly anticipate what it will do, producers will set their price as a markup over the expected wage that will prevail after the monetary authority moves. If the nominal interest rate turns out to be higher (lower) than expected, the real wage can be higher (lower) than expected. Essentially, an unanticipated move by the monetary authority allows a self-fulfilling fall in demand for goods. Lower demand for goods means producers don't need to hire as much labor, the real wage falls, and since agents earn less, demand for goods will indeed be lower. Such a surprise move by the monetary authority can change earnings  $e$ , just as an income tax or subsidy would. We will refer to any intervention at date 0 that reduces the earnings  $e_0$  at date 0 to below the level  $e$  that would have prevailed without intervention as a contractionary policy. We can deduce the implications of such a policy from the comparative statics of changing  $e_0$  in our original endowment economy. The next proposition, based on our analysis in Appendix B, summarizes these effects.

**Proposition 4** *An unanticipated monetary intervention at date 0 that reduces earnings  $e_0$  below the earnings  $e$  that would have prevailed absent any intervention leads to a lower asset price  $p_0^D$  and a higher real interest rate on loans  $R_0^D$  than would have prevailed absent any intervention.*

Given this result, we can discuss the welfare implications of a contractionary monetary intervention at date 0. Since prices are set each period, an intervention at date 0 will have no impact on any cohorts born at dates  $t = 1, 2, 3, \dots$ . The expected amount available for agents born at date 0 to consume is

$$[(1 - \pi)(D + p_1^D) + \pi(d + p_1^d)] - \pi\Phi p_0^D + \int_{R_0^D}^{\infty} (1 + y)n(y)dy \quad (19)$$

The first term in (19) represents the expected payout on the asset at date 1 and is unaffected by what the monetary authority does at date 0. The next term represents expected default costs. A contractionary policy at date 0 drives down the price  $p_0^D$  and lowers the expected costs of default. The last term represents the proceeds from production by entrepreneurs in this cohort. Since tighter monetary policy increases  $R_0^D$ , fewer entrepreneurs produce. A contractionary monetary policy thus mitigates excessive borrowing against assets but exacerbates underproduction by entrepreneurs. The impact of the intervention on this cohort is ambiguous, although for sufficiently large  $\Phi$  the first effect will dominate and it will be better off. Finally, the old at date 0 will be worse off given their earnings  $p_0^D$  fall. However, since the effect of policy on  $p_0^D$  and  $R_0^D$  is independent of  $\Phi$ , then when  $\Phi$  is sufficiently large it should be possible for the cohort at date 0 to leave the old at date 0 whole and still be better off on account of the lower default costs. Hence, although contractionary monetary policy has generally ambiguous effects on welfare, it can potentially lead to a Pareto improvement. This result is reminiscent of Svensson (2017), who argues tighter monetary policy is generally costly but can lower the odds of a financial crisis. In our framework, the probability the boom ends is fixed at  $\pi$ , but tighter monetary policy mitigates the severity of the output decline if the boom ends.

Given that a contractionary intervention mitigates excessive borrowing during the boom but exacerbates the underprovision of production, in Appendix B we consider an alternative that avoids this tradeoff. Suppose the monetary authority does nothing at date 0 but can credibly promise to be contractionary at date 1 if the boom continues. Since producers set prices anticipating the average outcome at date 1, this means the monetary authority will be expansionary at date 1 if the boom ends by date 1. This is equivalent to promising a temporarily high endowment  $e_1^d > e$  at date 1 if  $d_1 = d$  and a temporarily low endowment  $e_1^D < e$  if  $d_1 = D$ . Per Proposition 4, the contractionary policy at date 1 will depress  $p_1^D$  and increase  $R_1^D$ . However, as the next result states, this will reduce both  $p_0^D$  and  $R_0^D$  at date 0.

**Proposition 5** *A commitment by the monetary authority at date 0 to set  $e_1^d > e > e_1^D$  leads to a lower asset price  $p_0^D$  and a lower interest rate on loans  $R_0^D$  at date 0 than would have prevailed absent any intervention.*

A promise to tighten if a boom continues (and consequently ease if the boom ends) mitigates both excessive leverage and insufficient entrepreneurial activity at date 0, in contrast to tightening immediately. Not surprisingly, this policy can raise welfare under more general circumstances than immediate tightening. Formally, cohorts born at  $t = 2, 3, \dots$  after the intervention will be unaffected. We show in Appendix B that the cohort born at date 1 will be better off if  $d_1 = d$ . Although they work more than without the intervention, the monopoly power we need to allow for price setting implies employment is too low in the absence of intervention, and so higher employment raises welfare. Whether this cohort will be better off if  $d_1 = D$  is ambiguous, just as a direct intervention at date 0 was ambiguous: This cohort will fund less entrepreneurial activity given  $R_1^D$  is higher but will incur smaller default costs  $\Phi p_1^D$ . Even if  $\Phi$  is small so default isn't very costly, as long as the probability  $\pi$  that dividends fall is close to 1, this cohort will be better off *ex ante*. The cohort born at date  $t = 0$  will be strictly better off, since both expected default costs  $\Phi p_0^D$  are lower and more entrepreneurial activity is financed when  $R_0^D$  is lower. Finally, the old at date 0 will be worse off given the amount they earn from the assets they sell  $p_0^D$  will be lower. But the young at date 0 would be better off even if they had to fully compensate the old when  $\Phi > 0$ . Hence, this intervention can be Pareto improving even when  $\Phi$  is small and tightening at date 0 is not Pareto improving.

The advantage of a commitment to tighten in the future is that it discourages speculation by punishing those who buy risky assets but not those who produce. Lenders would do the same if we allowed them to write contingent financial contracts. We ruled this out on the grounds that enforcing contingent contracting is too costly. But contingent monetary policy does not involve enforcement and may serve as a substitute.

## 5 Macprudential Regulation

We now turn to interventions that involve regulating credit. As we already anticipated, this will require us to relax our assumption that entrepreneurs are endowed with no resources. When entrepreneurs have no wealth, any arbitrarily small down-payment requirement would shut down all credit. This would eliminate speculation, but it would also end all trade between savers and entrepreneurs. To analyze interventions

that only restrict rather than eliminate leverage, we need borrowers to be able to produce and speculate even when leverage is restricted. We therefore allow for entrepreneurs to be endowed with some resources. But this modification introduces a complication. When agents have no wealth, they can only be infinitely levered. When they have wealth, they can choose how much leverage to take on. This requires multiple markets that can accommodate a choice of leverage rather than a single market as we have analyzed so far.

Since our analysis no longer involves monetary policy, we return to assuming agents are endowed with goods rather than inputs used to produce goods. As before, each cohort consists of savers endowed with  $e$  goods who want to save them and entrepreneurs who can convert goods at date  $t$  into goods at date  $t + 1$ . Up to now, we assumed entrepreneurs were all endowed with no resources but varied in productivity  $y$ . We now consider the opposite case: Entrepreneurs vary in their endowment but all have the same productivity  $y^*$ . We discuss the case where entrepreneurs vary in both wealth and productivity at the end of this section.

We assume a uniform distribution for wealth  $w$ . That is, for each  $w \in [0, 1]$ , the density of entrepreneurs with wealth  $w$  is equal to  $2\varphi e$ , where  $\varphi$  is a constant such that  $0 < \varphi < 1$  and  $e$  is the endowment of savers. The total endowment of all entrepreneurs is therefore

$$\int_0^1 w (2\varphi e) dw = \varphi e$$

Together, savers and entrepreneurs are endowed with  $(1 + \varphi) e$ . To produce at capacity, entrepreneurs need

$$\int_0^1 (1 - w) (2\varphi e) dw = \varphi e$$

Since  $\varphi < 1$ , entrepreneurs require fewer resources than savers have, in contrast to what we assumed in (2).

As for the common productivity  $y^*$ , we assume it is large enough to exceed the maximal return on the asset. To see that the maximal return on the asset is finite, observe that the asset price  $p_t$  is at least  $(1 - \varphi) e$ , the amount of resources available to spend on the asset if all entrepreneurs produce at capacity, and at most  $(1 + \varphi) e$ , the total resources each cohort is endowed with. The maximal return on the asset occurs when  $d_{t+1} = D$ , the price of the asset at date  $t$  assumes its lowest value  $(1 - \varphi) e$ , and the price at  $t + 1$  assumes its maximum value  $(1 + \varphi) e$ . We assume  $1 + y^*$  exceeds this return, i.e.,

$$1 + y^* > \frac{D + (1 + \varphi) e - (1 - \varphi) e}{(1 - \varphi) e} = \frac{D + 2\varphi e}{(1 - \varphi) e} \quad (20)$$

Our assumptions ensure all entrepreneurs can and will produce at capacity in equilibrium. By contrast, up to now we have assumed only an endogenously determined fraction of entrepreneurs is funded in equilibrium. Assuming all entrepreneurs are fully funded allows us to avoid solving for the endogenous fraction of entrepreneurs funded in each of a continuum of markets, which greatly simplifies the analysis.

Now that entrepreneurs have positive wealth, they can help finance their own investments. We assume lenders can observe the resources borrowers use to finance their investment but not what borrowers choose

to invest in. Verifying how much borrowers invest is different than understanding what they invest in. By paying for a share of their investment, the borrower commits resources that can be used to repay the lender in case of default, since a lender who knows about the borrower's share of the project can go after its proceeds. However, we continue to assume lenders cannot observe any resources the agent has beyond what she invests in her project, i.e., borrowers can use shell entities to hide any additional wealth they have.

Formally, borrowers choose the fraction  $\lambda \in [0, 1)$  of their investment to finance. We model this as a continuum of markets indexed by  $\lambda \in [0, 1)$ . An agent who borrows in market  $\lambda$  can borrow  $\frac{1-\lambda}{\lambda}$  units for each unit of her own wealth that she invests. She can thus leverage her endowment of  $w$  to finance an investment of size  $\frac{w}{\lambda}$ . When  $w > 0$ , the choice of leverage is non-trivial: By going to a market with a lower  $\lambda$ , an entrepreneur can borrow more and produce at a larger scale, but this will leave their lender with a smaller cushion to go after in case of default. Back when we assumed all entrepreneurs had no wealth, agents had no choice. They could only borrow in market  $\lambda = 0$  and choose infinite leverage. Now that agents have wealth, we need a market for each  $\lambda \in [0, 1)$  to accommodate any leverage they might choose.

We now define and solve for an equilibrium when there is a continuum of markets. To anticipate our results, we describe an equilibrium in which entrepreneurs with wealth  $w$  go to market  $\lambda = w$  and borrow  $1 - w$  to produce at capacity. Thus, entrepreneurs sort into different markets. Intuitively, entrepreneurs are happy to invest all of their wealth in production and reassure their lender about any potential losses in case of default. At the same time, other agents will borrow to speculate, but all such borrowing will be confined to markets with low  $\lambda$  (alternatively, to markets with high leverage). This motivates us to consider the effect of leverage restrictions, i.e., shutting down markets where  $\lambda$  is below some floor  $\underline{\lambda}$ .

## 5.1 Equilibrium with Multiple Markets

An equilibrium now consists of a path of interest rates  $\{R_t(\lambda)\}_{t=0}^{\infty}$  for each market  $\lambda \in [0, 1)$ , together with a path of asset prices  $p_t$  and amounts borrowed in each market. Rather than track the share of lending used to buy assets in each market, we must now track the amount borrowed for each purpose in each market. Let  $f_t^a(\lambda)$  and  $f_t^p(\lambda)$  denote the rate at which agents borrow in market  $\lambda$  to buy assets and to produce, respectively, and define  $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$  as total borrowing in market  $\lambda$  for any purpose. We can integrate these rates to obtain the total amounts borrowed in all markets,  $\int_0^1 f_t^a(\lambda) d\lambda$  and  $\int_0^1 f_t^p(\lambda) d\lambda$ . Although we refer to borrowing rates, we are not ruling the possibility that agents borrow non-infinitesimal amounts in some markets. Indeed, once we introduce leverage restrictions, there will be a market that will attract a mass of borrowers. We discuss how to deal with this formally in Appendix C, but, loosely, we can think of such markets as featuring infinite borrowing rates. We will refer to market  $\lambda$  as *inactive* if  $f_t(\lambda) = 0$  and *active* if  $f_t(\lambda) > 0$ . The price  $p_t$ , interest rates  $R_t(\lambda)$ , and amounts borrowed  $f_t^a(\lambda)$  and  $f_t^p(\lambda)$  must ensure all markets clear when agents act optimally.

To determine if lenders are optimizing, we need to know what they expect to earn from lending in any market  $\lambda \in [0, 1)$ . Building on our previous notation, let  $\bar{R}_t(\lambda)$  denote the expected return to lending at

date  $t$  in market  $\lambda$ . If market  $\lambda$  is active, the expected return  $\bar{R}_t(\lambda)$  to lending in this market depends on the amounts agents borrow in this market to buy assets and produce that determine the probability a borrower defaults. We can thus deduce  $\bar{R}_t(\lambda)$  from the interest rate  $R_t(\lambda)$  and the amounts  $f_t^a(\lambda)$  and  $f_t^p(\lambda)$  agents borrow. But if market  $\lambda$  is inactive, there is nothing to guide lenders on what to expect if they were to lend to a market where no borrowers show up. Instead, we need to assign an expected return  $\bar{R}_t(\lambda)$  to each inactive market as part of our definition of an equilibrium. In what follows, we first look for an equilibrium in which all markets are active to avoid the question of how to assign  $\bar{R}_t(\lambda)$  in inactive markets. We then discuss equilibria in which markets can be inactive. This naturally leads into our analysis of regulatory interventions in which some markets are inactive by decree rather than because of beliefs agents hold.

As in Section 1, we proceed as if equilibrium prices are deterministic within each regime and verify there are no stochastic equilibria in Appendix C. We begin with the case where  $d_t = d$  for all  $t$ . With no uncertainty there is no risk of default, so the expected return to lending  $\bar{R}_t(\lambda)$  in each active market  $\lambda$  is equal to the interest rate on loans  $R_t(\lambda)$  in that market. The expected return in all active markets must be the same for lenders to agree to lend in all markets. Hence, there exists a value  $\bar{R}_t$  such that  $\bar{R}_t(\lambda) = \bar{R}_t$  for all  $\lambda \in [0, 1)$ . It follows that the interest rate on loans  $R_t(\lambda)$  also equals  $\bar{R}_t$  for all  $\lambda \in [0, 1)$ . In equilibrium, this common interest rate on all loans  $1 + \bar{R}_t$  must equal the return on the asset  $1 + r_t \equiv \frac{d+p_t+1}{p_t}$ . For suppose the interest rate on loans  $\bar{R}_t$  exceeded the return on the asset  $r_t$ . Then no agent would borrow to buy the asset, nor would any buy the asset given they can earn a higher return from lending. But old agents who own assets will want to sell them, so this cannot be an equilibrium. Suppose instead that the interest rate on loans  $\bar{R}_t$  were lower than the return on the asset  $r_t$ . Then no agent would agree to lend given she can buy the asset and earn a higher return. But this contradicts the fact that all markets are active. The only possible equilibrium in which all markets are active is the one where  $R_t(\lambda) = r_t$  for all  $\lambda \in [0, 1)$ , i.e., where the interest rate on loans in all markets equals the return on the asset.

Given these interest rates, entrepreneurs with productivity  $y^*$  can earn more producing than from buying the asset or lending. Moreover, they will have an incentive to borrow enough to produce at capacity, and will be indifferent about which markets to borrow in as long as they borrow enough to reach capacity. One possibility is to let entrepreneurs with wealth  $w$  borrow an additional  $1 - w$  in market  $\lambda = w$ . This arrangement ensures all markets will be active, confirming such an equilibrium exists.

With all entrepreneurs producing at capacity, total resources invested in production will equal  $2\varphi e$ . Since young agents want to consume when old, any of the total endowment  $(1 + \varphi)e$  of a given cohort that is not used to produce will be spent on the asset. This implies

$$p_t + 2\varphi e = (1 + \varphi)e \tag{21}$$

It follows that  $p_t = (1 - \varphi)e$  for all  $t$ . The return to buying the asset  $r_t$  and the interest rate on loans  $R_t(\lambda)$  in all markets  $\lambda$  will then be  $\frac{d}{(1-\varphi)e}$ . This leads to the following analog to our earlier Proposition 1:

**Proposition 6** *When  $d_t = d$  for all  $t$ , there exists an equilibrium in which all markets are active. In any*

such equilibrium,  $p_t = (1 - \varphi)e \equiv p^d$  for all  $t$ ,  $R_t(\lambda) = \frac{d}{(1-\varphi)e} \equiv R^d$  for all  $\lambda \in [0, 1)$  and all  $t$ , and all entrepreneurs borrow enough to produce at capacity.

Next, we turn to the case where  $d_t = D$  at date 0 but switches permanently to  $d$  with probability  $\pi$  each period. Once again, we use a superscript  $D$  to refer to an equilibrium object at date  $t$  when  $d_t = D$  and a superscript  $d$  when  $d_t = d$ . We already know the values  $p_t^d$  and  $R_t^d(\lambda)$  in any equilibrium where all markets are active from Proposition 6. We only need to solve for the case where  $d_t = D$ .

We begin with interest rates. For each active market  $\lambda$ , either agents borrow to buy assets in that market, i.e.,  $f_t^a(\lambda) > 0$ , or they do not, i.e.,  $f_t^a(\lambda) = 0$ . In the latter case, there will be no default and the expected return to lending  $\overline{R}_t^D(\lambda)$  will equal the interest rate on loans  $R_t^D(\lambda)$ . Since  $\overline{R}_t^D(\lambda)$  is the same in all active markets, we have  $R_t^D(\lambda) = \overline{R}_t^D(\lambda) = \overline{R}_t^D$  in any market  $\lambda$  where  $f_t^a(\lambda) > 0$  but  $f_t^a(\lambda) = 0$ .

Next, consider an active market  $\lambda$  in which agents do borrow to buy assets, i.e.,  $f_t^a(\lambda) > 0$ . In equilibrium, agents must expect to earn the same from borrowing in market  $\lambda$  to buy assets as from lending out the resources they would need to spend on assets if they borrowed. If the return to lending were higher, nobody would borrow to buy assets in market  $\lambda$ . Conversely, if the return to buying assets were higher, no agent would lend in any market given they can speculate in market  $\lambda$ . But then market  $\lambda$  would be inactive. So the two returns must be equal. The expected payoff to lending out  $\lambda$  units of resources is  $(1 + \overline{R}_t^D)\lambda$ . Leveraging these resources to buy assets and defaulting if returns are low yields an expected payoff of

$$(1 - \pi) \left[ \frac{p_{t+1}^D + D}{p_t^D} - (1 - \lambda)(1 + R_t^D(\lambda)) \right]$$

Equating the two, we can solve for the interest rate on loans  $R_t^D(\lambda)$  in an active market  $\lambda$  with speculation:

$$1 + R_t^D(\lambda) = \frac{1}{1 - \lambda} \left[ \frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda(1 + \overline{R}_t^D)}{1 - \pi} \right] \quad (22)$$

The next lemma, derived in Appendix C, shows there exists a cutoff  $\Lambda_t^D$  such that the interest rate in markets  $\lambda < \Lambda_t^D$  is given by (22) and in markets  $\lambda \geq \Lambda_t^D$  is equal to  $\overline{R}_t^D$ .

**Lemma:** *If all markets are active, then there exists a cutoff  $\Lambda_t^D \in [0, 1)$  such that*

$$1 + R_t^D(\lambda) = \begin{cases} \frac{1}{1 - \lambda} \left[ \frac{p_{t+1}^D + D}{p_t^D} - \frac{\lambda(1 + \overline{R}_t^D)}{1 - \pi} \right] & \text{if } \lambda \in [0, \Lambda_t^D) \\ 1 + \overline{R}_t^D & \text{if } \lambda \in [\Lambda_t^D, 1) \end{cases} \quad (23)$$

Our proof also shows  $R_t^D(\lambda) > \overline{R}_t^D$  when  $\lambda < \Lambda_t^D$ . This means some in markets  $\lambda < \Lambda_t^D$  there is some risk of default, since the interest rate exceeds the expected return to lending, but in markets  $\lambda \geq \Lambda_t^D$  there is no such risk. Intuitively, when  $\lambda$  is low, borrowers can shift enough losses to creditors that they would be willing to speculate. But when they have enough skin in the game, they will not engage in speculation.

Figure 2 shows the schedule of interest rates in (23). In market  $\lambda = 0$ , where agents are infinitely levered, the interest rate  $R_t^D(0)$  is equal to the maximal return on the asset,  $\frac{p_{t+1}^D + D}{p_t^D}$ . This is the same as we saw in



Section 2, where  $\lambda = 0$  was the only possible market. The logic is the same: When agents put no resources down, they must hand over all of the returns from the asset to the lender to ensure they earn no profits. For  $0 < \lambda \leq \Lambda_t^D$ , the interest rate on loans  $R_t(\lambda)$  decreases with  $\lambda$ . We prove this in Appendix C, but intuitively, when the borrower pledges more of her own resources, the lender need not charge as high of an interest rate on those who do not default to keep the expected return to lending equal to  $\bar{R}_t^D$ . For  $\lambda \geq \Lambda_t^D$  where default is no longer a concern, the interest rate  $R_t(\lambda)$  is the same for all  $\lambda$ . There is a set of markets where agents speculate, those where  $\lambda < \Lambda_t^D$ , and a set where they don't, those where  $\lambda \geq \Lambda_t^D$ .

Given the schedule of interest rates in (23), what would entrepreneurs choose? Their productivity  $y^*$  exceeds the maximum return on the asset, which recall is equal to  $R_t^D(0)$ . We also know that  $R_t^D(0)$  exceeds  $\bar{R}_t^D$ , the expected return to lending. Entrepreneurs can therefore earn a higher return investing in production than from buying assets or from lending. So they should use their endowment  $w$  to produce. But they can leverage their endowment to produce at a larger scale. They must therefore choose which market  $\lambda \in [0, 1]$  to go to, where we include the option  $\lambda = 1$  to allow them to take on no leverage.

Consider first an entrepreneur with wealth  $w > \Lambda_t^D$ . If she borrowed in market  $\lambda = w$ , she could borrow up to  $1 - w$  at an interest rate of  $\bar{R}_t^D$ , the lowest available interest rate on loans. If she borrowed in some market  $\lambda < w$ , she would be able to borrow more than  $1 - w$ . But she has no use for this extra borrowing given her capacity. Moreover, the interest rate in this market would be the same or higher than  $\bar{R}_t^D$ . So there is no benefit to going to markets  $\lambda < w$  over going to market  $\lambda = w$ . If she borrowed in some market  $\lambda > w$ , she would have to borrow less than  $1 - w$ , and she would face the same interest rate  $\bar{R}_t^D$ . This too offers no benefit over going to market  $\lambda = w$ . The best these entrepreneurs can do is go to market  $\lambda = w$  to borrow  $1 - w$ , although they can achieve the same payoff going to any market  $\lambda \in [\Lambda_t^D, w]$ .

Next, consider an entrepreneur with wealth  $w \leq \Lambda_t^D$ . If she borrowed in market  $\lambda = w$ , she could borrow up to  $1 - w$  at an interest rate of  $R_t^D(w)$ . If she borrowed in some market  $\lambda < w$ , she would be able to borrow more than  $1 - w$ , but she has no use for this extra borrowing. Moreover, the interest rate in this market would be higher than  $R_t^D(w)$ . If she borrowed in some market  $\lambda > w$ , she would have to borrow less than  $1 - w$ . But she would face a lower interest rate. The question is whether it is worth reducing capacity to obtain a lower rate. Her payoff from borrowing in market  $\lambda \in [w, \Lambda_t^D]$  would be  $\frac{w}{\lambda} [1 + y^* - (1 - \lambda)(1 + R_t(\lambda))]$ . Substituting in from (23), this is equal to

$$\frac{w}{\lambda} \left[ 1 + y^* - \frac{p_{t+1}^D + D}{p_t^D} + \frac{\lambda(1 + \bar{R}_t^D)}{1 - \pi} \right]$$

This payoff is decreasing in  $\lambda$ , so there is no advantage to borrowing in these markets instead of  $\lambda = w$ . Borrowing in any market  $\lambda \in (\Lambda_t^D, 1)$  is dominated by borrowing in market  $\lambda = \Lambda_t^D$ , which we already argued was worse than borrowing in  $\lambda = w$ . So borrowing  $1 - w$  in market  $\lambda = w$  is uniquely optimal.

In any equilibrium where all markets are active, then, entrepreneurs with wealth  $w \in [0, \Lambda_t^D)$  will choose market  $\lambda = w$ , while entrepreneurs with  $w \in (\Lambda_t^D, 1)$  will choose borrow in markets in the set  $[\Lambda_t^D, 1)$ . Formally,  $f_t^p(\lambda) = 2\varphi e$  for  $\lambda \in [0, \Lambda_t^D)$ , while  $f_t^p(\lambda)$  for  $\lambda \in [\Lambda_t^D, 1)$  is indeterminate. Just as in the case

where  $d_t = d$  for all  $t$ , entrepreneurs with enough wealth are indifferent about exactly which markets offering the interest rate  $\bar{R}_t$  they borrow their  $1 - w$  resources. The indeterminacy of  $f_t^p(\lambda)$  for  $\lambda > \Lambda_t^D$  is thus irrelevant for allocations or welfare, since total borrowing to produce  $\int_0^1 f_t^p(\lambda) d\lambda = 2\varphi e$  in any equilibrium where all markets are active. Just as before, we can ensure all markets are active by assuming entrepreneurs with wealth  $w \geq \Lambda_t^D$  borrow in market  $\lambda = w$ . In that case,  $f_t(\lambda) = 2\varphi e$  for all  $\lambda \in [0, 1]$ .

Once again, any resources the young do not use to produce will be spent on the asset. This implies

$$p_t^D + 2\varphi e = (1 + \varphi) e \quad (24)$$

It follows that  $p_t^D = (1 - \varphi) e$  for all  $t$ . This is the same price as when  $d_t = d$ . Although the price is the same, the expected return to buying the asset when  $d_t = D$  is higher at  $1 + \bar{r}^D = \frac{(1-\pi)D + \pi d}{(1-\varphi)e}$ .

We now turn to the amount agents borrow to buy assets,  $f_t^a(\lambda)$ . Recall that the expected return to lending  $\bar{R}_t^D(\lambda) = \bar{R}_t^D$  for all  $\lambda$ . Let  $\alpha_t(\lambda) \equiv f_t^a(\lambda) / f_t(\lambda)$  denote the fraction of lending in market  $\lambda$  that is used to buy assets, which is well defined given all markets are active. Equating  $\bar{R}_t^D(\lambda)$  with  $\bar{R}_t^D$  implies

$$(1 - \pi\alpha_t(\lambda)) R_t^D(\lambda) + \pi\alpha_t(\lambda) \left[ \frac{d}{(1-\varphi)e} - \Phi \right] = \bar{R}_t^D \quad (25)$$

Consider first markets  $\lambda < \Lambda_t^D$ . Since  $R_t^D(\lambda) > \bar{R}_t^D$ , it is indeed the case that  $f_t^a(\lambda) > 0$  for  $\lambda < \Lambda_t^D$ . This confirms agents speculate in these markets. Using the value of  $R_t(\lambda)$  in (23), we can solve for  $\alpha_t(\lambda)$ . We can then derive  $f_t^a(\lambda)$  from  $\alpha_t(\lambda)$  using the fact that  $f_t^p(\lambda) = 2\varphi e$ . Given that the interest rate on loans  $R_t^D(\lambda)$  is decreasing in  $\lambda$  for  $\lambda \in [0, \Lambda_t^D)$ , then  $\alpha_t(\lambda)$ , and consequently  $f_t^a(\lambda)$ , will be decreasing in  $\lambda$  for  $\lambda < \Lambda_t^D$ . That is, there will be more borrowing to finance speculation in markets with more leverage. This is not because leverage makes speculation more attractive, but because there has to be just enough speculation in equilibrium to ensure the return to lending is equal to  $\bar{R}_t^D$  in all markets. Speculators borrow more to speculate in markets where lending to entrepreneurs is more profitable.

Next, we turn to markets  $\lambda \geq \Lambda_t^D$ . Since  $R_t(\lambda) = \bar{R}_t$  for these  $\lambda$ , we know there is no default in these markets. This could be because agents don't borrow to buy assets in these markets, or because they contribute enough of their own resources to avoid default. In Appendix C, we confirm agents do not borrow to buy assets in these markets, i.e.,  $f_t^a(\lambda) = 0$  for  $\lambda \in [\Lambda_t^D, 1]$ . Intuitively, interest rates on loans will have to cover the expected losses from those borrowers who do speculate. This makes borrowing costly, which discourages agents from borrowing to buy assets they would otherwise be indifferent to holding.

To recap, the amount agents borrow to buy assets  $f_t^a(\lambda)$  is uniquely determined for all  $\lambda \in [0, 1]$  in any equilibrium in which all markets are active. We can also say something about which agents borrow to speculate. Recall that before, when entrepreneurs had no wealth, any agent could borrow to buy assets. This remains true here for market  $\lambda = 0$ . But in markets  $\lambda > 0$ , borrowers must invest their own wealth to speculate. We argued above that entrepreneurs invest all of their resources in production, so only savers will be able to borrow in markets  $\lambda > 0$  to buy assets. In equilibrium, then, savers will be indifferent between lending and speculating, and some will borrow in markets  $\lambda \in (0, \Lambda_t^D)$  and speculate.

We have thus solved for the equilibrium price  $p_t^D$ , the amounts  $f_t^a(\lambda)$  and  $f_t^p(\lambda)$  agents borrow to buy assets and produce, and the interest rates on loans  $R_t^D(\lambda)$  for all  $\lambda$ . However, some of these objects are defined in terms of the expected return to lending  $\bar{R}_t^D$ , which we have yet to derive. To solve for  $\bar{R}_t^D$ , we can track what savers earn in equilibrium. First, they lend to entrepreneurs, on which they earn

$$\int_0^1 (1 + R_t^D(w)) (1 - w) (2\varphi e) dw$$

Second, all of the funds used to purchase assets comes from savers, either as lenders or as speculators who help finance their asset purchases. Hence, they also collect all the returns on these assets, which equal  $(1 + \bar{r}_t^D) p_t^D$ . However, we need to net from this expected default costs. Let  $\gamma_t^D$  denote the fraction of spending on assets that is financed with some debt. The agents who buy these assets will default if returns are low. Since default is proportional to the size of the borrower's project, expected default costs are  $\gamma_t^D \Phi p_t^D$ . Substituting in for the price  $p_t^D = (1 - \varphi) e$  and equating what savers earn with  $(1 + \bar{R}_t^D) e$  yields

$$(1 + \bar{R}_t^D) e = [1 + \bar{r}^D - \pi \gamma_t^D \Phi] (1 - \varphi) e + \int_0^1 (1 + R_t^D(w)) (1 - w) (2\varphi e) dw \quad (26)$$

We need an additional equation to characterize  $\gamma_t^D$ . When the expected return to lending  $\bar{R}_t^D$  exceeds the expected return to buying the asset  $\bar{r}^D$ , only agents who borrow in market  $\lambda < \Lambda_t^D$  buy the asset. In that case,  $\gamma_t^D = 1$ . If instead  $\bar{R}_t^D = \bar{r}^D$ , then  $\gamma_t^D$  would have to ensure that  $\bar{R}_t^D$  is indeed equal to  $\bar{r}^D$ . We can combine the two conditions into a single equation:

$$1 + \bar{R}_t^D = \max \left\{ 1 + \bar{r}^D, [1 + \bar{r}^D - \pi \Phi] (1 - \varphi) + \int_0^1 (1 + R_t^D(w)) (1 - w) (2\varphi) dw \right\} \quad (27)$$

It is easy to verify that when  $\bar{R}_t^D > \bar{r}^D$ , equations (26) and (27) imply  $\gamma_t^D = 1$ , and when  $\bar{R}_t^D = \bar{r}^D$  we can find a unique value of  $\gamma_t^D$  that will equate the two. Since  $\bar{r}^D$  is time invariant, the solutions to these equations,  $\bar{R}^D$  and  $\gamma^D$ , are also time invariant. Given  $\bar{R}^D$ , we can solve for the time invariant cutoff  $\Lambda^D$  as the smallest value of  $\lambda$  for which  $R^D(\lambda) = \bar{R}^D$ . This completes the characterization of an equilibrium when all markets are active, which we can summarize as follows.

**Proposition 7** *There exists an equilibrium in which all markets are active while  $d_t = D$ . In any such equilibrium, the asset price is given by*

$$p_t^D = (1 - \varphi) e \equiv p^D$$

and, in the limit as  $\phi \rightarrow 0$ , the interest rates on loans in different markets are given by

$$1 + R_t^D(\lambda) = \max \left\{ 1 + \bar{R}^D, \frac{1}{1 - \lambda} \left[ 1 + \frac{D}{p^D} - \frac{\lambda(1 + \bar{R}^D)}{1 - \pi} \right] \right\}$$

where  $\bar{R}^D$  is the value that solves (26) and (27) together with  $\gamma^D$ . Borrowing for production is given by  $f_t^p(\lambda) = 2\varphi e$  for  $\lambda \in [0, \Lambda^D]$  and for  $\lambda \in [\Lambda^D, 1]$  is any distribution for which  $\int_{\Lambda^D}^1 f_t^p(\lambda) d\lambda = (1 - (\Lambda^D)^2) \varphi e$ . Borrowing for buying assets  $f_t^a(\lambda)$  ensures  $\bar{R}^D(\lambda) = \bar{R}^D$  for  $\lambda \in [0, \Lambda^D]$  and  $f_t^a(\lambda) = 0$  for  $\lambda \in [\Lambda^D, 1]$ .

The equilibrium above is similar to the one in Proposition 2 when entrepreneurs were endowed with nothing. Poor entrepreneurs still take on high leverage, although this is now by choice rather than because

they can only take on infinite leverage. Markets with low  $\lambda$  allow for shifting risk, and in equilibrium agents borrow to speculate in these markets. Rich entrepreneurs who do not need much leverage can avoid being lumped in with speculators and obtain low interest rates. Although we don't show it explicitly, the high dividend regime can once again give rise to credit booms and, if  $\Phi$  isn't too large, bubbles. One difference worth noting is that since in this specification all entrepreneurs produce at capacity, there is no sense in which resources spent on assets could have been better deployed in production. In other words, there is no misallocation during the boom. However, if  $\Phi > 0$ , borrowing to buy assets remains socially wasteful, and an intervention might still improve welfare by curbing excessive leverage.

So far, we have only considered equilibria where all markets are active. But we can always construct an equilibrium in which for any  $\lambda$ , the interest rate on loans  $R_t(\lambda)$  exceeds  $y^*$  to ensure no agent would want to borrow in that market, and then set the expected return  $\bar{R}_t(\lambda)$  arbitrarily low to ensure no one would want to lend in market  $\lambda$ . Such equilibria are essentially coordination failures where markets that could sustain trade are instead inactive. Inactivity in some markets will generally affect prices and interest rates in remaining active markets, and so characterizing equilibria with inactive markets requires deriving interest rates, asset prices, and amounts borrowed. Below, we study the effects of interventions that shut down markets with low  $\lambda$ . This illustrates how inactivity in particular markets affects an equilibrium, since inactive markets have the same effect whether they are inactive by decree or because of the beliefs of agents about that market. Given our interest is in the effect of policy interventions that shut down markets that would have otherwise been open, it seems natural to focus on an equilibrium where all markets are active as the benchmark and limit our analysis of inactive markets to those consistent with our intervention.

## 5.2 Leverage Restrictions

To introduce leverage restrictions, consider a restriction that shuts down all markets with  $\lambda$  below some floor  $\underline{\lambda}$ . This is equivalent to a cap on leverage. Agents with wealth  $w < \underline{\lambda}$  could only undertake projects of size  $w/\underline{\lambda} < 1$ , while entrepreneurs with wealth  $w \geq \underline{\lambda}$  could operate at full capacity. For simplicity we consider a permanent floor, although one could equally consider a floor that is only in effect while  $d_t = D$ .

We will continue to focus on equilibria in which all markets  $\lambda \geq \underline{\lambda}$  are active to avoid assigning expected returns  $\bar{R}_t(\lambda)$  to inactive markets. The equilibrium where all markets are active can then be viewed as a special case in which  $\underline{\lambda} = 0$ . When  $\underline{\lambda} > 0$ , we can apply a similar analysis to conclude that interest rates  $R_t(\lambda)$  will be given by (23) when  $d_t = D$ , although the expected return to lending  $\bar{R}_t^D$  will depend on  $\underline{\lambda}$ . Given interest rates still satisfy (23), entrepreneurs will want to produce at full capacity. But entrepreneurs with  $w < \underline{\lambda}$  can no longer do so. Since their profits decrease with  $\lambda$  for  $\lambda > w$ , these entrepreneurs will all flock to  $\underline{\lambda}$  and produce at scale  $w/\underline{\lambda}$ . The total inputs entrepreneurs will use to produce is then

$$\begin{aligned} \int_{w=0}^{\underline{\lambda}} 2\varphi e \left( \frac{w}{\underline{\lambda}} \right) dw + \int_{w=\underline{\lambda}}^1 (2\varphi e) dw &= \frac{\varphi e}{\underline{\lambda}} w^2 \Big|_0^{\underline{\lambda}} + 2\varphi e (1 - \underline{\lambda}) \\ &= \underline{\lambda}\varphi e + (1 - \underline{\lambda}) 2\varphi e \end{aligned}$$

The amount that remains to spend on the asset is  $(1 + \varphi)e$  minus the above, which pins down its price:

$$p_t^D = (1 - \varphi(1 - \underline{\lambda}))e \quad (28)$$

Increasing  $\underline{\lambda}$  will lead to a higher asset price. Intuitively, leverage restrictions force poor entrepreneurs to operate at a smaller scale. Since savers want to save a fixed amount  $e$  regardless of  $\underline{\lambda}$ , the decline in production will release resources to buy assets, pushing  $p_t^D$  up. Assuming  $\underline{\lambda}$  is imposed permanently, the same logic implies  $p_t^d = (1 - \varphi(1 - \underline{\lambda}))e$ . The expected return on the asset when  $d_t = D$  is thus

$$1 + \bar{r}_t^D = \frac{(1 - \pi)(D + p_{t+1}^D) + \pi(d + p_{t+1}^d)}{p_t^D} = 1 + \frac{(1 - \pi)D + \pi d}{(1 - \varphi(1 - \underline{\lambda}))e}$$

This reveals that leverage restrictions reduce the expected return to buying the asset. While it is hard to describe the effects of increasing  $\underline{\lambda}$  on the entire schedule of interest rates  $R_t(\underline{\lambda})$ , we show in Appendix C that the expected return to lending  $\bar{R}^D$  declines with  $\underline{\lambda}$ . We summarize the effects of raising  $\underline{\lambda}$  as follows.

**Proposition 8** *The asset price  $p_t^D = (1 - \varphi(1 - \underline{\lambda}))e$  is increasing in  $\underline{\lambda}$ , while the expected returns from the asset  $\bar{r}^D = \frac{(1 - \pi)D + \pi d}{(1 - \varphi(1 - \underline{\lambda}))e}$  and from lending  $\bar{R}^D$  when  $d_t = D$  are decreasing in  $\underline{\lambda}$  for a permanent floor  $\underline{\lambda}$ .*

Note the contrast between the effects of contractionary monetary policy we discussed in the previous section and the effects of restrictions on leverage above. Both policies reduce output: Monetary policy reduces what is produced today, while leverage restrictions reduce the amount entrepreneurs today produce for next period. However, tighter monetary policy dampens asset prices and raises the returns to saving while leverage restrictions increase asset prices and lower the return to saving. This suggests leverage restrictions may be counterproductive, stoking asset prices rather than dampening them. This counterproductive aspect of leverage restrictions is new as far as we know. Stein (2013) argues leverage restrictions may be limited and ineffective, but his point was that borrowers can often circumvent them, not that regulations might contribute to more speculation. The logic for our result is that in risk-shifting models there must be an investment activity cross-subsidizing speculation. If this other investment is particularly sensitive to leverage restrictions, restricting leverage may redirect resources toward speculation. We anticipate that the same would hold true in risk-shifting models of housing in which speculators buy the same asset that liquidity constrained households buy. If the demand for housing by constrained households is particularly sensitive to leverage restrictions but the amount of funds available for mortgage lending is relatively inelastic with respect to interest rates, leverage restrictions could end up encouraging speculation on housing.

Proposition 8 establishes that tighter leverage restrictions drive up asset prices. But that does not necessarily mean that total borrowing against assets must rise. Even if a higher  $\underline{\lambda}$  increases  $p^D$ , it could still lower the share of assets purchased with debt  $\gamma^D$ . Our next result shows that when  $\underline{\lambda}$  is low and all assets are purchased with debt, or when  $\underline{\lambda}$  is high enough to discourage speculation altogether, increasing  $\underline{\lambda}$  will increase  $p^D$  without changing  $\gamma^D$ . In these cases, raising  $\underline{\lambda}$  will make agents worse off. For intermediate values, increasing  $\underline{\lambda}$  at some point must reduce both the share of assets purchased with debt and expected default costs. On its own this would make society better off, but increasing  $\underline{\lambda}$  also reduces what poor entrepreneurs can produce. For large  $\Phi$ , however, reducing default costs would be paramount.

**Proposition 9** *There exist cutoffs  $\Lambda_0 \leq \Lambda_1 < 1$  in  $[0, 1)$  such that*

1. *If  $\underline{\lambda} < \Lambda_0$ , increasing  $\underline{\lambda}$  leaves  $\gamma^D = 1$ , increases expected default costs  $\pi\gamma^D\Phi p^D$ , and leaves fewer goods for cohorts to consume from date  $t = 1$  on.*
2. *If  $\underline{\lambda} \geq \Lambda_1$ ,  $\gamma^D = 0$  and there is no default. Increasing  $\underline{\lambda}$  then leaves fewer goods for cohorts to consume from date  $t = 1$  on.*
3. *If  $\Lambda_0 < \underline{\lambda} < \Lambda_1$ , increasing  $\underline{\lambda}$  there exist values of  $\underline{\lambda}$  at which increasing  $\underline{\lambda}$  lowers  $\gamma^D$  and expected default costs  $\pi\gamma^D\Phi p^D$ . In this case, increasing  $\underline{\lambda}$  while  $d_t = D$  can be Pareto improving for large  $\Phi$ .*

In contrast to monetary policy, a threat to tighten credit conditions in the future rather than tighten them immediately will not mitigate this counterproductive aspect. Raising  $\underline{\lambda}$  next period will increase  $p_{t+1}^D$ , and regardless of how it affects  $\gamma_{t+1}^D$ , a higher  $p_{t+1}^D$  at date  $t + 1$  makes speculation at date  $t$  more attractive. However, our finding that increasing  $\underline{\lambda}$  raises asset prices contemporaneously will not necessarily hold in general. Suppose that instead of the cases we considered in which entrepreneurs are identical in either wealth or productivity, we allow the wealth and productivity of entrepreneurs to follow some general distribution  $n(w, y)$ . Entrepreneurs with positive wealth and low productivity would behave like savers. An increase in  $\underline{\lambda}$  that lowers the return to saving could induce some of the entrepreneurs who are on the margin between lending and producing to switch from lending to borrowing in order to produce. If enough entrepreneurs switch from lending to producing, the fall in lending and the increase in demand for borrowing to produce may leave fewer resources to spend on the asset, and its price will fall. We confirm numerically that there exist distributions  $n(w, y)$  for which increasing  $\underline{\lambda}$  reduces  $p_t^D$ .<sup>8</sup>

While increasing  $\underline{\lambda}$  can lower asset prices when  $d_t = D$  under some circumstances, this will only be true when there is risk-shifting. When  $d_t = d$ , increasing  $\underline{\lambda}$  will raise  $p_t^d$  regardless of the distribution  $n(w, y)$ . To see this, note that when  $d_t = d$  there is no default.  $R_t^d(\lambda)$  is then equal to a constant  $R_t^d$  for all  $\lambda$ . This common rate  $R_t^d$  and the asset price  $p_t^d$  satisfy two equilibrium conditions similar to (3) and (4). First, since all the resources of savers and entrepreneurs will be used to produce or buy the asset, we have

$$\int_{R^d} \int_0^1 \min\left\{1, \frac{w}{\underline{\lambda}}\right\} n(w, y) dw dy + p_t^d = \int_0^\infty \int_0^1 wn(w, y) dw dy + e \quad (29)$$

This defines  $R_t^d$  as a function  $\rho_\lambda(p_t^d)$  of the price  $p_t^d$  which is increasing in  $p_t^d$  for a fixed  $\underline{\lambda}$  and decreasing in  $\underline{\lambda}$  for a fixed  $p_t^d$ . Second, the interest rate on loans must equal the return on the asset, and so

$$(1 + R_t^d) p_t^d = d + p_{t+1}^d \quad (30)$$

Substituting in  $R_t^d = \rho_\lambda(p_t^d)$  implies  $p_{t+1}^d = \rho_\lambda(p_t^d) - d$ . Figure 3 illustrates the effect of increasing  $\underline{\lambda}$  graphically. Since  $\rho_\lambda(p_t^d)$  is decreasing in  $\underline{\lambda}$  for a fixed  $p_t$ , the curve that plots  $p_{t+1}^d$  as a function of  $p_t^d$  is

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<sup>8</sup>Even without relying on a more general distribution  $n(w, y)$ , our results are in part due to our assumption that savers only like to consume when old. This means their saving is inelastic to the interest rate. If we modified this, tighter leverage constraints that reduce the returns to savings could lead them to save less, which might lead to lower asset prices.

lower for all  $p_t^d > 0$ , which implies a higher steady state  $p^d$ . Intuitively, the return on the asset is negatively related to its price. In the absence of risk-shifting, the return on the asset is equal to the interest rate on loans. Increasing  $\underline{\lambda}$  requires the interest rate on loans to fall so that credit markets clear even after demand for borrowing by poor entrepreneurs falls. Hence, the return on the asset must fall and the price of the asset must rise. By contrast, with risk-shifting, the return on the asset and the interest rate on loans are different, so it will be possible for interest rates on loans to fall but the return on the asset to rise. The fact that tighter leverage restrictions only reduce asset prices when there is risk-shifting suggests a possible way for policymakers to detect its presence even when there is no other evidence that it is taking place.

## 6 Conclusion

This paper examines the role of policy in a risk shifting model of asset pricing. As in previous work on risk-shifting, we show that our model can capture many observable features of asset booms, including high asset prices that may exceed fundamentals, accompanying credit booms, and an eventual crisis. The general equilibrium framework we use allows us to go beyond this and analyze policy and welfare. We show that risk-shifting leads to misallocation and excessive leverage, opening up a possible role for intervention. We then look at whether the leading policy proposals of contractionary monetary policy and leverage restrictions can help mitigate these distortions. In our model, tighter monetary policy increases interest rates and lowers asset prices, which reduces excessive leverage but further inhibits investment that is already underfunded. Leverage restrictions have the opposite effect, lowering interest rates and, at least under certain conditions, increasing asset prices. But they also discourage borrowing against assets. Both policies turn out to have ambiguous welfare implications. Whether a policy improves welfare depends on how it affects speculators vis-a-vis the productive activity that cross-subsidize them. It will also depend on how it is implemented; credibly promising to tighten if a boom persists may improve welfare even when tightening immediately does not. Finally, we find that when default costs are large, risk shifting can occur without giving rise to bubbles, something previous work, has overlooked by ignoring default costs. This has the important implication that there may be no need for policymakers contemplating intervening against high asset prices to first determine if there is a bubble given evidence of risk-shifting.

We focus on risk shifting because asset booms tend to be associated with opaque assets that make it difficult for lenders to judge the risks they face with any given borrower. There is a large literature on bubbles that seeks to explain asset booms without risk shifting. However, these models should not be viewed as competing explanations, since the mechanisms they consider are complementary to the risk-shifting we study. For example, there is a large literature which shows how bubbles can arise with fully rational agents because of dynamic inefficiency as in Galí (2014, 2017) or binding credit market frictions as in Martin and Ventura (2012), Hirano and Yanagawa (2017), Miao and Wang (forthcoming). Recent work by Bengui and Phan (2018) showed it is possible combine risk-shifting and dynamic inefficiency. One can similarly combine risk-shifting and borrowing constraints by replacing our assumption that entrepreneurs have a finite capacity with the assumption that an entrepreneur's scale is bounded by how much they can

borrow. In this case, the distortions from risk-shifting that we emphasize would have to be balanced against the fact that overvalued assets may help entrepreneurs relax their borrowing constraints. There is also a literature that shows how disagreement can give rise to bubbles, e.g. Scheinkman and Xiong (2003), Hong, Scheinkman, and Xiong (2006), Simsek (2013), and Barberis, Greenwood, Jin, and Shleifer (2018). This too can be combined with risk-shifting. In the last section of the paper, savers lend to otherwise identical savers who then speculate on assets. Differences in beliefs can help down who lends to whom, since savers who are pessimistic about assets will presumably end up lending to savers who are optimistic about them. Whether risk-shifting interacts with these forces in interesting ways remains an open question.

Our model also suggests directions for future research on risk-shifting models of asset prices. For example, we assumed lenders suffer a cost  $\Phi$  when their borrowers default. In practice, the main costs associated with the collapse of asset prices involve a decline in output due to the way agents respond when asset prices fall. To get at these channels would require introducing financial intermediaries or borrowing constraints for individual households. These may have important implications for what type of interventions are best during booms, since how interventions affect outcomes once asset prices collapse will likely matter for welfare. In terms of applications, we have described the analog between our setup and the housing market. However, cross-subsidization in the housing market works differently, since there both types of agents buy the same asset. By contrast, in our model only speculators buy an asset. This raises the question of whether an intervention that shifts resources from illiquid home buyers to speculators still drives house prices up as in our setting. It is also not obvious whether the policy implications we deduce in our model would hold in open economy settings. For example, we argued that a contractionary monetary policy raises interest rates and dampens asset prices. But if contractionary monetary policy leads to higher real rates that attract capital inflows, it is not clear whether asset prices will still fall. Extending our framework to deal with these issues is essential for figuring out its relevance and limitations for real world scenarios.



## Appendix A: Proof of Proposition 1

**Proof of Proposition 1:** In the text, we showed there is a unique deterministic equilibrium. Here we allow for stochastic equilibrium paths for  $\{p_t, R_t\}_{t=0}^{\infty}$  and confirm that the equilibrium is in fact deterministic.

First, note that for any date  $t$ , in equilibrium it must be the case that  $0 < p_t \leq e$ . If the price  $p_t \leq 0$  there would be infinite demand for the asset given its dividend  $d > 0$  and there is free disposal. But the supply of assets is finite, so this cannot be an equilibrium. At the same time, the most any cohort can spend to buy the assets is  $e$ . Let  $z_t$  denote the return to buying the asset, i.e.,  $z_t \equiv \frac{d+p_{t+1}}{p_t}$ . This can be random if  $p_{t+1}$  is random. Let  $G_t(z)$  denote the (possibly degenerate) distribution of the return  $z_t$ . Since  $0 < p_t \leq e$  for all  $t$ , the maximum return  $z_t^{\max}$  is finite, since  $z_t^{\max} = \frac{d+p_{t+1}^{\max}}{p_t} \leq \frac{d+e}{p_t} < \infty$ , where  $p_{t+1}^{\max}$  is the maximum possible realization of the price at date  $t+1$ .

The equilibrium satisfies two conditions. First, as in (3), all resources will be used either to buy assets or to initiate production:

$$\int_{R_t}^{\infty} n(y) dy + p_t = e \quad (31)$$

This implies  $R_t = \rho(p_t)$  where  $\rho'(\cdot) > 0$ . Second, the interest rate on loans  $R_t$  must satisfy

$$(1 + R_t) p_t = d + p_{t+1}^{\max} \quad (32)$$

If the interest rate on loans  $1 + R_t$  exceeded  $\frac{d+p_{t+1}^{\max}}{p_t}$ , no agent would want to buy assets, which cannot be an equilibrium. If interest rate on loans  $1 + R_t$  exceeded  $\frac{d+p_{t+1}^{\max}}{p_t}$ , agents could earn positive profits from borrowing, so demand for credit would be infinite. Substituting  $R_t = \rho(p_t)$  into (32) implies

$$p_{t+1}^{\max} = (1 + \rho(p_t)) p_t - d$$

Suppose  $p_t > p^d$ . Consider the sequence  $\{\tilde{p}_\tau\}_{\tau=t}^{\infty}$  that comprises the upper support of prices at each date given the history of previous prices, starting from  $p_t$ . Formally, set  $\tilde{p}_t = p_t$  and define

$$\tilde{p}_{\tau+1} = (1 + \rho(\tilde{p}_\tau)) \tilde{p}_\tau - d$$

Since  $p_t > p^d$ , the sequence  $\tilde{p}_t$  would shoot off to infinity and would exceed  $e$  in finite time. This means there is a state of the world in which the price exceeds  $e$ , which cannot be an equilibrium. So  $p_t \leq p^d$ .

Next, suppose  $p_t < p^d$ . Again, we can construct the sequence  $\{\tilde{p}_\tau\}_{\tau=t}^{\infty}$  that comprises the upper support of prices at each date given the history of previous prices, starting from  $p_t$ . That is, we set  $\tilde{p}_t = p_t$  and then

$$\tilde{p}_{\tau+1} = (1 + \rho(\tilde{p}_\tau)) \tilde{p}_\tau - d$$

Since  $p_t < p^d$ , the sequence  $\tilde{p}_t$  would turn negative. Hence, there is a state of the world in which the price is negative, which cannot be an equilibrium. The distribution of the price at date  $t$  is degenerate with full support at  $p^d$ . From (31),  $R_t = \rho(p_t)$  is uniquely determined as well. ■

## Appendix B: Monetary Policy

This appendix introduces within-period production, a monetary authority, and nominal price rigidity into our setup as in our discussion in Section 4. We set up the model and derive the results that underlie Propositions 4 and 5 in the text.

### B.1 Agent Types and Endowments

Our approach largely follows Galí (2014) in how we incorporate production, nominal price rigidity, and monetary policy into an overlapping generations economy with assets. As in our benchmark model, agents live two periods and care only about consumption when old. Each cohort still consists of two types – savers who are endowed with resources but cannot produce intertemporally and entrepreneurs endowed with no resources who can convert goods at date  $t$  into goods at date  $t + 1$ . We continue to model entrepreneurs as in the benchmark model, but we now assume savers are endowed with the inputs to produce goods rather than with the goods themselves. This allows for an endogenous quantity of goods that can potentially vary with the stance of monetary policy.

More precisely, we assume two types of savers, each of mass 1. Half are workers, endowed with 1 unit of labor each who must choose how to allocate it. The other half are producers, endowed with the knowledge of how to convert labor into output but not with labor itself.<sup>9</sup> Producers set the price of the goods they produce and then hire the labor needed to satisfy their demand. Although producers and entrepreneurs both produce output, they differ in when and how they produce it. Producers born at date  $t$  convert labor into goods at date  $t$ . Entrepreneurs then convert the goods producers created at date  $t$  into goods at date  $t + 1$ . Producers operate within the period; entrepreneurs operate across periods.

### B.2 Production, Pricing, and Labor Supply

Workers allocate their one unit of labor to home and market production. Home production yields the same good as the market, but using a technology  $h(\ell)$  that is concave in the amount of labor  $\ell$  allocated to home production. We assume  $h'(0) = 1$  and  $h'(1) = 0$  for reasons that will become clear below.

Workers who sell their labor on the market earn a wage  $W_t$  per unit labor. Their labor services are hired by producers, whom we index by  $i \in [0, 1]$ . Each producer can produce a distinct intermediate good according to a linear technology. In particular, if producer  $i$  hires  $n_{it}$  units of labor, she will produce  $x_{it} = n_{it}$  units of intermediate good  $i$ . The different intermediate goods can then be combined to form final

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<sup>9</sup>This setup borrows from Adam (2003) rather than Galí (2014). The latter assumes agents are homogeneous, selling labor when young and hiring labor when old. We want income to only accrue to the young as in our benchmark model.

consumption goods according to a constant elasticity of substitution (CES) production function available to all agents. That is, given  $x_{it}$  of each  $i \in [0, 1]$ , the amount of final goods  $X_t$  that can be produced is

$$X_t = \left( \int_0^1 x_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \quad (33)$$

Let  $P_t$  denote the price of the final good and  $P_{it}$  denote the price of intermediate good  $i$ . At these prices, the  $x_{it}$  that maximize the profits of a final goods producer solve

$$\max_{x_{it}} P_t \left( \int_0^1 x_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} x_{it} di$$

The first-order condition with respect to  $x_{it}$  yields

$$x_{it} = X_t \left( \frac{P_{it}}{P_t} \right)^{-\frac{1}{\sigma}} \quad (34)$$

If we set  $X_t = 1$ , we can compute the price of the cost of the optimal bundle of intermediate goods  $x_{it} = \left( \frac{P_{it}}{P_t} \right)^{-1/\sigma}$  needed to produce one unit of the final good:

$$\int_0^1 P_{it} x_{it} di = \int_0^1 P_{it}^{1-\frac{1}{\sigma}} P_t^{\frac{1}{\sigma}} di$$

Since any agent can produce final goods, the price  $P_t$  must equal the per unit cost of producing a good in equilibrium. Equating the two yields the familiar CES price aggregator:

$$P_t = \left( \int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \quad (35)$$

Each intermediate goods producer chooses their price  $P_{it}$  to maximize expected profits given demand (34) and wage  $W_t$ . To allow producers to move either before or after the monetary authority, we condition producer  $i$ 's choice on their information  $\Omega_{it}$  when choosing their price. Each producer will set  $P_{it}$  to solve

$$\max_{P_{it}} E \left[ (P_{it} - W_t) X_t \left( \frac{P_{it}}{P_t} \right)^{-1/\sigma} \middle| \Omega_{it} \right]$$

The optimal price is then

$$P_{it} = \frac{E[W_t X_t | \Omega_{it}]}{(1-\sigma) E[X_t | \Omega_{it}]} \quad (36)$$

By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e.,  $n_{it} = n_t$  for all  $i \in [0, 1]$ . The output of consumption goods is thus

$$X_t = \left( \int_0^1 n_t^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = n_t$$

Workers receive  $(W_t/P_t)n_t$  of these goods and producers get the remaining  $(1 - W_t/P_t)n_t$ . Workers also produce goods at home. Their income is thus  $(W_t/P_t)n_t + h(1 - n_t)$ , which is maximized at

$$h'(1 - n_t) = W_t/P_t \quad (37)$$

By contrast, the total resources available to young agents is  $e_t = n_t + h(1 - n_t)$ , which is maximized at

$$h'(1 - n_t) = 1$$

Our assumption that  $h'(0) = 1$  implies total resources are maximized when  $n_t = 0$  and all goods are produced in the market, and  $e_t = n_t + h(1 - n_t)$  is increasing in  $n_t$  for all  $n_t \in [0, 1]$ .

### B.3 Assets, Credit, and Money

Since agents want to consume when old, they will wish to save their earnings  $e_t = n_t + h(1 - n_t)$ . As in the benchmark model, they can buy assets and make loans. Without money, this specification would be equivalent to our benchmark model, the only difference being that the income of savers  $e_t$  which before was exogenous and fixed is now endogenous and potentially time-varying. Equilibrium in the asset and credit markets involves the same conditions as in the benchmark model. First, regardless of the income they earn, the young will spend all of their resources either funding entrepreneurs or buying assets, and so we still have

$$\int_{R_t}^{\infty} n(y) dy + p_t = e_t$$

where  $p_t$  is the real price of the asset and  $R_t$  is the real interest rate on loans. The interest rate  $R_t$  must still ensure agents cannot earn profits by borrowing and buying assets. When  $d_t = d$ , this requires

$$(1 + R_t^d) p_t^d = d + p_{t+1}^d$$

and when  $d_t = D$ , this requires

$$(1 + R_t^D) p_t^D = D + p_{t+1}^D$$

We can then use  $R_t$  and  $p_t$  to solve for the expected return on loans:

$$\bar{R}_t = \begin{cases} R_t^d & \text{if } d_t = d \\ \max \left\{ \bar{r}_t^D, \left(1 - \frac{p_t^D}{e_t}\right) R_t^D + \frac{p_t^D}{e_t} (\bar{r}_t^D - \pi\Phi) \right\} & \text{if } d_t = D \end{cases} \quad (38)$$

where  $\bar{r}_t^D$  is the expected real return to buying the asset. Below, we show that when prices are flexible or money is absent altogether, the equilibrium real wage  $W_t/P_t$  will be constant over time. Employment  $n_t$  and total earnings of all savers  $e_t = n_t + h(1 - n_t)$  will then also be constant. The reduced form of our model in the absence of money thus coincides with our benchmark model.

To introduce money, we follow Galí (2014) in assuming money does not circulate in equilibrium. That is, money is the numeraire, and  $P_t$  and  $W_t$  denote the price of goods and labor relative to money. However, no agent actually holds money in equilibrium. The monetary authority announces a nominal interest rate  $i_t$  at each date  $t$ . The monetary authority commits to pay this rate at date  $t + 1$  to those who lend to it (with money it can always issue), and will charge  $i_t$  to those who borrow from it with full collateral. This is roughly in line with what central banks do in practice, paying interest on reserves and lending at the discount window against collateral. To ensure money doesn't circulate, the real return on lending to the monetary authority must equal the expected return on savings. Let  $\Pi_t = P_{t+1}/P_t$  denote the gross inflation rate between dates  $t$  and  $t + 1$ . Since agents always lend to entrepreneurs, the expected return on savings will equal  $\bar{R}_t$ , the expected return on loans. This implies

$$1 + i_t = (1 + \bar{R}_t) \Pi_t \quad (39)$$

When the monetary authority changes the nominal interest rate  $i_t$ , either inflation  $\Pi_t$  or the expected return  $1 + \bar{R}_t$  or both will have to adjust to ensure agents will neither borrow nor lend to the monetary authority.

## B.4 Defining an Equilibrium

Given a path of nominal interest rates  $\{1 + i_t\}_{t=0}^{\infty}$ , an equilibrium consists of a path of prices  $\{P_t, W_t, p_t, R_t\}_{t=0}^{\infty}$  and a path of employment  $\{n_t\}_{t=0}^{\infty}$  such that agents behave optimally and markets clear. Collecting the relevant conditions from above yields the following five equations for these five variables:

$$\begin{aligned}
 \text{(i) Optimal pricing:} & \quad P_t = \frac{E[W_t X_t | \Omega_t]}{(1 - \sigma) E[X_t | \Omega_t]} \\
 \text{(ii) Optimal labor supply:} & \quad h'(1 - n_t) = W_t / P_t \\
 \text{(iii) Optimal saving:} & \quad \int_{R_t}^{\infty} n(y) dy + p_t = e_t \\
 \text{(iv) Credit market clearing:} & \quad 1 + R_t = \begin{cases} \frac{D + p_{t+1}^D}{p_t^D} & \text{if } d_t = D \\ \frac{d + p_{t+1}^d}{p_t^d} & \text{if } d_t = d \end{cases} \\
 \text{(v) Money market clearing:} & \quad \Pi_t = \frac{1 + i_t}{1 + \bar{R}_t}
 \end{aligned}$$

where the expected return on loans  $\bar{R}_t$  in the last condition is given by (38).

## B.5 Equilibrium with Flexible Prices

We begin with the case where producers set their prices  $P_{it}$  after observing the wage  $W_t$ . This corresponds to the case where prices are fully flexible, or alternatively where there is no money and so no sense in which nominal prices are set in advance. Producers can deduce what other producers will do and the labor workers will supply, they can perfectly anticipate total output  $X_t$ . Hence, their information set  $\Omega_t = \{W_t, X_t\}$ . It follows that  $E[W_t X_t | \Omega_t] = W_t X_t$  and  $E[X_t | \Omega_t] = X_t$ . The optimal pricing rule (i) then implies

$$P_t = \frac{W_t}{1 - \sigma}$$

The real wage is thus constant and equal to  $1 - \sigma$ . Substituting this into (ii) yields

$$h'(1 - n_t) = 1 - \sigma \tag{40}$$

Since  $h(\cdot)$  is concave,  $n_t$  is equal to some constant  $n^*$  for all  $t$ . It follows that  $e_t = n^* + h(1 - n^*)$  is also constant for all  $t$ . We can then use (iii) and (iv) to solve for  $p_t$  and  $R_t$  as in the benchmark model, and then use (38) to compute  $\bar{R}_t$ . Finally, given  $\bar{R}_t$  we can use the implied  $\Pi_t$  from (v) to derive  $\{P_t\}_{t=1}^{\infty}$  for any initial value for  $P_0$ . The initial price level  $P_0$  is indeterminate, in line with the Sargent and Wallace (1975) result on the price level indeterminacy of pure interest rate rules. The nominal wage  $W_t = (1 - \sigma) P_t$ .

## B.6 Equilibria with Rigid Prices

We now turn to the case where producers set the price of their intermediate good  $P_{it}$  before the monetary authority moves. That is, producers set prices, the monetary authority sets  $1 + i_t$ , and then producers hire workers at a nominal wage  $W_t$ . This formulation implies prices are only rigid for one period.

If monetary policy is deterministic, producers can perfectly anticipate the nominal interest rate and the equilibrium nominal wage  $W_t$ , and so  $\Omega_t = \{W_t, X_t\}$  and  $W_t/P_t = 1 - \sigma$  as before.

Next, suppose monetary policy is contingent on some random variable, i.e.,  $i_t = i(\xi_t)$  where  $\{\xi_t\}_{t=0}^\infty$  is a sequence of random variables. For simplicity, consider the case where  $\xi_t$  is only random at  $t = 0$ , i.e.,

$$\xi_0 = \begin{cases} H & \text{w/prob } \chi \\ L & \text{w/prob } 1 - \chi \end{cases}$$

$\xi_t$  is deterministic for  $t = 1, 2, \dots$

From date  $t = 1$  on, we know from the optimal price-setting condition (i) that  $W_t/P_t = 1 - \sigma$ . It then follows that  $n_t = n^*$  and  $e_t = e^* \equiv n^* + h(1 - n^*)$  for all  $t \geq 1$ , and we can determine  $p_t$ ,  $R_t$ , and  $\bar{R}_t$  for  $t \geq 1$  just as in the case where prices are flexible. All we need is to solve for the equilibrium at date 0.

We use a superscript  $\xi \in \{H, L\}$  to denote the value of a variable as for a given realization of  $\xi_0$ . Assume wlog that  $i_0^H > i_0^L$ . The optimal price setting condition (1) is now

$$\frac{\chi n_0^H \frac{W_0^H}{P_0} + (1 - \chi) n_0^L \frac{W_0^L}{P_0}}{\chi n_0^H + (1 - \chi) n_0^L} = 1 - \sigma \quad (41)$$

That is, the output-weighted average real wage over the two values of  $\xi$  is equal to  $1 - \sigma$ . Optimal labor supply (ii) then implies

$$\begin{aligned} h'(1 - n_0^H) &= \min \left\{ \frac{W_0^H}{P_0}, 1 \right\} \\ h'(1 - n_0^L) &= \min \left\{ \frac{W_0^L}{P_0}, 1 \right\} \end{aligned}$$

These are three equations for four unknowns, meaning the set of all equilibria can be parameterized by a single parameter. Wlog, we choose the real wage when  $\xi = H$  to be this parameter. The three equations above yield values for  $W_0^L/P_0$ ,  $n_0^H$ , and  $n_0^L$  given  $W_0^H/P_0$ . From these, we can deduce earnings  $e_0^\xi = n_0^\xi + h(1 - n_0^\xi)$  for each  $\xi \in \{H, L\}$ . We can then use (iii) and (iv) to derive  $p_0^\xi$  and  $R_0^\xi$  by solving

$$\int_{R_0^\xi}^\infty n(y) dy + p_0^\xi = e_0^\xi \quad (42)$$

$$(1 + R_0^\xi) p_0^\xi = D + p^D \quad (43)$$

and then compute the expected return on loans  $\bar{R}_0^\xi$  using (38), and, via (v), the inflation rate  $\Pi_0^\xi$  for each  $\xi \in \{H, L\}$ . As before, the price level  $P_0$  is indeterminate. Optimal pricing only restricts the average real wage across states but not the real wage for each realization of  $\xi_0$ , introducing an indeterminacy. The equilibrium real wage can exceed  $1 - \sigma$  for one realization of  $\xi_0$  if it falls below  $1 - \sigma$  for the other realization.

There case where monetary policy has no effect on real variables at date 0 remains an equilibrium. In this case,  $W_0^H/P_0 = W_0^L/P_0 = 1 - \sigma$ . But price rigidity expands the set of equilibria to include ones in which real variables vary with the nominal interest rate. Since the nominal interest rate only serves as a signal

to coordinate real activity rather but does not directly affect it, there are equilibria in which  $W_0^H > W_0^L$  as well as equilibria in which  $W_0^H < W_0^L$ .<sup>10</sup> Since higher nominal interest rates seem to be contractionary in practice, we focus on equilibria in which  $W_0^H/P_0 < 1 - \sigma < W_0^L/P_0$ , i.e., real wages are lower when the monetary authority unexpectedly raises the nominal interest rate. In this case, from condition (ii) we know that a higher nominal interest rate will be associated with lower employment ( $n_0^H < n^* < n_0^L$ ) and hence lower earnings ( $e_0^H < e^* < e_0^L$ ). From (42), we can infer that  $R_0^\xi = \rho^\xi(p_0^\xi)$  where  $\rho^H(x) > \rho^L(x)$  for the same value  $x$ . As is clear from Figure 1, this implies a higher nominal interest rate will be associated with a lower real asset price ( $p_0^H < p^D < p_0^L$ ). This also implies a higher real interest rate on loans ( $R_0^H > R^D > R_0^L$ ). The real expected return to buying assets will also be higher ( $\bar{r}_0^H > \bar{r}^D > \bar{r}_0^L$ ). However, whether the real expected return to lending  $\bar{R}_0^H$  will be higher is ambiguous. (38) implies  $\bar{R}_0^\xi$  is either equal to  $\bar{r}_0^\xi$  or to a weighted average of  $R_0^\xi$  and  $\bar{r}_0^\xi$ . In the latter case, although both terms are higher when  $\xi = H$  the weight on  $\bar{r}_0^\xi$ , which is  $p_0^\xi/e_0^\xi$ , can be higher or lower for  $\xi = H$ . These results are summarized in Proposition 4 in the paper.

## B.7 Promises of Future Intervention

Our last point concerns the effects of a promise at date 0 to be contractionary at date 1 if the boom continues into that date. In this case,  $\xi_0$  and  $\xi_t$  for  $t \geq 2$  are deterministic, while  $\xi_1 = d_1 \in \{d, D\}$ . That is, we assume producers set prices each period before  $d_t$  is revealed. Solving for equilibrium at date 1 is identical to how we solved for the equilibrium at date 0 when we assumed  $\xi_0$  was random. Consider equilibria in which the real wage is lower if the boom continues, so

$$W_1^D/P_1 < 1 - \sigma < W_1^d/P_1.$$

This implies  $n_1^D < n^* < n_1^d$  and so  $e_1^D < e < e_1^d$ . In other words, if dividends fall and the boom ends, monetary policy must be expansionary. By the same logic as above, such a policy would imply  $p_1^D < p^D$  and  $p_1^d > p^d$ , as well as  $R_1^D < R^D$  and  $R_1^d > R^d$ . Turning back to date 0, conditions (iii) and (iv) imply

$$\begin{aligned} \int_{R_0^D}^{\infty} n(y) dy + p_0^D &= e \\ (1 + R_0^D) p_0^D &= D + p_1^D \end{aligned}$$

Comparative statics of this system with respect to  $p_1^D$  reveals that  $p_0^D < p^D$  and  $R_0^D < R^D$ . That is, while contractionary monetary policy at date 0 dampens  $p_0^D$  but raises  $R_0^D$  at date 0, a threat to enact contractionary monetary policy at date 1 if dividends remain high will dampen both  $p_0^D$  and  $R_0^D$  at date 0. These results are summarized in Proposition 5 in the paper.

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<sup>10</sup>One way to avoid such multiplicity is to assume dynamic monetary policy rules that are conditioned on past economic variables. This allows a central bank to take actions that are unsustainable if a high interest rate today leads to certain outcomes, eliminating equilibria with those outcomes. See Cochrane (2011) for a discussion of these issues.

## Appendix C: Macroprudential Regulation

In this appendix, we define an equilibrium for an economy with multiple markets as in Section 5. We then show that for an equilibrium in which all markets are active, various aspects of the equilibrium are uniquely determined. We then discuss some comparative static results with respect to the set of active markets.

### C.1 Defining an Equilibrium

We begin with some notation. Let  $p_t$  denote the price of the asset at date  $t$ . Given asset prices, we can define the return to buying the asset at date  $t$  as

$$z_t \equiv \frac{d_{t+1} + p_{t+1}}{p_t}$$

The return  $z_t$  can be random both because  $d_{t+1}$  might be uncertain (if  $d_t = D$ ) and because  $p_{t+1}$  might in principle be stochastic. Let  $G_t(z)$  denote the (possibly degenerate) cumulative distribution of the return  $z_t$ , i.e.,  $G(z) \equiv \Pr(z_t \leq z)$ . Let  $1 + r_t^{\max}$  denote the maximum possible return on the asset. As discussed in the text,  $1 + r_t^{\max}$  is finite, since  $r_t^{\max} \leq \frac{D+2\varphi e}{(1-\varphi)e}$ . We will use  $\bar{r}_t$  to denote the expected return to buying the asset at date  $t$ , i.e.,

$$1 + \bar{r}_t \equiv \int_0^{1+r_t^{\max}} z_t dG_t(z)$$

We now define variables for the different markets  $\lambda \in [0, 1)$  agents can borrow in. Let  $R_t(\lambda)$  denote the interest rate on loans in market  $\lambda$ , so an agent who agrees to pay a share  $\lambda$  of the project she undertakes will promise to pay back  $1 + R_t(\lambda)$  for each unit she borrows. Since agents may default, let  $\bar{R}_t(\lambda)$  denote what lenders expect to earn from lending in market  $\lambda$  given the possibility of default. Finally, we represent borrowing in markets with density functions  $f_t^a(\lambda)$  and  $f_t^p(\lambda)$  for all  $\lambda \in [0, 1)$  such that the total amount of resources borrowed to buy assets and produce are given by  $\int_A f_t^a(\lambda) d\lambda$  and  $\int_A f_t^p(\lambda) d\lambda$ , respectively. Let  $f_t(\lambda) \equiv f_t^a(\lambda) + f_t^p(\lambda)$  denote the density of borrowing for any purpose in market  $\lambda$ .

Representing the quantities agents borrow in each market as a density function ignores the possibility that there may be equilibria in which agents borrow a positive mass of resources in certain markets. More generally, we can allow for a set  $\Delta \subset [0, 1)$  with countably many elements such that each market  $\lambda \in \Delta$  is associated with a positive mass of borrowing  $m_t^x(\lambda) > 0$ . The amount borrowed in any market  $\lambda \in [0, 1) \setminus \Delta$  can still be represented with a density function. Heuristically, we can appeal to the Dirac-delta construction and represent the amount borrowed in any market as if it were a density. That is, for any  $\lambda \in \Delta$ , we set the density  $f_t^x(\lambda) = m_t^x(\lambda) \delta_\lambda(\lambda)$ , where  $\delta_\lambda(q)$  is the Dirac-delta function defined so that  $\delta_\lambda(q) = 0$  for  $q \neq \lambda$  and  $\int_0^1 \delta_\lambda(q) dq = 1$ . This convention treats markets  $\lambda \in \Delta$  as essentially having an infinite density. We will refer to a market  $\lambda$  as *inactive* if  $f_t(\lambda) = 0$  and *active* if  $f_t(\lambda) > 0$  or if  $\lambda \in \Delta$ .

Given these preliminaries, we define an equilibrium as a path  $\{p_t, f_t^p(\lambda), f_t^a(\lambda), R_t(\lambda), \bar{R}_t(\lambda)\}_{t=0}^\infty$  that satisfies conditions (44)-(49) below to ensure that all markets clear when agents are optimizing.



Our first three conditions stipulate that agents act optimally. We begin with lenders. Optimality requires that agents will only invest their wealth where the expected return is highest. Let  $\bar{R}_t$  denote the maximal expected return to lending in any market  $\lambda$ , i.e.,

$$\bar{R}_t \equiv \sup_{\lambda \in [0,1]} \bar{R}_t(\lambda)$$

Optimal lending requires that agents lend in market  $\lambda'$  only if they expect to earn  $\bar{R}_t$  and if this rate exceeds the expected return to buying the asset, i.e.,

$$f_t(\lambda') > 0 \text{ only if } \bar{R}_t(\lambda') = \bar{R}_t \text{ and } \bar{R}_t \geq \bar{r}_t \quad (44)$$

Next, entrepreneurs must act optimally. We first argue this means they should use their endowment to produce. Recall entrepreneurs have productivity  $y^*$  where  $y^* > r_t^{\max} \geq \bar{r}_t$  from (20), so producing is better than buying assets. But  $y^*$  must also exceed the expected return to lending  $\bar{R}_t$ . For suppose  $\bar{R}_t$  were higher than  $y^*$ . Since  $y^* > r_t^{\max}$ , then  $\bar{R}_t$  must also exceed  $r_t^{\max}$ . In that case, no agent would use their endowment to buy assets, nor would any agent borrow to buy assets given the interest rate on loans in any active market must be at least  $\bar{R}_t$ . Yet assets must trade in equilibrium: Owners sell their assets whenever the asset price is positive, while demand for the asset would be infinite if its price were nonnegative. Since production offers the highest return, entrepreneurs should use their endowment  $w$  to produce.

Since entrepreneurs can leverage their endowment to produce at a larger capacity, we also need to characterize their borrowing. If they borrow in market  $\lambda$  where  $\lambda < w$ , they can borrow enough to reach full capacity. Optimality requires that there will be borrowing to produce in market  $\lambda'$  only if some entrepreneur finds it optimal to borrow in that market from all  $\lambda \in [0, 1]$ , including  $\lambda = 1$  for no borrowing. This implies

$$f_t^p(\lambda') > 0 \text{ only if } \lambda' \in \arg \max_{\lambda \in [0,1]} \left\{ \begin{array}{ll} [1 + y - (1 - w)(1 + R_t(\lambda))] & \text{if } \lambda \leq w \\ \frac{w}{\lambda} [1 + y - (1 - \lambda)(1 + R_t(\lambda))] & \text{if } \lambda > w \end{array} \right\} \text{ for some } w \quad (45)$$

Third, agents who borrow to buy assets must act optimally. They will agree to borrow in market  $\lambda \in [0, 1]$  to buy assets only if doing so yields a higher expected return than lending out the same resources. Define

$$x_t(\lambda) \equiv (1 + R_t(\lambda))(1 - \lambda)$$

The expected profits from borrowing in market  $\lambda$  to buy one consumption unit's worth of assets is

$$\int_{x_t(\lambda)}^{\infty} (z_t - x_t(\lambda)) dG(z_t) \quad (46)$$

Agents will borrow in market  $\lambda$  to buy assets only if (46) equal  $(1 + \bar{R}_t)\lambda$ , the return on what they must spend on assets. If (46) were lower than  $(1 + \bar{R}_t)\lambda$ , no agent would borrow to buy assets. If (46) were higher than  $(1 + \bar{R}_t)\lambda$ , then no one would ever lend given they can borrow in market  $\lambda'$ , and so  $f_t(\lambda') = 0$ . But this contradicts the fact that  $f_t^a(\lambda) > 0$ . Optimality implies

$$f_t^a(\lambda') > 0 \text{ only if } \int_{x_t(\lambda')}^{\infty} (z_t - x_t(\lambda')) dG(z_t) = (1 + \bar{R}_t)\lambda' \quad (47)$$

Fourth, savers will not waste any resources. Since entrepreneurs use their endowment to produce, all the resources spent to buy the asset must come from savers. This implies that  $e$  must be either lent to entrepreneurs to produce or be spent on assets:

$$\int_0^1 f_t^p(\lambda) d\lambda + p_t = e \quad (48)$$

Finally, we turn to equilibrium beliefs. In any active market  $\lambda'$ , lenders must expect the return on lending  $\bar{R}_t(\lambda')$  to conform with the actual fraction of borrowers who borrow in market  $\lambda'$  with the intent to produce and to buy assets, respectively. That is,

$$\bar{R}_t(\lambda') = \frac{f_t^p(\lambda')}{f_t(\lambda')} R_t(\lambda') + \frac{f_t^a(\lambda')}{f_t(\lambda')} E_t \max \left\{ R_t(\lambda'), \frac{d_{t+1} + p_{t+1}}{p_t} - 1 \right\} \text{ if } f_t(\lambda') > 0 \quad (49)$$

In a market  $\lambda \in \Delta$  with a positive mass of borrowing, the expression  $\frac{f_t^x(\lambda')}{f_t(\lambda')}$  will be replaced by  $\frac{m_t^x(\lambda)}{m_t(\lambda)}$ . Condition (49) does not impose any restrictions on expectations in inactive markets where  $f_t(\lambda') = 0$ .

## C.2 Solving for Equilibrium

We now proceed to solve for an equilibrium. As in the text, we restrict attention to equilibria in which all markets  $\lambda \in [0, 1)$  are active. Such equilibria are natural given we focus on the effects of interventions to shut down markets. Our first result characterizes the schedule of interest rates in such an equilibrium.

**Proposition C1:** *In an equilibrium where all markets are active, there exists a value  $\Lambda_t \in [0, 1]$  such that the equilibrium interest rate schedule will be given by*

$$1 + R_t(\lambda) = \begin{cases} \frac{\tilde{x}_t(\lambda)}{1-\lambda} & \text{if } \lambda \in [0, \Lambda_t) \\ 1 + \bar{R}_t & \text{if } \lambda \in [\Lambda_t, 1) \end{cases} \quad (50)$$

where  $\tilde{x}_t(\lambda)$  is the value of  $x$  that solves

$$\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z) = (1 + \bar{R}_t) \lambda \quad (51)$$

The schedule of interest rates  $R_t(\lambda)$  is a decreasing and continuous function of  $\lambda$  for  $\lambda \in [0, \Lambda_t]$ .

**Proof of Proposition C1:** Our proof relies proceeds as two lemmas.

**Lemma C1:** In an equilibrium where all markets are active,  $1 + R_t(\lambda) = \max \left\{ \frac{\tilde{x}_t(\lambda)}{1-\lambda}, 1 + \bar{R}_t \right\}$ , where  $\tilde{x}_t(\lambda)$  equals the  $x$  that solves (51) and  $\bar{R}_t$  is the expected return to lending in any market  $\lambda$ .

**Proof of Lemma C1:** Recall we defined  $x_t(\lambda) \equiv (1 + R_t(\lambda))(1 - \lambda)$  as the equilibrium debt obligation for an agent who invests one unit of resources in assets. As we argued above, for all  $\lambda$  we have

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z - x_t(\lambda)) dG_t(z) \leq (1 + \bar{R}_t) \lambda \quad (52)$$

since otherwise agents would refuse to lend, which is incompatible with  $f_t(\lambda) > 0$  for all  $\lambda \in [0, 1)$ . The expression  $\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z)$  is strictly decreasing in  $x$ . It also tends to  $+\infty$  as  $x \rightarrow -\infty$  and to 0 as  $x \rightarrow 1+r_t^{\max}$ . Hence, for any  $\lambda \in [0, 1)$  and any  $\bar{R}_t \geq 0$ , there exists a unique  $x \in (-\infty, 1+r_t^{\max}]$  for which

$$\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z) = (1 + \bar{R}_t) \lambda \quad (53)$$

Denote  $\tilde{x}_t(\lambda)$  as the unique solution to equation (53). By contrast,  $x_t(\lambda)$  refers to the value of  $(1 + R_t(\lambda))(1 - \lambda)$  evaluated at the equilibrium interest rate  $R_t(\lambda)$ .

For any  $\lambda' \in [0, 1)$  in which (52) holds with equality, we have  $\tilde{x}_t(\lambda') = x_t(\lambda')$ , and so

$$1 + R_t(\lambda') = \frac{\tilde{x}_t(\lambda')}{1 - \lambda'}$$

For any remaining values of  $\lambda' \in [0, 1)$ , condition (52) holds as a strict inequality. This means borrowing in market  $\lambda'$  and buying assets yields a lower payoff than lending out the resources needed to borrow in market  $\lambda'$ . Hence, no agent will borrow in market  $\lambda'$  to buy assets, implying  $f_t^a(\lambda') = 0$ . In an equilibrium where all markets are active,  $f_t^p(\lambda') > 0$ . From (44) we know that  $\bar{R}_t(\lambda') = \bar{R}_t$ , and from (49) we know that since  $f_t^a(\lambda') = 0$  then  $\bar{R}_t(\lambda') = R_t(\lambda')$ . Combining these implies  $R_t(\lambda') = \bar{R}_t$ .

Hence, in an equilibrium where all markets are active, we must have either  $R_t(\lambda) = \bar{R}_t$  or  $R_t(\lambda) = \frac{\tilde{x}_t(\lambda)}{1-\lambda}$  for all  $\lambda \in [0, 1)$ . To further show that  $1 + R_t(\lambda) = \max\left\{1 + \bar{R}_t, \frac{\tilde{x}_t(\lambda)}{1-\lambda}\right\}$ , consider a value of  $\lambda$  for which  $\frac{\tilde{x}_t(\lambda)}{1-\lambda} > 1 + \bar{R}_t$ , i.e., for which  $\tilde{x}_t(\lambda) > x_t(\lambda)$ . Since  $\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z)$  is decreasing in  $x$ , this means

$$\int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z-x_t(\lambda)) dG_t(z) > \int_{z=\tilde{x}(\lambda)}^{1+r_t^{\max}} (z-\tilde{x}(\lambda)) dG_t(z) = (1 + \bar{R}_t) \lambda$$

Since in equilibrium we must satisfy (52), it follows that in this case we have  $1 + R_t(\lambda) = \frac{\tilde{x}_t(\lambda)}{1-\lambda}$ .

Next, consider a value of  $\lambda$  for which  $\frac{\tilde{x}_t(\lambda)}{1-\lambda} < 1 + \bar{R}_t$ , i.e., for which  $\tilde{x}_t(\lambda) < x_t(\lambda)$ . Then we would have

$$(1 + \bar{R}_t) \lambda = \int_{z=\tilde{x}(\lambda)}^{1+r_t^{\max}} (z-\tilde{x}(\lambda)) dG_t(z) > \int_{z=x_t(\lambda)}^{1+r_t^{\max}} (z-x_t(\lambda)) dG_t(z)$$

In this case, (52) can only hold as a strict inequality. But we already know that in this case  $R_t(\lambda) = \bar{R}_t$ . This establishes the lemma. ■

Our next lemma establishes that  $\frac{\tilde{x}_t(\lambda)}{1-\lambda}$  is a weakly decreasing and continuous function of  $\lambda$ . Combined with Lemma C1, this implies there exists a cutoff  $\Lambda_t$  such that  $R_t(\lambda) = \bar{R}_t$  for  $\lambda \geq \Lambda_t$ .

**Lemma C2:** In any equilibrium where all markets are active,  $\frac{\tilde{x}_t(\lambda)}{1-\lambda}$  is nonincreasing and continuous in  $\lambda$ .

**Proof of Lemma C2:** The function  $\tilde{x}_t(\lambda)$  corresponds to the value of  $x$  which solves (51). Although the distribution  $G_t(z)$  can contain mass points, the integral  $\int_{z=x}^{1+r_t^{\max}} (z-x) dG_t(z)$  is still continuous in  $x$ .

This implies  $\tilde{x}_t(\lambda)$  is a continuous function of  $\lambda$ . However,  $\tilde{x}_t(\lambda)$  may exhibit kinks, meaning its directional derivatives need not be equal at all values. To show that  $\tilde{x}_t(\lambda)$  is decreasing, it will suffice to show that all of its directional derivatives are nonpositive for all  $\lambda \in [0, 1)$ . Totally differentiating (51) implies

$$\frac{d\tilde{x}_t(\lambda)}{d\lambda} = -\frac{1 + \bar{R}_t}{\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)}$$

For any  $\lambda$  where  $\tilde{x}_t(\lambda)$  is a mass point of  $G_t(z)$ ,  $\lim_{\lambda' \rightarrow \lambda^+} \int_{\tilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z) \neq \lim_{\lambda' \rightarrow \lambda^-} \int_{\tilde{x}_t(\lambda')}^{1+r_t^{\max}} dG_t(z)$ . Nevertheless, both  $\lim_{\lambda' \rightarrow \lambda^+} \frac{d\tilde{x}_t(\lambda')}{d\lambda'}$  and  $\lim_{\lambda' \rightarrow \lambda^-} \frac{d\tilde{x}_t(\lambda')}{d\lambda'}$  are negative, so  $\tilde{x}_t(\lambda)$  is strictly decreasing in  $\lambda$ .

Next, define  $\tilde{R}_t(\lambda) \equiv \frac{\tilde{x}_t(\lambda)}{1-\lambda} - 1$ . The function  $\tilde{R}_t(\lambda)$  is also continuous in  $\lambda$  with possible kink-points. Differentiating the equation  $\tilde{x}_t(\lambda) = (1-\lambda)(1 + \tilde{R}_t(\lambda))$  implies

$$\frac{d\tilde{x}_t(\lambda)}{d\lambda} = -(1 + \tilde{R}_t(\lambda)) + (1-\lambda) \frac{d\tilde{R}_t(\lambda)}{d\lambda}$$

Rearranging and using the expression for  $\frac{d\tilde{x}_t(\lambda)}{d\lambda}$  above yields

$$\begin{aligned} \frac{d\tilde{R}_t(\lambda)}{d\lambda} &= \frac{1}{1-\lambda} \left[ 1 + \tilde{R}_t(\lambda) + \frac{d\tilde{x}_t(\lambda)}{d\lambda} \right] \\ &= \frac{1}{1-\lambda} \left[ 1 + \tilde{R}_t(\lambda) - \frac{1 + \bar{R}_t}{\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)} \right] \\ &= \frac{1}{(1-\lambda) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z)} \left[ (1 + \tilde{R}_t(\lambda)) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) - (1 + \bar{R}_t) \right] \end{aligned} \quad (54)$$

Once again,  $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$  is discontinuous at  $\lambda$  where  $\tilde{x}_t(\lambda)$  is a mass point of  $G_t(z)$ .

To evaluate the sign of  $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$ , we must consider two cases. First, suppose  $\tilde{R}_t(\lambda) < \bar{R}_t$ . Then

$$\begin{aligned} (1 + \tilde{R}_t(\lambda)) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) &< (1 + \bar{R}_t) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} dG_t(z) \\ &\leq 1 + \bar{R}_t \end{aligned}$$

In that case, we have  $\frac{d\tilde{R}_t(\lambda)}{d\lambda} < 0$  from (54) regardless of the direction we take the derivative. Next, suppose  $\tilde{R}_t(\lambda) \geq \bar{R}_t$ . From Lemma C1, in this case (52) holds with equality. Rearranging this equation, we get

$$\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[ z - (1 + \tilde{R}_t(\lambda)) \right] dG_t(z) = \lambda \left[ (1 + \bar{R}_t) - \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1 + \tilde{R}_t(\lambda)) dG_t(z) \right]$$

We can establish that  $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$  in (54) is nonnegative for  $\lambda > 0$  if we can show that

$$\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \left[ z - (1 + \tilde{R}_t(\lambda)) \right] dG_t(z) \geq 0$$

Towards this, observe that the expected profits from borrowing in market  $\lambda$  to buy assets are given by

$$\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z)$$

Since these are equal to  $(1+\bar{R}_t)\lambda$  when  $\tilde{R}_t(\lambda) \geq \bar{R}_t$ , we have

$$\begin{aligned} (1+\bar{R}_t)\lambda &= \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} \lambda z_t dG(z) \\ &\leq \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + \int_0^{1+r_t^{\max}} \lambda z_t dG(z) \\ &= \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (1-\lambda)(z_t - (1+R_t)) dG(z) + (1+\bar{r}_t^D)\lambda \end{aligned} \quad (55)$$

But in an equilibrium where all markets are active, we must have  $\bar{R}_t^D \geq \bar{r}_t^D$ . This implies

$$0 \leq (\bar{R}_t - \bar{r}_t^D)\lambda \leq (1-\lambda) \int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1+R_t)) dG(z)$$

This confirms  $\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} (z_t - (1+R_t)) dG(z) \geq 0$ . All directional derivatives  $\frac{d\tilde{R}_t(\lambda)}{d\lambda}$  are nonnegative. ■

From Lemmas C1 and C2, set  $\Lambda_t$  to be either 1 or the minimum value in  $[0, 1]$  for which  $R_t(\lambda) = \bar{R}_t$ . It follows that  $R_t(\lambda) > \bar{R}_t$  for  $\lambda < \Lambda_t$  and  $R_t(\lambda) = \bar{R}_t$  for all  $\lambda \geq \Lambda_t$ . This establishes the proposition. ■

We can use the schedule of interest rates in Proposition C1 to determine how much entrepreneurs should produce and in which markets to borrow if they do.

**Proposition C2:** *In an equilibrium where all markets are active, entrepreneurs with wealth  $w$  will borrow  $1-w$  units to produce, in a market with an interest rate equal to  $R_t(w)$ .*

**Proof of Proposition C2:** Consider an entrepreneur with wealth  $w$ . If she borrows in a market  $\lambda$  where  $\lambda \leq w$ , she can produce at full capacity and would only need to put down  $\lambda \left( \frac{1-w}{1-\lambda} \right)$  resources to borrow  $1-w$  to reach full capacity. This would earn her an expected profit of

$$1 + y^* - (1 + R_t(\lambda))(1 - w)$$

This value is maximized by choosing  $\lambda$  to minimize  $R_t(\lambda)$ . From Proposition C1, we know  $R_t(\lambda)$  is weakly decreasing in  $\lambda$  and is therefore maximized at  $\lambda = w$ .

Next, suppose she borrows in a market  $\lambda$  where  $\lambda > w$ . In that case, she could not produce at full capacity. Since  $y^* > r_t^{\max} = R_t(0) \geq R_t(\lambda)$  for all  $\lambda \in [0, 1]$ , it will be optimal to borrow enough to produce at the maximal capacity possible. For  $\lambda > w$ , this maximum is  $\frac{w}{\lambda}$ . Her profits would thus equal

$$\frac{w}{\lambda} (1 + y^* - x_t(\lambda)) \quad (56)$$

where recall  $x_t(\lambda) = (1 - \lambda)(1 + R_t(\lambda))$  is the amount a borrower is required to repay per each unit of resource she borrows. Since  $R_t(\lambda) = \bar{R}_t$  for all  $\lambda \in (\Lambda_t, 1)$ , there would be no benefit to going to market  $\lambda > \Lambda_t$ : She would have to produce less at the same interest rate as in market  $\Lambda_t$ . The only case that remains is the interval of markets  $\lambda \in [w, \Lambda_t]$ . In that case, we can differentiate profits in (56) to get

$$\begin{aligned} \frac{d}{d\lambda} \left( \frac{w}{\lambda} (1 + y^* - x_t(\lambda)) \right) &= -\frac{w}{\lambda^2} \left[ (1 + y^* - x_t(\lambda)) + \lambda \frac{dx_t(\lambda)}{d\lambda} \right] \\ &= -\frac{w}{\lambda^2} \left[ (1 + y^* - x(\lambda)) - \frac{\lambda(1 + \bar{R}_t)}{\int_x^{1+r_t^{\max}} dG_t(z)} \right] \\ &= -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)} \left[ \int_x^{1+r_t^{\max}} (1 + y^* - x_t(\lambda)) dG_t(z) - \lambda(1 + \bar{R}_t) \right] \end{aligned}$$

Since  $y^* > \frac{D+2\varphi e}{(1-\varphi)e} > r_t^{\max}$ , we have

$$\frac{d}{d\lambda} \left( \frac{w}{\lambda} (1 + y^* - x_t(\lambda)) \right) < -\frac{w}{\lambda^2 \int_x^{1+r_t^{\max}} dG_t(z)} \left[ \int_x^{1+r_t^{\max}} (z - x_t(\lambda)) dG_t(z) - \lambda(1 + \bar{R}_t) \right]$$

But for  $\lambda \leq \Lambda_t$ , the expression in brackets is equal to 0. Hence, borrowing in a market with  $\lambda > w$  will be strictly dominated by borrowing in the market with  $\lambda = w$ . At the optimum, each entrepreneur borrow  $1 - w$  at a rate of  $R_t(w)$ . ■

**Proposition C3:** *In an equilibrium where all markets are active, the equilibrium price of the asset will be given by  $p_t = (1 - \varphi)e$*

**Proof of Proposition C3:** Condition (48) implies that all the resources of the young in cohort  $t$  will be used to either produce or to buy assets. From Proposition C2, we know that all entrepreneurs will produce at capacity, so the total amount used to produce is given by

$$\int_0^1 (2\varphi e) dw = 2\varphi e$$

This implies

$$p_t + 2\varphi e = (1 + \varphi)e$$

and so  $p_t = (1 - \varphi)e$  as claimed. ■

Propositions C1-C3 do not require any restrictions on the distribution of  $d_t$ . When  $d_t = d$ , the return on the asset  $1 + r_t$  will have a degenerate distribution with full mass at  $\frac{d}{(1-\varphi)e}$ . Substituting this into (51) reveals that  $\tilde{x}(\lambda) = (1 - \lambda) \left( 1 + \frac{d}{(1-\varphi)e} \right)$  for all  $\lambda$ , that  $\frac{d\tilde{R}(\lambda)}{d\lambda} = 0$  for all  $\lambda$ , and the cutoff  $\Lambda_t = 0$ . Hence, when all markets are active,  $R_t(\lambda) = \bar{R}_t = \frac{d}{(1-\varphi)e}$  for all  $\lambda \in [0, 1)$  as described in the text. One equilibrium in which all markets are active if it entrepreneurs with wealth  $w$  borrow in market  $\lambda = w$ . But other equilibria in which all markets are active also exist.

When dividends follow a regime-switching process, then if  $d_t = D$  at date  $t$ ,  $z_t$  would be distributed as

$$z_t = \begin{cases} \frac{D}{(1-\varphi)e} & \text{w/prob } 1 - \pi \\ \frac{d}{(1-\varphi)e} & \text{w/prob } \pi \end{cases}$$

We can verify that this distribution implies that  $\frac{dR_t(\lambda)}{d\lambda} < 0$  when  $\lambda \leq \Lambda_t^D$ . In particular, observe that (55) in Lemma C2 relies on the fact that  $\int_{\tilde{x}_t(\lambda)}^{1+r_t^{\max}} z_t dG(z) \leq \int_0^{1+r_t^{\max}} z_t dG(z)$ . But for the above distribution, the first expression is equal to  $(1-\pi) \left(1 + \frac{D}{(1-\varphi)e}\right)$ , which is strictly less than  $1 + \frac{(1-\pi)D+\pi d}{(1-\varphi)e}$  which corresponds to the second expression. Hence, we can replace (55) with a strictly inequality, implying  $\frac{dR_t(\lambda)}{d\lambda}$  is strictly negative for  $\lambda \leq \Lambda_t^D$ . This is in line with what we discuss in the text and depict in Figure 2.

Since  $\Lambda_t^D$  is the minimum value of  $\lambda$  at which  $\frac{\tilde{x}_t(\lambda)}{1-\lambda} = 1 + \bar{R}_t^D$ , we have

$$\frac{1}{1-\Lambda_t} \left[ 1 + \frac{D}{(1-\varphi)e} - \left(1 + \bar{R}_t^D\right) \frac{\Lambda_t}{1-\pi} \right] = 1 + \bar{R}_t^D$$

which, upon rearranging, yields

$$\Lambda_t^D = \frac{1-\pi}{\pi(1+\bar{R}_t^D)} \left( \frac{D}{(1-\varphi)e} - \bar{R}_t^D \right)$$

Since  $R_t^D(\lambda)$  is decreasing in  $\lambda$  for  $\lambda \in [0, \Lambda_t^D)$ , Proposition C2 implies only borrowers with wealth  $w$  borrow in market  $\lambda = w$  for  $w \in [0, \Lambda_t^D)$ . Hence,  $f_t^p(\lambda) = 2\varphi e$  for  $\lambda \in [0, \Lambda_t^D)$ . By contrast,  $f_t^p(\lambda)$  is indeterminate for  $\lambda \in [\Lambda_t^D, 1)$ . However, we know that  $f_t^p(\Lambda_t) > 0$ , since borrowers with wealth  $w = \Lambda_t^D$  will have to borrow in this market to borrow  $1-w$ . As for the amount borrowed to buy assets,  $f_t^a(\lambda)$ , we can deduce  $f_t^a(\lambda)$  for  $\lambda \in [0, \Lambda_t^D]$  from  $R_t^D(\lambda)$ ,  $\bar{R}_t^D$ , and  $f_t^p(\lambda)$  using (49). For  $\lambda > \Lambda_t^D$ , the fact that  $\frac{dR_t^D(\lambda)}{d\lambda} < 0$  at  $\lambda = \Lambda_t^D$ , combined with the fact that  $\frac{dR_t^D(\lambda)}{d\lambda} < 0$  for  $\lambda > \Lambda_t^D$  from Lemma C2, implies that no agent would want to borrow to buy assets. So  $f_t^a(\lambda) = 0$  for all  $\lambda \geq \Lambda_t^D$ . We can solve for  $\bar{R}_t^D$  as in the text.

### C.3 Comparative Statics

Next, we consider equilibria where all markets above some floor  $\underline{\lambda}$  are active. These results correspond to Propositions 8 and 9 in the text. The first result concerns how the equilibrium changes with  $\underline{\lambda}$ .

**Proof of Proposition 8:** In the text, we show that  $p^D$  and  $\bar{r}^D$  are increasing and decreasing in  $\underline{\lambda}$ , respectively. Here, we show that  $\bar{R}^D$  is decreasing in  $\underline{\lambda}$ . For any  $\underline{\lambda}$ , either  $\bar{R}^D$  equals  $\bar{r}^D$  or exceeds  $\bar{r}^D$ . Since the expected return on loans  $\bar{R}^D$  is continuous in  $\underline{\lambda}$ , it will suffice to show that  $\bar{R}^D$  is decreasing in  $\underline{\lambda}$  when  $\bar{R}^D > \bar{r}^D$ .

When  $\bar{R}^D > \bar{r}^D$ , we have  $\gamma^D = 1$ , and the equilibrium conditions for  $R^D$  and  $\Lambda^D$  are given by

$$(1 - \Lambda^D) \bar{R}^D = \left[ \frac{D}{(1-(1-\underline{\lambda})\varphi)e} - \Lambda^D \left( \frac{1+\bar{R}^D}{1-\pi} - 1 \right) \right] \quad (57)$$

$$1 + \bar{R}^D = (1 - \varphi(1 - \underline{\lambda})) [1 + \bar{r}^D - \pi\Phi] + 2\varphi \int_0^1 \left[ \min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] [1 + R^D(\max \{w, \underline{\lambda}\})] dw \quad (58)$$

If  $\bar{R}^D > \bar{r}^D$ , the floor  $\underline{\lambda}$  must be below the cutoff  $\Lambda^D$ . For suppose  $\underline{\lambda} \geq \Lambda^D$ . Then all markets where agents might default will be shut down. But without default, the expected return on lending and the expected

return on the asset must be equal to ensure both the credit market and asset market clear. Since  $\underline{\lambda} < \Lambda^D$ , we can expand the integral term in (58) to obtain

$$\int_0^1 \left[ \min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] [1 + R^D(\max\{w, \underline{\lambda}\})] = (1 + R(\underline{\lambda})) \left( \frac{1}{\underline{\lambda}} - 1 \right) \int_0^{\underline{\lambda}} w dw + \int_{\underline{\lambda}}^{\Lambda^D} (1 + R(w))(1 - w) dw + (1 + \bar{R}^D) \int_{\Lambda^D}^1 (1 - w) dw$$

We use the fact that  $1 + R^D(\lambda) = \frac{1}{1-\lambda} \left[ 1 + \frac{D}{(1-(1-\lambda)\varphi)e} - \frac{\lambda(1+\bar{R}_t^D)}{1-\pi} \right]$  to express the three integrals above as

$$(1 + R(\underline{\lambda})) \left( \frac{1}{\underline{\lambda}} - 1 \right) \int_0^{\underline{\lambda}} w dw = \left[ 1 + \frac{D}{(1-(1-\lambda)\varphi)e} - \underline{\lambda} \left( \frac{1+\bar{R}^D}{1-\pi} - 1 \right) \right] \frac{\underline{\lambda}}{2} \quad (59)$$

$$\int_{\underline{\lambda}}^{\Lambda^D} (1 + R(w))(1 - w) dw = \int_{\underline{\lambda}}^{\Lambda^D} \left[ 1 + \frac{D}{(1-(1-\lambda)\varphi)e} - \frac{w(1+\bar{R}_t^D)}{1-\pi} \right] dw \quad (60)$$

$$(1 + \bar{R}^D) \int_{\Lambda^D}^1 (1 - w) dw = \frac{1}{2} (1 + \bar{R}^D) (1 - \Lambda^D)^2 \quad (61)$$

We can write (57) and (58) more compactly as

$$\begin{aligned} h_1(\bar{R}^D, \Lambda^D) &= 0 \\ h_2(\bar{R}^D, \Lambda^D) &= 0 \end{aligned}$$

Totally differentiating this system of equations gives us the comparative statics of the equilibrium  $\bar{R}^D$  and  $\Lambda^D$  with respect to any variable  $a$  as

$$\begin{bmatrix} \frac{\partial h_1}{\partial \bar{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \bar{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix} \begin{bmatrix} d\bar{R}^D/da \\ d\Lambda^D/da \end{bmatrix} = \begin{bmatrix} -\frac{\partial h_1}{\partial a} \\ -\frac{\partial h_2}{\partial a} \end{bmatrix}$$

Differentiating (57) and (58) using expressions (59)-(61) yields

$$\begin{aligned} \frac{\partial h_1}{\partial \bar{R}^D} &= 1 - \Lambda^D + \frac{\Lambda^D}{1-\pi} & \frac{\partial h_1}{\partial \Lambda^D} &= \frac{\pi(1+\bar{R}^D)}{1-\pi} \\ \frac{\partial h_2}{\partial \bar{R}^D} &= 1 + \varphi \left[ \frac{1}{1-\pi} (\Lambda^D)^2 - (1 - \Lambda^D)^2 \right] & \frac{\partial h_2}{\partial \Lambda^D} &= 0 \end{aligned}$$

When we evaluate comparative statics with respect to  $\underline{\lambda}$ , we now have

$$\begin{aligned} \begin{bmatrix} d\bar{R}^D/d\underline{\lambda} \\ d\Lambda^D/d\underline{\lambda} \end{bmatrix} &= \begin{bmatrix} \frac{\partial h_1}{\partial \bar{R}^D} & \frac{\partial h_1}{\partial \Lambda^D} \\ \frac{\partial h_2}{\partial \bar{R}^D} & \frac{\partial h_2}{\partial \Lambda^D} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dh_1}{d\underline{\lambda}} \\ \frac{dh_2}{d\underline{\lambda}} \end{bmatrix} \\ &= \frac{\varphi}{\kappa} \begin{bmatrix} 0 & \frac{\pi}{1-\pi} (1 + \bar{R}^D) \\ 1 + \varphi \frac{(\Lambda^D)^2}{1-\pi} - \varphi (1 - \Lambda^D)^2 & - \left( 1 - \Lambda^D + \frac{\Lambda^D}{1-\pi} \right) \end{bmatrix} \begin{bmatrix} -\frac{D}{(1-(1-\lambda)\varphi)^2 e} \\ -\frac{2D(1+\Lambda^D\varphi)}{(1-(1-\lambda)\varphi)^2 e} - (1 + \pi\Phi) \end{bmatrix} \end{aligned}$$

where  $\kappa = \frac{\pi(1+\bar{R}^D)}{1-\pi} \left( 1 + \varphi \frac{(\Lambda^D)^2}{1-\pi} - \varphi (1 - \Lambda^D)^2 \right) > 0$ . It follows that

$$\frac{d\bar{R}^D}{d\underline{\lambda}} = -\varphi \left( 1 + \varphi \left[ \frac{1}{1-\pi} (\Lambda^D)^2 - (1 - \Lambda^D)^2 \right] \right)^{-1} \left[ \frac{2D(1+\Lambda^D\varphi)}{(1-(1-\lambda)\varphi)^2 e} + (1 + \pi\Phi) \right] < 0$$



Since  $\bar{R}^D$  is decreasing in  $\underline{\lambda}$  whether  $\bar{R}^D > \bar{r}^D$  or  $\bar{R}^D = \bar{r}^D$ , the claim follows. ■

Proposition 9 concerns how changing  $\underline{\lambda}$  affects the expected costs of default  $\gamma^D \Phi p^D$ . Since we already know  $p^D$  is increasing in  $\underline{\lambda}$ , any changes in expected default costs occur entirely through  $\gamma^D$ . Our next result argues that there exists cutoffs  $\Lambda_0$  and  $\Lambda_1$  such that  $d\gamma^D/d\underline{\lambda} = 0$  when  $\underline{\lambda} < \Lambda_0$  or  $\underline{\lambda} > \Lambda_1$ . When  $\Lambda_0 < \underline{\lambda} < \Lambda_1$ , we only claim it must be decreasing for some  $\underline{\lambda}$  in this interval.

**Proof of Proposition 9:** Define

$$\rho(\underline{\lambda}) = \frac{\bar{R}^D}{(1 - (1 - \underline{\lambda}) \varphi)}$$

Using the fact that  $\frac{d\bar{R}^D}{d\underline{\lambda}} < 0$ , we have

$$\frac{d\rho(\underline{\lambda})}{d\underline{\lambda}} = \frac{d\bar{R}^D/d\underline{\lambda} - \varphi\rho(\underline{\lambda})}{1 - (1 - \underline{\lambda}) \varphi} < 0$$

Since

$$\bar{R}^D/\bar{r}^D = [(1 - \pi) D + \pi d] \rho(\underline{\lambda})$$

it follows that the ratio  $\bar{R}^D/\bar{r}^D$  is decreasing in  $\underline{\lambda}$ . Hence, there exists a value  $\Lambda_0 \geq 0$  such that  $\bar{R}^D > \bar{r}^D$  for  $\underline{\lambda} < \Lambda_0$  and  $\bar{R}^D = \bar{r}^D$  for  $\underline{\lambda} \geq \Lambda_0$ . Since  $\bar{R}^D > \bar{r}^D$  when  $\underline{\lambda} < \Lambda_0$ , then  $\gamma^D = 1$  for  $\underline{\lambda} < \Lambda_0$ . It follows that expected default costs  $\pi\gamma^D\Phi p^D = \pi\Phi p^D$  are increasing in  $\underline{\lambda}$  in this region. A higher  $\underline{\lambda}$  for  $\lambda < \Lambda_0$  reduces the amount entrepreneurs produce and increases the foregone output when dividends fall. Each cohort will therefore be left with fewer goods to consume.

We next turn to the case where  $\underline{\lambda} \geq \Lambda_0$ . Here, we know  $\bar{R}^D = \bar{r}^D$ . Substituting this into (57) yields

$$(1 - \Lambda^D) (1 + \bar{r}^D) = \left[ 1 + \frac{D}{(1 - (1 - \underline{\lambda}) \varphi) e} - \frac{\Lambda^D}{1 - \pi} (1 + \bar{r}^D) \right]$$

which, upon rearranging,

$$\Lambda^D = \frac{(1 - \pi)(D - d)}{(1 - (1 - \underline{\lambda}) \varphi) e + (1 - \pi) D + \pi d}$$

From this, we can conclude that  $\Lambda^D \geq \underline{\lambda}$  if

$$\frac{(1 - \pi)(D - d)}{(1 - (1 - \underline{\lambda}) \varphi) e + (1 - \pi) D + \pi d} \geq \underline{\lambda}$$

or, upon rearranging, if

$$(1 - \pi)(D - d) \geq \underline{\lambda} [(1 - (1 - \underline{\lambda}) \varphi) e + (1 - \pi) D + \pi d] \quad (62)$$

The RHS of (62) is a quadratic in  $\underline{\lambda}$  with a positive coefficient on the quadratic term. The inequality is satisfied when  $\underline{\lambda} = 0$  and violated when  $\underline{\lambda} = 1$ . This implies there exists a cutoff  $\Lambda_1 \in (0, 1)$  such that  $\Lambda^D > \underline{\lambda}$  if  $\underline{\lambda} \in [0, \Lambda_1)$  and  $\Lambda^D < \underline{\lambda}$  if  $\underline{\lambda} \in (\Lambda_1, 1)$ . We can deduce that  $\Lambda_1 \geq \Lambda_0$  since by definition  $\Lambda_0$  is the cutoff such that  $\bar{R}^D = \bar{r}^D$  when  $\underline{\lambda} \geq \Lambda_0$ , yet at  $\underline{\lambda} = \Lambda_1$  we have

$$\bar{R}^D = R(\Lambda^D) = R(\underline{\lambda}) = R(\Lambda_1)$$

By construction, we know that  $R(\underline{\lambda})$  when  $\underline{\lambda} = \Lambda_1$  is equal to  $\bar{r}^D$ . This implies  $\Lambda_1 \geq \Lambda_0$ .

When  $\underline{\lambda} > \Lambda_1$  no agent will borrow to buy the asset, so  $\gamma^D = 0$ . Expected default costs are 0, and so the only effect of increasing  $\underline{\lambda}$  is to reduce production. This will leave fewer goods for each cohort to consume.

Finally, we turn to the case where  $\Lambda_0 < \underline{\lambda} < \Lambda_1$ . We do not analyze this case in general. However, when  $\Lambda^D = \underline{\lambda}$ , the interest rate in all active markets would equal  $\bar{R}^D$ , since the only active markets are those with  $\lambda \geq \underline{\lambda} = \Lambda^D$ . Since  $\underline{\lambda} \geq \Lambda_0$ , we know that  $\bar{R}^D = \bar{r}^D$  and so the interest rate in all active markets is  $\bar{r}^D$ . The equilibrium condition that determines  $\gamma^D$  is given by

$$\begin{aligned}
(1 + \bar{r}^D) &= (1 - (1 - \underline{\lambda})\varphi) [1 + \bar{r}^D - \gamma^D \pi \Phi] + 2\varphi \int_0^1 \left[ \min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] [1 + R^D(\max\{w, \underline{\lambda}\})] dw \\
&= (1 - (1 - \underline{\lambda})\varphi) [1 + \bar{r}^D - \gamma^D \pi \Phi] + 2\varphi (1 + \bar{r}^D) \int_0^1 \left[ \min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] dw \\
&= (1 - (1 - \underline{\lambda})\varphi) [1 + \bar{r}^D - \gamma^D \pi \Phi] + 2\varphi (1 + \bar{r}^D) [\underline{\lambda}/2 + (1 - \underline{\lambda}) - 1/2] \\
&= 1 + \bar{r}^D - \gamma^D (1 - (1 - \underline{\lambda})\varphi) \pi \Phi
\end{aligned}$$

Hence, when  $\underline{\lambda} = \Lambda_1$ , we have  $\gamma^D = 0$ . For  $\underline{\lambda} < \Lambda_1$ , however,  $\gamma^D > 0$ , since

$$\int_0^1 [1 + R^D(\max\{w, \underline{\lambda}\})] \left[ \min \left\{ \frac{w}{\underline{\lambda}}, 1 \right\} - w \right] dw$$

will be strictly greater than  $\frac{1}{2} (1 + \bar{r}^D) (1 - \underline{\lambda})$ . Hence, in the limit as  $\underline{\lambda} \uparrow \Lambda_1$ , we have  $d\gamma^D/d\underline{\lambda} < 0$  expected default costs  $\pi\gamma^D\Phi p^D$  must be decreasing in  $\underline{\lambda}$  since this expression goes from a positive value to 0.

Finally, to show that this can generate a Pareto improvement, observe that increasing  $\underline{\lambda}$  while dividends are high will make the initial old at date 0 better off given  $p_0^D$  increases. Cohorts born after dividends have fallen will be unaffected if  $\underline{\lambda}$  is only increased while dividends are high. Cohorts who are born while dividends are high expect to consume the dividends from the asset net of default costs  $E[d_{t+1}] - \Phi\pi\gamma^D p_t^D$  as well as the output produced by entrepreneurs. If  $\Phi$  is sufficiently large and  $\varphi$  is small, we can promise these agents a higher expected consumption. ■

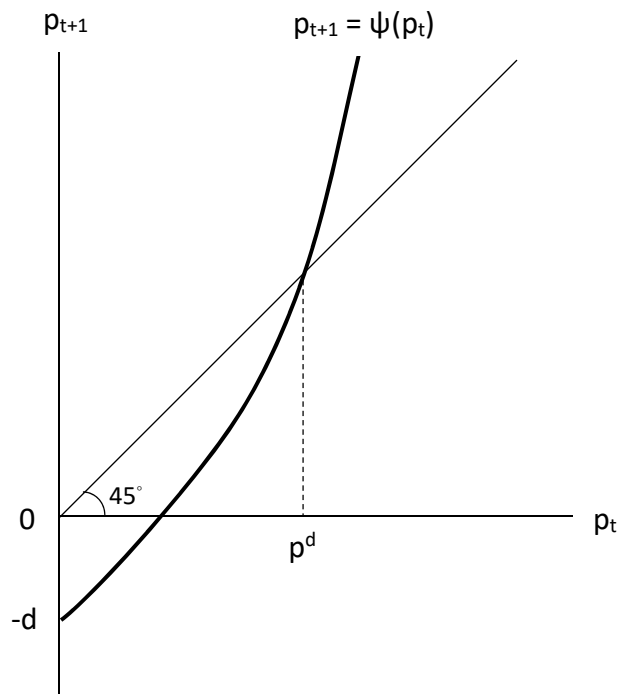


Figure 1: Determination of equilibrium price  $p^d$  with deterministic dividends

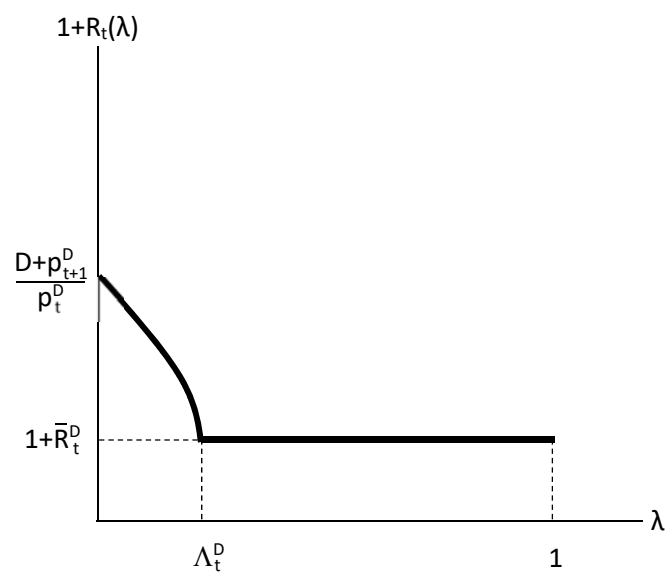


Figure 2: Interest rates as a function of share  $\lambda$  of investment that borrowers pay

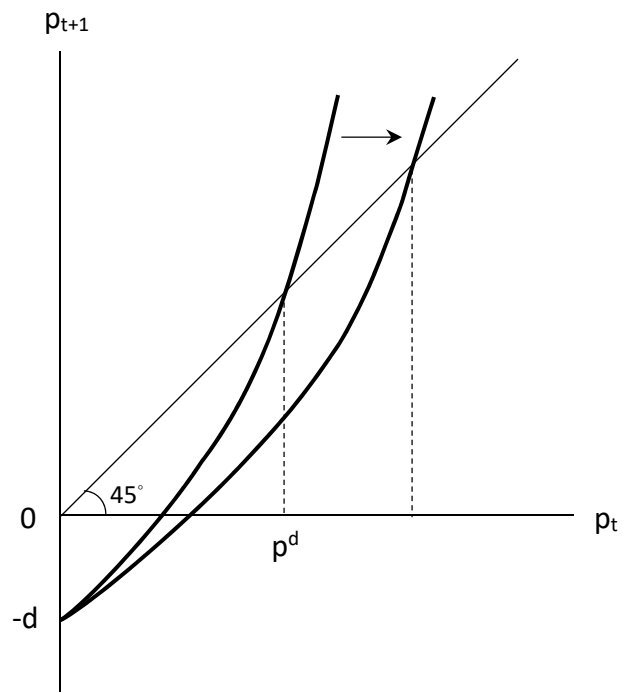


Figure 3: Effect of increasing floor  $\underline{\lambda}$  with deterministic dividends

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