ALTERNATING-OFFER BARGAINING WITH INCOMPLETE INFORMATION AND MECHANISMS

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ABSTRACT. We study an alternating offer bargaining over heterogeneous pie with N parts, with one-sided uncertainty about the preferences over different parts of the pie. Players can offer general mechanisms to determine the allocation. When $N = 2$ and offers are frequent, there is a unique limit of Perfect Bayesian Equilibrium outcomes: the uninformed player proposes the optimal screening menu subject to the constraint that each of the types of the informed player gets at least her complete information payoff. When $N > 2$, there is an equilibrium in which the informed player may receive strictly less than her complete information benchmark.

1. Introduction

Incomplete information about preferences is an important feature of many bargaining situations. Its presence in the game-theoretic models of bargaining typically leads to two problems. Due to a screening problem, a player's offer may be acceptable for some, but not for all types of the opponent. This may lead to a delay, and a new offer for the remaining types, which may change the incentives to accept the original one. Due to a signaling problem, an agent may accept or make an unfavorable offer because of the

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threat of being punished with beliefs that lead to a very bad continuation payoff. The signaling problem typically leads to the multiplicity of equilibria that can sometimes be resolved by looking at equilibrium refinements.

In this paper, we show that both screening and signaling problem have a satisfactory solution when the incomplete information is about relative value of the components of the bargaining object and players have enough flexibility in choosing offers. Two players, Alice and Bob, want to divide a heterogeneous pie with $N \geq 2$ parts. Bob's preferences over the relative value of different parts of the pie are known. Bob has beliefs about Alice's preferences. In alternating periods, each player offers a procedure, or a mechanism to determine an allocation, which the other player accepts or rejects. If the offer is rejected, the game moves to the next period, with the other player making an offer. When the offer is accepted, the mechanism is implemented. A mechanism is defined as an arbitrary finite (extensive-form) game with perfectly observable actions, where players choices determine final allocations. Examples include single allocations, menus, or menus of menus, where one of the player chooses a menu of acceptable allocations for the other player. A well-known example of the latter is "I divide and you choose".

The inclusion of general mechanisms in a strategic bargaining is an important contribution of this paper. (Mechanisms have been considered as the natural object of bargaining under uncertainty in the axiomatic theory since Harsanyi and Selten (1972) and Myerson (1984).) Sophisticated offers like menus or proposals to move to arbitration are common tools in the real world bargaining.[1] From the theoretical perspective,

[1] See Jackson et al. (2018) for real-world and experimental examples. The author of this study had an opportunity to observe the bargaining over a pension plan reform that took place in 2016-18 between three Ontario universities and the representatives of faculty and staff. Among others, the parties negotiated the size of the spousal benefit, early retirement options, inflation indexation, etc. It was understood that the universities care only about the total actuarial cost, but the preferences of the labor side were uncertain, mostly due to the heterogeneity of the labor side (for instance, the staff, but

the mechanisms expand the set of outcomes beyond what is achievable under a standard bargaining protocol. Due to the Myerson's Inscrutability Principle, mechanisms allow to condition the allocation on private information without making the agents worry about strategic consequences of information revelation.

An important assumption of our model is that once the mechanism is offered and accepted, the players are committed to its implementation. However, players cannot commit to any offer in the subsequent periods[2]. Thus, our assumption resembles the recent literature on the dynamic mechanism design without commitment (Skreta (2006), Doval and Skreta (2018), and others). The main difference is that we allow both the uninformed and the informed player to offer mechanisms. As far as we know, ours is the first paper that studies the informed principal problem in the dynamic setting with limited commitment.

Our main result shows that, when $N = 2$, and players become patient, there is a unique limit of outcomes in perfect Bayesian equilibria (PBE): Bob proposes the optimal (for him) screening menu m^* subject to the constraint that each of Alice's types receives at least her complete information payoff. The final allocation is *ex ante*, but not *ex post* efficient. The solution has natural comparative statics with respect to information: Bob is better off when his information improves. When Bob's beliefs converge to certainty, the outcome converges to the complete information Nash solution.

The proof has two parts: first, we show that any Bob's offer that gives Alice types their Nash payoffs is going to be accepted, and second, we show that each Alice type

not the faculty, valued the early retirement more than the spousal benefit). In the end, the universities proposed a menu of options, and the labor side chose an option from this menu.

[2]More precisely, they cannot commit to any future offer unless that commitment is accepted by the opponent. In principle, the space of mechanisms is rich enough so that a player can offer to alter the bargaining protocol in an arbitrary way; if such offer is accepted, it is implemented. For example, the players may agree to settle the division of one part of the pie first and divide the other part later.

can ensure the Nash payoff. Two types of mechanisms play role in the proof. Bob's ability to offer menus (of allocations, for Alice to choose) allows him to screen among Alice's types without them worrying about revealing information. On the other hand, by offering menus (for Bob to choose) of menus (of allocations, for Alice to choose), Alice can protect herself from "punishments with beliefs".

The main result can be contrasted with the Coase conjecture, which predicts that the informed player has all the advantage of the interaction, the equilibrium is efficient and it corresponds to the worst outcome for the uniformed player across all possible types of the informed player. A companion paper Peski (2019) studies a war-of-attrition bargaining in a similar environment, but where players have additional ability to commit to their offers due to the reputational types. (There are other differences as well; including the fact that players can only propose menus rather than arbitrary mechanisms.) Interestingly, more commitment leads to a Coasian-type result: in the unique (limit) equilibrium, Bob proposes a menu $m^{1/2}$ of all allocations that give him his worst possible Nash payoff (equal to $\frac{1}{2}$). Bob is typically strictly worse-off than under m^*; Alice types are better-off, some of them strictly.

If $N > 2$, the main result does not hold. We construct an equilibrium, where some Alice types receive a payoff strictly lower than her Nash payoff. This is of interest in itself, as Maskin and Tirole (1990) claim that, in the private value case, the informed principal must benefit from incomplete information due to the collapse of the agent incentive and individual rationality constraints[3]. They mention that this observation may fail with interdependent values. Although we work with private values, in the dynamic setting, the continuation payoffs typically depend on the belief of the uninformed agent and there is an endogenous interdependence.

[3] I am grateful to V Bhaskar for this observation.

The uniqueness of equilibrium outcome without any equilibrium refinement is a surprising result in the informed principal literature, where, typically, there are many equilibria supported by the belief punishment threats. The type of the uncertainty considered here is very important. Because there no best or worst types, but simply different, the threat of being punished with beliefs can be tested by mechanism that are acceptable for both Bob with his punishment beliefs and the true Alice's type. The availability of sophisticated offers plays an important role. If players are only able to offer simple allocations, there might multiple equilibria, including an Anti-Coasian one, where each of Alice's types receives her worst possible payoff across all possible (probability 1) Bob's beliefs about Alice; Bob receives his best possible payoff. The construction of such an equilibrium involves punishing Alice's deviations with beliefs that she is the worst (for her, but best for Bob) type.

Almost all related papers work with two dimensions and two types. Sen 2000 studies an alternating offer game with two types, where players can offer menus, but not general mechanisms. The author shows that there is a unique PBE subject to relatively weak refinement (perfect sequential equilibrium due to Grossman and Perry (1986)). The equilibrium depends on whether the high type prefers her own complete information Nash payoff, or the one of the low type (the incentives of the low type go in the right direction). Inderst 2003 studies a similar setting, but assumes that the two types have incentives to separate to complete information payoffs. In the context of the Coasian bargaining, Wang 1998 studies a similar bargaining environment with two types for Alice, and with Bob making all offers. He shows that, in the unique equilibrium, Bob separates the two-type of Alice with an optimal screening contract. In particular, the Coase conjecture fails as Bob keeps all power subject to the incentive compatibility constraints. More recently, Strulovici 2017 assumes that, instead of ending the game, any accepted offer becomes a status quo for future bargaining. In this setting, the Coase conjecture holds and the uninformed player is unable to offer an inefficient payoff to type u_1' in order to screen out the more extreme type u_1''.

2. Model

2.1. **Bargaining.** Two players, Alice and Bob, bargain over a heterogeneous pie with $N \geq 2$ parts. An allocation is defined as a tuple $x = (x_{i,n}) \in X = \left\{x \in [0,1]^{2N} : \sum_{i,n} x_{i,n} \leq 1\right\}$, where $x_{i,n}$ is player i's share of the nth part of the pie. We allow for allocations with waste, but it does not affect our results. The main result is about case $N = 2$, in which case, we refer to the two parts of the pie as chocolate and strawberry, $n = c, s$. (We comment on the case $N > 2$ in Section 4.2.)

Each player i has a linear preference over allocations $u_i(x) = u_i \cdot x_i$, where $u_i \in \mathcal{U} = \left\{u \in [0,1]^N : \sum u_n = 1\right\}$. We normalize the preferences so that the coefficients add up to 1; because the multiplication of payoffs by a constant does not change the strategic behavior, the normalization is w.l.o.g.. The extreme preferences are denoted as $\omega^n \in \mathcal{U}$, where $\omega^n_m = 1_{n=m}$ for each n, m. Bob's preferences, denoted as v, are commonly known. Alice's preferences, denoted as u, are privately known by her; Bob's beliefs are denoted by $\mu \in \Delta\mathcal{U}$.

In alternating periods, one player offers to choose an allocation with a mechanism m; the other player either accepts or reject. The first offer is made by player j. If the offer is accepted, the mechanism m is implemented, the allocation is determined in a continuation equilibrium, and the game ends with players receiving payoffs from the allocation. If the offer is rejected, the game moves to the next period, with the other player making an offer. A *mechanism* is formally defined as an extensive form, finite-horizon game with observable actions $m = \left((S_i^t)_{i=A,B}^{t \leq T}, \chi\right)$, where $T < \infty$ is a finite length, S_i^t is a finite set of actions for player i in period t, , and $\chi : \prod_i^T S_i^t \to X$ is an allocation function.[4] Let

$$\mathcal{M}_F = \bigcup_{T<\infty, \left(S_i^t\right)_{i=A,b}^{t\leq T}:\text{finite } S_i^t \subseteq \mathbb{N}} X^{\prod_i^T S_i^t}$$

denote the space of all mechanisms. We do not allow for transfers, but otherwise we allow for arbitrary (finite) mechanisms, including:

- *simple offers:* $T = |S_A^1| = |S_B^1| = 1$. Each simple offer can be identified with a single allocation $\chi \in X$,
- *(Alice's)menus*: $T = 1 = |S_B^1|$. Each menu m is characterized by a finite set of allocations $Y_m = \{\chi(s_A) : s_A \in S_A^1\} \in CX$, where CX is the space of closed subsets of X with Hausdorff distance.
- *(Bob's) menus of (Alice's) menus*: $T = 2, |S_A^1| = |S_B^2| = 1$. Menu m of menus are characterized by a finite set of finite sets of allocations: $W_m = \{Y(s_B) : s_B \in S_B^1\} \in C^2X$, where $Y(s_B) = \{\chi(s_A, s_B) : s_A \in S_A^2\}$. Here, first Bob chooses a menu $Y \in W$, and then Alice chooses an allocation in Y. An example is "Bob divides, Alice chooses" mechanism.

All the results go through as long as the space of available mechanisms contains menus and menus of menus.

Because we do not have the existence of equilibrium for infinite action games, we choose to work with finite approximations.[5] Because $\mathcal{M}_F$ is separable (as a countable

[4]Nothing would change if we allow mechanisms with possibly infinite length $T = \infty$ as long as the allocation function is continuous in the Tychonoff topology on $\prod_i^T S_i^t$ (for example, a player could offer to continue bargaining under an altered protocol). In principle, because each extensive-form can be presented as a normal form, nothing would change if we restricted the mechanisms to $T = 1$. However, the latter restriction would complicate the statement of the finite approximations below; for this reason, we do not use it.

[5]There are two main reasons for the lack of existence result in our model. First, the space of mechanisms is not compact, hence, the existence of a best response is not guaranteed. More importantly,

union of closed subsets of Euclidean spaces), it can be approximated with an increasing sequence of finite subsets $\mathcal{M}_k \subseteq \mathcal{M}_{k+1} \subseteq ...\mathcal{M}_F$ such that $\mathrm{cl}\bigcup_k \mathcal{M}_k = \mathcal{M}_F$. We write $\mathcal{M}_k \to \mathcal{M}_F$.

The players discount with a common factor $\delta < 1$. We are interested in the case of frequent offers, or $\delta \to 1$. For some results (that we explicitly state), we assume that the players observe the outcome of a public randomization device before taking any action. Let $\Gamma^j(\delta, \mathcal{M}_k, \mu)$ denote the bargaining game in which player j makes the first offer, Bob's initial beliefs are given by μ, and the players choose their offers from set $\mathcal{M}_k$.

2.2. **Strategies and equilibrium.** Let $T_j = \{t \in \mathbb{N} : t \text{ odd}\}$ be the periods in which the initial player j makes the offer. Let $T_{-j} = \{t \in \mathbb{N} : t \text{ even}\}$. For $t \geq 1$, let $H_t = \mathcal{M}_k^{t-1}$ be the set of histories in the beginning of period t. A (complete information) pure strategy of player i is a tuple $\sigma = \left(\sigma^M, \sigma^D, \sigma^m\right)$, where $\sigma^M : \bigcup_{t\in T_i} H_t \to \Delta\mathcal{M}_k$ describes the choice of mechanism when player i makes an offer, $\sigma^D : \bigcup_{t\in T_{-i}} H_t \times \mathcal{M}_k \to \Delta\{A, R\}$ is the decision about player $-i$'s offer , and $\sigma^m : \bigcup_t H_t \to \Delta\Sigma_i^m$ describes the behavior in the proposed and accepted mechanism m. Here, Σ^m is the set of pure strategies in the mechanism m. Let Σ_i be the set of complete information strategies of player i.

An *assessment* is defined as a tuple of $(\sigma_A, \sigma_B, \mu)$, where measurable mapping $\sigma_A : \mathcal{U} \to \Delta\Sigma_A$ is Alice's strategy, $\sigma_B \in \Delta\Sigma_B$ is Bob's strategy, and $\mu_t : \bigcup_{t\in T_A} H_t \times \mathcal{M}_k \cup \bigcup_{t\in T_B} H_t \times \mathcal{M}_k \times \{A, R\} \to \Delta\mathcal{U}$ is a *belief function* that specifies Bob's beliefs about Alice's types either after she offers a mechanism or after she makes a decision about Bob's offer. The implicit restriction is that the beliefs get updated only after Alice's actions.

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there are well-known problems with the existence of sequential equilibrium in signaling games with infinitely many actions (Myerson and Reny (2015)).

A *Perfect Bayesian equilibrium* (or, simply, equilibrium) is an assessment such that (a) the players best respond to their strategies and the beliefs, and (b) $\mu = \mu_0$ and the beliefs are updated through Bayes formula after each Alice's decision (mechanism choice or acceptance) that has a positive probability given history and strategies. Because the action choices are finite at each decision node, the PBE exists by the standard argument due to Selten (1975).

An equilibrium outcome is a pair of (measurable) function $e_A : \mathcal{U} \to [0,1]$ and a payoff $e_B \in [0,1]$, with the interpretation that $e_A(u)$ is the expected payoff of type u of Alice, and e_B is the expected Bob's payoff. Let $E^j(\delta, \mathcal{M}_k, \mu)$ be the set of expected equilibrium outcomes in game $\Gamma^j(\delta; \mathcal{M}_k, \mu)$. We are interested in the equilibrium outcomes as, first, the space of mechanism becomes well approximated as $k \to \infty$, and next, the offers become more and more frequent as $\delta \to 1$:

$$E^j(\delta, \mu) = \sup_{(\mathcal{M}_k):\mathcal{M}_k \to \mathcal{M}_F} \limsup_{k\to\infty} E^j(\delta, \mathcal{M}_k, \mu) = \bigcup_{(\mathcal{M}_k):\mathcal{M}_k \to \mathcal{M}_F} \bigcap_n \operatorname{cl} \bigcup_{k \geq n} E^j(\delta, \mathcal{M}_k, \mu),$$

$$E^j(\mu) = \limsup_{\delta \to 1} E^j(\delta, \mu) = \bigcap_n \operatorname{cl} \bigcup_{\delta \geq 1 - \frac{1}{n}} E^j(\delta, \mu).$$

The closure is taken with respect to the topology of uniform convergence. We show below (see the comment after Lemma 2) that each of the closed sets in the above definitions is compact, and the intersections of compact sets are not empty.

3. Main result

In this section, we assume that $N = 2$.

3.1. Complete information benchmark.

A special case of our model is when Alice's preferences are commonly known to be u. The argument from Rubinstein (1985) implies that our game has a unique subgame perfect equilibrium payoffs $\left(R_A^{j,\delta}(u), R_B^{j,\delta}(u)\right) \in$

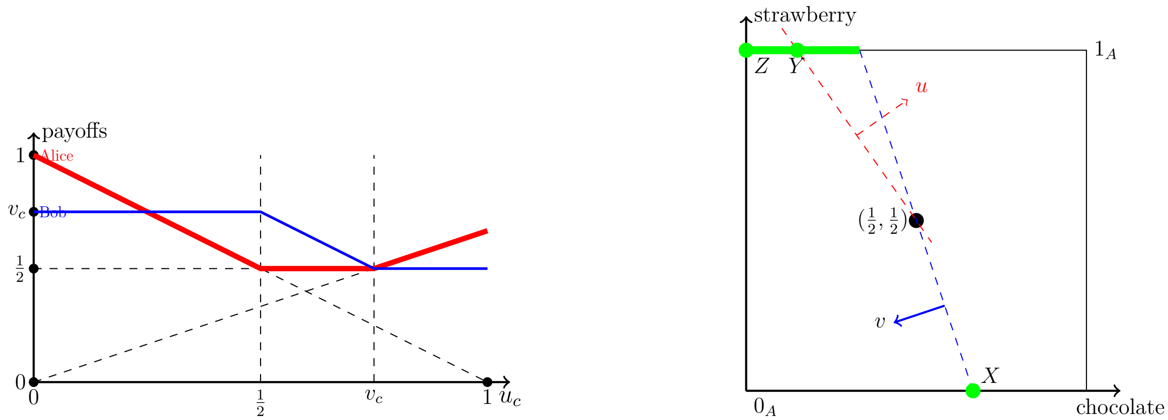

FIGURE 1. Nash payoffs and allocations.

$[0,1]^2$ of Alice type u and Bob. When $\delta \to 1$, the payoffs converge to the Nash bargaining payoffs of the two agent with preferences (u, v):

$$\left(R_A^{j,\delta}(u), R_B^{j,\delta}(u)\right) \to \left(\mathcal{N}_A(u), \mathcal{N}_B(u)\right).$$

When $v_c \geq v_v$, the Nash payoffs are given by function

$$\mathcal{N}_A(u_c, u_s) = \max\left(\frac{1}{2v_c}u_c, \frac{1}{2}, \frac{1}{2}(1-u_c)\right). \tag{1}$$

The Nash payoffs and allocations are illustrated on Figure 1. Bob likes strawberry, but he prefers chocolate. If Alice likes chocolate more, she is going to get her favorite allocation subject to the constraint that Bob's payoff is at least $\frac{1}{2}$ (allocation X). Bob's payoff is $\frac{1}{2}$; Alice gets higher payoff. If Alice has the same preference as Bob, allocation $\left(\frac{1}{2}, \frac{1}{2}\right)$ or any other allocation which gives both of them payoff of $\frac{1}{2}$ is a Nash allocation. If Alice prefers chocolate to strawberry, but she likes chocolate less than Bob likes chocolate, Bob receives his favorite allocation subject to the constraint that Alice's payoff is at least $\frac{1}{2}$ (allocation Y if Alice preferences are u). The allocation and

Bob's payoff depends on Alice's preference; Alice's payoff is $\frac{1}{2}$. Finally, if Alice likes strawberry more, each player receives his or her favorite part of the pie (allocation Z).

The Figure makes clear that Alice does not always have incentives to honestly reveal her type. If she likes chocolate less than Bob likes chocolate, she is best-off if Bob thinks that Alice preferences are almost like his, but with a slightly stronger preference for strawberry.

3.2. **Menus.** Finite and infinite (Alice's) menus play an important role in the analysis. Formally, a menu is any compact subset $Y \in \mathcal{C}X$. Each menu has its dual characterization through the payoff function $y\left(u;Y\right) = \max_{x\in Y} u\left(x\right)$. Any payoff function obtained from a menu is called a *menu function*. Let $\mathcal{Y}$ be the set of all menu functions.

For each u, and payoff function $y : \mathcal{U} \to [0,1]$, let $D_u y$ be the set of all affine functions $l : \mathcal{U} \to [0,1]$ such that $l\left(u\right) = y\left(u\right)$ and $\forall_{u'} y\left(u'\right) \geq l\left(u'\right)$.

Lemma 1. *Payoff function y is a menu function if and only if y is convex, continuous, and for each $u \in \mathcal{U}$, $D_u y$ is non-empty and closed. The set of menu functions $\mathcal{Y}$ is compact under the topology of the uniform convergence.*

In the dual approach, the "derivative" set $D_u y$ can be interpreted as the set of optimal choices l for Alice type u, where Alice's share of the nth part of the pie is equal to $l\left(\omega^n\right)$. Given Alice's choice l, Bob's payoff is equal to $1 - l\left(v\right)$. This leads to the following definition: for each menu function y, each belief μ, let

$$\Pi\left(y,\mu\right) = \int \left(\max_{l\in D_u y}\left(1 - l\left(v\right)\right)\right) d\mu\left(u\right) = 1 - \int \left(\min_{l\in D_u y} l\left(v\right)\right) d\mu\left(u\right)$$

be a tight upper bound on Bob's payoff in a menu associated with payoff function y.

The next result states a simple but important property of equilibrium outcomes.

Lemma 2. *For each δ, μ, k, if $(e_A, e_B) \in E^j(\delta, \mathcal{M}_k, \mu)$ is an equilibrium outcome, then e_A is a menu function, and $e_B \leq \Pi(e_A, \mu)$.*

The Lemma implies that $E^j(\delta, \mathcal{M}_k, \mu) \subseteq \mathcal{Y} \times [0,1]$. The standard arguments imply that the set of equilibrium outcomes is closed under the uniform topology. Hence, the equilibrium payoffs sets are closed subsets of a compact space. In particular, for each sequence $e_k \in E^j(\delta, \mathcal{M}_k, \mu)$, there exists a convergent subsequence with a limit $e_k \to e \in E^j(\delta, \mu)$. Similarly, for each sequence $e_{\delta_k} \in E^j(\delta_k, \mu)$ st. $\delta_k \to 1$, there exists a convergent subsequence with a limit $e_k \to e \in E^j(\mu)$.

3.3. **Equilibrium payoffs.** For each belief μ and each function $c : \mathcal{U} \to [0,1]$, define

$$\Pi_{\text{opt}}(c, \mu) = \max_{y \in \mathcal{Y}, y \geq c} \Pi(y, \mu) \text{ and } \mathcal{M}_{\text{opt}}(c, \mu) = \arg\max_{y \in \mathcal{Y}, y \geq c} \Pi(y, \mu).$$

Here, $\Pi_{\text{opt}}(c, \mu)$ is the largest payoff that Bob can attain with a menu that ensures that each Alice's type u gets at least $c(u)$. Because of the compactness of $\mathcal{Y}$, the set of optimal menus $\mathcal{M}_{\text{opt}}(c, \mu)$ is non-empty. The optimal menu is unique for generic beliefs.

Theorem 1. *Suppose that $N = 2$. Then, $E^j(\mu) \subseteq \mathcal{M}_{opt}(\mathcal{N}_A, \mu) \times \{\Pi_{opt}(\mathcal{N}_A, \mu)\}$.*

In the limit, as the space of mechanisms available becomes dense, and the offers become more and more frequent, regardless who makes the first offer, Bob's equilibrium payoff is equal to the expected payoff from the optimal screening menu subject to the constraint that each type of Alice receives her Nash (i.e., her complete information) payoff. If the optimal screening menu is unique, the payoff of each type of Alice is also unique. The outcome is ex ante efficient, but not ex post efficient. Bob's payoff is equal to the payoff that he would obtain if he was able to commit to an optimal

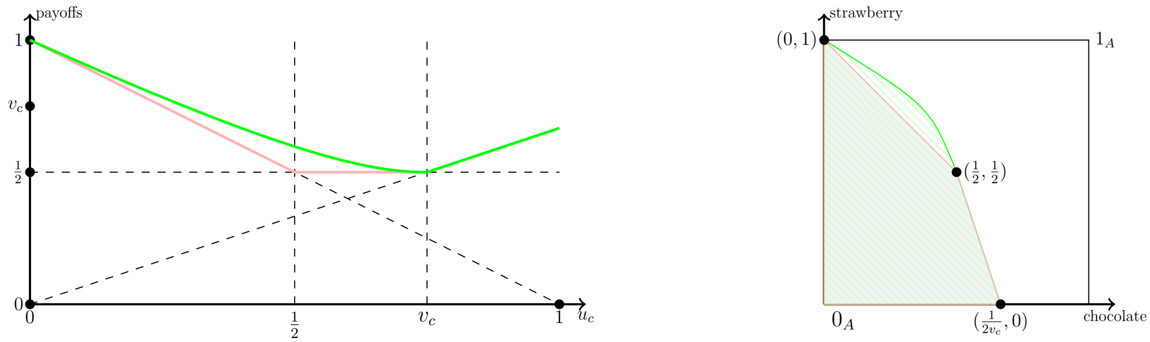

FIGURE 2. Payoffs and allocations in the constrained optimal menu.

mechanism (subject to the complete information constraint). This opposite to what one could expect from the literature on the Coase conjecture.

The optimal menu is illustrated on Figure 2. If Alice likes chocolate more than Bob, or if Alice likes only strawberry, the equilibrium allocation is equal to the complete information Nash Allocation. Otherwise, for generic beliefs, she receives a higher payoff.

We are not able to construct any equilibrium in this game, and in particular, no equilibrium with payoffs close to the limit. However, because the optimal screening menu significantly depends on Bob's beliefs, the equilibrium behavior cannot be too much different than as if Bob offered the optimal screening menu, and Alice accepted it. In particular, the mechanisms has to be accepted without too much delay, and, prior to that moment, there cannot be any substantial revelation of information.

Bob's optimal payoff, $\mathcal{M}_{\text{opt}}(\mathcal{N}_A, \mu)$, is convex in μ, which implies that it has a natural comparative statics with respect to information: Bob is better-off if his information improves. When $\mu \to \delta_u$ for some Alice's type, Bob's payoff converges to the Nash outcome of bargaining against type u, $\mathcal{N}_B(u)$.

We briefly explain the structure of the proof, with the emphasis on the role played by mechanisms. The proof has two parts. In the first part, we show that if the payoffs of Alice are too large (in a certain sense defined below), then Bob has a profitable deviation in the form of a menu mechanism. The deviation is chosen so that it is acceptable to all types of Alice. In order to find such a deviation, it is crucially important that Bob has access to a sufficiently large set of menus that well-approximates all menus. In general, it is not possible to find an optimal deviation in the form of a simpler mechanisms, like single-offers. The reason is that, typically, a single offer is not acceptable to some of the types.

In the second part, we show that Alice's equilibrium payoffs cannot be significantly lower than the Nash payoffs. If not, then in the game in which Bob makes the first offer, each type u of Alice has a deviation to reject the Bob's offer, and propose a new mechanism in the next period. We choose such a deviation so that it be accepted by Bob, and that it improves type u's payoffs.In the proof, Alice's counteroffer is a particular menu of menus: for each $y_u \in [0, 1]$, let

$$W_{u,y_u} = \{\text{all menus with menu functions } y \text{ st. } y(u) \geq y_u\}. \tag{2}$$

An offer of W_{u,y_u} can be interpreted as Alice's request to Bob: "I am type u and I want y_u. You can design any menu as long as you give me that." Formally, W_{u,y_u} consists of infinitely many menus, hence it is not a mechanism in the sense of our definitions. However, we show that as $k \to \infty$, W_{u,y_u} can be well-approximated by a sequence of menus of menus $W^k_{u,y_u} \in \mathcal{M}_k$.

The menu of menus helps Alice to address the signaling problems. When Alice deviates from an equilibrium path (first by rejecting, and then, possibly, by making a particular counteroffer), Bob can have arbitrary beliefs about her types that are not constrained in any way by the solution concept. In general, the Bob's equilibrium beliefs may be such to maximize his incentives to reject the deviation. If the set of mechanisms that Alice chooses from is small enough, it is possible that any counteroffer can be "punished" with Bob's beliefs that make waiting for the next period continuation equilibrium more attractive. (We discuss this issue in a more detail in Section 4.1.) By offering an approximate version of W_{u,y_u}, Alice allows Bob to choose an optimal mechanism (subject to the constraint $y(u) \geq y_u$), *whatever are his beliefs.*

As the above argument makes clear, the Theorem holds as long as the limit set of available mechanisms $\mathcal{M}$ contains all menus and menus of menus. As we argue in Section 4.1, the thesis of the Theorem fails, if $\mathcal{M}$contains only simple offers. We do not know if the Theorem holds if $\mathcal{M}$ contains menus, but not menus of menus, as is the setting studied in Sen 2000 and Inderst 2003. However, in such a case, one can show that $\Pi_{\text{opt}}(\mathcal{N}_A, \mu)$ is a lower bound on Bob's equilibrium payoffs.

4. Comments

4.1. **Simple offers.** We consider a special case of our model, when players are only allowed to make simple offers. Let $\mathcal{S} \subseteq \mathcal{M}$ be the collection of all single-offer mechanisms (all mechanisms in which no player chooses any action). Any such a mechanism can be identified with a single allocation x. Let $X(\mathcal{S})$ be the collection of all such offers. Let $\mathcal{S}_1 \subseteq \mathcal{S}_2 \subseteq ... \subseteq \mathcal{S}$ be an approximation sequence with finite sets, where $\mathcal{S}_k \to S$ in the Hausdorff distance sense.

Proposition 1. *Suppose that $v_c > v_s$. Fix $u^* \in \mathcal{U}$ st. $u^*_c < v_c$. There exists δ_0 and k_0 such that for each $\delta \geq \delta_0$, $k \geq k_0$, and any belief μ st. $u^*_c = \inf\{u_c : u_c \in \operatorname{supp}\mu\}$, there is $(e_A, e_B) \in E^B(\delta, \mathcal{S}_k, \mu_0)$ such that $\forall_u e_A(u) \leq \max_{x:v(x)\geq\delta\mathcal{N}_B(u^*)} u(x)$ and $e_B \geq \delta\mathcal{N}_B(u^*)$*

Let u^* be the type with the strongest preference for strawberry in the support of the type distribution. If players are sufficiently patient, then there is an equilibrium, in which Bob receives his complete information payoff $\mathcal{N}_B(u^*)$ as if facing the type u^*,

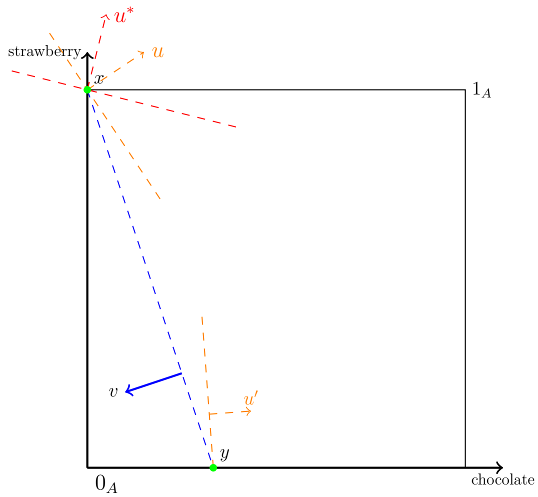

FIGURE 3. Allocations in the Anti-Coasian equilibrium.

regardless of his beliefs. This is also his best complete information payoff across all types in Alice's support. Alice types receive the best payoffs subject to the constraint that Bob's payoff is at least $\mathcal{N}_B(u^*)$. The Lemma is illustrated on Figure 3. The Nash allocation of type u^* gives her the strawberry part of the pie; the chocolate goes to Bob (allocation x) . The blue line is Bob's indifference curve. Alice types, generically choose between two allocation: x and y.

The proof constructs such an equilibrium with required properties. The idea is that, roughly, Alice must offer either the allocations x or y (or anything in-between). If she deviates, she is punished with a belief that she is type u^*. From now on, Bob expects nothing less than allocation x.

The punishment with beliefs has been always available in our equilibrium constructions. However, if mechanisms like (2) are available, the "punishment" can be challenged by Alice by a sophisticated counter-offer.

The equilibrium has Anti-Coasian flavor, as Bob receives his best possible complete information payoff across all Alice's types. A positive fraction of Alice types (including, among others, all types that like chocolate more than Bob) is strictly worse than under complete information; the other types are not better-off. Alice would benefit from being able to credibly reveal her type. This observation might be surprising to the reader familiar with the informed principal literature. Maskin and Tirole (1990) claim that, in the private value case, the informed principal must benefit from incomplete information due to the collapse of the agent incentive and individual rationality constraints. This observation does not necessarily hold with interdependent values. We work with private values, but, in the dynamic setting, the continuation value depends on the belief of the uninformed agent. Hence, there is endogenous interdependence.

There are potentially other equilibria. For some beliefs, it is easy to extend the construction from Proposition 1, in which Alice's types that like strawberry receive allocation x and the types that like chocolate receive allocation with more chocolate than y.

4.2. **Case $N > 2$.** The thesis of Theorem 1 does not hold when $N = 3$. We show it with an example. Let $v = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$. Figure 4 presents the Nash payoffs for all types who only care about the first two dimensions. Let $\tau_1 = \left(\frac{2}{3}, \frac{1}{3}, 0\right), \tau = \left(\frac{1}{2}, \frac{1}{2}, 0\right), \tau_2 = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$ be three distinguished types of Alice. We have $\mathcal{N}_A(\tau_1) = \mathcal{N}_A(\tau_2) = \frac{2}{3}$ and $\mathcal{N}_A(\tau) = \frac{3}{4}$. Notice that the Nash payoffs are not convex.

Proposition 2. *Fix $\delta < 1$, and suppose that players have an access to a public randomization. Suppose that $\mathcal{M}' \subseteq \mathcal{M}$ is a finite set of mechanisms that contains two menus:*

$$Y^B = \left\{(1,0,0), \left(\frac{2}{3}, \frac{2}{3}, 0\right), (0,1,0)\right\},$$
$$Y^A = \left\{\left(1, 2\frac{1-\delta}{\delta}, 0\right), \left(\frac{1}{\delta}\frac{2}{3}, \frac{1}{\delta}\frac{2}{3}, 0\right), \left(2\frac{1-\delta}{\delta}, 1, 0\right)\right\}.$$

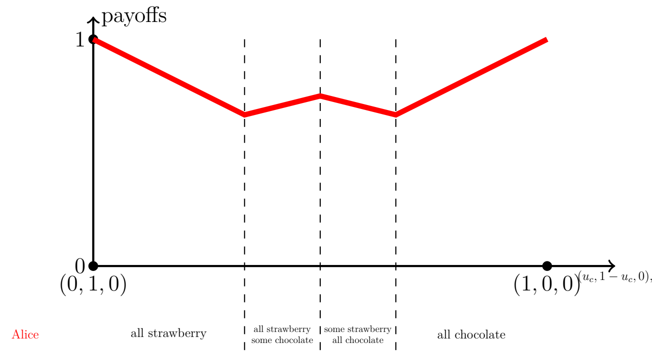

FIGURE 4. Nash payoffs when $N = 3$.

(The tuples in the menus correspond to Alice shares; Bob receives the complementary shares.) Then, for any belief $\Delta\{\tau_1, \tau_2, \tau\}$ *that assigns a strictly positive probability to types* τ_1, τ_2*, there exists* $\left(e^j_A.e^j_B\right) \in E^j(\delta, \mathcal{M}', \mu)$ *such that* $e^A_A(\tau) = \frac{1}{\delta}\frac{2}{3}, e^B_A(\tau) = \frac{2}{3}$*. In particular, if* $\delta < 1$ *is sufficiently high, type* τ *receives a payoff substantially lower than her Nash payoff of* $\frac{3}{4}$*.*

We construct a (sequential) equilibrium with the required features. In the equilibrium, player j always offers menu Y^j, and the offer is accepted. If Alice proposes some other mechanism (as a possible deviation), we show that there are Bob's beliefs such that either the mechanism has an equilibrium where all Alice's types receive payoffs smaller than $\frac{1}{\delta}\frac{2}{3}$, or Bob's payoffs are smaller than the discounted continuation equilibrium payoff of $\delta\frac{2}{3}$, in which case Bob rejects it. Similarly, we show that if Bob deviates, his proposal either has an equilibrium that gives him a payoff less that his equilibrium payoff of $e^B_B = \frac{2}{3}$, or it gives less than $\frac{2}{3}$ to all types of Alice, in which case Alice rejects it.

Although we are not able to fully characterize the set of payoffs when $N \geq 3$, we have the following bound.

Theorem 2. *Suppose that $N \geq 2$. Then, for any $j = A, B$, any belief $\mu \in \Delta U$, any limit payoff $(e_A, e_B) \in E^j(\mu)$, any type $u \in U$, we have $e_A(u) \geq \frac{1}{2}, e_B \geq \frac{1}{2}$.*

In any limit of equilibria, Bob and each type of Alice receive higher payoff than their worst possible complete information payoff (in case of Alice types, the worst possible across all Bob's preferences v).

5. Proof of Theorem 1

The proof is divided into two parts. In the first part, we show that any menu that with payoffs for each Alice's type strictly above her complete information payoffs is going to be accepted. In the second part, we show that each type of Alice will reject a menu that gives her less than her complete information payoff.

5.1. **Upper bound.**

We show that if Alice's payoffs when she makes the first offer are too high, then Bob has a profitable deviation.

We proceed in two steps. The first step can be understood as a generalization of the method from Rubinstein (1982) to situations when player's payoffs are described by a function rather than a single number. We define a property of a payoff function, and we show that the payoff of any Alice's type in the belief support cannot be larger than a value of a payoff function with the property. Take any $\gamma < 1$ and $\delta < 1$. We say that menu function $h : \mathcal{U} \to [0,1]$ *has $UB(\gamma, \delta)$-property* if $\inf h > 0$ and, for each menu function $y \geq h$, each belief μ, there exists a menu function y' such that

$$y' \geq_{\text{supp}\mu} (1 - \gamma(1 - \delta))\, y \text{ and } \gamma\delta\Pi(y', \mu) \geq \Pi(y, \mu).$$

Lemma 3. *For all $\gamma, \delta_0 < 1$, there exists k_0 such that if function h has $UB(\gamma, \delta)$-property, then if $(e_A, e_B) \in E^A(\delta, \mathcal{M}_k, \mu)$ for some $k \geq k_0$ and μ, then $\inf_{u \in supp\mu} e_A(u) < h(u)$.*

The Lemma says that for any equilibrium, there must be an Alice's type in the belief support such that her payoff is strictly smaller than the value of a function with UB-property. The proof goes by contradiction. Suppose that there is an equilibrium with Alice's payoffs above h. We show that we can find maximal equilibrium payoffs which such a property. Let y be Alice's payoff function in such an equilibrium. Because y is larger than h, the UB-property implies the existence of a menu function y' such that each Alice's type receives more than δy and such that Bob receives more that $\frac{1}{\delta}\Pi(y,\delta)$, or (by Lemma 2), more than $\frac{1}{\delta}$ times his equilibrium payoff. If Bob rejects the current Alice's offer, and proposes menu with payoffs y', such an offer is accepted by all types in the support. (If not, because payoffs y are maximal, some rejecting types have to receive less than y in the continuation equilibrium, and the rejection of more than $y' > \delta y$ is not profitable.) Because the deviation leads to payoffs that are higher than $\frac{1}{\delta}$ times the current equilibrium payoff, this contradicts the existence of equilibrium with payoffs y.

The need to use $\gamma < 1$ in the above definition and the result is due to the fact that we work with approximate spaces of mechanisms rather than all mechanisms.

Lemma 4. *Suppose that function h has property $UB(\gamma,\delta)$ for some $\gamma,\delta < 1$. Then, if $(e_A,e_B) \in E^B(\delta,\mu)$, then $e_B \geq \Pi_{opt}(\delta h,\mu)$.*

Proof. We present the proof here, because it is short and intuitive. Let k_0 be as in Lemma 3 . We are going to show that, in game $\Gamma^B(\delta,\mathcal{M}_k,\mu)$ for any $k \geq k_0$, Alice accepts any menu $m \in \mathcal{M}_k$ with payoffs strictly higher than δh for each type in the support with probability 1. On the contrary, if a positive probability set of types rejects such a menu, in the next period they face a continuation equilibrium described in Lemma 3. By Lemma 3, at least some of those types receive a continuation payoff that is strictly lower than h. But then, their rejection of m could not have been a best response.

For each $k \geq k_0$, let $\mathcal{M}_k^* \subseteq \mathcal{M}_k$ be the set of such menus. The argument implies that, if $(e_A, e_B) \in E^B(\delta, \mu, \mathcal{M}_k)$, it must that $e_B \geq \max_{m \in \mathcal{M}_k:\mathrm{m}} \Pi(y_m, \mu)$. By the approximation results (Lemma 12 from the Appendix), the RHS off the above inequality converges to $\Pi_{\mathrm{opt}}(\delta h, \mu)$ as $k \to \infty$. □

In the second step, we show that an approximation to the Nash payoffs has the UB-property.

Lemma 5. *Suppose that $N = 2$. Then, for each $\varepsilon > 0$, there is a function h such that $\sup_u |h(u) - \mathcal{N}_A(u)| \leq \varepsilon$ and $\gamma, \delta_0 < 1$ such that h has property $UB(\gamma, \delta)$ for each $\delta \geq \delta_0$.*

We sketch the intuition. We want to show that menu function $h > \mathcal{N}_A$ has the UB-property. Suppose that $y > h$ is a menu function associated with menu Y and such that $y > h$. Let $x(u) \in \arg\max_{x \in Y} u(x)$ be an optimal choice of type u in menu Y. We divide the space of allocations. into the area below and above the 45° diagonal. For each u such that $x(u)$ is below the diagonal, define a wasteless allocation $x'(u)$ so that

$$x'_A(u) = \delta x_A(u) + (1 - \delta) x_A(u).$$

Similarly, for each u such that $x_Y(u)$ is above diagonal, define a waste-less allocation $x'(u)$ so that

$$x'_A(u) = x_A(u) + \rho(x_A(u) - \mathbf{1}_A),$$

where constant $\rho = (1 - \delta)\frac{\alpha}{1-\alpha}$ is chosen so that the two definitions agree for u^* st. $x(u^*) = \alpha \mathbf{1}_A + (1 - \alpha)\mathbf{0}_A$ lies exactly on the diagonal. (See Figure 5). Because $y > \mathcal{N}_A$, we check that $\alpha > \frac{1}{2}$, and $\rho > 1 - \delta$. (In the latter case, so defined $x'(u)$ will

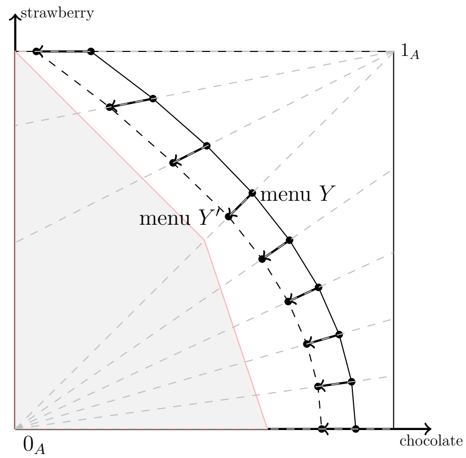

FIGURE 5. Menus Y and Y'.

exist if $x(u)$ is sufficiently inside X; or, alternatively, if $1-\delta$ is small and $u(x(u)) \geq h(u)$ is bounded away from $\mathcal{N}_A(u)$. The details can be found in the Appendix.) Let $Y' = \{x'(u) : u \in \mathcal{U}\}$. Because the new allocations are obtained by (partially) linear operations, it is easy to show that $x'(u)$ is the optimal choice of type u in menu Y'. Further, we check by direct calculations that $u(x'(u)) \geq \delta u(x(u))$ and $\delta v(x'(u)) \geq v(x(u))$ for each u. For instance, suppose that $x(u)$ is below the diagonal. Then, the second inequality follows easily from the fact that $\rho > 1$. For the first inequality, notice that

$$u(x'(u)) - \delta u(x(u)) = (1+\rho-\delta)\, u(x(u)) - \rho$$
$$= (1-\delta)\frac{1}{1-\alpha}(u(x(u)) - \alpha) \geq (1-\delta)\frac{1}{1-\alpha}(u(x(u^*)) - \alpha) = 0,$$

where the inequality comes from the fact that $x(u^*)$ is one of the choices available for type u. The two inequalities imply that the menu function y' satisfies $y' \geq \delta y$ and $\delta \Pi(y', \mu) \geq \Pi(y, \mu)$.

5.2. **Lower bound.** As in the upper bound case, we begin with a certain generalization of the Rubinstein's method. For $\gamma, \delta < 1$, say that payoff function $h : \mathcal{U} \to [0,1]$ has $LB(\gamma, \delta)$-property if for each type w, each constant $y_w \le h(w)$, any belief $\psi \in \Delta\mathcal{U}$,

$$\Pi_{\text{opt}}\left(\left(\frac{1}{\gamma\delta}y_w\right)\mathbb{1}_{.=w}, \psi\right) \ge (1 - \gamma(1-\delta))\,\Pi_{\text{opt}}\left(y_w\,\mathbb{1}_{.=w}, \psi\right). \tag{3}$$

Here, $c\,\mathbb{1}_{.=w}$ for some constant $c \in \mathbb{R}$ is a function of types defined as $c\,\mathbb{1}_{u=w} = \begin{cases} c, & \text{if } u = w \\ 0, & \text{otherwise} \end{cases}.$

Lemma 6. *For all $\gamma, \delta < 1$, there exists $k_0 < \infty$ such that if h has $LB(\gamma, \delta)$-property, then for each $k \ge k_0, \mu \in \Delta\mathcal{U}$, if $(e_A, e_B) \in E^B(\delta, \mathcal{M}_k, \mu)$, then, $e_A \ge h$.*

To see the intuition, take an arbitrary type $w \in U$ of Alice. Let y_w be the minimum of the equilibrium payoffs of type w across all equilibria and all beliefs in a game where Bob makes the first offer; we assume for simplicity that such an minimum is attained in some equilibrium. Suppose that $y_w < h$. Given such an equilibrium with payoffs $y_w = e_A(w)$, we consider Alice's deviation to reject any offer in the first period and to propose menu of menus $W_{w,\frac{1}{\gamma\delta}y_w}$ in the next period. Such an offer induces equilibrium (possibly, off-path) beliefs ψ. Bob's payoff from accepting the menu is equal to the left-hand side of (3). On the other hand, if Bob rejects Alice's offer, the continuation equilibrium in the next period yields at least y_w to Alice's type y_w. Thus, Bob's expected and discounted continuation payoff is equal to $\delta\Pi_{\text{opt}}(\mathbb{1}_{.=w}\,y_w, \psi)$. Inequality (3) implies that Bob prefers to accept the menu of menus. But, such a menu of menus leads to a payoff of $\frac{1}{\gamma\delta}y_w$ for Alice type w. Hence, the deviation is profitable, which contradicts the existence of equilibrium with payoffs y_w.

Lemma 7. *For any $\varepsilon > 0$, there exist $\gamma, \delta < 1$ such that the following functions h have $LB(\gamma,\delta)$-property:*

(1) $h^0(u) = (1-\varepsilon)\frac{1}{2}$,
(2) $h^k(u) = (1-\varepsilon)\min\left(1, \frac{1}{2v_k}\right)u_k$ *for any $k = c, s$.*

In both cases, we take arbitrary menu function y such that $y(w) \geq y_w$, where $y_w \leq h(w)$, and use it to construct a menu function y' such that $y'(w) \geq \frac{1}{\gamma\delta}y_w$ and such that the expected payoffs of Bob with beliefs ψ from accepting menu y' are higher than waiting for the next period and menu y, $\Pi(y', \psi) \geq \delta\Pi(y, \psi)$. For the first claim, it is enough to replace y by $y' = \delta y + 1 - \delta$, or, in other words, replace menu associated with y by its convex combination with allocation $\mathbf{1}_A$ with weight $1-\delta$ on the latter. Similarly, in the case of the second claim, we also replace the original menu, but with an appropriate convex combination with an allocation that gives the entire part k of the pie to Alice.

5.3. **Proof of Theorem 1.** Assume w.l.o.g. that $v_c \geq v_s$, or that Bob likes chocolate more. It follows from Corollary 4 and Lemma 5 that if $(e_A, e_B) \in E^B(\mu)$, then $e_B \geq \Pi_{\text{opt}}(\mathcal{N}_A, \mu)$. Similarly, it follows from Lemmas 6 and 7 that for each $(e_A, e_B) \in E^B(\mu)$, it must be that

$$e_A(u) \geq \max\left(\frac{1}{2}, \min\left(1, \frac{1}{2v_c}\right)u_c, \min\left(1, \frac{1}{2v_s}\right)u_s\right) = \max\left(\frac{1}{2}, \frac{1}{2v_c}u_c, u_s\right) = \mathcal{N}_A(u),$$

where the last equality is consequence of (1). The result follows from the definition of the value $\Pi_{\text{opt}}(.)$ and the solution $\mathcal{M}_{opt}(.)$ to Bob's optimization problem.

Appendix A. Menu functions

A.1. **Menu functions.**

Lemma 8. *Suppose that $N = 2$. Suppose that y is a convex function and types $u, u' \in \mathcal{U}$ such that $u_c < u'_c$. If $D_u y$ and $D_{u'} y$ are non-empty, then $D_w y$ is non-empty for any type w such that $u_c < w_c < u'_c$.*

Proof. Because y is convex, for any $u, u' \in \mathcal{U}$ st. $u_c < u'_c$, and any $l \in D_u y, l' \in D_{u'} y$, we have $l(0) > l'(0)$, and $l(1) < l'(1)$. □

Lemma 9. *For each menu function y_0, each belief μ, each $\alpha \in [0,1]$, we have*

$$\Pi_{opt}(\alpha y_0 + (1-\alpha)\mathbf{1}, \mu) \geq \alpha \Pi_{opt}(y_0, \mu).$$

Proof. Observe that if $y \in \mathcal{M}_{\text{opt}}(y_0, \mu)$, then $y' = \alpha y + (1-\alpha)\mathbf{1} \geq \alpha y_0 + (1-\alpha)\mathbf{1}$. Because $D_u y' = \{\alpha l(.) + 1 - \alpha : l \in D_u y\}$, we have

$$\Pi(y', \mu) = 1 - \int \left(\max_{l \in D_u(\alpha y + (1-\alpha)\mathbf{1})} l(v) \right) d\mu(u) = 1 - \int \left(\max_{l \in D_u(\alpha y + (1-\alpha)\mathbf{1})} (\alpha l(v) + 1 - \alpha) \right) d\mu(u)$$
$$= \alpha - \alpha \int \left(\max_{l \in D_u(\alpha y + (1-\alpha)\mathbf{1})} l(v) \right) d\mu(u) = \alpha \Pi(y_0, \mu).$$

□

Lemma 10. *For any two menu functions $y, y' \in \mathcal{Y}$, $\max(y, y')$ is also a menu function.*

Proof. The convexity and the payoff restriction are immediate. For the derivative bound, notice that $D_u \max(y, y') \subseteq D_u y \cup D_u y'$. □

A.2. **Payoff-dominance.** For each menu function y, each $u \in \mathcal{U}$, let

$$d(y, u) = \max_{l \in D_u y} l(v).$$

Say that menu y is payoff-dominated by menu y', $y \leq_d y'$ if and only if $d(y,.) \geq_{\mathcal{U}} d(y',.)$. It is an obvious consequence of the definitions that if $y \leq_d y'$ for two menu functions y, y', then $\Pi(y,\mu) \leq \Pi(y,\mu')$ for any belief $\mu \in \Delta\mathcal{U}$.

A.3. **Proof of Lemma 1.** For the "only if" direction, notice that

$$D_u y = \left\{ l : \forall_{u'} l(u') = u'(x) \text{ for some } x \in \arg\max_{z \in Y} u(z) \right\}.$$

For the "if" direction, for each affine function $l : \mathcal{U} \to [0,1]$ define an allocation $x(l)$ such that $x_{A,n}(l) = l(\omega^n)$ and $x_{B,n}(l) = 1 - x_{A,n}(l)$. Let $Y = \mathrm{cl}\{x(l) : l \in \bigcup_u D_u y\}$. Then, y is the payoff function for menu Y.

Because of the characterization, each menu function is Lipschitz with bounded payoffs. Hence, the set $\mathcal{Y}$ is equicontinuous, and the Arzelà–Ascoli theorem implies that it is precompact under the topology of the uniform convergence. Let $y_n \in \mathcal{Y}$ be a convergent sequence and, for each u, let $l_n^u \in D_u y_n$ be a supporting affine function.

A.4. **Proof of Lemma 2.** Any profile of (complete information) strategies $\sigma \in \prod_i \Sigma_i$ induces a probability distribution $\mu(\sigma) \in \Delta(\mathbb{N} \times X \cup \{\infty\})$, with the interpretation that $\mu(t, A|\sigma)$ for measurable $A \subseteq X$ is the probability that the game ends in period t with outcome in A, and $\mu(\infty|\sigma)$ is the probability that the bargaining never ends. Let $x(\sigma) \in X$ be defined so that $x_i(\sigma) = (1-\delta)\sum \delta^t \int x_i \mu(t, dx)$ is the expected and discounted award to player i.

Suppose that $(\sigma_A, \sigma_B, \mu)$ is an equilibrium with payoffs $(e_A, e_B) \in E^j(\delta, \mathcal{M}_k, \mu)$. For each u, let $x_u = x(\sigma_A(u), \sigma_B)$ be the expected and discounted equilibrium allocation. Then, $u(x_{u'})$ is the expected payoff of Alice's type u from mimicking the strategy of type u'. For each u, define an affine $l_u : \mathcal{U} \to [0,1]$ so that $l_u(u') = u'(x_u)$. The incentive compatibility implies that for each u and u', $e_A(u') = l_{u'}(u') \geq l_u(u')$. It follows that e_A is convex (as it is a supremum over affine functions l_u), and $l_u \in D_u e_A$. By Lemma 1, e_A is a menu function. Moreover,

$$e_B = \int v\left(x\left(u\right)\right) d\mu\left(u\right) \leq \int \left(1 - l_u\left(v\right)\right) d\mu\left(u\right) \leq \Pi\left(e_A, \mu\right).$$

Appendix B. Mechanisms and approximations

This part of the Appendix is devoted to approximations of mechanisms.

Let $\mathcal{M}_{\text{menu}}$ denote the class of menu mechanisms, and let $\mathcal{M}_{menu}\left(n\right)$ denote the subset of menus with n actions for Alice. Let $\mathcal{M}_{\text{mmenu}}$ denote the class of (Bob's) menus of (Alice's menus) and let $\mathcal{M}_{mmenu}\left(n_B, n_A\right)$ denote the subset of such mechanisms with n_i actions for player i. Let d_{CX} and d_{C^2X} be the Hausdorff distances on spaces, respectively, CX and C^2X induced by the Euclidean distance on X. In the interest of transparency, we drop the subscripts in the definition of the distance.

For any mechanism m, any beliefs $\mu \in \Delta\mathcal{U}$, let $E\left(m, \mu\right) \subseteq \mathcal{Y} \times [0,1]$ denote the set of outcomes $\left(e_A, e_B\right)$ that can be obtained in equilibrium.

B.1. Menus.

For each menu Y, each $\eta > 0$, define menu

$$Y^{\eta} = \left\{\left(1-\eta\right)x + \eta v\left(x\right)\mathbb{1}_A + \eta\left(1 - v\left(x\right)\right)\mathbb{0}_A : x \in Y\right\}.$$

Lemma 11. *For each $\eta > 0$, there exists $\varepsilon > 0$ such that for each menu Y, each menu Y' such that $d\left(Y^{\eta}, Y'\right) \leq \varepsilon$, for each $u \in \mathcal{U}$, if $x \in \arg\max_{x \in Y} u\left(x\right)$ and $x' \in \arg\max_{x \in Y'} u\left(x\right)$, then*

$$u\left(x'\right) \geq u\left(x\right) - 2\eta \text{ and } v\left(x'\right) \geq v\left(x\right) - 2\eta. \tag{4}$$

Proof. Choose $\varepsilon > 0$ such that $\eta > \varepsilon$ and $\eta\left(\eta - \varepsilon\right) > 2\varepsilon$. Take any two menus Y and Y' such that $d\left(Y^{\eta}, Y'\right) \leq \varepsilon$. Fix u.

For the first inequality in (4), notice that for each $x \in Y$, there exists $x \in Y'$ such that $\|(1-\eta)x + \eta(1-v(x))\mathbb{0}_A + \eta v(x)\mathbb{1}_A - x'\| \leq \varepsilon$. But

$$u(x') \geq u((1-\eta)x + \eta(1-v(x))\mathbb{0}_A + \eta v(x)\mathbb{1}_A) - \varepsilon$$
$$= u(x) + \eta(v(x) - u(x)) - \varepsilon \geq u(x) - 2\eta,$$

where the last inequality comes from the choice of ε.

For the second inequality in (4), suppose on the contrary that there is $x \in \arg\max_{x\in Y} u(x)$ and $x' \in \arg\max_{x\in Y'} u(x)$ such that $v(x') < v(x) - 2\eta$. Because $x' \in Y'$ and $d(Y^\eta, Y') \leq \varepsilon$, there exists $x_0 \in Y^\eta$ such that $u(x') \leq u(x_0) + \varepsilon$ and $v(x_0) \leq v(x)(1-2\eta) + \varepsilon$. Because $x_0 \in Y^\eta$, there exists $x_1 \in Y$ such that

$$x_0 = (1-\eta)x_1 + \eta(1-v(x_1))\mathbb{0}_A + \eta v(x_1)\mathbb{1}_A.$$

Recall that $v(\mathbb{0}_A) = 1 = 1 - v(\mathbb{1}_A)$. Hence,

$$v(x_1) \leq v(x_1) + \eta(1-2v(x_1)) + \eta = v(x_0) + \eta \leq v(x) - \eta + \varepsilon.$$

Moreover, because $x \in Y(u)$, we have $u(x_1) \leq u(x)$, which implies

$$\begin{aligned} u(x') &\leq u(x_0) + \varepsilon = u((1-\eta)x_1 + \eta(1-v(x_1))\mathbb{0}_A + \eta v(x_1)\mathbb{1}_A) + \varepsilon \\ &= (1-\eta)u(x_1) + \eta v(x_1) + \varepsilon \leq (1-\eta)u(x_1) + \eta(v(x) - \eta + \varepsilon) + \varepsilon \\ &\leq (1-\eta)u(x) + \eta v(x) - \eta(\eta - \varepsilon) + \varepsilon. \end{aligned} \tag{5}$$

On the other hand, because $d(Y^\eta, Y') \leq \varepsilon$, there is $x_2 \in Y'$ such that

$$u(x_2) \geq u((1-\eta)x + \eta(1-v(x))\,\mathbb{0}_A + \eta v(x)\,\mathbb{1}_A) - \varepsilon$$
$$= (1-\eta)u(x) + \eta v(x) - \varepsilon. \tag{6}$$

Because of the choice of ε, inequalities (5) and (6) contradict that $x' \in \arg\max_{x\in Y'} u(x)$. The contradiction demonstrates that $v(x') \geq v(x) - 2\eta$. □

Lemma 12. *For each $\eta > 0$, there exists k_0 such that for each $k \geq k_0$, each $y \in \mathcal{Y}$, there is a mechanism $m \in \mathcal{M}$ such that for each $\mu \in \Delta\mathcal{U}$, each $(e_A, e_B) \in E(m, \mu)$, we have $e_A(u) \geq y(u) - \eta$ for each u, and $e_B \geq \Pi(y, \mu) - \eta$.*

Proof. Because CX is compact in the Hausdorff metric, there is $n(\varepsilon) < \infty$ such that for each $Y \in CX$, there exists $m \in \mathcal{M}_{\text{menu}}(n(\varepsilon))$ such that $d(Y, Y_m) \leq \frac{1}{2}\varepsilon$. Further, because $\mathcal{M}_{menu}(n(\varepsilon))$ is a compact subset of Euclidean space, for each $\varepsilon > 0$, there exists $k_0(\varepsilon)$ such that for any $k \geq k_0(\varepsilon)$, any $m \in \mathcal{M}_{\text{menu}}(n(\varepsilon))$, there exists $m' \in \mathcal{M}_k \cap \mathcal{M}_{\text{menu}}(n(\varepsilon))$ such that $d(Y_m, Y_{m'}) \leq \frac{1}{2}\varepsilon$. It follows that for each $k \geq k_0(\varepsilon)$, any $Y \subseteq X$, there is $m \in \mathcal{M}_k \cap \mathcal{M}_{\text{menu}}(n(\varepsilon))$ such that $d(Y, Y_m) \leq \varepsilon$.

Let $\varepsilon > 0$ be small enough so that the thesis of Lemma 11 holds for $\frac{1}{2}\eta$. Take arbitrary menu function y and the associated menu Y. Find mechanism m such that $d\left(Y^{\frac{1}{2}\eta}, Y_m\right) \leq \varepsilon$. The claim follows from Lemma 11. □

B.2. **Menus of menus.**

Lemma 13. *For each $\eta > 0$, there exists k_0 such that for each $k \geq k_0$, each $(u, y_u) \in \mathcal{U} \times [0,1]$, there is a mechanism $m \in \mathcal{M}$ such that for each $\mu \in \Delta\mathcal{U}$, each $(e_A, e_B) \in E(m, \mu)$, we have $e_A(u) \geq y_u - \eta$, and $e_B \geq \max_{y\in\mathcal{Y}: y(u)\geq y_u} \Pi(y, \mu) - \eta$.*

Proof. Because the space of all menus of menus C^2X is compact in the Hausdorff metric, there is $n_B(\varepsilon) < \infty$ such that for each $W \in C^2X$, there exists $m \in \mathcal{M}_{\text{menu}}(n_B(\varepsilon), n(\varepsilon))$ (where $n(\varepsilon)$ is as in the proof of Lemma 12) such that $d(W, W_m) \leq \frac{1}{2}\varepsilon$. Further, because the $\mathcal{M}_{menu}(n_B(\varepsilon), n(\varepsilon))$ is a compact subset of Euclidean space, for each $\varepsilon > 0$, there exists $k_0(\varepsilon)$ such that for any $k \geq k_0(\varepsilon)$, any $m \in \mathcal{M}_{\text{mmenu}}(n_B(\varepsilon), n(\varepsilon))$, there exists $m' \in \mathcal{M}_k \cap \mathcal{M}_{\text{menu}}(n_B(\varepsilon), n(\varepsilon))$ such that $d(W_m, W_{m'}) \leq \frac{1}{2}\varepsilon$. It follows that for each $k \geq k_0(\varepsilon)$, any $W \subseteq C^2X$, there is $m \in \mathcal{M}_k$ such that $d(W, W_m) \leq \varepsilon$.

Let $\varepsilon > 0$ be small enough so that the thesis of Lemma 11 holds for $\frac{1}{2}\eta$. Fix $k \geq k_0(\varepsilon)$. Take arbitrary $(u, y_u) \in \mathcal{U} \times [0,1]$. Construct menu of menus W_{u,y_u} of all menus Y such that for each associated menu function $y(u) \geq y_u$. Further, construct menu of menus $W^{\eta}_{u,y_u} = \left\{Y^{\frac{1}{2}\eta} : Y \in W_{u,y_u}\right\}$. Find mechanism $m \in \mathcal{M}_k$ such that $d(W^{\eta}, W_m) \leq \varepsilon$. The latter means that:

- for each $s_B \in S^1_B$, there exists $Y \in W_{u,y_u}$ such that $d\left(Y^{\frac{1}{2}\eta}, Y_m(s_B)\right) \leq \varepsilon$. If y is the menu function associated with Y and $y_m(s_B)$ is associated with $Y_m(s_B)$, then by Lemma 11, we have that for any s_B,$y_m(u; s_B) \geq y(u) - \eta \geq y_u - \eta$,
- for each $Y \in W_{u,y_u}$, there exists $s_B \in S^1_B(m)$ such that $d\left(Y^{\frac{1}{2}\eta}, Y_m(s_B)\right) \leq \varepsilon$, and, if y is the menu function associated with Y and $y_m(s_B)$ is associated with $Y_m(s_B)$, then by Lemma 11, we have that $\Pi(y_m(s_B), \mu) \geq \Pi(y, \mu) - \eta$ for any belief $\mu \in \Delta U$. Therefore, $\max_{s_B} \Pi(y_m(s_B), \mu) \geq \max_{y \in \mathcal{Y}: y(u) \geq y_u} \Pi(y, \mu) - \eta$.

□

Appendix C. Proofs of upper bound of Theorem 1

C.1. Basic equilibrium bound.

Lemma 14. *For each* δ, k,

- $e_A(u) \geq 1 - \delta, e_B \geq \delta(1-\delta)$ for each u and each $(e_A, e_B) \in E^A(\delta, \mathcal{M}_k, \mu)$,

- $e_A(u) \geq \delta(1-\delta), e_B \geq 1-\delta$ *for each* u *and each* $(e_A, e_B) \in E^B(\delta, \mathcal{M}_k, \mu)$.

Proof. Notice that Alice is going to accept menu $\delta\,\mathbb{1}_A + (1-\delta)\,\mathbb{0}_A$, which puts the lower bound on Bob's payoff. Anticipating that, Bob is going to reject any continuation equilibrium that gives him less that $\delta(1-\delta)$. The other case is analogous. □

C.2. **Proof of Lemma 3.** Let $\eta = \frac{1}{5}(\inf h)(1-\gamma)(1-\delta) > 0$. Let k_0 be such that Lemma 12 holds. Suppose that h has $UB(\gamma,\delta)$ property. Fix a belief μ.

Let $E = \left\{(e_A, \mu) : (e_A, e_B) \in E^A(\delta, \mathcal{M}_k, \mu)\right\}$. Suppose that there exists $(e_A, e_B, \mu) \in E$ such that $\forall_{u \in \mathrm{supp}\mu} e_A(u) \geq h(u)$. By the remark after Lemma 2, E is compact, and we can find the equilibrium payoffs and the beliefs (e_A, μ) that are undominated in E the following sense: there is no (e'_A, μ') such that $e_A(u) + \eta < e'_A(u)$ for some $u \in \mathrm{supp}\mu' \subseteq \mathrm{supp}\mu$. Let $y = \max(h, e_A)$ and notice that y is a menu function by Lemma 10. Also, $y(u) = e_A(u)$ for each $u \in \mathrm{supp}\mu$, and, by Lemma 14, $\Pi(y,\mu) = \Pi(e_A,\mu) \geq e_B \geq \delta(1-\delta)$.

By the definition of the UB-property, there exists a menu function y' such that

$$y' \geq (1-\gamma(1-\delta))\,y \geq (1-\gamma(1-\delta))\,e_A, \tag{7}$$

and

$$\gamma\delta\Pi(y',\mu) \geq \Pi(y,\mu) = \Pi(e_A,\mu). \tag{8}$$

By Lemma 12, there exists a mechanism m such that for any $\mu \in \Delta\mathcal{U}$, each μ', each $(e'_A, e'_B) \in E(m,\mu')$, each u, we have

$$e'_A(u) \geq y'(u) - \eta \geq (1-\gamma(1-\delta))\,e_A(u) - \eta \geq \left(1 - \left(1 - \frac{1}{2}(1-\gamma)\right)(1-\delta)\right) e_A(u) - \eta.$$

The last inequality follows from the choice of

$$\eta \leq \frac{1}{4}(1-\gamma)(1-\delta)h(u) \leq \frac{1}{4}(1-\gamma)(1-\delta)e_A(u).$$

Additionally, we have

$$e'_B \geq \Pi(y',\mu) - \eta \geq \frac{1}{\gamma\delta}\Pi(e_A,\mu) - \eta = \frac{1}{\delta}\Pi(e_A,\mu) + \left(\frac{1-\gamma}{\gamma}\right)\frac{1}{\delta}\Pi(e_A,\mu) - \eta > \frac{1}{\delta}\Pi(e_A,\mu),$$

where the choice of η implies that $\frac{1-\gamma}{\gamma}\frac{\Pi(e_A,\mu)}{\delta} \geq \frac{1-\gamma}{\gamma}(1-\delta) > \eta$.

Consider an equilibrium that supports an outcome (y,μ). Let (f,ψ) be an continuation equilibrium outcome with beliefs starting from period 3 after a history such that in period 1, Bob rejects; in period 2, Bob proposes mechanism m with continuation payoffs $(e'_A, e'_B) \in E(m,\mu')$ that is rejected by Alice. (Here, μ' is a belief after the mechanism m is accepted.) Let $A_R \subseteq \text{supp}\mu$ be the set of Alice's types for whom the rejection in period 2 is a (possibly, weak) best response. For each $u \in A_R$, it must be that

$$f(u) \geq \frac{1}{\delta}e'_A(u). \tag{9}$$

Suppose that the rejection in period 2 occurs with a positive probability. If so, then ψ is absolutely continuous wrt. μ, and, in particular, $\text{supp}\psi \subseteq A_R \subseteq \text{supp}\mu$. Because (y,μ) is undominated in E, there must be $u_0 \in \text{supp}\psi$ such that $y(u_0) \geq f(u_0) - 2\eta$. By (7) and (9) ,

$$\begin{aligned} y(u_0) &\geq f(u_0) - 2\eta \geq \frac{1}{\delta}e'_A(u_0) - 2\eta \geq \frac{1}{\delta}\left(1-\left(1-\frac{1}{2}(1-\gamma)\right)(1-\delta)\right)e_A(u_0) - 2\eta \\ &\geq e_A(u_0) + \frac{1}{\delta}\frac{1}{2}(1-\gamma)(1-\delta)e_A(u_0) - 2\eta \\ &\geq e_A(u_0) + (1-\gamma)(1-\delta)\left[\frac{1}{2}e_A(u_0) - \frac{2}{5}(\inf h)\right] > e_A(u_0), \end{aligned}$$

where the last inequality follows from the fact that $e_A(u_0) \geq h(u_0)$.

The contradiction shows that period 2 offer of m is accepted with probability 1. By Lemma 2, $e_B \leq \Pi(e_A, \mu)$. On the other hand, Bob's strategy to reject any offer in period 1 and propose m leads to the expected discounted payoff of $\delta e'_B > \Pi(e_A, \mu)$. But this leads to a contradiction with a choice of (y, μ) as an equilibrium.

C.3. **Proof of Lemma 5.** We assume w.l.o.g. that $v_c \geq v_s$. We consider the following two cases separately:

- $v_c > v_s$, or Bob likes chocolate more than strawberry; this case is further divided into two sub-cases that depend on the offered menu,
- $v_c = v_s$, or Bob is indifferent between chocolate and strawberry.

Because we work with $N = 2$, it is possible and convenient to redefine any function of type $u = (u_c, 1 - u_c)$ (i.e., $l(u)$, $\mathcal{N}_A(u)$, etc.) as a function of the first coordinate (i.e., $l(u_c)$, $\mathcal{N}_A(u_c)$, etc.)

C.3.1. *Case A:* $v_c > v_s$. Assume that $\varepsilon > 0$ is sufficiently small that

$$\mathcal{N}_A(1)(1+\varepsilon) = \frac{1}{2v_c}(1+\varepsilon) \leq 1 \text{ and } v_c > \frac{1}{2-\varepsilon}. \tag{10}$$

Define function h as

$$h(u_c) = \begin{cases} (1-\varepsilon)\mathcal{N}_A(u_c) + \varepsilon, & \text{if } u_c \leq \frac{1}{2} \\ \mathcal{N}_A(1)(1+\varepsilon), & \text{if } u_c rR \geq \frac{1}{2}. \end{cases}$$

Notice that function h is continuous due to the fact that $\mathcal{N}_A\left(\frac{1}{2}\right) = \frac{1}{2}$.

Find $\gamma, \delta < 1$ such that

$$\frac{\gamma^2\delta(1-\delta)}{1-\gamma\delta} \geq \frac{1-\varepsilon}{1+\varepsilon},\ \frac{\varepsilon^2}{1+\varepsilon} > \gamma(1-\delta),\ v_c \geq \frac{1}{2-\varepsilon+\frac{1}{\varepsilon}\gamma(1-\delta)},\ \text{and } \varepsilon > \frac{1-\gamma\delta}{\gamma\delta}. \tag{11}$$

Take any menu function $y \geq h$. Find $u_c^* \in \arg\min_{u\in\mathcal{U}} y(u_c)$ and take $y^* := y(u_c^*) \geq \inf l = \frac{1}{2}(1+\varepsilon)$. We consider separately two sub-cases:

Subcase A1: $y^* < 1 - \frac{1}{\varepsilon}\gamma(1-\delta)$. Construct function

$$y'(u_c) = \begin{cases} 1 - \left(1 + \frac{y^*}{1-y^*}\gamma(1-\delta)\right)(1 - y(u_c)), & \text{if } u_c \leq u_c^*, \\ (1-\gamma(1-\delta))\, y(u_c), & \text{if } u_c \geq u_c^*. \end{cases}$$

We are going to show that (a) y' is a menu function, (b) $y' \geq (1-\gamma(1-\delta))\, y$ and (c) for each type u,

$$\gamma\delta(1 - d(u_c, y')) \geq 1 - d(u_c, y). \tag{12}$$

That shall verify that l has $B(\gamma, \mathcal{U}, \delta_0)$ property.

Ad (a). We start by checking that function y' is convex. First, notice that function y' is continuous at u_c^*, and, as y, minimized at u_c^*. It follows that for each $a \leq u_c^* \leq b$, we have $y'(u_c^*) \leq \frac{b-u_c^*}{b-a} y'(a) + \frac{u_c^*-a}{b-a} y'(b)$. Further, for any $\alpha < \frac{b-u_c^*}{b-a}$, we have

$$\begin{aligned}
& y'(\alpha a + (1-\alpha) b) \\
\leq & \frac{\alpha a + (1-\alpha) b - u_c^*}{b - u_c^*} y'(b) + \frac{b - \alpha a + (1-\alpha) b}{b - u_c^*} y'(u_c^*) \\
\leq & \frac{\alpha a + (1-\alpha) b - u_c^*}{b - u_c^*} y'(b) + \frac{b - \alpha a + (1-\alpha) b}{b - u_c^*} \left[\frac{b - u_c^*}{b-a} y'(a) + \frac{u_c^* - a}{b-a} y'(b)\right] \\
= & \left(\frac{\alpha a + (1-\alpha) b - u_c^*}{b - u_c^*} + \frac{b - \alpha a + (1-\alpha) b}{b - u_c^*} \frac{u_c^* - a}{b-a}\right) y'(b) + \frac{b - \alpha a + (1-\alpha) b}{b-a} y'(a) \\
= & \frac{1}{b - u_c^*} \left(\frac{(\alpha a + (1-\alpha) b)(b - u_c^*) - u_c^* a - ba}{b-a}\right) y'(b) + \frac{b - \alpha a + (1-\alpha) b}{b-a} y'(a) \\
= & \frac{\alpha a + (1-\alpha) b - a}{b-a} y'(b) + \frac{b - \alpha a + (1-\alpha) b}{b-a} y'(a).
\end{aligned}$$

An analogous calculations show the same inequality when $\alpha > \frac{b-u_c^*}{b-a}$. Finally, if $a, b \leq u_c^*$ and $a, b \geq u_c^*$, then $y'(\alpha a + (1-\alpha) b) \leq \alpha y'(a) + (1-\alpha) y'(b)$ for any $\alpha \in [0,1]$, due to the construction of y' as a piecewise-linear transformation of convex y.

Because $y'$is convex, $D_u y'$ is closed for each u. We show that it is non-empty. By Lemma 8, it is enough to check the non-emptiness for $u_c = 0, 1$.

- For any $l \in D_1 y$, we have $(1 - \gamma(1-\delta)) l \in D_1 y$. To see it, observe that the derivative of the convex function is multiplied by a constant $(1-\gamma(1-\delta)) \in (0,1)$ and so obtained affine function satisfy the payoff restriction.
- Let $l \in D_0 y$. Because $1 \geq y \geq h$ and $h(0) = 1$, it must be that $y(0) = l(0) = 1$ and $l(1) \geq \Delta_0 h(1)$, where $\Delta_0 h$ is the affine function tangent to h at 0. By the definition of h, we have $\Delta_0 h(1) = \varepsilon$. Consider affine l' defined by $l'(u_c) = 1 - \left(1 + \frac{y^*}{1-y^*}\gamma(1-\delta)\right)(1 - l(u_c))$. Clearly, l' supports y' at 0. By the construction, $l'(0) = 1$, and

$$1 \geq l'(1) \geq 1 - \left(1 + \frac{y^*}{1-y^*}\gamma(1-\delta)\right)(1-\varepsilon).$$

  Because we work with the case $y^* < 1 - \frac{1}{\varepsilon}\gamma(1-\delta)$, the above is not larger than

$$\geq 1 - (1 + \varepsilon - \gamma(1-\delta))(1-\varepsilon) \geq 1 - (1+\varepsilon)(1-\varepsilon) \geq 0.$$

Lemma 1 implies that y' is a proper menu function.

Ad (b). For $u_c \leq u_c^*$, we have

$$
\begin{aligned}
&1-\left(1+\frac{y^*}{1-y^*}\gamma(1-\delta)\right)(1-y(u_c))-(1-\gamma(1-\delta))\,y(u_c)\\
=&1-\left(1+\frac{y^*}{1-y^*}\gamma(1-\delta)\right)+\left(1+\frac{y^*}{1-y^*}\gamma(1-\delta)-1+\gamma(1-\delta)\right)y(u_c)\\
=&\gamma(1-\delta)\left[\frac{y(u)-y^*}{1-y^*}\right]\geq 0.
\end{aligned}
$$

The claim is immediate for $u_c \geq u_c^*$.

Ad (c). Fix u_c. We have

$$
d(u_c, y') = \begin{cases} 1-\left(1+\frac{y^*}{1-y^*}\gamma(1-\delta)\right)(1-d(u_c,y)), & \text{if } u_c \leq u_c^*,\\ (1-\gamma(1-\delta))\,d(u_c,y), & \text{if } u_c \geq u_c^*. \end{cases}
$$

For $u_c \leq u_c$, we check that

$$
\begin{aligned}
&\gamma\delta\left(1+\frac{y^*}{1-y^*}\gamma(1-\delta)\right)(1-d(u_c,y))-(1-d(u_c,y))\\
=&\left(\frac{y^*}{1-y^*}\gamma^2\delta(1-\delta)-(1-\gamma\delta)\right)(1-d(u_c,y))\\
=&(1-\gamma\delta)\left(\frac{y^*}{1-y^*}\frac{\gamma^2\delta(1-\delta)}{1-\gamma\delta}-1\right)(1-d(u_c,y))\geq 0,
\end{aligned}
$$

where the last inequality follows from the fact that $d(u_c,y) \leq 1$, $\frac{y^*}{1-y^*} \geq \frac{1+\varepsilon}{1-\varepsilon}$ and inequality (11).

For u such that $u_c \geq u_c^*$, notice first that $d(u_c,y) \leq \min(d(u_c^*), d(1))$ due to the fact that function $d(.,y)$ is increasing below v_c and decreasing above that. Moreover, $d(u_c^*,y) = y^* \geq \frac{1}{2}(1+\varepsilon)$ and,

$$
d(1,y) \geq v_c y(1) \geq v_c(1+\varepsilon)\mathcal{N}_A(1) = \frac{1}{2}(1+\varepsilon).
$$

(See (1).) Therefore,

$$\gamma\delta\left(1-\left(1-\gamma\left(1-\delta\right)\right)d\left(u_c,y\right)\right)-\left(1-d\left(u_c,y\right)\right)=d\left(u_c,y\right)\left(1-\gamma\delta\left(1-\gamma\left(1-\delta\right)\right)\right)-\left(1-\gamma\delta\right)$$

$$=\left(1-\gamma\delta\right)\left(\left(1+\frac{\gamma^2\delta\left(1-\delta\right)}{1-\gamma\delta}\right)d\left(u_c,y\right)-1\right)\geq\left(1-\gamma\delta\right)\left(\left(1+\frac{1-\varepsilon}{1+\varepsilon}\right)\frac{1}{2}\left(1+\varepsilon\right)-1\right)\geq 0.$$

Subcase A2: $y^*\geq 1-\frac{1}{\varepsilon}\gamma\left(1-\delta\right)$. Construct function $y'\left(u_c\right)=y\left(u_c\right)-\frac{1-\gamma\delta}{\gamma\delta}\left(1-\varepsilon\right)u_c$. As in the subcase A1, we are going to establish (a), (b), and (c).

Ad (a) Notice that function y' is convex because it is a sum of convex y and an affine function. We check that sets $D_{u_c}y'$ are non-empty for each u_c. Let $l_{u_c}\in D_{u_c}y$ and define $l'_{u_c}\left(u\right)=l_{u_c}\left(u\right)-\frac{1-\gamma\delta}{\gamma\delta}\left(1-\varepsilon\right)u_c$. Notice that $l'_{u_c}\left(0\right)=l_{u_c}\left(0\right)$. Moreover, $l_{u_c}\left(1\right)\geq l_0\left(1\right)$. As in the sub-case A1, we determine that $l_0\left(1\right)\geq\Delta_0 h\left(1\right)=\varepsilon$. Hence, due to (11), we have

$$l_{u_c}\left(1\right)\geq\varepsilon-\frac{1-\gamma\delta}{\gamma\delta}\left(1-\varepsilon\right)\geq\varepsilon-\frac{1-\gamma\delta}{\gamma\delta}>0$$

Ad (b). We have

$$y'\left(u_c\right)-\left(1-\gamma\left(1-\delta\right)\right)y\left(u_c\right)=y\left(u_c\right)\gamma\left(1-\delta\right)-\frac{1-\gamma\delta}{\gamma\delta}\left(1-\varepsilon\right)u_c$$

$$\geq\left(1-\frac{1}{\varepsilon}\gamma\left(1-\delta\right)\right)\gamma\left(1-\delta\right)-\frac{1-\gamma\delta}{\gamma\delta}\left(1-\varepsilon\right)=\gamma\left(1-\delta\right)\left[1-\frac{1}{\varepsilon}\gamma\left(1-\delta\right)-\frac{1-\gamma\delta}{\gamma^2\delta\left(1-\delta\right)}\left(1-\varepsilon\right)\right]$$

$$\geq\gamma\left(1-\delta\right)\left[1-\frac{1}{\varepsilon}\gamma\left(1-\delta\right)-\frac{1}{1+\varepsilon}\right]\geq\frac{\gamma\left(1-\delta\right)}{\varepsilon}\left[\frac{\varepsilon^2}{1+\varepsilon}-\gamma\left(1-\delta\right)\right]\geq 0.$$

where we used the fact that $y\left(u_c\right)\geq y^*\geq 1-\frac{1}{\varepsilon}\gamma\left(1-\delta\right)$, and inequalities (11).

Ad (c). Fix u_c. Notice that $d\left(u_c,y\right)$ is initially decreasing and, then, increasing, and $d\left(1,y\right)\geq v_c y^*$ and $d\left(0,y\right)\geq 1-v_c\left(1-\varepsilon\right)$. (For the first inequality, notice that if $l\in D_1 y$, then it must be that $l\left(1\right)\geq y^*$ and $l\left(0\right)\geq 0$. For the second inequality, notice that if $l\in D_0 y$, then $l\left(0\right)=1$ and, as in part (a) of this case, $l\left(1\right)\geq\Delta_u h=\varepsilon$.)

Because of (11), we have

$$v_c y^* - (1 - v_c(1-\varepsilon)) \geq v_c\left(1 - \frac{1}{\varepsilon}\gamma(1-\delta) + (1-\varepsilon)\right) - 1$$
$$\geq v_c\left(2 - \frac{1}{\varepsilon}\gamma(1-\delta) - \varepsilon\right) - 1 \geq 0,$$

which implies that $d(u_c, y) \geq \min(d(0,y), d(1,y)) = 1 - v_c(1-\varepsilon)$. Further, notice that

$$d(u_c, y') = d(u_c, y) - \frac{1-\gamma\delta}{\gamma\delta}(1-\varepsilon)v_c.$$

Therefore,

$$\gamma\delta\left(1 - \max_{l\in D_{u_c}y'} l(v)\right) - (1 - d(u_c,y)) = \gamma\delta\left(1 - d(u_c,y) + \frac{1-\gamma\delta}{\gamma\delta}(1-\varepsilon)v_c\right) - (1 - d(u_c,y))$$
$$= (1-\gamma\delta)(1-\varepsilon)v_c - (1-d(u_c,y))(1-\gamma\delta) \geq [(1-\gamma\delta)(1-\varepsilon)]v_c - v_c(1-\varepsilon)(1-\gamma\delta) \geq 0.$$

C.3.2. *Case B:* $v_c = v_s = \frac{1}{2}$. In such a case, we define $h(u_c) = (1-\varepsilon)\mathcal{N}_A(u_c) + \varepsilon$. Find $\gamma, \delta < 1$ such that

$$\varepsilon > 1 - \gamma\delta, \frac{\gamma^2\delta(1-\delta)}{1-\gamma\delta} \geq \frac{1-\varepsilon}{1+\varepsilon}. \tag{13}$$

Take any menu function $y \geq h$. In particular, for each u_c

$$y(u_c) \geq (1-\varepsilon)\frac{1}{2} + \varepsilon = \frac{1}{2}(1+\varepsilon). \tag{14}$$

Let

$$y'(u_c) = 1 - \frac{1}{\gamma\delta}(1 - y(u_c)) = \frac{1}{\gamma\delta}y(u_c) - \frac{1-\gamma\delta}{\gamma\delta}.$$

As above, we are going to show (a), (b), and (c).

Ad (a). Function y' is trivially convex as the sum of a (scaled-up) convex function y and a constant. We check that sets $D_{u_c}y'$ are non-empty for each u_c. By Lemma

8, it is enough to check for $u_c = 0, 1$. Let $l_{u_c} \in D_{u_c} y$ and define $l'_{u_c}(u) = \frac{1}{\gamma\delta} l_{u_c}(u) - \frac{1-\gamma\delta}{\gamma\delta}$. We check that affine l'_{u_c} satisfies the required payoff restriction. Notice that $l'_{u_c}(u) \leq l_{u_c}(u) \leq 1$ for each u, including $u = 0, 1$. Because y is convex, $l_{u_c}(0) \geq l_1(0)$ and $l_{u_c}(1) \leq l_0(1)$. Becuase $1 \geq y \geq h$ and $h(0) = h(1) = 1$, it must be that $y(0) = y(1) = 1$ as well. It follows that $l_0(1), l_1(0) \geq \Delta_0 h(1) = \Delta_1 h(0)$, where $\Delta_x h$ is the affine function tangent to h at x. By the definition of h, we have $\Delta_0 h(1) = \varepsilon$. To summarize, $l_{u_c}(0), l_{u_c}(1) \geq \varepsilon$, and

$$1 \geq l'_{u_c}(1) \geq \frac{1}{\gamma\delta}\varepsilon - \frac{1-\gamma\delta}{\gamma\delta} = \frac{1}{\gamma\delta}(\varepsilon - (1-\gamma\delta)) \geq 0,$$

where the last inequality comes from (11). Lemma 1 implies that y' is a proper menu function.

Ad (b). Notice that

$$y'(u_c) - (1-\gamma(1-\delta))\, y(u_c) = \left(\frac{1}{\gamma\delta} - (1-\gamma(1-\delta))\right) y(u_c) - \frac{1-\gamma\delta}{\gamma\delta}$$

$$= \frac{1-\gamma\delta}{\gamma\delta}\left[\left(1 + \frac{\gamma^2\delta(1-\delta)}{1-\gamma\delta}\right) y(u_c) - 1\right] \geq \frac{1-\gamma\delta}{\gamma\delta}\left[\left(1 + \frac{1-\varepsilon}{1+\varepsilon}\right)\frac{1}{2}(1+\varepsilon) - 1\right] \geq 0,$$

where we used (14) and (13).

Ad (c). Fix u_c and notice that $d(u_c, y') = \frac{1}{\gamma\delta} d(u_c, y) - \frac{1-\gamma\delta}{\gamma\delta}$. Hence,

$$\gamma\delta\left(1 - \max_{l \in D_{u_c} y'} l(v)\right) - (1 - d(u_c)) = \gamma\delta\left(1 - \frac{1}{\gamma\delta} d(u_c) + \frac{1-\gamma\delta}{\gamma\delta}\right) - (1 - d(u_c)) = 0.$$

Appendix D. Proofs of lower bound case of Theorem 1

D.1. **Proof of Lemma 6.** Let

$$\eta < \min\left(\frac{5}{11}\frac{1-\gamma}{\gamma}\frac{1-\delta}{\delta}, \frac{1}{2}(1-\gamma)\,\delta\,(1-\delta)^2\right) > 0. \tag{15}$$

Let k_0 be such that Lemma 13 holds for η. Fix $k \geq k_0$, and $\delta \geq \delta_0$. Define functions: for each $u \in \mathcal{U}$ and $\psi \in \Delta\mathcal{U}$,

- $e_A^{\min}(u) = \lim_{k\to\infty} \inf_{(e_A,e_B)\in E^B(\delta,\mathcal{M}_k,\mu) \text{ for some } \mu} e_A(u)$ be the lowest equilibrium payoff of type u across equilibria for any beliefs,
- $g(u,\psi,y_u) = \sup_{y\in\mathcal{Y}:y(u)\geq e_A^{\min}(u)} \Pi(y)$ be the largest possible Bob's payoff given beliefs ψ and subject to the constraint that Alice type u receives at least $e_A^{\min}(u)$.

By Lemma 14, for any ψ and $(f_A, f_B) \in E^B(\delta, \mathcal{M}_k, \psi)$, we have

$$1-\delta \leq f_B \leq g(u,\psi,y_u)\,, \text{ and } \delta(1-\delta) \leq e_A^{\min}(u) \leq f_A(u)\,. \tag{16}$$

Suppose that $h(u) > e_A^{\min}(u)$ for some type u. Find a belief μ and equilibrium outcome $(e_A, e_B) \in E^B(\delta, \mathcal{M}_k, \mu)$ such that $e_A(u) < \min\left(e_A^{\min}(u) + \frac{1}{10}\eta, y_u\right)$. By Lemma 13, there exists a mechanism $m \in \mathcal{M}_k$ such that for each ψ and each $(p_A, p_B) \in E(m,\psi)$,

$$\begin{aligned} p_A(u) &\geq \frac{1}{\gamma\delta} e_A^{\min}(u) - \eta \geq \frac{1}{\gamma\delta}\left(e_A(u) - \frac{1}{10}\eta\right) - \eta \\ &= \frac{1}{\delta}e_A(u) + \frac{1-\gamma}{\gamma}\frac{1}{\delta}e_A(u) - \frac{11}{10}\eta > \frac{1}{\delta}e_A(u)\,, \end{aligned} \tag{17}$$

where the last inequality follows from (15) and (16), and

$$\begin{aligned} p_B &\geq \max_{y'\in\mathcal{Y}:y'(u)\geq\frac{1}{\gamma\delta}e_A^{\min}(u)} \Pi(y',\mu) - \eta \geq (1-\gamma(1-\delta)) \max_{y\in\mathcal{Y}:y(u)\geq e_A^{\min}(u)} \Pi(y,\mu) - \eta \\ &= \delta g(u,\psi,y_u) + (1-\gamma)(1-\delta)\, g(u,\psi,y_u) - \eta > \delta g(u,\psi,y_u)\,, \end{aligned} \tag{18}$$

where the last inequality follows from (15) and (16).

Given equilibrium with payoffs $(e_A, e_B) \in E^B(\delta, \mathcal{M}_k, \mu)$, consider an deviation by Alice, where she rejects whatever was Bob's offer in the first period and proposes mechanism m in the subsequent period. Let ψ be the continuation beliefs and $(f_A, f_B) \in E^B(\delta, \mathcal{M}_k, \psi)$ be the period 3 continuation equilibrium outcome after Bob rejects Alice's offer. By (16), the period 2 present value of rejection is not larger than $\delta f_B \leq \delta g(u, \psi, y_u)$, which, by (18), is strictly smaller than the payoff from accepting m. Hence, Bob is going to accept in equilibrium. But then period 1 discounted Alice's payoff from her deviation to m is, by (17), strictly higher than her equilibrium payoff $e_A(u)$. This contradicts the definition of the equilibrium. The contradiction concludes the proof of the lemma.

D.2. **Proof of Lemma 7.** Choose $\gamma, \delta < 1$ so that

$$\frac{1-\gamma\delta}{\gamma\delta}, 2v_k\frac{1-\gamma\delta}{\gamma\delta} \leq \varepsilon, \frac{\gamma^2\delta(1-\delta)}{1-\gamma\delta} \geq \frac{1-\varepsilon}{1+\varepsilon}. \tag{19}$$

We consider the two cases separately. In each case, we take menu function y, type w st. $y(w) \leq h(w)$ and a belief $\mu \in \Delta\mathcal{U}$,and we define a new function y' such that (a) y' is a menu function, (b) $\gamma\delta y'(w) \geq y(w)$, and (c) $\Pi(y', \mu) \geq (1-\gamma(1-\delta))\Pi(y, \mu)$. For the purpose of the argument, we can assume that for any menu function $\hat{y}$ such that $\hat{y}(w) \geq y(w)$, we have $\Pi(\hat{y}, \mu) \leq \Pi(y, \mu)$ (otherwise, we can replace y by $\hat{y}$).

Part 1: $h(u) = (1-\varepsilon)\frac{1}{2}$. Take any w and any menu function such that $y(w) \leq \frac{1}{2}(1-\varepsilon)$. We can assume that $\Pi(y, \mu) \geq 1 - y(w_1)$; otherwise, we can replace y by $\hat{y}(u) = y(w)$ for each $u \in \mathcal{U}$. Define y': for each u,

$$y'(u) = 1 - (1-\gamma(1-\delta))(1-y(u)) = (1-\gamma(1-\delta))y(u) + \gamma(1-\delta)$$

Ad (a). y' is convex as an affine transformation of a convex function Let $l_u \in D_u y$ and define $l'_u(u') = (1-\gamma(1-\delta)) l_u(u') + \gamma(1-\delta)$ for each u'. Because $l_u(0), l_u(1) \in [0,1]$, and because $\gamma(1-\delta) \in (0,1)$, we have $l'_u(0), l'_u(1) \in [0,1]$. It follows that $l'_u \in D_u y'$ and the sets $D_u y'$ are non-empty.

Ad (b) Observe that $\gamma\delta y'(u) - y(u) = \gamma(1-\delta)[1-y(u)] \geq 0$.

Ad (c). We have $d(y', u) = (1-\gamma(1-\delta)) d(y,u) + \gamma(1-\delta)$. Hence,

$$\begin{aligned}&\Pi(y', \mu) - (1-\gamma(1-\delta))\Pi(y,\mu)\\ =& \int [1 - ((1-\gamma(1-\delta)) d(y,u) + \gamma(1-\delta)) - (1-\gamma(1-\delta))(1-d(y,u))] \, d\mu(u) = 0.\end{aligned}$$

Part 2: $h(u) = (1-\varepsilon)\min\left(\frac{1}{2v_k}, 1\right) u_i$ for some k. Take menu function y and type w st. $y(w) \leq h(w)$. We can assume that

$$\Pi(y,w) \geq 1 - (1-\varepsilon)\min\left(\frac{1}{2v_k}, 1\right) v_k = 1 - (1-\varepsilon)\min\left(\frac{1}{2}, 1\right) \geq \frac{1}{2}(1+\varepsilon);$$

otherwise, we can replace y by $\hat{y}(u) = h(u)$. Define function $y'(u) = y(u) + \frac{1-\gamma\delta}{\gamma\delta} y(w) u_k$ for each u.

Ad (a). y' is convex as a sum of convex y and an affine function. We check that sets $D_u y'$ are non-empty. Let $l \in D_u y$ and define $l'(x) = l(x) + \frac{1-\gamma\delta}{\gamma\delta} y(w) x_k$. Clearly, $l'(u) = y'(u)$ and $l'(x) \leq y'(x)$ for each $x \in \mathcal{U}$. We check that affine l' satisfies the restriction $l'(\omega^n) \in [0,1]$ for each n. Clearly, $l'(\omega^n) \geq 0$. Notice that $l'(\omega^n) = l(\omega^n) + \frac{1-\gamma\delta}{\gamma\delta} y(w) \mathbb{1}_{n=k}$. Thus it is enough to show the claim for $n = k$. But then,

$$l'\left(\omega^k\right) = l\left(\omega^k\right) + \frac{1-\gamma\delta}{\gamma\delta} y(w) \leq y\left(\omega^k\right) + \frac{1-\gamma\delta}{\gamma\delta} \leq 1 - \varepsilon + \frac{1-\gamma\delta}{\gamma\delta} \leq 1.$$

Ad (b) Immediate.

Ad (c). Notice that for each belief μ, $\Pi(y',\mu) = \Pi(y,\mu) - \frac{1-\gamma\delta}{\gamma\delta} y(w) v_k$. Hence, because $y(w) \leq (1-\varepsilon)\min\left(\frac{1}{2v_k},1\right) w_k \leq \frac{1}{2}(b1-\varepsilon)$ and because of (19),

$$\Pi(y',\mu) - (1-\gamma(1-\delta))\Pi(y,\mu) = \gamma(1-\delta)\left(\Pi(y,\mu) - \frac{1-\gamma\delta}{\gamma^2\delta(1-\delta)} y(w) v_k\right)$$

$$\geq \gamma(1-\delta)\left(\frac{1}{2}(1+\varepsilon) - \frac{1+\varepsilon}{1-\varepsilon}(1-\varepsilon)\min\left(\frac{1}{2},v_k\right) w_k\right) \geq \gamma(1-\delta)\left(\frac{1}{2}(1+\varepsilon) - \frac{1}{2}(1+\varepsilon)\right) = 0.$$

Appendix E. Proofs of Section 4

E.1. **Proof of Proposition 1.** The proof has three parts. The first part develops notation. The second part contains two intermediary steps. The last part constructs an equilibrium and verifies the equilibrium condition.

E.1.1. *Preliminaries.* Let $\eta^k = d(X, X(\mathcal{S}_k))$ be the quality of approximation of the space of simple offers with $\mathcal{S}_k$. Let $\left(x^{j,\delta}\right)$ be the Rubinstein's allocation for type u^*, i.e., the outcome of the complete information game of Bob and Alice type u^* with unrestricted (simple) offers $\Gamma^j(\delta, \mathcal{S}, \delta_{u^*})$. Let $\left(x^{j,k,\delta}\right)$ be the Rubinstein's allocation in the restricted game $\Gamma^j(\delta, \mathcal{S}_k, \delta_{u^*})$. Then, $x^{A,k,\delta} \in \arg\max_{x\in\mathcal{S}_k: v(x)\geq \delta v\left(x^{B,k,\delta}\right)} u^*(x)$ and $x^{B,k,\delta} \in \arg\max_{x\in\mathcal{S}_k: u^*(x)\geq\delta u^*\left(x^{A,k,\delta}\right)} v(x)$and $\lim_{k\to\infty} x^{j,k,\delta} = x^{j,\delta}$. It follows that $\lim_k v\left(x^{B,k,\delta}\right) = \mathcal{R}_B^{B,\delta}(u^*) \geq \delta\mathcal{N}_B(u^*)$.

We are going to construct an equilibrium in which Bob accepts any offer of Alice that gives him at least $\delta v\left(x^{B,k,\delta}\right)$. Let $A^{k,\delta} = \left\{x\in\mathcal{S}_k : v(x) \geq \delta v\left(x^{B,k,\delta}\right)\right\}$ be the set of allocations that are acceptable for Bob. For each u, each $\mu \in \mathcal{U}$, let

- $e_A^{A,k,\delta}(u) = \max_{x\in A^{k,\delta}} u(x)$ be the best payoff of Alice type u among all acceptable allocations,
- $x^{A,k,\delta}(u) = \arg\max_{x\in\mathcal{S}_k: u(x)\geq e_A^{A,k,\delta}(u)} v(x)$ be the Bob's optimal allocation among Alice's optimal choices,

- $e_B^{A,k,\delta}(u) = v\left(x^{A,k,\delta}(u)\right)$ the associated Bob's payoff given Alice type u, and
- $e_B^{A,k,\delta}(\mu) = \int e_B^{A,k,\delta}(u)\, d\mu(u)$ be the expected Bob's payoff given beliefs μ.

When Bob makes offer x in his turn, this offer is going to be rejected by each type u of Alice such that $u(x) < \delta e_A^{A,k,\delta}(u)$. For each belief μ, each x, let $p^{x,k}(\mu) = \mu\left(\left\{u : u(x) < \delta e_A^{A,k,\delta}(u)\right\}\right)$ and $\mu^{x,k} = \mu\left(.|u : u(x) < \delta e_A^{A,k,\delta}(u)\right)$ be respectively,the probability of rejection, and the updated belief after offer x is rejected. Let

- $e_B(x) = p^{x,k}(\mu)\, \delta e_B^{A,k,\delta}(\mu^x) + \left(1 - p^{x,k}(\mu)\right) v(x)$ be Bob's expected payoff from making offer x that can be rejected, in which case, he gets continuation payoff $\delta e_B^{A,k,\delta}(\mu^x)$,
- $x^{B,k,\delta}(\mu) = \arg\max_{x \in \mathcal{S}_k} e_B(x)$ be the payoff-maximizing Bob's offer, and
- $e_B^{B,k,\delta}(\mu) = \max_{x \in \mathcal{S}_k} e_B(x)$ be the optimal payoff.

E.1.2. *Intermediary steps.*

Lemma 15. *Fix k, δ and $r \in [0,1]$, $\rho > 0$. Let x^0 be the solution to equations $v(x^0) = r$ and $\frac{x^0_{A,c}}{x^0_{A,s}} = \rho$. Then, for each type u such that $u_c \leq v_c$, $x_0 \in \arg\max_{x: v(x) \geq r, \frac{x_{A,c}}{x_{A,s}} \geq \rho} u(x)$.*

Proof. The result has a simple intuition. Because Alice's u likes chocolate less than Bob, her optimal payoff is achieved when Bob's constrain binds, and the allocation is the least chocolatey as possible. □

The next result shows that Bob's optimal payoff is smaller than $v\left(x^{B,k,\delta}\right)$. In particular, when Alice makes an offer with an expected payoff that is not smaller than $\delta v\left(x^{B,k,\delta}\right)$, Bob will prefer to accept such an offer rather than wait for his period and receive $v\left(x^{B,k,\delta}\right)$

Lemma 16. *For sufficiently high k, any belief μ, $e_B^{B,k,\delta}(\mu) \leq v\left(x^{B,k,\delta}\right)$.*

Proof. To shorten the notation, we denote $x^A = x^{A,k,\delta}, x^B = x^{B,k,\delta}$. Notice that $\lim_k \sup_{u\in\mathcal{U}} \left|e_B^{A,k,\delta}(u) - \delta v\left(x^B\right)\right| = 0$. Let k be high enough so that $\sup_{u\in\mathcal{U}} e_B^{A,k,\delta}(u) \leq v\left(x^B\right)$.

We are going to show that, if k is sufficiently large than, for any offer $x \in \mathcal{S}_k$, $e_B(x) \leq v\left(x^B\right)$. First, suppose that $v(x) \leq v\left(x^B\right)$.Then, by the choice of k,

$$e_B(x) \leq p^{x,k}(\mu)\,\delta e_B^{A,k,\delta}(\mu^x) + \left(1 - p^{x,k}(\mu)\right) v(x) \leq v\left(x^B\right).$$

Next, suppose that $v(x) > v\left(x^B\right)$. We are going to show below that, in such a case, $p^{x,k}(\mu) = 1$, or the offer x is rejected μ-almost surely. If so, $e_B(x) \leq \delta e_B^{A,k,\delta}(\mu^x) \leq v\left(x^B\right)$.

Recall that the offer is rejected by type u if $u(x) < \delta e_A^{A,k,\delta}(u) = \delta u\left(x^{A,k,\delta}(u)\right)$. Let $\rho^A = \frac{x^A_{A,c}}{x^A_{A,s}}$ be the ratio of chocolate to strawberry in Alice's part of allocation x^A.

- Because $u^*(x) < \delta u^*\left(x^A\right)$ (recall that x^B was the largest Bob's payoff from an allocation that led to the payoff of at least $\delta u^*\left(x^A\right)$ for type u^*), offer x is rejected by type u^*.
- Take any u such that $u^*_c \leq u_c$ (which implies $u^*_s \geq u_s$) and suppose that x is less chocolatey for Alice than x^A:

$$\rho = \frac{x_{A,c}}{x_{A,s}} \leq \rho^A. \tag{20}$$

  Then, we check that

$$\frac{u(x)}{\delta e_A^{A,k,\delta}(u)} \leq \frac{u(x)}{\delta u(x^A)} \leq \frac{u^*(x)}{\delta u^*(x^A)} < 1.$$

  (Indeed, the first inequality comes from the fact that $e_A^{A,k,\delta}(u) \geq u\left(x^A\right)$. The second inequality is a consequence of the fact that u^*likes chocolate less than

u: after some algebra, it is equivalent to

$$\frac{u_c^*}{u_s^*}\rho+\frac{u_c}{u_s}\rho^A+\left(\frac{u_c}{u_s}\frac{u_c^*}{u_s^*}\rho\rho^A+1\right)\geq\frac{u_c^*}{u_s^*}\rho'+\frac{u_c}{u_s}\rho+\left(\frac{u_c}{u_s}\frac{u_c^*}{u_s^*}\rho\rho^A+1\right).$$

After subtracting the terms in the bracket, we obtain $\frac{u_c^*}{u_s^*}\rho+\frac{u_c}{u_s}\rho^A\geq\frac{u_c^*}{u_s^*}\rho^A+\frac{u_c}{u_s}\rho$ which holds due to the fact that $0\leq\frac{u_c^*}{u_s^*}<\frac{u_c}{u_s}$.) It follows that type u rejects x.

- From now on, we assume that x is strictly more chocolatey than x^A. Consider the case $u_c\leq v_c$, i.e., Alice likes chocolate less than Bob. Let x^0 be the solution to equations $v\left(x^0\right)=v\left(x^B\right)$ and $\frac{x^0_{A,c}}{x^0_{A,s}}=\rho^A$. Then, $v\left(x\right)\geq v\left(x^B\right)$ and $\frac{x_{A,c}}{x_{A,s}}>\rho^A$, and, by Lemma 15, we have $u\left(x\right)\leq u\left(x_0\right)$. But, then, $u\left(x\right)\leq u\left(x_0\right)\leq\delta e_A^{A,k,\delta}\left(u\right)$ due to the fact that x_0 satisfies (20), and it falls under the previous case.
- Finally, consider the case $u_c\geq v_c$, Alice likes chocolate more than Bob. Then, for all sufficiently high k, $\delta v\left(x^B\right)=v\left(x^A\right)+O\left(\eta^k\right)>\frac{1}{2}$. In the same time, because types A receive substantially less than their Rubinstein's payoffs, $\dot{\lim}_{k\to\infty}\frac{\max_{x:v(x)\geq v\left(x^B\right)}u(x)}{\max_{x:v(x)\geq v\left(x^A\right)}u(x)}<\delta$. It follows that fur sufficiently high k, $u\left(x\right)<\delta e_A^{A,k,\delta}\left(u\right)$ (uniformly across u st. $u_c\geq v_c$).

□

E.1.3. *Equilibrium.* Let Δ^* be the set of beliefs μ such that $u_s^*=\arg\max_{u\in\text{supp}\mu}u_s$. Then, $\delta_{u^*}\in\Delta^*$.For each $\mu\in\Delta^*$, we are going to construct equilibrium with payoffs $\left(e_A,e_B\right)\in E^B\left(\delta,\mathcal{M}_k,\mu\right)$ such that $e_B\left(u\right)\geq\delta v\left(x^{B,k,\delta}\right)$, and $e_A\left(u\right)\leq\max_{x\in\mathcal{S}_k:v(x)\geq\delta v\left(x^{B,k,\delta}\right)}u\left(x\right)$.

For any belief $\mu\in\Delta^*$, we construct the following strategies and belief-updating, and we verify that they form an equilibrium using the one-shot deviation strategy:

- We say that a history is good if Alice has always proposed in set $A^{k,\delta}$. After a not good history, Bob's beliefs are fixed at δ_{u^*}. The continuation behavior is expected to be as in the equilibrium of the complete information game against type u^*:

- If Alice's turn to make an offer, the expected payoffs are $\left(e_A^{A,k,\delta}(.), v\left(x^{A,k,\delta}\right)\right)$.
- If Bob's turn to make an offer, the expected payoffs are $\left(\left(\max\left(x^{B,k,\delta}, \delta e_A^{A,k,\delta}(.)\right)\right)_{u\in\mathcal{U}}, v\left(x^{B,k}\right.\right.$
- Alice always proposes in $A^{k,\delta}$. Any other offer is rejected. Type u of Alice rejects Bob's offer only if she strictly prefers to wait to the next period and receive $\delta e_A^{A,k,\delta}(u)$. (In particular, type u^* accepts the offer). Bob always accepts offer in $A^{k,\delta}$.
- Clearly, the behavior in the continuation game after not good history is an equilibrium.

• Let $\mu(h)$ be a belief after some good history h.
  - If it is his turn, Bob offers $x^{B,k,\delta}(\mu(h))$.
    * The expected payoffs from offer x are $p^{x,k}(\mu)\,\delta e_B^{A,k,\delta}(\mu^x)+\left(1-p^{x,k}(\mu)\right)v\left(x_B^{B,k,\delta}\right)$. Hence, the choice of $x^{B,k,\delta}(\mu(h))$ as the maximum is a best response.
  - After Bob's offer x, Alice type u accepts it only if $u(x) \leq \delta e_B^{A,k,\delta}(u)$. If she rejects the offer, the beliefs are updated to $\mu^x(h)$.
    * The expected payoffs after she accepts are $u(x)$. If she rejects, the payoffs are $\delta e_B^{A,k,\delta}(u)$. Hence, her choice is a best response.
  - If her turn, Alice type u makes an offer $x^{A,k,\delta}(u)$.
    * the expected payoff from offer $x \in A^{k,\delta}$ is $u(x)$.
    * the expected payoff from offer $x \notin A^{k,\delta}$ are $\delta u\left(x^{B,k,\delta}\right) < u\left(x^{B,k,\delta}\right) \leq \max_{x\in A^{k,\delta}} u(x)$. Hence, it is a best response for any type of Alice to choose (one of) the best(s) outcome in $A^{k,\delta}$.
    * If Alice's chooses $x \notin A^{k,\delta}$, the beliefs change to δ_{u^*} (this can be justified in a sequential equilibrium by appropriately chosen

- Bob accepts any offer in $A^{k,\delta}$ and rejects any other offer.
  - The expected payoff from accepting offer $x \in A^{k,\delta}$ are $v(x) \geq \delta v\left(x^{B,k,\delta}\right)$, and the beliefs are potentially updated to $\mu(h,x)$ following Alice's choice. The expected payoff from rejecting the offer is $\delta e_B^{B,k,\delta}(\mu(h,x))$. By Lemma 16, the latter is smaller than the former, and accepting is a best response.
  - The expected payoffs from accepting an offer $x \notin A^{k,\delta}$ are $v(x) < \delta v\left(x^{B,k,\delta}\right)$. Because any such an offer leads to beliefs δ_{u^*}, the left-hand side of the inequality is equal to the expected and discounted payoff from waiting till the next period. Thus, rejecting x is a best response.

E.2. **Proof of Theorem 2.** The lower bound on Alice payoffs is a consequence of Lemma 7.

We show the bound on Bob's payoff. For each Alice type u, define $u(v) = \max_{x:v(u)\geq v} u(x)$ be the largest payoff of type u that is consistent with Bob receiving at least v. For each $v \in [0,1]$, let $Y(v)$ be a menu $Y(v) = \{x : v(x) \geq v\}$.

The approximation $\mathcal{M}_k \to \mathcal{M}$ ensures that

$$\eta_k := \sup_{Y\in\mathcal{M}_{\text{menu}}} \min_{Y_k\in\mathcal{M}_{\text{menu}}\cap\mathcal{M}_k} d(Y, Y_k) \to 0.$$

Let $Y_k \in \mathcal{M}_k \cap \mathcal{M}_{\text{menu}}$ be the sequences of menus chosen in the minimum part of the above expression.

Let

$$e_B^{A,k,\delta} = \inf\left\{e_B : (e_A, e_B) \in E^A(\delta, \mathcal{M}_k, \mu) \text{ for any } \mu \in \Delta\mathcal{U}\right\}.$$

be the lowest equilibrium payoff across all possible beliefs in the game in which Alice makes the first offer. We are going to show that for each $\varepsilon > 0$, there is k_0 sufficiently high so that for all $k \geq k_0$, $e_B^{A,k,\delta} \geq \frac{1}{1+\delta} - \varepsilon$.

In any equilibrium of the game where Alice makes the first offer, Bob's expected payoff is not lower than $e_B^{A,k,\delta}$. It cannot be that all Alice's types u receive payoffs that are strictly higher than $u\left(e_B^{A,k,\delta}\right)$. Hence, a positive-measure fraction of them must accept any Bob's offer that is strictly higher than $\delta u\left(e_B^{A,k,\delta}\right)$. But, in a similar fashion to the proof of Lemma 4, we can show that all Alice types should accept any menu with payoffs described by menu function $y(u) > \delta u\left(e_B^{A,k,\delta}\right)$. (If it is rejected by some types, than positive fraction of them would receive tomorrow's payoffs that are lower than $u\left(e_B^{A,k,\delta}\right)$. But then, a rejection would not be a best response.) Due to linearity,

$$\begin{aligned}\delta u\left(e_B^{A,k,\delta}\right) &= \delta \max_{x:v(x)\geq e_B^{A,k,\delta}} u(x) = \max_{x:v(x)\geq e_B^{A,k,\delta}} u\left(\delta x + (1-\delta)\,\mathbf{0}_A\right) \\ &= \max_{x:v(\delta x+(1-\delta)\mathbf{0}_A)\geq \delta e_B^{A,k,\delta}+1-\delta} u\left(\delta x + (1-\delta)\,\mathbf{0}_A\right) \\ &\leq \max_{x:v(x)\geq \delta e_B^{A,k,\delta}+1-\delta} u(x) = u\left(\delta e_B^{A,k,\delta} + 1 - \delta\right).\end{aligned}$$

(The inequality comes from the fact that the set of allocations is convex, and for each $x \in X$, $\delta x + (1-\delta)\,\mathbf{0}_A \in X$.) We conclude that Alice accepts any menu that contains menu $Y\left(\delta e_B^{A,k,\delta} + 1 - \delta\right)$ in its interior.

On the contrary, suppose that $e_B^{A,k,\delta} < \frac{1}{1+\delta} - \varepsilon$. Then, there exists an equilibrium of the game where Alice makes the first offer with Bob's expected payoffs $e_B \leq e_B^{A,k,\delta} + \eta_k$. Consider a deviation, where Bob rejects any Alice's offer, and, instead, proposes a menu $Y_k(x)$. The above paragraph implies that such menu is accepted for sure if $x \geq \delta e_B^{A,k,\delta} + 1 - \delta - 2\eta_k$. Bob's deviation is profitable if $\delta(x - \eta_k) \geq e_B \geq e_B^{A,k,\delta} + \eta_k$.

The two inequalities can be satisfied simultaneously if $e_B^{A,k,\delta} \leq \frac{\delta}{1+\delta} - 3\frac{1}{1-\delta^2}\eta_k$. Take k_0 such that for all $k \geq k_0$, $\eta_k (1-\delta)^2 \leq \varepsilon$.

E.3. **Proof of Proposition 2.**

E.3.1. *Preliminary observations.*

Lemma 17. *For each $y \in \mathcal{Y}$ and $\mu \in \Delta\{\tau_1, \tau_2\}$, if $\Pi(y, \mu) > \frac{2}{3}$, then there is i such that $y(\tau_i) \leq \frac{2}{3}$ and if $\Pi(y, \mu) > \delta\frac{2}{3}$, then, there is i such that $y(\tau_i) \leq \frac{1}{\delta}\frac{2}{3}$.*

Proof. Let $b(a, u) = \max_{x \in X: u(x) \geq a} v(x)$ be the maximal payoff of Bob given that Alice type y gets payoff a. Then, $b(., u)$ is concave and decreasing and $b\left(\frac{2}{3}, \tau_i\right) = \frac{2}{3}$ for each i and $b\left(\frac{2}{3}, \tau\right) = 1 - \frac{4}{9}$. Also, $\Pi(y, \mu) \leq \sum_u \mu(u) b(y(u), u)$.The first claim follows from the fact that, if $\mu \in \Delta\{\tau_1, \tau_2\}$, and $y(\tau_i) \geq \frac{2}{3}$ for both i, then the above implies that $\Pi(y, \mu) \leq \frac{2}{3}$. The second claim is analogous. □

Lemma 18. *For each $y \in \mathcal{Y}$, if $y(\tau_i) \leq \frac{2}{3}$ for $i = 1, 2$, then $y(\tau) \leq \frac{2}{3}$. Similarly, if $y(\tau_i) \leq \frac{1}{\delta}\frac{2}{3}$ for $i = 1, 2$, then $y(\tau) \leq \frac{1}{\delta}\frac{2}{3}$.*

Proof. The claims follow from the fact that, due to the concavity of menu function y, we have $y(\tau) \leq \frac{1}{2}(y(\tau_1) + y(\tau_2))$. □

Recall that $E(m, .) : \Delta\mathcal{U} \rightrightarrows \mathcal{Y} \times [0, 1]$ is the correspondence of equilibrium outcomes $(e_A, e_B) \in E(\mu; m)$ of m with initial beliefs μ. Standard arguments show that $E(m, .)$ is a non-empty-valued, and u.h.c. correspondence. Additionally, because of the public randomization, $E(m, \mu)$ is convex. Hence, $E(m, .)$ is a Kakutani correspondence.

The next two result presents two binary divisions of the space of all mechanisms.

Lemma 19. *For each mechanism m, there are $\mu^A(m) \in \Delta\{\tau_1, \tau_2\}$ and $\left(e_A^A(m), e_B^A(m)\right) \in E(m, \mu(m))$ such that either*

(1) $e_B^A(m) \leq \delta\frac{2}{3}$, *or*

(2) $e_A^A(\tau_1; m), e_A^A(\tau; m), e_A^A(\tau_2; m) \leq \frac{1}{\delta}\frac{2}{3}$.

Let $\mathcal{M}_1^A \subseteq \mathcal{M}$ denote the set of mechanisms that satisfy the first condition.

Proof. Take mechanism $m \notin \mathcal{M}_1^A$, and define set $E \subseteq \Delta\{\tau_1, \tau_2\} \times \mathcal{Y} \times [0,1]$ of tuples (μ, e_A, e_B) such that $(e_A, e_B) \in E(m, \mu)$. Set E is compact and connected as a graph of a Kakutani correspondence. Let $P_i \subseteq E$ be the set of all tuples $(\mu, e_A, e_B) \in E$ such that $e_A(\tau_i) \leq \frac{1}{\delta}\frac{2}{3}$. Set P_i is a closed subset of a compact set, hence compact. Moreover, by the first part of Lemma 17, $E = P_1 \cup P_2$. Because E is connected, the intersection of the two sets is non-empty. Take $(\mu, e_A, e_B) \in P_1 \cap P_2$. By the construction, $e_A(\tau_1), e_A(\tau_2) \leq \frac{1}{\delta}\frac{2}{3}$. Lemma 18 implies that $e(\tau) \leq \frac{1}{\delta}\frac{2}{3}$. □

Lemma 20. *For each mechanism m and each belief $\mu \in \Delta\{\tau_1, \tau, \tau_2\}$, there is an acceptance probability $\alpha^B(m, \mu)$, beliefs $\mu_\alpha^B(m, \mu), \mu_r^B(m, \mu) \in \Delta\{\tau_1, \tau, \tau_2\}$ and $\left(e_A^B(m, \mu), e_B^B(m, \mu)\right) \in E(m, \mu_\alpha(m, \mu))$ such that $\alpha^B \mu_\alpha^B + \left(1 - \alpha^B\right)\mu_r^B = \mu$, and for each $u \in \{\tau_1, \tau, \tau_2\}$,*

(1) *if $e_A^B(u; m, \mu) > \frac{2}{3}$, then $\left(1 - \alpha^B(m, \mu)\right)\mu_r(u; m, \mu) = 0$,*

(2) *if $e_A^B(u; m, \mu) < \frac{2}{3}$, then $\alpha^B(m, \mu)\mu_\alpha(u; m, \mu) = 0$.*

Proof. Consider a mechanism m' in which, first, Alice chooses whether to play mechanism m or menu Y^B, and second, the chosen mechanism is implemented. Such a mechanism has an equilibrium strategies. The probability of choosing m is denoted as $\alpha^B(m, \mu)$. Let μ_α^B denote the conditional beliefs after choosing m, and let μ_r^B denote the conditional beliefs after choosing Y^B. Finally, let $\left(e_A^B(.), e_B^B\right)$ denote the payoffs in the (sub) mechanism m. □

E.3.2. *Proof of Proposition 2.* We describe the equilibrium, and beliefs. We need to consider four type of histories h. In each case, we denote the beliefs as $\mu(h)$:

- Alice turn to make an offer:
  - In equilibrium, Alice offers Y^A. The continuation payoffs are $\frac{1}{\delta}\frac{2}{3}$ for each type of Alice $u \in \{t_1, \tau, \tau_2\}$, and $\left(\mu\left(\tau_1, \tau_2|h\right)\frac{1}{\delta}\frac{2}{3} + \mu\left(\tau|h\right)\left(1 - \frac{1}{\delta}\frac{4}{9}\right)\right)$ for Bob.
  - If Alice offers a mechanism $m \neq Y^A, m \notin \mathcal{M}_1^A$, the beliefs are updated to $\mu\left(m\right)$. The continuation payoffs are $e_A^A\left(.|m\right) \leq \frac{1}{\delta}\frac{2}{3}$ for all types of Alice.
  - In all other cases, the beliefs are updated to $\mu\left(m\right)$ and Alice's continuation payoffs are $\delta\frac{2}{3} < \frac{1}{\delta}\frac{2}{3}$.
  - Hence, offeringY^A is Alice's best response.
- Bob's turn to accept:
  - If Alice proposed Y^A, Bob accepts with continuation payoffs $\frac{1}{\delta}\frac{2}{3}$ for each type of Alice $u \in \{t_1, \tau, \tau_2\}$, and $\left(\mu\left(\tau_1, \tau_2|h\right)\frac{1}{\delta}\frac{2}{3} + \mu\left(\tau|h\right)\left(1 - \frac{1}{\delta}\frac{4}{9}\right)\right)$ for Bob.
    * If Bob rejects, his discounted continuation payoffs are equal to $\mu\left(\tau_1, \tau_2|h\right)\frac{2}{3} + \mu\left(\tau|h\right)\delta\left(1 - \frac{4}{9}\right)$. Notice that $\delta\left(1 - \frac{4}{9}\right) < 1 - \frac{1}{\delta}\frac{4}{9}$ for sufficiently high δ. Hence, Bob's behavior is a best response.
  - If Alice proposes a mechanism $m \neq Y^A, m \notin \mathcal{M}_1^A$, Bob accepts and the continuation equilibrium has payoffs $e_A^A\left(.|m\right)$ for types of Alice and $e_B^A \geq \delta\frac{2}{3}$ for Bob.
    * If Bob rejects, his discounted continuation payoffs from the game with beliefs $\mu\left(m\right)$ are equal to $\delta\frac{2}{3}$. Hence, Bob's behavior is a best response.
  - In all other cases, Bob rejects the offer, which leads to the discounted payoffs $\delta\frac{2}{3}$.

      * If Bob accepts m, an arbitrary equilibrium of m is played, which leads to payoffs $e_B \leq \delta \frac{2}{3}$. Thus, accepting is a best response.

- Bob's turn to make an offer:
  - In equilibrium, Bob offers Y^B. The continuation payoffs are $\frac{2}{3}$ for each type of Alice $u \in \{t_1, \tau, \tau_2\}$, and $\left(\mu(\tau_1, \tau_2|h) \frac{2}{3} + \mu(\tau|h)\left(1 - \frac{4}{9}\right)\right)$ for Bob.
  - If Bob offers a mechanism $m \neq Y^B$, the continuation payoffs are for Alice are $y(u) = \max\left(\frac{2}{3}, e_A^B(u; m, \mu(h))\right)$ for each $u \in \{\tau_1, \tau, \tau_2\}$ and note more than $\Pi(y, \mu)$ for Bob. By the first part of Lemma 17, $\Pi(y, \mu) \leq \frac{2}{3}$.
  - Hence, offeringY^B is Bob's best response.
- Alice's turn to accept:
  - If Bob proposed Y^B, Alice accepts it with continuation payoffs $\frac{2}{3}$ for each type of Alice $u \in \{t_1, \tau, \tau_2\}$, and $\left(\mu(\tau_1, \tau_2|h) \frac{2}{3} + \mu(\tau|h)\left(1 - \frac{4}{9}\right)\right)$ for Bob.
      * If Alice rejects, the discounted continuation payoffs are $\frac{2}{3}$ for each of her type. Hence, accepting is a best response.
  - If Bob proposes a mechanism $m \neq Y^B$, each type of Alice decides whether to accept or reject, depending on which choice maximizes her payoffs. If she accepts, the beliefs are updated to $\mu(m, \mu(h))$ and an equilibrium of m with payoffs $\left(e_A^B(m, \mu(h)), e_B^B(m, \mu(h))\right)$ is played. If she rejects, her discounted continuation payoffs are equal to $\frac{2}{3}$ for each type. Thus, her best response payoffs are equal to $\max\left(\frac{2}{3}, e_A^B(u; m, \mu(h))\right)$.

References

Doval, L. and V. Skreta (2018): "Mechanism Design with Limited Commitment," SSRN Scholarly Paper ID 3281132, Social Science Research Network, Rochester, NY.

Grossman, S. J. and M. Perry (1986): "Perfect sequential equilibrium," *Journal of Economic Theory*, 39, 97–119.

Harsanyi, J. C. and R. Selten (1972): "A Generalized Nash Solution for Two-Person Bargaining Games with Incomplete Information," *Management Science*, 18, 80–106.

Inderst, R. (2003): "Alternating-offer bargaining over menus under incomplete information," *Economic Theory*, 22, 419–429.

Jackson, M. O., H. Sonnenschein, Y. Xing, C. Tombazos, and O. Al-Ubaydli (2018): "Negotiation as the Art of the Deal," SSRN Scholarly Paper ID 3153853, Social Science Research Network, Rochester, NY.

Maskin, E. and J. Tirole (1990): "The principal-agent relationship with an informed principal: The case of private values," *Econometrica: Journal of the Econometric Society*, 379–409.

Myerson, R. B. (1984): "Two-Person Bargaining Problems with Incomplete Information," *Econometrica*, 52, 461–487.

Myerson, R. B. and P. J. Reny (2015): "Sequential equilibria of multi-stage games with infinite sets of types and actions," *manuscript, University of Chicago.*

Peski, M. (2019): "Bargaining over Heterogeneous Good with Structural Uncertainty," .

Rubinstein, A. (1982): "Perfect equilibrium in a bargaining model," *Econometrica*, 50.

——— (1985): "A Bargaining Model with Incomplete Information About Time Preferences," *Econometrica*, 53, 1151–1172.

Selten, R. (1975): "Reexamination of the perfectness concept for equilibrium points in extensive games," *International Journal of Game Theory*, 4, 25–55.

SEN, A. (2000): "Multidimensional Bargaining Under Asymmetric Information," *International Economic Review*, 41, 425–450.

SKRETA, V. (2006): "Sequentially Optimal Mechanisms," *The Review of Economic Studies*, 73, 1085–1111.

STRULOVICI, B. (2017): "Contract Negotiation and the Coase Conjecture: A Strategic Foundation for Renegotiation-Proof Contracts," *Econometrica*, 85, 585–616.

WANG, G. H. (1998): "Bargaining over a Menu of Wage Contracts," *The Review of Economic Studies*, 65, 295–305.