UNDERSTANDING THE EFFECT OF THE ONE-CHILD POLICY ON THE WEALTH DISTRIBUTION-A THEORETICAL APPROACH

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Introduction

The Chinese One-Child Policy, one of the most radical and extreme measure of population planning in the world, has been widely debated since its introduction in 1979. Many have argued the severe costs of such a policy, including disproportionately high number of male births (Wang, 2005), the doting behaviour of Chinese parents on the single child (Cameron et al., 2013) etc. However, little has been discussed on the effect of the policy on the wealth distribution of the country, through intergenerational bequests. In this paper, we shall construct a theoretical model to understand the bequest mechanism in shaping the wealth distribution. The model attempts to simulate the wealth precipitation process passing from one generation to the next. A simplified version of the model, which is solely designed to match couples and produce the second-generation wealth distribution, will be firstly given, then various factors including class segregations and consumption and savings behaviours will be introduced to the model to better mimic reality. To further stretch our findings, we attempt to make comparisons in order to capture the scope of wealth inequality under the policy. The purpose of our research is to render insight into the possible effect of such a policy on the wealth distribution in any countries who are considering implementing the policy.

Model Construct

Matching with Population Proportions

In order to simulate the couple matching process, we first create a male population of 10,000 people, each characterised only by the amount of life-time wealth the individual holds. This process is completed by randomly assigning a pool of numbers generated from a normal distribution with mean *50,000* and standard deviation *3,890,630*, and then filtering out the negative values to produce a fairly right-skewed distribution of data (Figure 1). The reason being, in reality, poor people indeed account for a relatively sizable proportion in most countries (D'Ambrosio & Wolff, 2008).

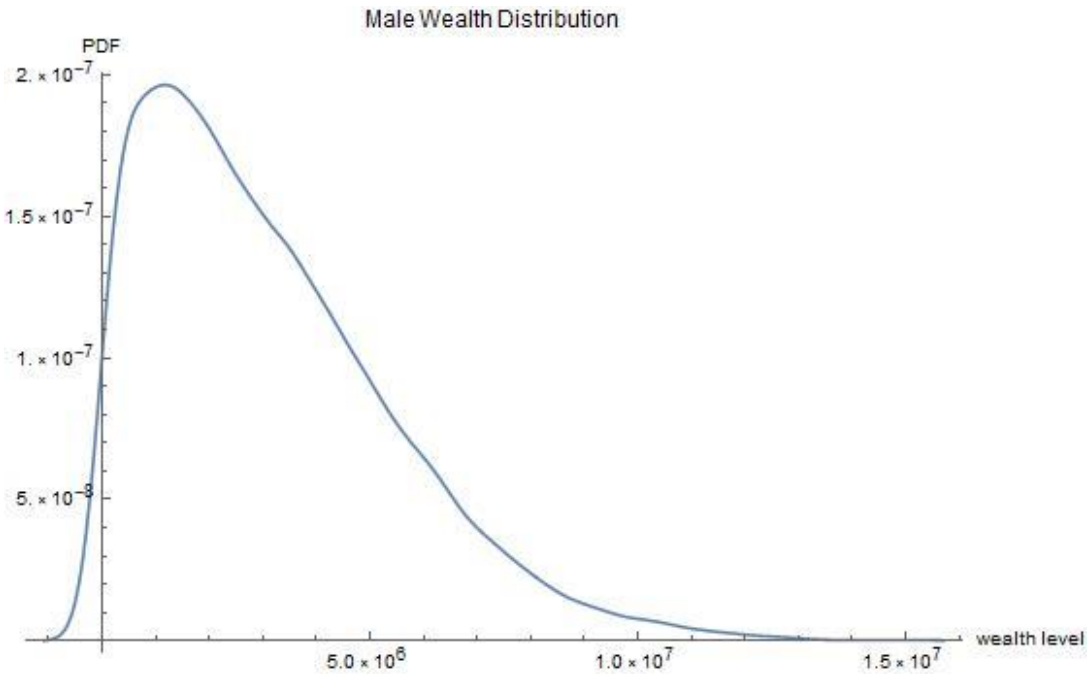


Figure 1

Then, assuming the female population has exactly the same size and wealth distribution, we can derive the wealth distribution for the full population of 20,000 people (Figure 2).

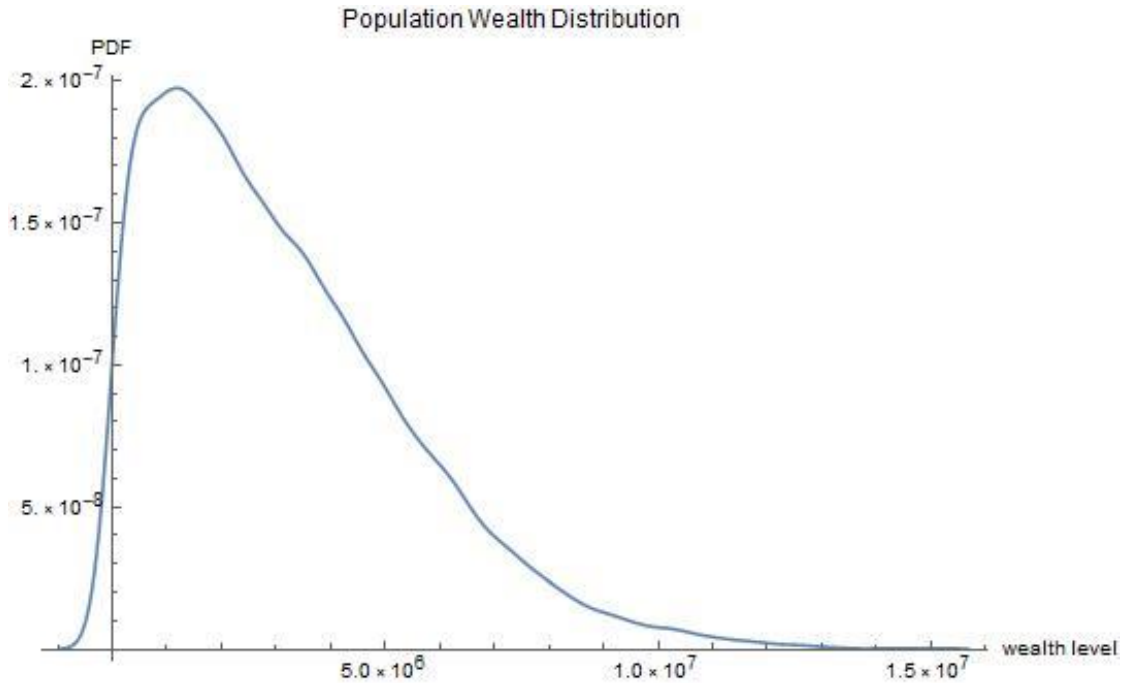


Figure 2

The matching process of couples starts by randomly selecting one male and one female from the population, with the life-time wealth of their only child given as the sum of their wealth. This process is repeated 10,000 times to generate a wealth distribution for the second generation. To reduce coincidental selection bias, we run the whole simulation 1000 times and take expected values for the second generation's wealth distribution (A1.1). For the purpose of comparison, we also generate the wealth distributions of the second generation when every family has two or three kids in exactly the same manner, assuming that the inheritable wealth is divided equally between the kids. The results are shown below in Figure 3.

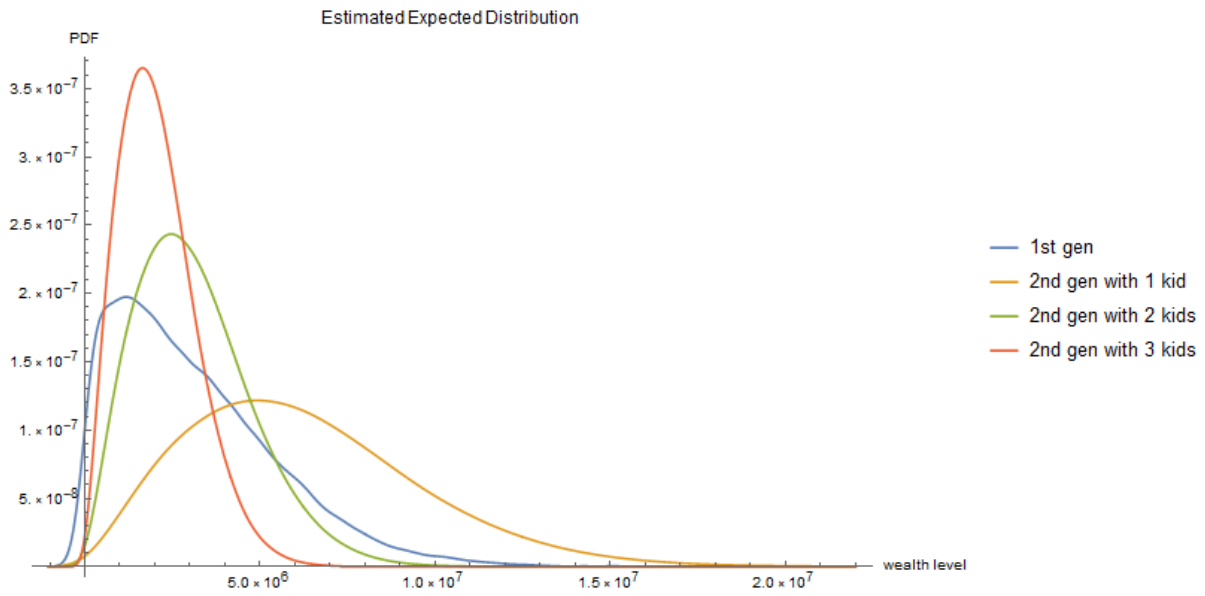


Figure 3

It can be observed that under the One-Child policy, the second-generation wealth distribution is more skewed and spread-out than the first generation. This forms a great contrast to the distributions under 2 and 3 children, where division of the wealth within the family acts as an equalising mechanism, producing a more centred and less spread-out wealth distribution. A more precise inference can be drawn from Figure 4, a table of numerical indicators describing various distributional features. The skewness and the dispersion of the distribution is measured by the value of mean-median and standard deviation correspondingly. As each family has more kids, both the value of mean-median and the standard deviation decreases, indicating a more centred and less dispersed distribution, which means that the gap between the rich and the poor is smaller. Therefore, we can establish a decision rule which states that as the values of mean-median and the standard deviation of a wealth distribution shrink, the level of wealth inequality also shrinks in the economy, whereas the same cannot be said when values are going in different directions. Under this rule, we can conclude that the One-Child Policy indeed increases the wealth inequality. However, when we compute the Gini coefficients for the three second generations, they turn out to be very similar. This is because the proportion of wealth is still held by the same proportion of people, despite changes in the size of population. Therefore, it is certain that the Gini coefficient is not an efficient indicator of wealth inequality in this case.

DISTRIBUTIONS	MEAN	MEDIAN	MEAN-MEDIAN	STANDARD DEVIATION	GINI COEFFICIENT
1ST GEN	3.11648×10^6	2.64611×10^6	0.47037×10^6	2.34201×10^6	0.411593
2ND GEN WITH 1 KID	6.23391×10^6	5.81889×10^6	0.41502×10^6	3.3126×10^6	0.296210
2ND GEN WITH 2 KIDS	3.11727×10^6	2.90928×10^6	0.20799×10^6	1.65669×10^6	0.296253

2ND GEN WITH 3 KIDS	2.07795×10^{6}	1.93963×10^{6}	0.13832×10^{6}	1.10381×10^{6}	0.296111

Figure 4

Class Segregation

Class segregation is introduced to the model through marriage preferences, where we assign different probabilities for an individual to marry into different social classes. To define social classes, lists of males and females are ordered according to their wealth then divided into 3 sub-lists, with each containing a third of the population. The three sub-lists represent different classes – the poor, the middle-class and the wealthy. A male is randomly selected, which in turn faces 60% chance of selecting a female from the same class as him and 20% chance each for selecting one from the rest of the two classes. Once potential spouse's wealth class is selected, it then randomly selects a female from the corresponding list. This process can again be generalised by repetition and the cases where the families have two or three kids can also be computed, assuming that wealth is divided equally among the kids. As shown by Figure 5 below, once preferences are introduced, the distributions are now multi-peaked. This is largely due to the fact that it bears higher probability for one to select a spouse from its own class, thereby creating more second generations within the middle wealth level of that class.

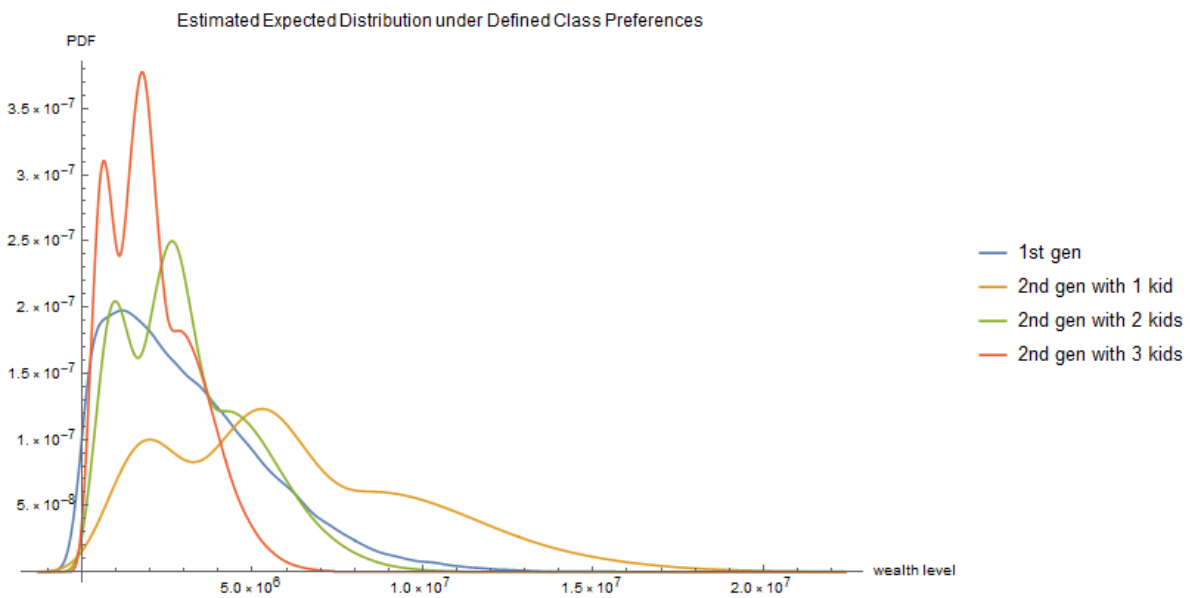


Figure 5

However, the preference factor does not interfere with the comparison between having different number of kids. As families have more kids, the wealth distribution of the second generation becomes more centred and less spread-out. This means that the same conclusion can be drawn – the One-Child Policy indeed increases wealth inequality. However, note that both the values of mean-median and standard deviation are larger comparing to the previous

case, as both the rich and the poor are more likely to marry within their classes, producing more extreme values.

DISTRIBUTIONS	MEAN	MEDIAN	MEAN-MEDIAN	STANDARD DEVIATION	GINI COEFFICIENT
1ST GEN	3.11648×10^6	2.64611×10^6	0.47037×10^6	2.34201×10^6	0.411593
2ND GEN WITH 1 KID	6.23011×10^6	5.60164×10^6	0.62847×10^6	3.78512×10^6	0.339808
2ND GEN WITH 2 KIDS	3.11531×10^6	2.80032×10^6	0.31499×10^6	1.89247×10^6	0.339828
2ND GEN WITH 3 KIDS	2.07687×10^6	1.86701×10^6	0.20986×10^6	1.26206×10^6	0.339782

Figure 6

Matching with Preferences under Utility Optimisation

Consumption and savings behaviour of individuals can also be built on top of the model (A1.2). As our matching process is only applicable to the first generation, we restrain ourselves to a two-generation model, where the wealth of the first generations is partly consumed by themselves and partly passed down to the second generation. This two-generation construct allows us to assume further that the second generations consume all of the inherited wealth for utility, without having to consider future generations. The first generations derive utility both from their own consumption and the consumption of their kids, whilst the second-generation utility function only considers second generation consumption, which is the inherited wealth. As we focus on analysing the wealth precipitation process, income is conveniently neglected. Additionally, we assume the same preference ordering across all generations.[2] Derived from the utility maximisation problem, the final result is given in the form of second-generation wealth (w_{t+1}) as a function of first-generation wealth (w_t), nominal interest rate (r), number of the kids per family (n) and an arbitrary constant (a).

$$w_{t+1} = \frac{w_t^2(1+r)}{n(a+2w_t)}$$

Fix the nominal interest rate at 4% and a at $\sqrt{1st\ generation\ mean}$, one could plot the wealth distributions under utility maximisation (Figure 7).

[2] Details can be found in Appendix A2.1

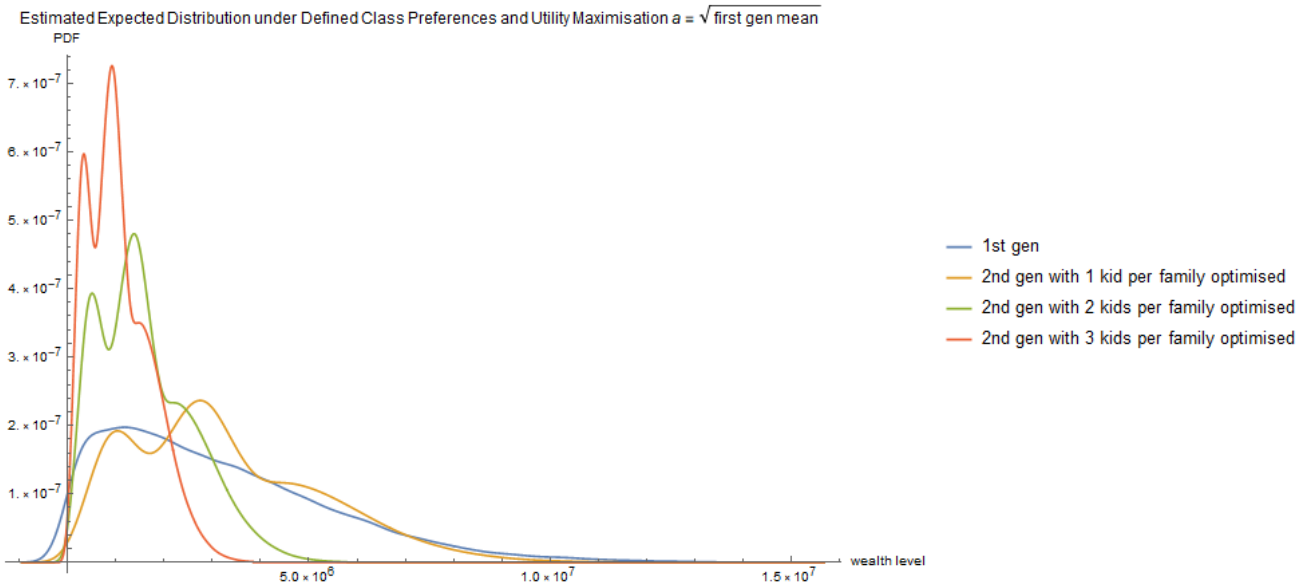


Figure 7

It can be observed that the wealth distribution of the second generation becomes more centred and less spread-out when the number of kids within the family increases. This means that the same conclusion can be drawn – the One-Child Policy indeed increases wealth inequality. Comparing to the graph under full inheritance, one may also observe that all the distributions are squeezed to the right since the inherited wealth with utility optimisation is strictly less than the one without consumption analysis. However, such comparison may not be very meaningful.

Uneven Wealth Divide

Previously, we impose an important assumption that the wealth of a family is divided equally between the kids. Here, we relax that assumption to see how the wealth distribution of the second-generation changes under different proportions of division. Given that all families will have two kids and the utility optimisation problem and marriage preferences remain unchanged, we could firstly compute the wealth distribution of the second generation under the 1:3 division ratio, which means one of the kids takes a quarter of the wealth and the other takes the rest.

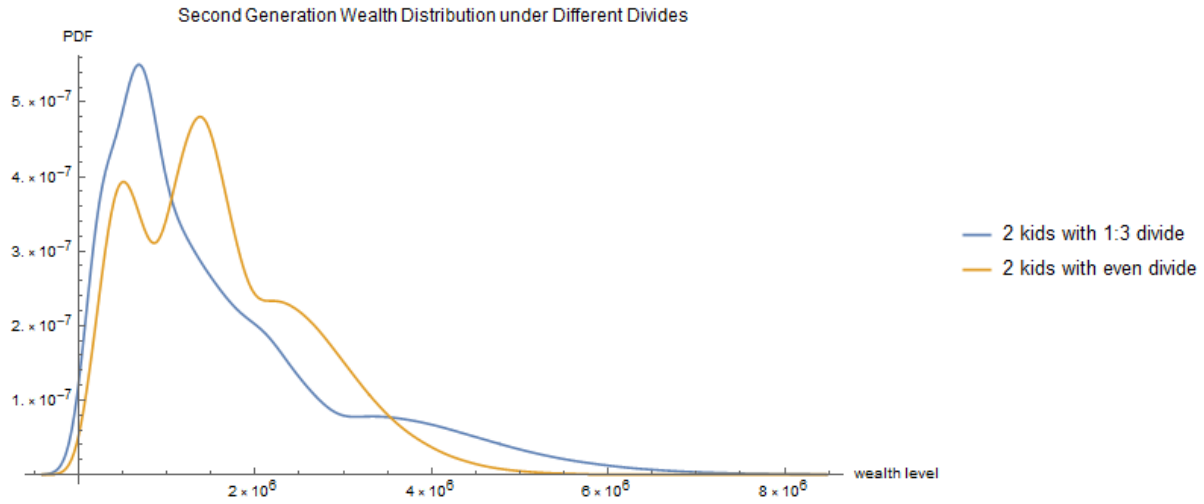


Figure 8

One would notice that the humps in the case of even division is smoothed out by uneven division. This is due to the process that some kids born into the well-off class, who are given a quarter of the family wealth, move down the wealth ladder, whilst some middle-class kids, given a higher proportion of wealth, move up. Without further explanation, one could conclude the uneven divide prompted the wealth distribution to be more unequal than the case with even divide. As in the previous section, we have established that, under even divide, the second-generation wealth distribution under one kid per family is more disperse and less centred than with 2 or 3 kids. To gain further insight on the One-Child policy, we are very much interested in how extreme the ratio of uneven divide should be to match the dispersion of distribution in the 1 kid scenario.

RATIO OF DIVIDE	MEAN	MEDIAN	MEAN-MEDIAN	STANDARD DEVIATION
1 KID	3.24005×10^6	2.91171×10^6	0.32834×10^6	1.96873×10^6
1:1	1.62002×10^6	1.45585×10^6	0.16417×10^6	0.98434×10^6
1:2	1.62002×10^6	1.36309×10^6	0.25693×10^6	1.16970×10^6
1:3	1.62002×10^6	1.17352×10^6	0.44650×10^6	1.36650×10^6
1:4	1.62002×10^6	1.02466×10^6	0.59536×10^6	1.50419×10^6
1:9	1.62002×10^6	0.65129×10^6	0.96873×10^6	1.80798×10^6
1:14	1.62002×10^6	0.48474×10^6	1.13528×10^6	1.91522×10^6
1:19	1.62002×10^6	0.38771×10^6	1.23231×10^6	1.96969×10^6

Figure 9

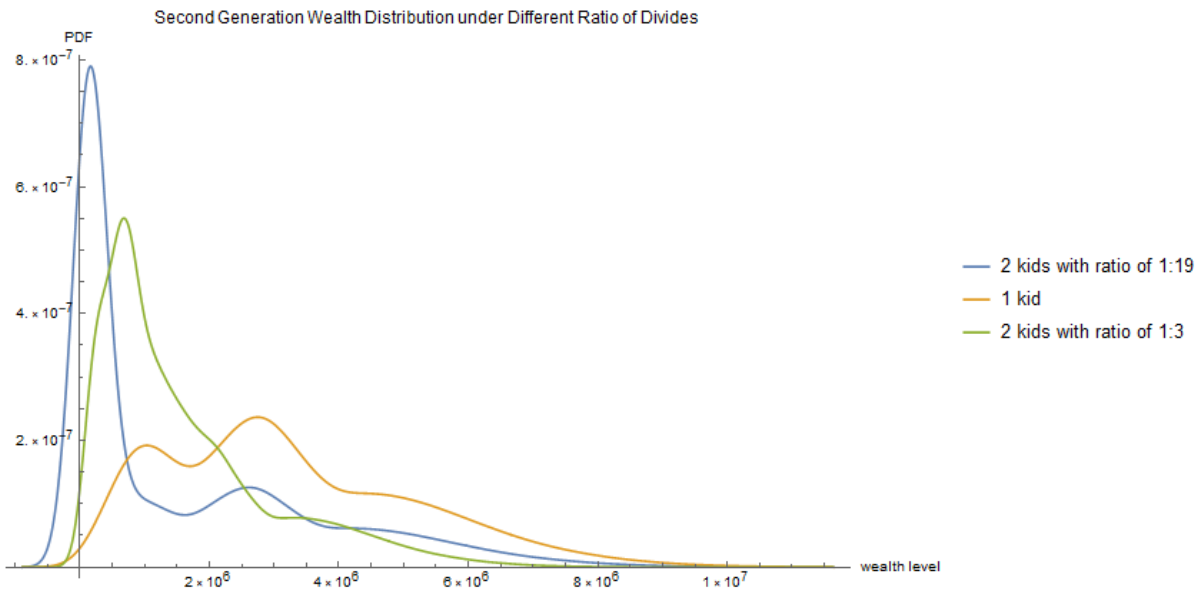


Figure 10

Figure 9 compiled a table of statistical measurements that one could utilise to make comparisons. To match the skewness of the distribution under the One-Child policy, which is measured by the difference between mean and median, the uneven divide between the second generations is estimated to be between 1:2 and 1:3. More interestingly, for roughly the same level of dispersion of distribution, measured by standard deviation, the divide is estimated to be around 1:19 to be comparable with the one-child distribution. By plotting the distributions together on the same graph, one could grasp how extreme the uneven division has to be in order to match the dispersion and skewness of the distribution under the One-Child policy. We also applied the same method to class segregations, but the level of distributional features cannot be achieved, thereby moved to Appendix A3.1.

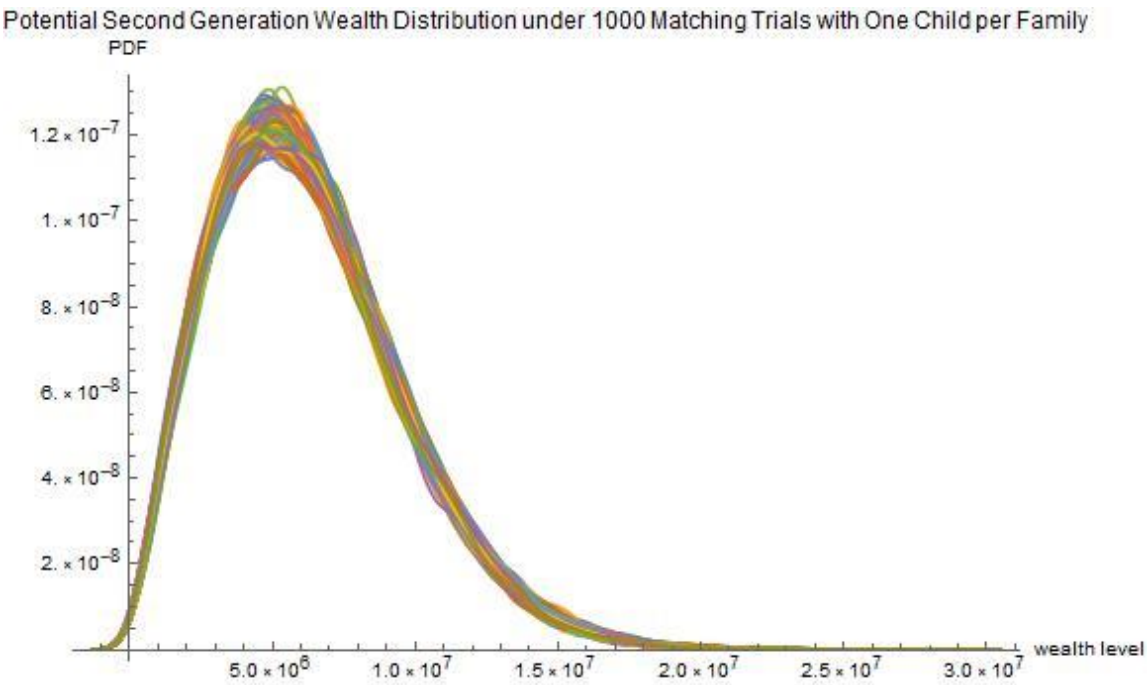
Conclusion

As discussed above, this essay attempts to model the second-generation wealth distribution under the One-Child policy. We have established, based on our decision rule using measures of skewness and spread of the distributions, that One-Child Policy increases the level of wealth inequality in the economy. Furthermore, we also attempted to quantify the change in the wealth inequality by varying the degree of uneven split of wealth between siblings to match the one-child distribution. In the meantime, we acknowledge the limitations of this much simplified model. Future elaborations could be built on multiple generations with more intricate utility optimisation solutions and a subtler approach to capture the preference factor. Additionally, it is absolutely essential to evaluate the model with empirical data whenever possible. The main purpose of this model, like many other macro-frameworks, is to mimic reality and provide complementary economic analyses and insights for policymakers.

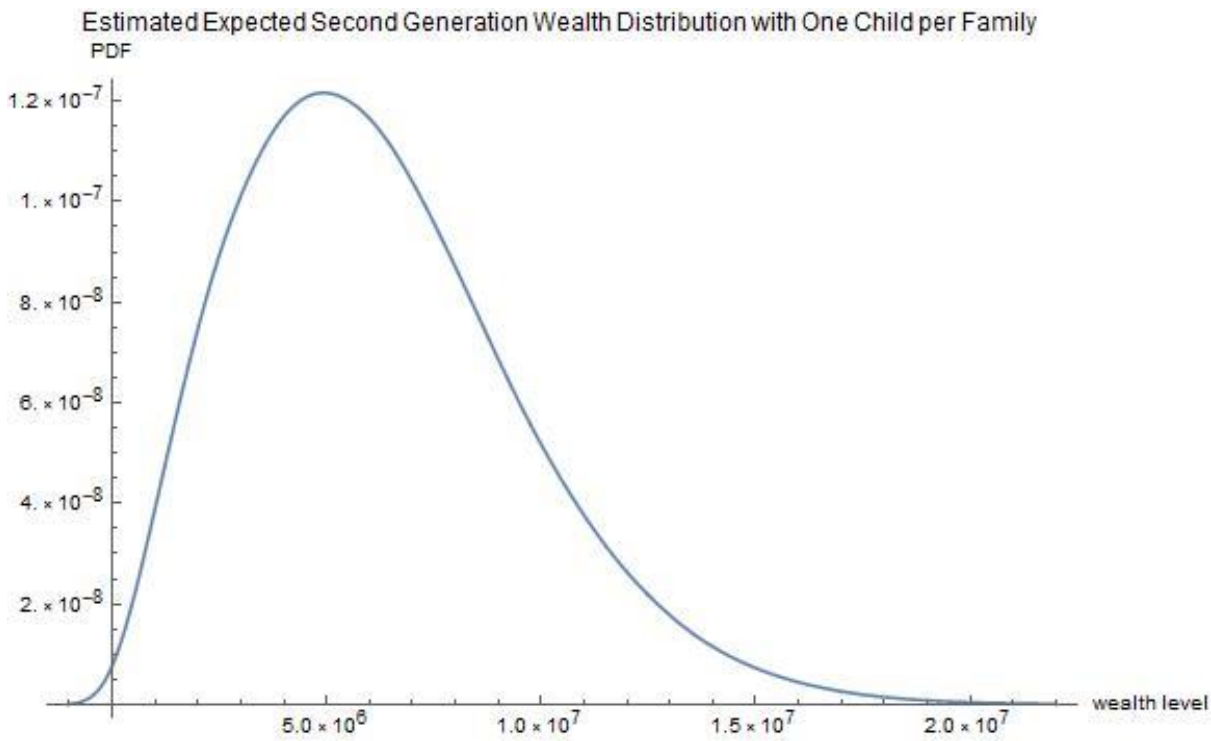
Appendix 1

(A1.1)

As there might be selection repetitions and coincidental selection bias for each generated wealth distribution, we run the whole trial for 1,000 times and plot all the potential distributions on the same graph (Figure A1). Each colour represents a trial of matching process, illustrating an array of distributions. Appealing to the Law of Large Numbers, we intend to estimate the expected second-generation wealth distribution by taking the mean PDF of all the generated distributions at every given wealth level throughout the spectrum, producing an expected distribution for the second-generation (Figure A2). This is similar as taking expectations on all the matching trials with each having equal probability of occurrence, reducing the coincidental matching bias.



(Figure A1)



(Figure A2)

(A1.2)

To further elaborate on the bequest motives, instead of simply using addition to produce second generation inherited wealth level, introducing consumption and savings pattern to the wealth precipitation process would greatly enhance the intricacies of the model construct. To establish the relationship between consumption decision and bequest motives, one could draw inspirations from Andreoni (1989)'s approach to model altruism, where the utility function is defined by the level of income and other altruistic factors. Though it was defined very broadly, one could apply the basic construct of altruism to intergenerational bequests, suggesting that the utility function of one generation could be modelled as dependent on the utility of other generations. Based on this insight, De Nardi (2003) theorised a model where generations overlap so that the utility of the first generation is determined by their own consumption and the expected utility of second-generation consumption, which in turn, is related to the probability of inheriting wealth at a particular time period. Though we might not be able to fully construct a time series model in order to let the generations overlap, it is, nevertheless, constructive to incorporate the so-called "warm glow" effect into the utility of the first generation. Similar to De Nardi's model, Laitner (1979) also incorporated the expected utility of the second generation into the utility function of the first. Rather than building a generation-overlapping construct, Laitner takes the utility function of the first generation as partly dependent on the expected utility of the second-generation consumption, which is solely dependent on second generations' income and expected bequests. Though De Nardi's model is more intricate in terms of time series analysis, following Laitner's generation-separated construct might be more applicable to our modelling purposes.

Nevertheless, both of the models capture an important element of discount factor. It considers the willingness and forward-looking behaviour of the first generation having to forgo their current own consumption in order to pass savings down to the second generation. Both of them consider the discount factor as an exogenous variable, unrelated to the amount of wealth the first generation is assigned with, implying constant saving behaviour across the entire population. This is in great contrast to the empirical analysis done by Carroll (1998), which shows that the rich saves a higher proportion of their wealth and income than less wealthy people. Based on this evidence, we intend to model the rate of saving as an increasing and concave function of wealth level. Differed to Laitner's model, Becker & Tomes (1979)'s attempt to capture the effect of the second generations' characteristics on the first generations' bequest motives by modelling characteristics variables into the utility function of the first remains debatable. However, taking into account of the return on capital on the first generations' savings could in turn introduce detailed elaborations on transfer of wealth and bequest motives.

Based on the above discussion, we would like to incorporate the utility optimising factor into our wealth precipitation process, using approachable elements in previous literature, instead of simply combining the wealth of the two parents in a family through addition.

Appendix 2

(A2.1)

As we establish the two-generation model, we could explore further on the numerical relationship between the constraints. The utility functions for the two generations are given as:

$$U_t = V_t(c_t, U_{t+1}(c_{t+1}))$$

$$U_{t+1} = V_{t+1}(c_{t+1}), \text{ where t=1}$$

We maximise the utility functions subject to the following constraint,

$$c_t = w_t - s_t \qquad (1)$$

$$c_{t+1} = \frac{(1+r)}{n} s_t = w_{t+1} \qquad (2);\ n \in 1,2,3$$

$$(2) \Leftrightarrow \frac{n}{1+r} c_{t+1} = s_t$$

$$(1) \Rightarrow w_t = c_t + \frac{n}{1+r} c_{t+1} \qquad (*)$$

As noted above, we model the discount factor β as follows.

$$U_t = f(c_t) + \beta(w_t) \cdot f(c_{t+1});\ \beta \in [0,1]$$

$$\beta(w_t) = \frac{w_t}{w_t + a};\ \forall a > 0$$

To solve the maximisation problem, we use the increasing and concave feature of natural logarithm in our utility function, while modelling the discount factor as an increasing concave function taking values between and including 0 and 1. Note that a is an arbitrary constant, giving the function its concave feature within the designated boundary. The substituted utility function is then given as,

$$U_t = ln(c_t) + \frac{w_t}{w_t + a} \cdot ln(c_{t+1})$$

As the first generation takes into account of the second, one could simply maximise the first generation utility function subject to the budge constraint (*).

$$L = ln(c_t) + \frac{w_t}{w_t + a} \cdot ln(c_{t+1}) - \mu(c_t + \frac{n}{1+r} c_{t+1} - w_t)$$

The first-order condition is computed as follows,

$$\frac{\partial L}{\partial c_t} = \frac{1}{c_t} - \mu = 0$$

$$\frac{\partial L}{\partial c_{t+1}} = \frac{w_t}{c_{t+1}(w_t + a)} - \mu \frac{n}{1+r} = 0$$

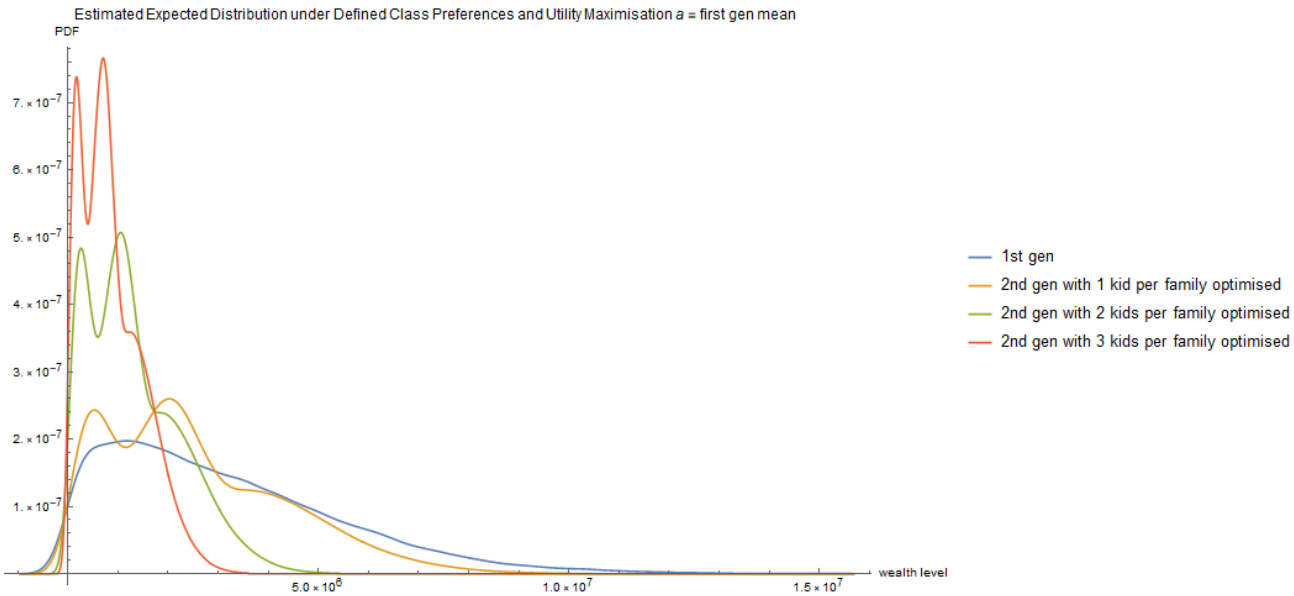
By solving the equations above, we could get the level of inherited wealth of the second generation as a function of wealth level of the first generation.

$$\Rightarrow w_{t+1} = \frac{w_t^2(1+r)}{n(a+2w_t)}$$

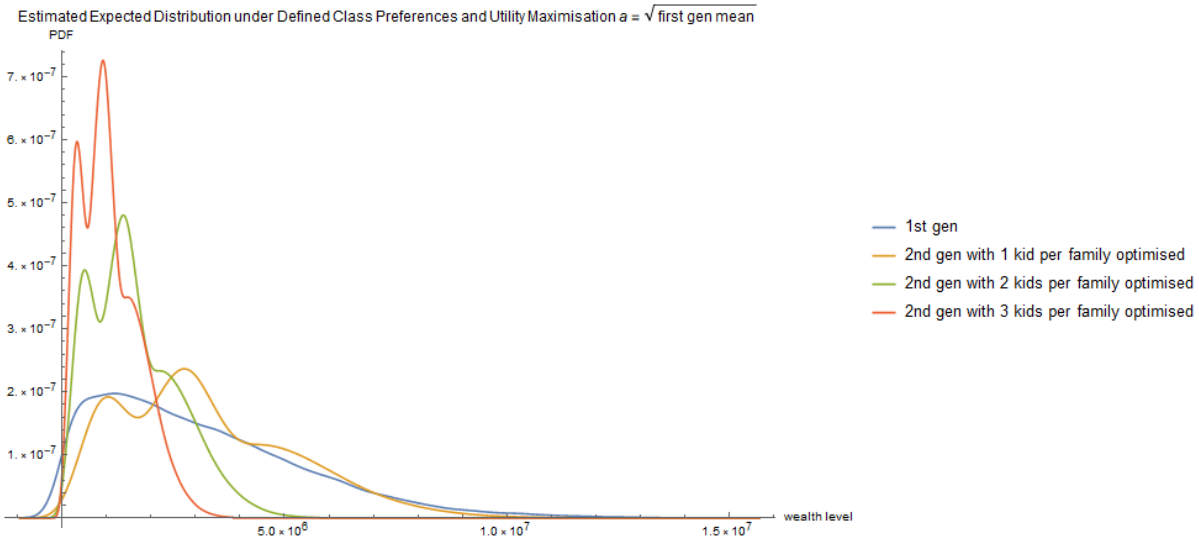
For the nominal interest rate, we intend to apply 4% per generation. The arbitrary constant a is a key determinant of the shape of the discount factor function. In this case, in order make the discount factor more significant in the utility function, we take a to be the square root of the mean first generation wealth.

(A2.2)

Based on the defined class preference factor, we could extend our model using the results above. One could assign the value of w_t for all of the generated wealth level of the first generation, producing a new distribution of w_{t+1}, which is interpreted as the inherited wealth of the second generation. For different number of kids per family (n), we repeat the same process, giving three different wealth distributions (Figure A3).



(Figure A3)



(Figure A4)

One may compare this newly generated graph with the one under full inheritance, but it is important to note that it is not entirely meaningful to do so, due to the fact that consumptions are not relevant in the distribution without utility optimisation. It is, thus, not surprising to see that all the utility optimised distributions under all number of kids are squeezed to the right, as the inherited wealth with utility optimisation is strictly less than the one without consumption analysis.

Meanwhile, we could also manipulate the value of a, which captures the tendency of the first generation to save money for the benefit of the second. As the value of a gets larger, for a given level of assigned wealth w_t, the first generation would save at a lesser proportion, due to the reduction in the discounted utility of the second-generation consumption. This interesting result can be observed directly by comparing Figure A3 and A4, where the latter is given a lower value of a. To make strict comparison, Figure A5 shows how different values of a might affect the overall distribution of the second generation. Intuitively speaking, if one roughly assumes that in a certain neighbourhood of the peaks of distributions lies the general distribution of different designated wealth classes, with lower levels of a, the peak, moves further to the right as the class is wealthier. To be more precise, one could compute the quantile cut-off values and compare the difference between them for all quantiles (Figure A6). As one moves up the quantile thresholds, the difference becomes wider but at a lower margin, the more rightward deviation one could observe, which is reflected in Figure A5. Given that we set the value of a as the first generation mean, a table of measurements is compiled in Figure A7 to help understand the distribution quantitatively. With various distributions with different number of kids, it is important to note that Figure A7 confirms what we have concluded above that the Gini coefficient might be an ineffective measure in our model. In addition, by comparing cases under different numbers of kids under utility optimisation, it coincides with the observation made in the previous sections, that distributions with more kids per family tend to be more centred and less disperse.

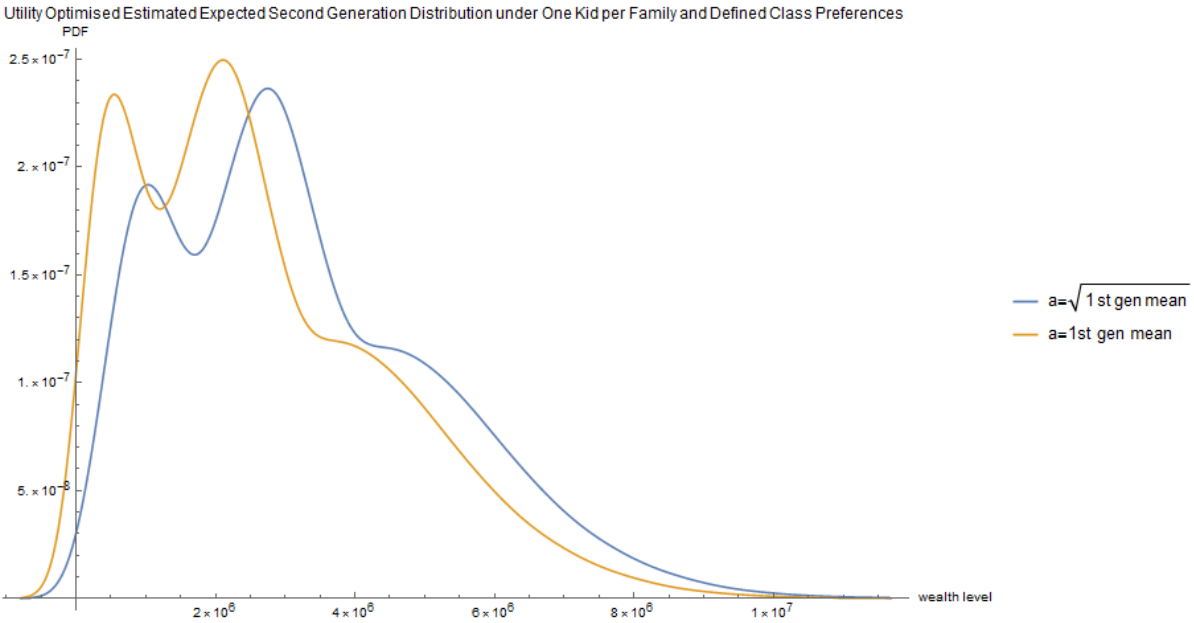


Figure A5

Quantile Values of the Utility Optimised 2nd Generation Distribution under 1 Kid per Family

QUANTILES	1/5	2/5	3/5	4/5	1
$a = \sqrt{1st\ gen\ mean}$	1.37123×10^6	2.51189×10^6	3.37743×10^6	4.96917×10^6	1.22758×10^7
$a = 1st\ gen\ mean$	0.86230×10^6	1.89967×10^6	2.72436×10^6	4.27294×10^6	1.15162×10^7
DIFFERENCE	0.50893×10^6	0.61222×10^6	0.65307×10^6	0.69623×10^6	0.75960×10^6

Figure A6

Matching by Defined Class Preferences and Utility Maximisation, a=1st gen mean

DISTRIBUTIONS	MEAN	MEDIAN	MEAN-MEDIAN	STANDARD DEVIATION	GINI COEFFICIENT
1ST GEN	3.11648×10^6	2.64611×10^6	0.47037×10^6	2.34201×10^6	0.411593
2ND GEN WITH 1 KID	2.64105×10^6	2.27845×10^6	0.36260×10^6	1.86786×10^6	0.392921
2ND GEN WITH 2 KIDS	1.32052×10^6	1.13922×10^6	0.18130×10^6	0.93391×10^6	0.392993
2ND GEN WITH 3 KIDS	0.88035×10^6	0.75948×10^6	0.12087×10^6	0.62260×10^6	0.392921

Figure A7

Appendix 3

(A3.1)

Class Segregation

One could also explore from the perspective of class segregation, where the probability of an individual marrying a spouse from its wealth class is different from the chance of marrying into other classes. In the model construction phase, we set the default probabilities as {60%, 20%, 20%}. Here, one could manipulate the level of class segregation to draw inferences from our distribution. As an example, we computed the class segregation level to be {80%, 10%, 10%}, {40%, 30%, 30%} and completely equal, and illustrate the comparison between the two wealth distributions in Figure A8. As expected, higher level of segregation can be reflected not only through the rise in the number of people in the poor class and the rich class, but also from the deepened drops between the hikes.

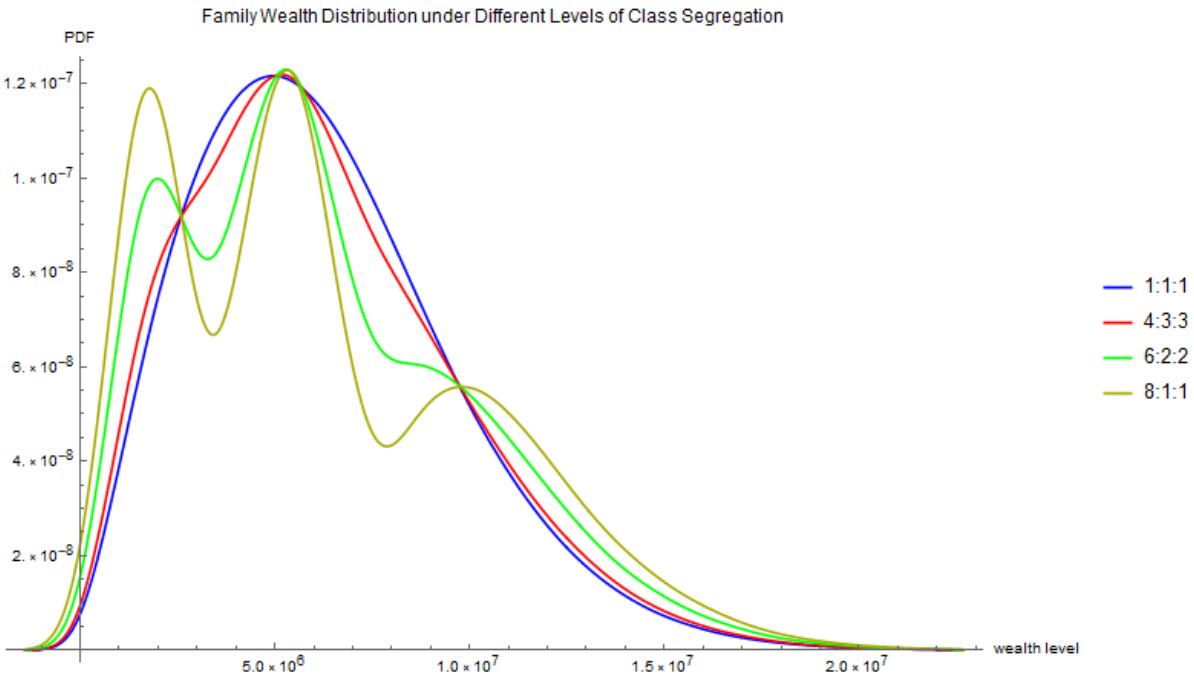


Figure A8

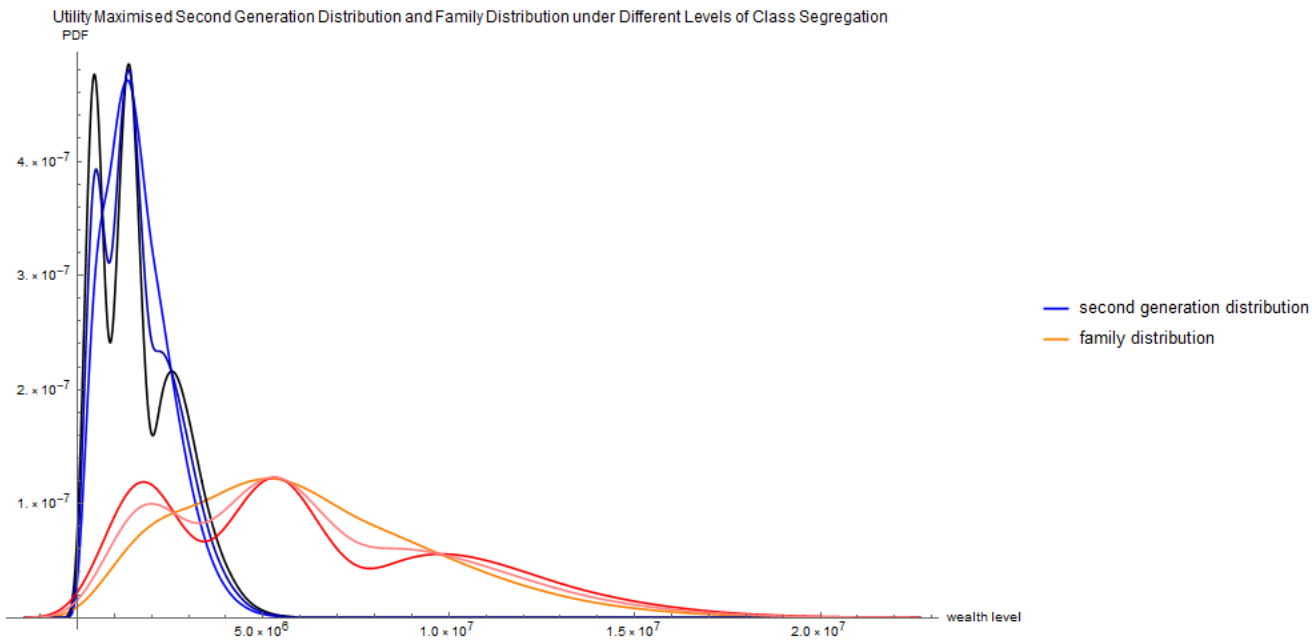


Figure A9

Given that the value of a in the discount factor is fixed as $\sqrt{1st\ generation\ mean}$, one would expect that the utility optimised second generation wealth distribution with 2 kids per family would appear with much less numerical values, due to consumption of the first generation and even split between siblings, as shown in Figure A9.

As comparison, we take the case without class segregation and undergo formulating the second generation wealth distribution under the One-Child policy, following from the same utility maximisation process. Meanwhile, assuming that the wealth is equally divided between kids, given that a family is allowed to have two children, we intend to find, using the same mind-set above, how segregated classes have to be under the case of two kids per family in order to match the level of centeredness and dispersion of distribution under the One-Child policy without class segregation.

LEVELS OF CLASS SEGREGATION	MEAN	MEDIAN	MEAN-MEDIAN	STANDARD DEVIATION
1 KID, NO SEG	3.24054×10^6	3.02480×10^6	0.32834×10^6	1.72150×10^6
4:3:3	1.61980×10^6	1.49764×10^6	0.16417×10^6	0.89264×10^6
6:2:2	1.62005×10^6	1.45626×10^6	0.16379×10^6	0.98430×10^6
8:1:1	1.61964×10^6	1.42068×10^6	0.19896×10^6	1.06793×10^6
18:1:1	1.61978×10^6	1.40518×10^6	0.21460×10^6	1.10758×10^6
1:0:0	1.61995×10^6	1.39005×10^6	0.22990×10^6	1.14617×10^6

Figure A10

Figure A10 lists the same statistical instruments as in the previous section, only with different levels of segregation instead of uneven divide of wealth between kids. One could see that the skewness and dispersion of distribution under the One-Child policy without class segregation is so extreme that under complete class segregate, which is computed as {100%, 0%, 0%} in the last row of Figure A10, the case with even wealth divide between two kids fails to match. To visualise the distributional features, Figure A11 illustrates the scale of variations between different distributions. Based on this idea, one would expect that the distribution under the One-Child policy with defined class preferences tends to be more disperse, further shrinking the feasibility of the case with 2 kids being able to match the distributional features. Given the level of dispersion and centeredness, one could infer that the distribution under the One-Child policy is more unequal than any level of class segregation with two kids evenly split family wealth.

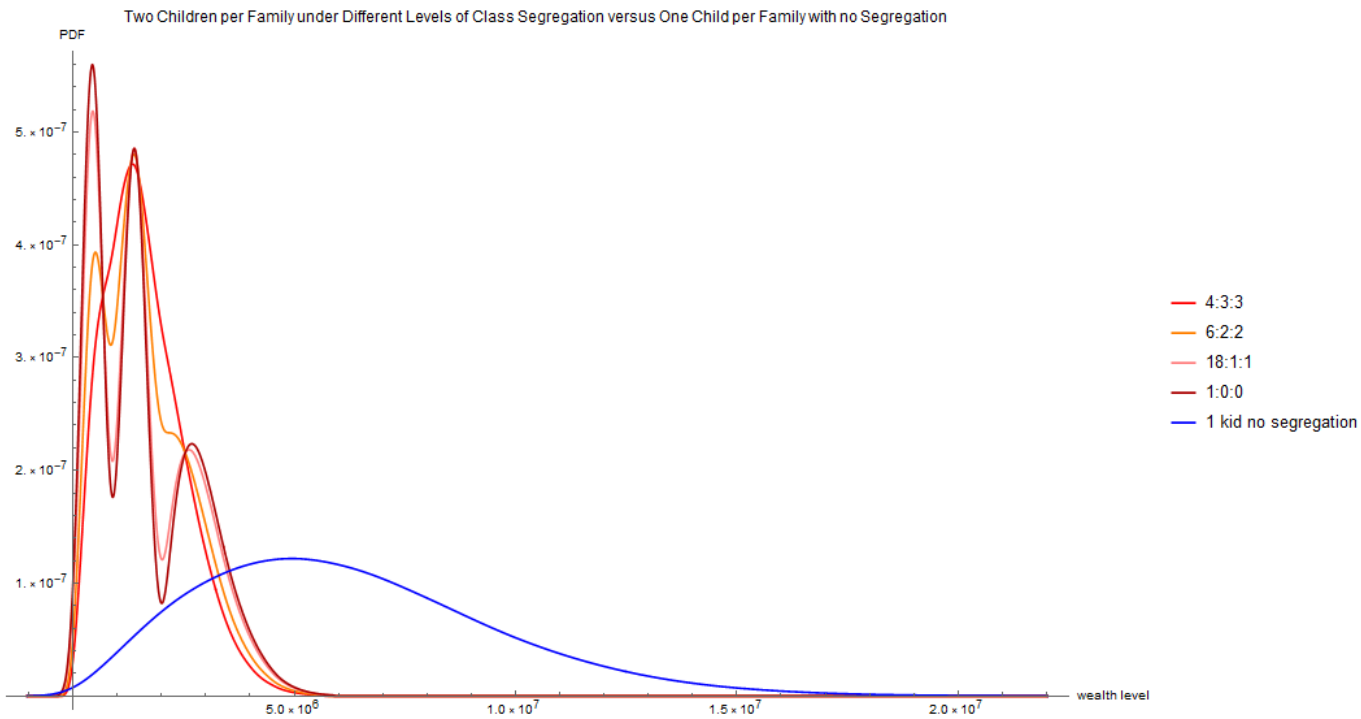


Figure A11

References

Andreoni, J. (1989). “Giving with Impure Altruism: Application to Charity and Ricardian Equivalence.” *Journal of Political Economy*, 97(6): pp. 1447-1458

C, D’Ambrosio. & E, Wolff. (2008). “The Distribution of Wealth in the United States from 1983 to 2004: Inequality and Polarization.” *Journal of Economic Literature.* (Paper prepared for the workshop “Income Polarization: Measurement, Determinants and Implications”, May 26-28 2008, Israel)

Cameron *et al*. (2013). “Little Emperors: Behavioural Impacts of China’s One-Child Policy.” *Science*, 339(6122): pp. 953-957

D. Carroll, C. (1998). “Why Do the Rich Save so much?” *NBER Working Paper, National Bureau of Economic Research*, No.6549

D. Nardi, M. (2004). “Wealth Inequality and Intergenerational Links.” *Review of Economic Studies*, 71(1): pp. 743-768

P. Laitner, J. (1979). “Bequests, Golden-age Capital Accumulation and Government Debt.” *Economica*, New Series, 46(184): pp. 403-414

S. Becker, G. & Tomes, N. (1979). “An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility.” *Journal of Political Economy*, 87(6): pp. 1153-1189

Wang, F. (2005). “Can China afford to continue its one-child policy?” *Honolulu: East-West Center AsiaPacific.* Issue No. 77