Optimal vaccine allocation: modelling population interaction and administrative constraints.

Jakub Terlikowski

BSc Philosophy, Politics and Economics with Social Data Science

2nd Year

University College London

Introduction

The number of deaths worldwide caused by COVID-19 has exceeded 6 million (Ritchie et al., 2020). Scientists developed vaccines several months after the first cases of COVID-19 were confirmed (AJMC Staff, 2021), and many countries have faced vaccine shortages (Feinmann, 2021). Given this reality, in order to limit human suffering, it is imperative to understand how to allocate vaccines optimally. In this paper, I look at how the interaction between populations and constraints on the number of vaccines administered daily affect the optimal vaccine allocation. Using a simple pandemic model and an optimisation algorithm, I show that both factors affect the optimal vaccine allocation differently, especially when the number of vaccines available is small.

Literature Review

Major pandemics have had a significant influence on the history of humankind (Piret and Boivin, 2021). To understand them, scholars have developed mathematical models that allow us to describe and predict the flow of a pandemic with a set of equations. SIR model, developed in the 1920s (Kermack and McKendrick, 1927), is one of the simplest ways of modelling pandemics. Its basic version divides the population into three compartments (Susceptible, Infected, Recovered) but scholars have extended the model to account for quarantines, lockdowns, vaccinations, and deaths (Fosu et al., 2020).

One of the reasons why scholars are interested in modelling pandemics is the need to understand how to fight them effectively. In particular, one of the questions discussed in the literature is how to allocate vaccines optimally between different populations, given the constraints policymakers face. This problem can be formulated as a constrained optimisation problem and has attracted the attention of economists (Kitagawa and Wang, 2021). Dujizer et al. (2017) showed the importance of the herd effect in determining the optimal allocation.

This paper attempts to extend the discussion of the optimal vaccine allocation problem in two ways. Firstly, I look at how administrative constraints (the number of people that can be vaccinated daily) affect the optimum. Secondly, I investigate how different patterns of interaction between populations (allowing for infections in population A to affect population B) affect the optimum.

The model

I use an extended version of the SIR model to understand how administrative constraints and population interconnectedness affect the optimal allocation. I assume that in each population, there are four different compartments: Susceptible, Infected, Recovered, and Dead. At any point in time, each person from every society is in one of them. Once someone is in the Recovered compartment, they can no longer move back to being infected (they are immune). Lastly, I assume it is possible to identify susceptible individuals, and vaccination (which is fully effective and takes no time) transfers people from the Susceptible to the Recovered compartment.

The number of susceptible, infected, recovered, and dead people in population *i* are denoted by S_i, I_i, R_i, and D_i. Recovery and death rates are denoted by γ_i and α_i, respectively. The number of people vaccinated in each period in population *i* is denoted by V_i (this is a fixed number for every population). Lastly, β_i and ω_{ij} capture the transmission of the disease. The former coefficient describes the increase in infections resulting from contact inside population *i*, whereas the latter describes the increase in infections in population *i* resulting from the contact with population *j* (note that ω_{ij} is not necessarily equal to ω_{ji}). Ω denotes the matrix containing all possible ω_{ij}. Figure 1 summarises the model.

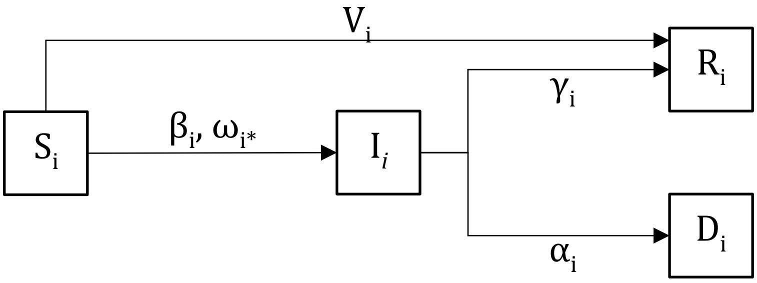

Figure 1*: A diagram of a SIR model for population i with four compartments, vaccination, and interaction between populations. ω_{i*} denotes the ith row of matrix Ω.*

The pandemic flow for *k* societies can be described as a system of differential equations (N_i denotes the size of population *i*):

$$\frac{dS_i}{dt} = -\frac{S_i}{N_i}\left(\beta_i I_i + \sum_{j\neq i}^{k} \omega_{ij} I_j\right) - V_i$$

$$\frac{dI_i}{dt} = \frac{S_i}{N_i}\left(\beta_i I_i + \sum_{j\neq i}^{k} \omega_{ij} I_j\right) - \alpha_i I_i - \gamma_i I_i$$

$$\frac{dR_i}{dt} = \gamma_i I_i + V_i$$

$$\frac{dD_i}{dt} = \alpha_i I_i$$

Minimisation algorithm

The problem of optimal vaccine allocation can be described as a constrained minimisation problem:

$$\min_{W} \sum_{i=1}^{k} D_i^T \ subject\ to \sum_{i=1}^{k} W_i \leq P$$

Where D_i^T denotes the number of deaths in society *i* at the end of the pandemic, W_i denotes the number of vaccines allocated to society *i,* and *P* denotes the number of vaccines available.

Instead of solving this problem explicitly, I designed a simple greedy minimization algorithm. It builds on the presumption that marginal gains from adding a small number of vaccines to every population are equal for all populations at the optimum. The algorithm works as follows:

Algorithm flow

1: **Input:** initial conditions (S_i, I_i, R_i, and D_i), and rates of change (α_i, β_i, γ_i, ω_i, and V_i) for every society, total vaccine stockpile P;

2: **Initialisation:** start from a random allocation of vaccines such that $\sum_{i=1}^{k} W_i = P$;

while the decrease in total deaths > ε:

> 3: compute marginal gain (decrease in total deaths from marginally increasing vaccine stockpile) for each society;
>
> 4: transfer 10 vaccines from the society with the lowest marginal gain to the one with the highest;

return W_i for all i

This algorithm ensures arriving at the locally optimal solution, but not necessarily the globally optimal one. To address this problem, for each of the cases described below, I ran the algorithm 100 times. Afterwards, from all allocations suggested by the algorithm, I chose the one with the smallest total number of deaths.

The four scenarios

The two phenomena I would like to analyse using the model are the presence of administrative constraints and interaction between populations. To understand them better, I look at the optimal vaccine allocation between three populations under four scenarios: the base scenario (with no interaction and no administrative constraint, corresponding to the case discussed by Duijzer et al. (2017)), the administrative constraint scenario (only 1% of the population can be vaccinated daily), the first interaction scenario (all populations interact with each other), and the second interaction scenario (Population 2 is affected by Population 1 and Population 3 is affected by Population 3, but not vice-versa). Table 1 summarises the parameter values used for each scenario.

	Population 1	*Population 2*	*Population 3*
Base scenario			
Population size	10,000	20,000	40,000
Number of people initially infected	150	240	400
β_i	2	2	2
γ_i	1	1	1
α_i	0.035	0.035	0.035
V_i	∞	∞	∞
ω_{i*}	[0.0, 0.0, 0.0]	[0.0, 0.0, 0.0]	[0.0, 0.0, 0.0]
Administrative constraint scenario*			
V_i	100	200	400
Interaction scenario 1 (network)*			
ω_{i*}	[0.0, 0.2, 0.2]	[0.2, 0.0, 0.2]	[0.2, 0.2, 0.0]
Interaction scenario 2 (chain)*			
ω_{i*}	[0.0, 0.0, 0.0]	[0.2, 0.0, 0.0]	[0.0, 0.2, 0.0]

*all other parameters are the same as in the base scenario

Table 1*: Summary of the parameters used for the simulations. ω_{i*} denotes the ith row of the Ω matrix.*

Scenario	*Base scenario*			*Administrative constraint scenario*			*Interaction scenario 1 (network)*			*Interaction scenario 2 (chain)*		
Population	*Pop 1*	*Pop 2*	*Pop 3*	*Pop 1*	*Pop 2*	*Pop 3*	*Pop 1*	*Pop 2*	*Pop 3*	*Pop 1*	*Pop 2*	*Pop 3*
Vaccine stockpile												
2000	2000	0	0	200	600	1200	0	0	2000	2000	0	0
5000	0	5000	0	700	1400	2900	0	0	5000	3500	1500	0
8000	1200	6800	0	1100	2300	4600	0	0	8000	3600	4400	0
10000	0	0	10000	1400	2800	5800	0	0	10000	3900	6100	0
15000	0	0	15000	2100	4300	8600	0	3000	12000	4200	7400	3400
20000	0	6200	13800	2800	5700	11500	700	5500	13800	4100	7500	8400
25000	3000	7000	15000	3700	7000	14300	2900	6900	15200	4200	8100	12700
30000	4000	8500	17500	5000	9000	16000	4300	8600	17100	4600	9100	16300

Table 2 *: Optimal vaccine allocations between three populations under four scenarios with different vaccine stockpiles. Table 1 contains a detailed description of the parameters used for this table.*

Results

I computed the optimal vaccine allocation for each of the four scenarios with different vaccine stockpiles. Table 2 summarises the results. Optimal vaccine allocations for the base scenario are, to a certain degree, similar to those presented by Duijzer et al. (2017, p. 2). The differences arise because my model incorporates deaths, and my objective is different (minimising the number of deaths rather than people infected). Building on their insights, I interpret my results as consistent with the finding that the herd effect is maximised at the optimum.

Compared to the base scenario, the optimal allocation is more equitable under the administrative constraint scenario. That may be because a more equitable distribution allows vaccination of more people in the early days of the pandemic. Hence, under administrative constraints, maximising the number of people vaccinated in the early days of the pandemic may be the optimal strategy.

When we include interaction between all societies (the network scenario), the largest population receives more vaccines. The larger the population size, the more a population can potentially affect populations interacting with it (since the impact of interaction is proportional to the total number of people infected). Hence, when focusing on larger populations, policymakers can slow the spread of the pandemic by limiting the effects of interaction. Secondly, under the chain interaction scenario, the population earlier in the chain receives more vaccines. This should not be a surprise since the earlier a population is in the chain, the more influence it effectively has on other groups, given the interaction structure. That would suggest policymakers should prioritise populations earlier in the chain, other things being equal.

Lastly, the optimal allocation is similar under all four scenarios when the number of vaccines available is high. Moreover, under all four scenarios, we can see the importance of the herd effect. In every case, populations receive vaccines until a certain threshold, after which the marginal gains from adding more vaccines diminish.

The results suggest four main hypotheses about the optimal vaccine allocation: (1) when faced with administrative constraints, policymakers should aim to maximise the total number of people vaccinated in the early days of the pandemic; (2) when several populations are equally interconnected, policymakers should prioritise the largest population; (3) when there is a chain of populations, policymakers should give more weight to those earlier in the chain; (4) given the herd effect, policymakers should not vaccinate whole populations when faced with a vaccine shortage.

Conclusion

In this paper, I examined how the interaction between societies and constraints on the number of vaccines administered daily might affect the optimal vaccine allocation. Both of these factors influence the optimum differently. The presented approach and analysis can be easily extended by allowing other parameters (death rates, recovery rates) to vary. Lastly, the hypotheses presented in this paper should be verified empirically.

References

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