Brexit provides us with an interesting example: by considering the options on GBP/USD and GBP/EUR and comparing these to the odds markets, it is possible to measure the fear that the referendum generated.

### INTRODUCTION

Risk-neutral density functions can be estimated from volatility smiles. According to Breeden and Litzenberger (1978), these risk-neutral density functions are equal to:

\[ g(K) = e^{\frac{1}{2} \sigma^2 \Delta t} \]

where \( g(K) \) is the probability of the strike \( K \) being in the money.

### FROM IMPLIED VOLATILITY TO RISK-NEUTRAL PROBABILITY DISTRIBUTIONS

The implied volatility is low for at-the-money options and greater for out-of-the-money or in-the-money options. From the volatility smile given by options with a given maturity, it is possible to deduce the risk-neutral probability distribution for the exchange rates:

\[ \frac{\partial c}{\partial R} = \frac{1}{2} \sigma^2 \]

This implies that the implied volatility is low for at-the-money options and greater for out-of-the-money or in-the-money options. Options are a particular class of financial derivatives contracts and they are distinguished between "call options", which give the holder the right to buy the underlying asset by a certain date for a certain price, and "put options", which give the holder the right to sell the underlying asset by a certain date for a certain price. Most of the options traded are either American- or European-style: the first type can be exercised at any time up to maturity, while the second only on the maturity date.

An option can be "at-the-money" if the stock price \( S \) equals the strike price \( K \), "in-the-money" if \( S > K \), "out-of-the-money" if \( S < K \). Hedges use options as insurance against the risk that they face from future movements of the asset price. The prices of European-style options are determined by the Black-Scholes-Merton (BSM) formula (1973):

\[ \text{Call} = S N(d_1) - K e^{-rT} N(d_2) \]
\[ \text{Put} = K e^{-rT} N(-d_2) - S N(-d_1) \]

where \( N() \) is the cumulative probability distribution function for \( d_1 = \frac{\ln(S/K) + (r - \sigma^2/2) T}{\sigma \sqrt{T}} \) and \( d_2 = d_1 - \sigma \sqrt{T} \).

BSM formula states that the price of the European option depends on five factors: (i) current underlying price, \( S_0 \); (ii) strike price, \( K \); (iii) time until expiration, \( T \); (iv) risk-free interest rate, \( r \); (v) volatility, \( \sigma \).

The Black-Scholes-Merton formula assumes that there are no transaction costs, no arbitrage opportunities, continuous hedging, unlimited liquidity, normally distributed returns and a geometric Brownian motion for the underlying assets, which results in \( S_0 \) and \( K \) being normally distributed with mean \( \mu \) and standard deviation \( \sigma \).

### VOLATILITY SMILE CURVE

One of the parameters of the Black-Scholes-Merton pricing equation that cannot be observed directly is the volatility of the stock price, \( \sigma \), also known as "implied volatility". The formula assumes that implied volatility is constant; however, \( \sigma \) is observed directly as the volatility of the stock price.

From a hedger perspective, options are a form of insurance; therefore, in light of these polls, market practitioners should not have worried too much about a Brexit outcome because it was deemed to be unlikely. However, traders perceived Brexit as a "tail event" and, as such, a very risk event, this higher risk pushed the insurance premium up, leading to a higher price for the options. The volatility smile curves and the implied distribution of the options on GBP/USD and GBP/EUR two days before the referendum with maturity on June 26, 2016 indeed show greater uncertainty about the outcome of the referendum as well as more fear of market participants who insuring themselves against the tail risk.

### FOREIGN CURRENCY OPTIONS

Black-Scholes-Merton formula can be extended to price foreign exchange options. Foreign currency options will display a pronounced volatility smile: volatility is not constant as predicted by Black-Scholes-Merton’s equation and therefore jumps are likely to occur. By looking at the options on GBP/EUR and GBP/USD, it is possible to infer this relationship.