Inverse Problems in Imaging with Uncertainty Quantification

Supervisors: Prof Marta Betcke & Prof Jason McEwen

The goal of this project is to tackle the dimensionality challenge and develop efficient numerical methods for uncertainly quantification for large scale imaging inverse problems. We will focus on the frontier of the reconstruction methods which combine the state of the art hybrid model and data driven reconstruction techniques and generative modelling and will explore insights from Monte Carlo methods, (stochastic) differential equations, optimisation, multi-scale methods and numerical analysis to develop efficient methods with provable guarantees. We will be motivated by applications in medical and preclinical imaging such as X-ray computed tomography, Photoacoustic tomography and MRI as well as applications in Astronomy such as large radio telescope surveys.

One way of overcoming the dimensionality challenge is to use deterministic bounds and optimisation [3]. We have recently extended this approach to learned priors. However, the deterministic bounds are limited to log concave likelihoods and rather pessimistic. Therefore, there is need for efficient sampling based approaches. An interesting line of work, sitting in between, was recently presented by Mallat and collaborators, which exploits multi scale structure of images and its statistical properties to factorise their distribution into a product of strongly log-concave conditional probabilities [5]. We will explore extension of these ideas to non-orthogonal directional frames which allow for more flexible decompositions. In particular, for micro local problems (like X-ray of Photoacoustic), we will investigate use of anisotropic diffusion to construct priors preserving the visible singularities and construct their strongly log concave factorisations in directional frames.

[3] Xiaohao Cai, Marcelo Pereyra, Jason D McEwen, Uncertainty quantification for radio interferometric imaging: II. MAP estimation, Monthly Notices of the Royal Astronomical Society, Volume 480, Issue 3, November 2018, Pages 4170–4182

[4] Matthew Holden, Marcelo Pereyra and Konstantinos C. Zygalakis, Bayesian Imaging With Data-Driven Priors Encoded by Neural Networks: Theory, Methods, and Algorithms, 2021 arXiv 2103.10182

[5] Florentin Guth, Etienne Lempereur, Joan Bruna, Stephane Mallat, Conditionally Strongly Log-Concave Generative Models, 2023, arXiv:2306.00181

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Marta Betcke's interests span numerical analysis, inverse problems, optimisation and machine learning. Relevant work includes multi-scale transforms like Wavelets, Curvelets and their multi-scale/micro-local representations of the forward and inverse problems with connections to deep learning [1,2]. She is also an expert on tomography (X-ray, acoustic) and PDE-based forward modelling and efficient solvers.

Jason McEwen's interests encompass a wide range of areas within astroinformatics and astrostatistics, including Bayesian inference, harmonic analysis, optimisation, computational techniques, and machine learning and artificial intelligence, with a focus on application to cosmology and radio interferometry.