COMPARATIVE STATICS ON PRODUCT QUALITY FOR OLIGOPOLY

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Section 1 Introduction

The purpose of this paper is to explore the effects of demand shocks on product quality in oligopoly by developing a model where firms set prices and quality. One may be interested in studying the effects of demand shocks on quality due to its possible welfare and policy implications. For instance, in order to predict the trends of COVID-19, policymakers may wish to know the relationship between the resulting soar in demand for face masks and the selling of low-quality or even fake face masks (Fan, 2020), which raises the risk of catching the disease. One may be interested in conducting the study in oligopoly since it is probably one of the more realistic market structures.

While characterizing the equilibrium in different market structures, papers such as Brems (1957), De Vany and Saving (1983) and Chioveanu (2012) directly or indirectly included quality as one of firms' choice variables, but without performing analysis with demand shift parameters. Papers such as Dixit (1986), Quirmbach (1988) and Jinji (2014) did the latter for oligopoly but without doing the former. Sweeny (1974) studied the equilibrium price changes at different quality levels in response to changes in the quality distribution of stocks. However, I would like to see how such quality changes happened in the first place while assuming demand shocks being the origin. Hence, to my knowledge, the research question implied in this paper is not explicitly tackled in the literature.

Section 2 presents the model. Section 3 shows whether firms' price and quality decisions are strategic complements or substitutes, which is helpful in decomposing the effects of demand shocks in Section 4. Section 4 then analyses the direction and magnitude of the change in product quality due to demand shocks. Section 5 discusses how demand shocks affect average product quality in the market.

Section 2 The model[2]

The following assumptions are made:

- Asm.1. Products in a market J are substitutes.
- Asm.2. Consumers agree over the preference ordering of some characteristics of a product in J and of the mix of those characteristics[3], which is quantified as a quality index s[4] and $s \in \mathbb{R}_+$. A product with a higher s is preferable to one with a lower s, *ceteris paribus*.
- Asm.3. There are $N-$firms in J and firm i, for $i = 1, \dots, N$ sets one[5] price $p_i, \forall p_i \in \mathbb{R}_+$ and choose one combination of characteristics for its products such that the quality index of its products is s_i.
- Asm.4. Price and quality are independent of each other on the demand and supply side[6].
- Asm.5. Firms behave non-cooperatively.

[2] The model is an extension of the differentiated product model proposed by Dixit (1979).
[3] This is vertical differentiation. See Tirole, 2015, p.96 for details.
[4] Hereafter, "quality" or "product quality" means quality index. See (Wolinsky, 1983)
[5] Price or quality discrimination in oligopoly is not considered but possible. For the former, see (Asplund, Eriksson, and Strand, 2008)
[6] This might not be the case. Consumers use prices as a signal for product quality.

Asm.6. Some technical assumptions.[7]

Following the above assumptions, let the demand equation of firm i be

$$q_i = f_i(\boldsymbol{p}, \boldsymbol{s}) \tag{1}$$

where q_i is quantity demanded for firm i's products, $\boldsymbol{p} = [p_1, \ldots, p_i, \ldots, p_N]$ and $\boldsymbol{s} = [s_1, \ldots, s_i, \ldots, s_N]$. A firm's quantity demanded is decreasing (increasing) in its own price (quality) and is increasing (decreasing) in other firms' prices (quality) by Asm.1. Hence, $\partial f_i/\partial p_i < 0, \partial f_i/\partial s_i > 0$ and $\partial f_i/\partial p_j > 0, \partial f_i/\partial s_j < 0$, for $i \neq j$. Firm i's cost c_i is

$$c_i = g_i(q_i, s_i) \tag{2}$$

where $\partial g_i/\partial q_i \geq 0$ and $dc_i/ds_i \geq 0$[8]. Following Asm.5, firm i maximises its profit $\Pi_i = p_i q_i - c_i$ by choosing p_i and s_i such that (3) ∩ (4) where

$$\partial \Pi_i/\partial p_i = 0 \tag{3}$$

$$\partial \Pi_i/\partial s_i = 0 \tag{4}$$

Solving a system of $2N$ equations, i.e. $(\partial\Pi_1/\partial p_1 = 0 \cap \partial\Pi_1/\partial s_1 = 0) \cap (\partial\Pi_2/\partial p_2 = 0 \cap \partial\Pi_2/\partial s_2 = 0) \cap \ldots \cap (\partial\Pi_N/\partial p_N = 0 \cap \partial\Pi_N/\partial s_N = 0)$ yields optimal choices p_i^* and s_i^* where $p_i^* = \arg\max_{p_i} \Pi_i$, $s_i^* = \arg\max_{s_i} \Pi_i$.

Section 3 Strategic complements and substitutes

Analyzing how firm i respond to firm j's changing its price or quality, for $i \neq j$, comes in useful later.

For firm i, Marginal cost *w.r.t.* output ("MC_i^q"), price ("MC_i^p") and quality ("MC_i^s") are respectively the effects on cost of increasing output, price and quality by one unit. Marginal revenue *w.r.t.* output ("MR_i^q"), price ("MR_i^p") and quality ("MR_i^s") are respectively the effects on revenue of increasing output, price and quality by one unit.

From (3) and (4),

$$\underbrace{\underbrace{p_i^*\, \partial f_i/\partial p_i{}^*}_{loss} + \underbrace{q_i^* \times 1}_{gain}}_{MR_i^p} = \underbrace{\partial g_i/\partial q_i{}^*\, \partial f_i/\partial p_i{}^*}_{MC_i^p} \tag{5}$$

$$\underbrace{p_i^*\, \partial f_i/\partial s_i{}^*}_{MR_i^s} = \underbrace{\partial g_i/\partial q_i{}^*\, \partial f_i/\partial s_i{}^* + \partial g_i/\partial s_i{}^*}_{MC_i^s} \tag{6}$$

where $\partial y/\partial x^* = z(x^*)$if $\partial y/\partial x = z(x)$. Thus, when firms are profit-maximizing, $MR_i^p = MC_i^p$ and $MR_i^s = MC_i^s$[9].

[7] See Appendix A.
[8] $dc_i/ds_i \geq 0 \Leftrightarrow \partial g_i/\partial q_i\, \partial f_i/\partial s_i + \partial g_i/\partial s_i \geq 0$.
[9] $MC_i^q = MC_i^q$ which is a condition for usual profit maximisation models is not necessarily true here since firms do not choose output.

Proposition 1. *Firm* i*'s optimal price and quality are strictly increasing in firm* j*'s price and are strictly decreasing in firm* j*'s quality if for firm* i*, Asm.7.* MC_i^q *is constant; Asm.8.* MC_i^s *is strictly increasing in quality and weakly decreasing in price; Asm.9, the decrease in quantity demanded when its price increases by one unit, which is* $\partial f_i/\partial p_i{}^*$ *in* (5) *is constant. Asm.10, the increase in quantity demanded when its quality increases by one unit, which is* $\partial f_i/\partial s_i{}^*$ *in* (6) *is constant.*

Mind that by Asm.7 and Asm.9, MC_i^p is constant since it is the effect on quantity demanded or output of increasing price multiplied by MC_i^q, or $\partial g_i/\partial q_i{}^* \, \partial f_i/\partial p_i{}^*$ in (5).

MR_i^p consists of two parts. If firm i increases its price by one unit, some consumers will buy less products from firm i. This creates a loss in revenue, which is $p_i^* \, \partial f_i/\partial p_i{}^*$ in (3). Some consumers continue to buy the same quantity of products from firm i as before, which is q_i^* in (3). This generates a gain in revenue, which is $q_i^* \times 1$ in (3) since they will pay an additional one unit of price.

Consider firm j's increasing its price. Quantity demanded for firm i, which is q_i^* in (3), increases as a result since $\partial f_i/\partial p_j > 0 \Rightarrow \partial q_i^*/\partial p_j > 0$. Thus, MR_i^p increases since more consumers are willing to pay the additional price. In order to have $MR_i^p = MC_i^p$ given that MC_i^p is fixed, firm i must increase its price, which makes the loss part in MR_i^p bigger, hence MR_i^p smaller. Given that firm i's price increases, MR_i^s which is $p_i^* \, \partial f_i/\partial s_i{}^*$ increases. In order to have $MR_i^s = MC_i^s$, firm i must increase quality, which increases MC_i^s.

The proof for other cases in the proposition is corollary of the previous arguments.

Section 4 Comparative statics in a near-linear case

The outcomes of further comparative statics are difficult to interpret. I shall thus turn to a near-linear case involving two firms with linear demands[10]

$$q_i^L = f_i^L(\boldsymbol{p}, \boldsymbol{s}) = a_i - b_i p_i + c p_j + d_i s_i - e_i s_j \tag{7}$$

for $i, j = 1,2, i \neq j$ and quadratic costs

$$c_i^L = g_i^L(q_i, s_i) = m_i q_i + n_i s_i^2 \tag{8}$$

which yields some clear results. Mind that in this near-linear case, Asm7-10 holds thus the results in Section 3 can be utilised. According to the assumptions in Section 2, $a_i, b_i, c, d_i, e_i, m_i, n_i > 0$. The intercept $a_i > 0$ as if not, $p_i, s_i = 0 \Rightarrow q_i < 0$.

A positive (negative) demand shock is defined as an increase (decrease) in quantity demanded at every possible price and quality index at a given point in time. By definition, a_i is a demand shock parameter since $\partial f_i^L/\partial a_i > 0$, i.e. increasing (decreasing) a_i is a positive (negative) demand shock. A change in a_i (a_j) is an individual demand shock for firm $i(j)$ and an asymmetric market demand shock. A change in a_i and a_j is a symmetric market demand shock.

[10] See Appendix B for proving that linear demands are possible.

Only the former case is analysed. Moreover, only the effects relating to an increase in a_i and a_j is analysed since effects relating to a decrease in a_i and a_j is the opposite of that.

Let $l_i = d_i e_j - 2cn_i \lesseqgtr 0, \varphi = k_1 k_2 - l_1 l_2$ and $\varphi > 0$[11]. The following results are derived:

$$p_i^* = \left(-d_i^2 m_i + 2\left(cp_j^* + a_i - s_j^* e_i + b_i m_i\right) n_i\right)/k_i$$

$$s_i^* = \left(-d_i\left(-cp_j^* - a_i + s_j^* e_i + b_i m_i\right)\right)/k_i$$

$$p_i^* = \left(\begin{array}{c}\left(-d_i^2 m_i + 2a_i n_i + 2m_i n_i b_i + cm_j n_i\right)k_j + \left(-d_i e_j m_i - 2n_i a_j\right)l_j + \\ d_j m_j n_i\left(2b_j e_i - cd_j\right)\end{array}\right)/\varphi$$

$$s_i^* = \left((a_i d_i - b_i d_i m_i)k_j - \left(a_j d_i + cd_i m_i\right)l_j - cd_i d_j^2 m_j + b_j d_i m_j\left(d_j e_i + 2cn_j\right)\right)/\varphi$$

$$q_i^* = \left(\begin{array}{c}\left(\left(2a_i - 2b_i m_i + cm_j\right)n_i b_i\right)k_j - 2b_i n_i\left(a_j + cm_i\right)l_j + \\ 2b_i d_j m_j n_i\left(cd_j + b_j e_i\right) - 3cb_i m_j n_i d_j{}^2\end{array}\right)\Big/\varphi$$

$$dp_i^*/da_i = 2k_j n_i/\varphi \text{ (9i)}, \quad dp_i^*/da_j = -2l_j n_i/\varphi \text{ (9ii)}, \quad ds_i^*/da_i = d_i k_j/\varphi \text{ (9iii)},$$
$$ds_i^*/da_j = -d_i l_j/\varphi \text{ (9iv)}, \quad dq_i^*/da_i = 2b_i k_j n_i/\varphi \text{ (9v)}, \quad dq_i^*/da_j = -2b_i l_j n_i/\varphi \text{ (9vi)}$$

$$\frac{\partial p_i^*}{\partial p_j} = \frac{2cn_i}{k_i} \text{ (10i)}, \quad \frac{\partial p_i^*}{\partial s_j} = \frac{-2e_i n_i}{k_i} \text{ (10ii)}, \quad \frac{\partial s_i^*}{\partial p_j} = \frac{cd_i}{k_i} \text{ (10iii)}, \quad \frac{\partial s_i^*}{\partial s_j} = \frac{-d_i e_i}{k_i} \text{ (10iv)}$$

$$\frac{\partial s_i^*}{\partial a_i} = \frac{d_i}{k_i}, \frac{\partial s_i^*}{\partial a_j} = 0, \frac{\partial p_i^*}{\partial a_i} = \frac{2n_i}{k_i}, \frac{\partial p_i^*}{\partial a_j} = 0$$

(9i) to (9iv) could be written in the following way:

$$\frac{ds_i^*}{da_i} = \frac{\partial s_i^*}{\partial a_i} + \frac{\partial s_i^*}{\partial p_j}\frac{dp_j^*}{da_i} + \frac{\partial s_i^*}{\partial s_j}\frac{ds_j^*}{da_i} = \frac{d_i}{k_i} + \frac{cd_i}{k_i}\frac{-2l_i n_j}{\varphi} + \frac{-d_i e_i}{k_i}\frac{-d_j l_i}{\varphi} \tag{11}$$

$$\frac{ds_i^*}{da_j} = \frac{\partial s_i^*}{\partial a_j} + \frac{\partial s_i^*}{\partial p_j}\frac{dp_j^*}{da_j} + \frac{\partial s_i^*}{\partial s_j}\frac{ds_j^*}{da_j} = 0 + \frac{cd_i}{k_i}\frac{2k_i n_i}{\varphi} + \frac{-d_i e_i}{k_i}\frac{d_j k_i}{\varphi} \tag{12}$$

$$\frac{dp_j^*}{da_j} = \frac{\partial p_j^*}{\partial a_j} + \frac{\partial p_j^*}{\partial p_i}\frac{dp_i^*}{da_j} + \frac{\partial p_j^*}{\partial s_i}\frac{ds_i^*}{da_j} = \frac{2n_j}{k_j} + \frac{2cn_j}{k_j}\frac{-2l_j n_i}{\varphi} + \frac{-2e_j n_j}{k_j}\frac{-d_i l_j}{\varphi} \tag{13}$$

$$\frac{dp_j^*}{da_i} = \frac{\partial p_j^*}{\partial a_i} + \frac{\partial p_j^*}{\partial p_i}\frac{dp_i^*}{da_i} + \frac{\partial p_j^*}{\partial s_i}\frac{ds_i^*}{da_i} = 0 + \frac{2cn_j}{k_j}\frac{2k_j n_i}{\varphi} + \frac{-2e_j n_j}{k_j}\frac{d_i k_j}{\varphi} \tag{14}$$

The effect of an one unit increase in a_j on s_i^*, or ds_i^*/da_j, is the effect of a change in p_j, s_j, due to an increase in a_j, on s_j. Since firms' decisions are interdependent as shown in Section 3, the demand shock in this case indirectly influences firm i's decisions through affecting firm j's decisions. ds_i^*/da_i can be positive, negative or zero as shown in (9iv) because of the existence of a negative component, which is $\partial s_i^*/\partial s_j$ in (12). $\partial s_i^*/\partial s_j$ is explained in Section 3. This implies that if firm j reacts stronger by decreasing its quality when s_i increases, it is

[11] See proof in Appendix C.

highly likely that ds_i^*/da_i turns negative and if ds_i^*/da_i is known to be positive, that decreases the magnitude of ds_i^*/da_i.

Moreover, dp_j^*/da_j is present in (12), which means that the magnitude of the shock on quality of firm i, represented by ds_i^*/da_i, depends on the effect of the shocks on price of firm j. The detail of dp_j^*/da_j is shown in (13).

The total effect of an increase in a_i on s_i^*, or ds_i^*/da_i is positive according to (9iii). It can be decomposed into the direct effect $\partial s_i^*/\partial a_i$ and the indirect effect $\partial s_i^*/\partial p_j\, dp_j^*/da_i + \partial s_i^*/\partial s_j^*\, ds_j^*/da_i$ as shown in (11). The former is positive since $\partial f_i^L/\partial a_i > 0$ and according to arguments similar to those in section 2. The latter is the effect of a change in p_j, s_j, due to an increase in a_i, on s_i. The latter can be negative since dp_j^*/da_i, which represents firm j's reacting to an increase in a_i by changing its price, can be negative, as shown in (9ii). Investigating in the reason of that being possibly negative then requires us to decompose dp_j^*/da_i.

As shown in (14), dp_j^*/da_i is negative if the size of $\partial p_j^*/\partial s_i$ is large. This means that if firm j decreases its price to a large extent when firm i increases its quality, then the magnitude of ds_i^*/da_i is smaller. How the shock in this case affects firm i's decisions depends on how firm j reacts to firm i's decisions.

Section 5 On average quality of products in the market

Besides considering changes in individual firms' quality, the change of average quality of products in the market should be analysed. The quantity-weighted average of quality of products in market J is defined by $\bar{S}_J = \sum_{i=1}^{N} \left(s_i (q_i / \sum_{i=1}^{N} q_i) \right)$.

More simplifying assumptions are made. Let $b_i = b_j = b, d_i = d_j = d, e_i = e_j = e, m_i = m_j = m$ and $n_i = n_j = n$ in (7) and (8). As a result, $k_i = k_j = k, l_j = l_j = l$.

Proposition 2. *Let $\bar{S}_J = \hbar(a_i)$. The sufficient condition for $\partial^2 \hbar/\partial a_i \partial a_i > 0$ is $l > 0$ and a_2 being sufficiently large such that $2(-b + c)m + a_j > 0$.*

Proposition 2 is saying that if the sufficient condition is satisfied, average quality of products will first decrease, then increase in a_i in a convex manner. This means the magnitude of the shock determines whether average quality will increase or decrease in the market. The intuition is that, when an individual shock for firm i occurs, firm j will decrease its quality and decrease its output since from (9iv) and (9vi), $l > 0$ means $ds_j^*/da_i , dq_j^*/da_i < 0$. a_j being sufficiently large means that quantity demanded, hence output share for firm j is large enough to drag down average quality of products when firm j produce lower quality products. However, when the shock is too large and firm j at the same time decreases its output to a large extent, the increasing output share of firm i, who is also producing higher quality products since $ds_i^*/da_i > 0$, will pull up average quality of products. Proof in Appendix D.

Section 6 Conclusion

This paper shows how firms react to other firms' decisions when quality is one of firms' choice variables and equipped with that, the effects of demand shocks on quality of products is broken down into direct and indirect effects. This leads to analysis regarding the direction and the magnitude of the effects of demand shocks on quality, which show the rich qualitative possibilities. In addition, some analysis related to average quality of products in the market is provided.

Further research associated with the following limitations of this study should be conducted. 1, The assumptions in the model could be relaxed, which possibly leads to different results. For instance, consumers might have different tastes, which violates Asm.2. 2, Supply shocks and symmetric demand shocks is neglected in this study. 3, The model assumes firms' choosing price and quality. However, market can determine the prices while firms are choosing outputs and quality. 3, The model is a conjectural variations model, which means that every firm treats the choices of other firms as given and is "subject to some well merited criticism[12]". 4, The model is also static, which ignores the dynamic adjustments when then market equilibrium changes.

(2012 words)

[12] (Dixit, 1986, p,107)

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Appendix A

The technical assumptions are having negative definite Hessian matrices $D^2\Pi_i(\boldsymbol{p},\boldsymbol{s}) = \begin{bmatrix} \partial^2\Pi_i/\partial p_i\partial p_i & \partial^2\Pi_i/\partial p_i\partial s_i \\ \partial^2\Pi_i/\partial s_i\partial p_i & \partial^2\Pi_i/\partial s_i\partial s_i \end{bmatrix}$ and the uniqueness of the equilibrium corresponding to an interior solution of the system of $2N$ equations exist. If costs are linear $s.t. \forall h_i, m_i, n_i \geq 0, c_i = h_i + m_i q_i + n_i s_i$, then $D^2\,\Pi_i(\boldsymbol{p},\boldsymbol{s}) = \begin{bmatrix} -2b_i & d_i \\ d_i & 0 \end{bmatrix}$, $\det D^2\,\Pi_i(\boldsymbol{p},\boldsymbol{s}) = -d_i^2 < 0$ and Π_i does not have a maximum.

Appendix B

Assume a representative consumer maximises $U = u(q_1,q_2,s_1,s_2) - \sum_{i=1}^{2} p_i q_i$ [13] where

$$u(q_1,q_2,s_1,s_2) = \alpha_1 q_1 + \alpha_2 q_2 - \begin{pmatrix} \beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2 \\ -2\delta_1 q_1 s_1 - 2\epsilon_2 q_2 s_1 - 2\epsilon_1 q_1 s_2 - 2\delta_2 q_2 s_2 \end{pmatrix}/2$$

and $D^2U(\boldsymbol{p},\boldsymbol{s}) = \begin{bmatrix} -\beta_1 & -\gamma \\ -\gamma & -\beta_2 \end{bmatrix}$ is negative definite *s.t.* $\beta_1,\beta_2 > 0$ and $\det D^2U(\boldsymbol{p},\boldsymbol{s}) = \zeta = \beta_1\beta_2 - \gamma^2 > 0$, yielding demands:

$$q_1 = f_1(\boldsymbol{p},\boldsymbol{s}) = a_1 - b_1 p_1 + c p_2 + d_1 s_1 - e_1 s_2$$
$$q_2 = f_2(\boldsymbol{p},\boldsymbol{s}) = a_2 + c p_1 - b_2 p_2 - e_2 s_1 + d_2 s_2$$

for $a_i = (\alpha_i\beta_j - \gamma\alpha_j)/\zeta\,, b_i = \beta_j/\,\zeta\,, c = \gamma/\,\zeta\,, d_i = \left(\beta_j\delta_i - \gamma\epsilon_j\right)/\zeta\,, e_i = (\gamma\delta_j - \beta_j\epsilon_i)/\zeta.$

Appendix C

Proof. $\varphi = k_1k_2 - l_1l_2 > 0$: Given demands (1) and (2) for $N = 2$, change c_i in (4) to $c_i = n_i s_i^2, \forall i \in \{1,2\}, \Rightarrow 4b_i n_i - d_i^2 = k_i > 0$ and $D^2\,\Pi_i(\boldsymbol{p},\boldsymbol{s}) = \begin{bmatrix} -2b_i & d_i \\ d_i & -2n_i \end{bmatrix}$, which is the same $D^2\,\Pi_i(\boldsymbol{p},\boldsymbol{s})$ for c_i in (4). Solving $FOC_1 \cap FOC_2$ yields $p_1^* = \frac{2n_1(a_2l_2 - a_1k_2)}{k_1k_2 - l_1l_2}, p_2^* = \frac{2n_2(a_1l_1 - a_2k_1)}{k_1k_2 - l_1l_2}, s_1^* = \frac{d_1(a_1k_2 - a_2l_2)}{k_1k_2 - l_1l_2},\ s_2^* = \frac{d_2(a_2k_1 - a_1l_1)}{k_1k_2 - l_1l_2}$ where $p_i^*, s_i^* \geq 0$ and $\varphi \neq 0$. Recall $l_i \gtreqless 0$ and $d_i, n_i > 0$. By the definition of demand, in (1) and (2) $\forall a_i \in \mathbb{R}_+, \exists b_i, c, d_i, e_i\ s.t.\ q_i^*, p_i^*, s_i^* \geq 0$. Assume $a_1 = a_2$. There are three cases to consider: case 1: $l_1 or\ l_2 = 0$; case 2: $\{l_1 > 0\ and\ l_2 < 0\}\ or\ \{l_1 > 0\ and\ l_2 < 0\}$; case 3: $l_i > 0\ or\ l_i < 0$. It is easy to see that case 1 and 2$\Rightarrow \varphi > 0$. For case 3, take $l_i > 0$:

Proof by contradiction. Suppose $\varphi = k_1k_2 - l_1l_2 < 0$ and either i) $l_1 > k_1$, ii) $l_1 < k_1 or$ iii) $l_1 = k_1$. If inequality i is true, $p_2^* < 0$, which contradicts with $p_i^* \geq 0$. If inequality ii is true, $s_2^* < 0$, which contradicts with $s_i^* \geq 0$. If inequality iii is true, $l_1 = k_1 \cap \varphi < 0 \Rightarrow l_2 > k_2 \Rightarrow p_1^* < 0$, which contradicts with $p_i^* \geq 0$. Hence, $\varphi \nless 0$. Without loss of generality, for $l_i < 0$, $\varphi \nleq 0$. *Q.E.D.*

[13] This is similar to the maximisation problem in Singh and Vives (1984) which excludes quality.

Appendix D

For $\bar{S}_J = \mathscr{k}(a_i)$, consider $i = 1, j = 2$. $\frac{\partial \bar{S}_J}{\partial a_1 a_1} = \frac{4d(d(-d+e)+4bn-2cn)((-b+c)m+a_2)^2}{(d(d+e)-2(2b+c)n)^2(2(-b+c)m+a_1+a_2)^3} > 0$, where $2(-b+c)m + a_2 > 0 \Rightarrow 2(-b+c)m + a_1 + a_2 > 0$ and $(-b+c)m + a_2 > 0$. $l, k > 0 \Rightarrow d(-d+e) + 4bn - 2cn > 0$. The case of $i = 2, j = 1$ could then be easily derived.