# Towards Bayesian Full-Waveform Source Inversion using Simulation-Based Inference

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## 1. Motivation – Improving Bayesian Inversion

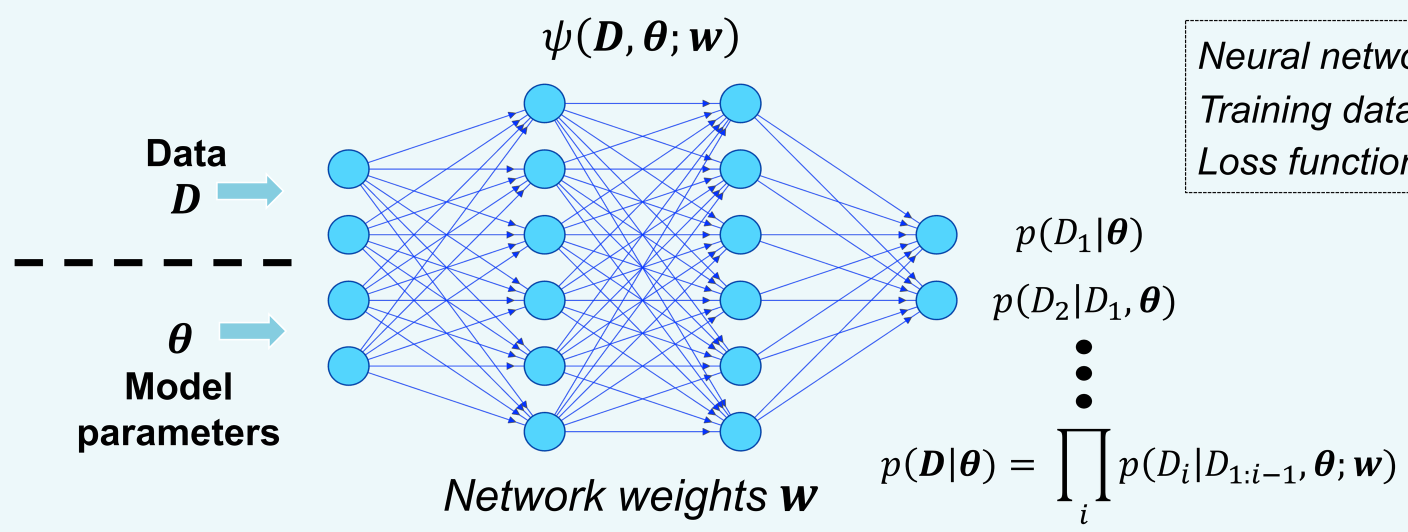
Given a forward model $f(\boldsymbol{\theta}) \to \boldsymbol{D}$ over parameters of interest $\boldsymbol{\theta}$ (e.g. source location):

$$\underbrace{p(\boldsymbol{\theta}|\boldsymbol{D})}_{\text{Posterior}} \propto \underbrace{p(\boldsymbol{D}|\boldsymbol{\theta})}_{\text{Likelihood}} \times \underbrace{p(\boldsymbol{\theta})}_{\text{Prior}}$$

- **Gaussian** likelihood function is often used to perform Bayesian inversions on seismic data.
- This methodology can introduce **bias** in the presence of non-Gaussian noise.

## 2. Simulation-Based Inference (SBI)

- **Learned** likelihood function replaces user-specified (e.g. Gaussian) likelihood function.
- This likelihood function is modelled by a Machine Learning (ML) model known as a Neural Density Estimator (NDE), which is trained on simulated data.




| | |
|---|---|
| *Neural network:* | $\psi(\boldsymbol{D},\boldsymbol{\theta};\boldsymbol{w})$ |
| *Training data:* | $\mathcal{D} = \{\boldsymbol{\theta}_i, \boldsymbol{D}_i\}$ |
| *Loss function:* | $\mathcal{L} = -\ln p(\boldsymbol{D}|\boldsymbol{\theta})$ |

*Fig. 1:* Overview of a **M**asked **A**utoencoder for **D**ensity **E**stimation (**MADE**) [1], a class of NDE that has shown great success in modeling probability densities by ensuring its output is a normalised probability distribution.

## 3. Data Compression

- The very high dimensionality of the displacement data $\boldsymbol{D}$, which represents full-waveforms at each station, makes training a NDE **infeasible**.
- Data $\boldsymbol{D}$ must be compressed to $\dim(\boldsymbol{\theta})$ summary statistics $\boldsymbol{t}$, replacing $\boldsymbol{D}$ in *Fig. 1*.

This study investigates two compression methods $\phi$ for seismic data:

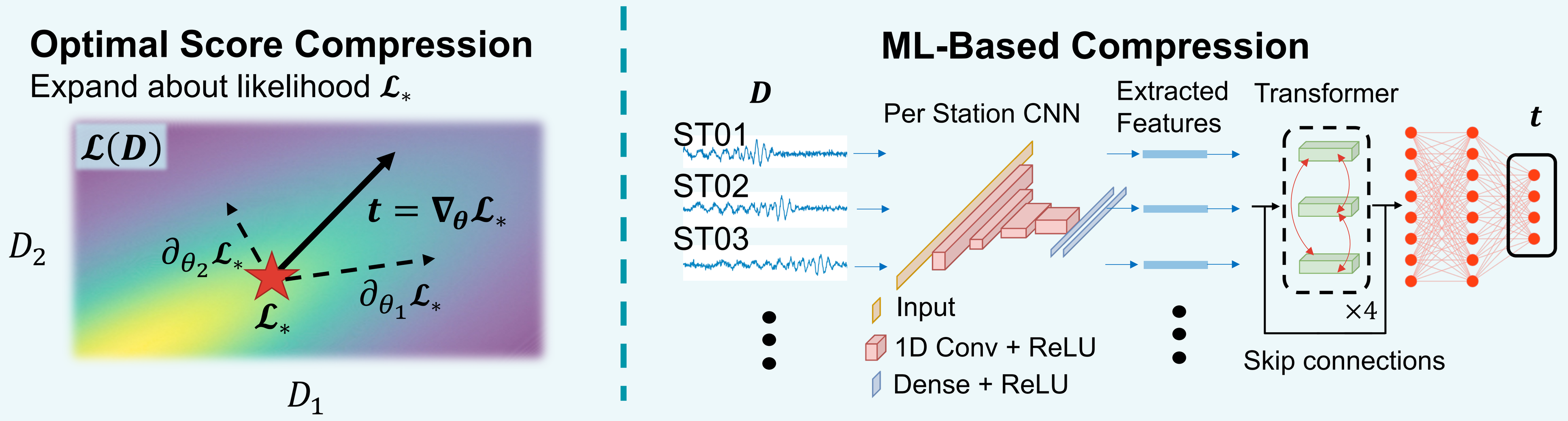



*Fig. 2: Left*: Classical technique for optimal compression using a first-order Taylor expansion, see [2]. *Right*: Neural network architecture, inspired by [3], trained to perform compression by learning to map $\phi : \boldsymbol{D} \to \boldsymbol{\theta}$.

## 4. Example Simulation-Based Study

- We demonstrate SBI on a simplified problem of source-location inversion, i.e. $\boldsymbol{\theta} = \{x, y, z, \Delta t\}$, a four parameter source inversion.
- Study region of the 2021 São Jorge crisis in the Azores, using the **UPFLOW 2021-22** [4] Ocean Bottom Seismometer (OBS) array.
- Entirely synthetic study, simulating events using the isotropic 1-D model PREM [5]. The simulations are accurate down to wave periods of $T \sim 2$ s.

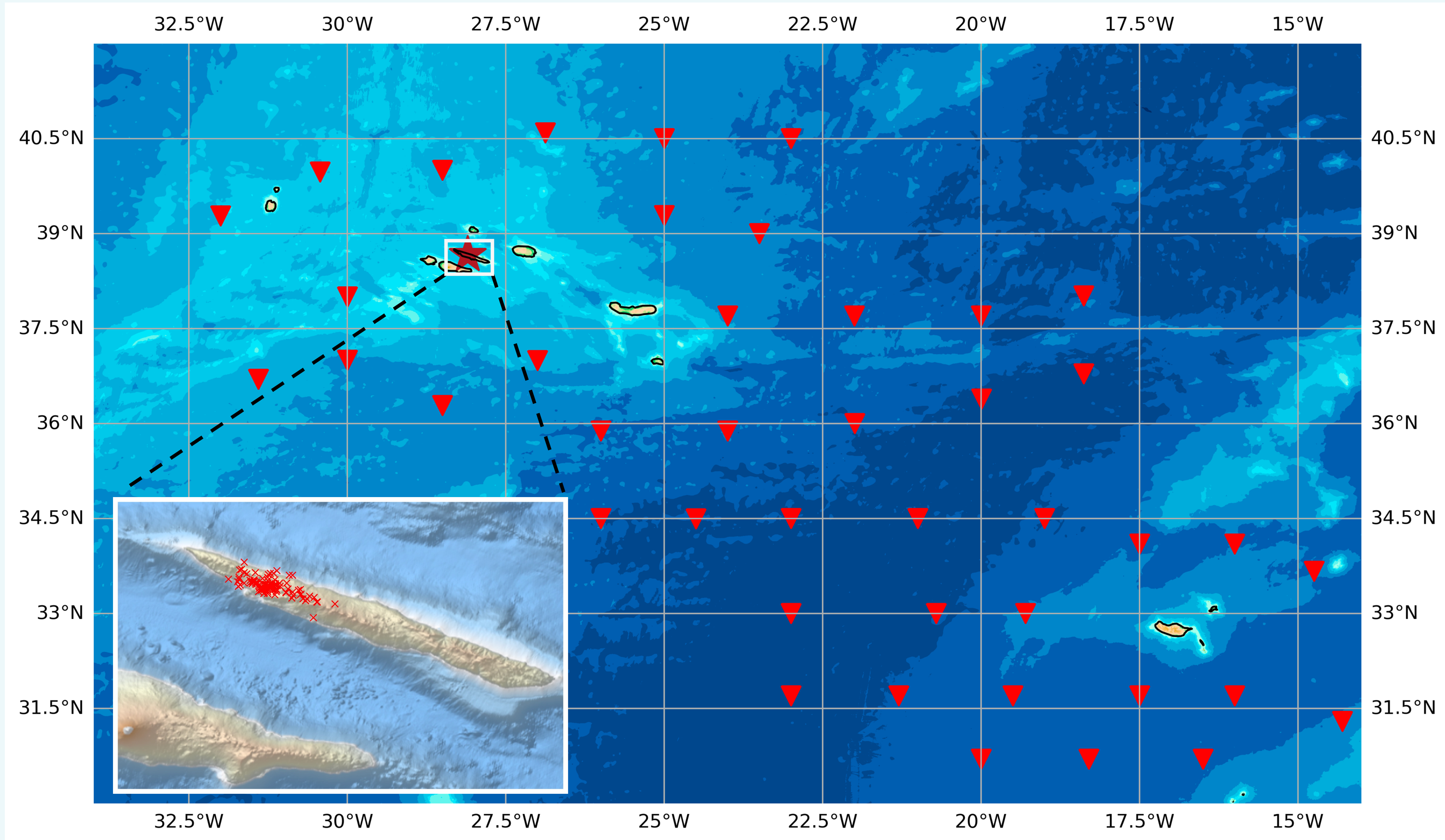



*Fig. 3:* A schematic showing the experimental setup for this simulation-based study. We use OBS stations from the UPFLOW array (*red triangular markers*) to study the recent seismic swarm crisis located on São Jorge island (*brown star*). *Bottom left*: zoom in on the São Jorge crisis, with a small selection of events from the *IPMA* [6] catalogue.

## 5. Simulation-Based Inference Workflow

### Generate a Compressed Dataset

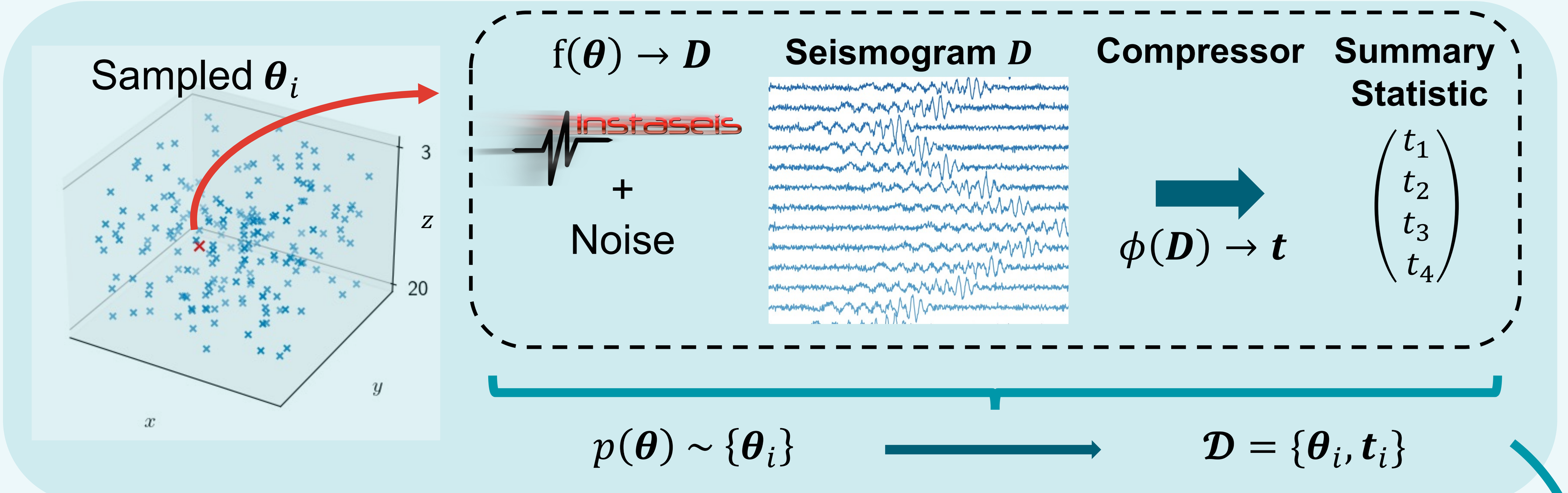



*Fig 4.* Events simulated with *Instaseis* [7]. Events are centred on the seismic swarm, sampled within a cube $(x, y, z) = 3\,\text{km} \times 3\,\text{km} \times 17\,\text{km}$, between depths $3 - 20\,\text{km}$. The time shift $\Delta t$ is sampled between $[-1,1]$ s. Synthetic noise is added to each waveform before compression.

### Train NDE to Estimate Likelihood

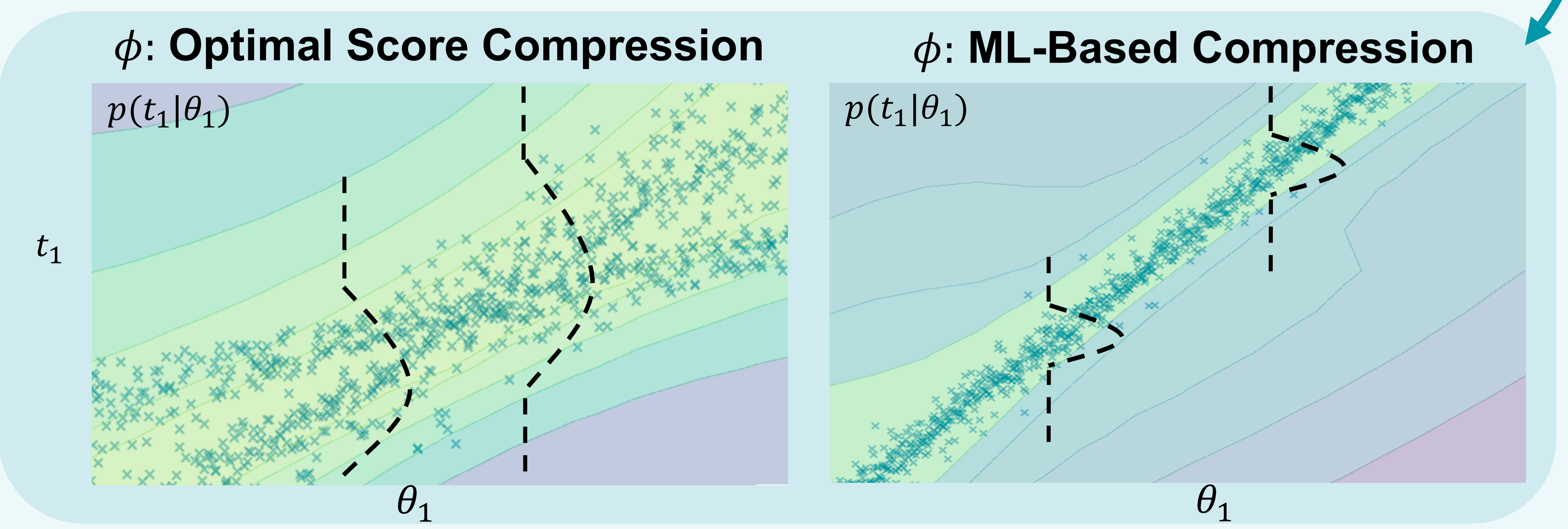



*Fig 5.* NDE likelihood contours for each compression method. Score compression performs poorly due to displacement $\boldsymbol{D}$ non-linearity with respect to location parameters $\boldsymbol{\theta}$. ML-based compression gives sharper likelihood contours, corresponding to less lossy compression.

## 6. Inversion Results

Samples from the posterior $p(\boldsymbol{\theta}\,|\boldsymbol{t}_{\text{obs}})$ are drawn using MCMC once the likelihood NDE is trained.

| | Compression | |
|---|---|---|
| | Score | ML |
| Compression MSE ↓ | $2.6 \times 10^{-2}$ | $\mathbf{8.2 \times 10^{-4}}$ |
| Posterior CRPS* ↓ | $2.6 \times 10^{-2}$ | $\mathbf{1.3 \times 10^{-2}}$ |
| Calibration Error ↓ | 0.35 | 0.31 |

*Table 1.* ML-based method improves compression and yields tighter posteriors (lower CRPS). Both methods are relatively well calibrated.
**Continuous Ranked Probability Score*

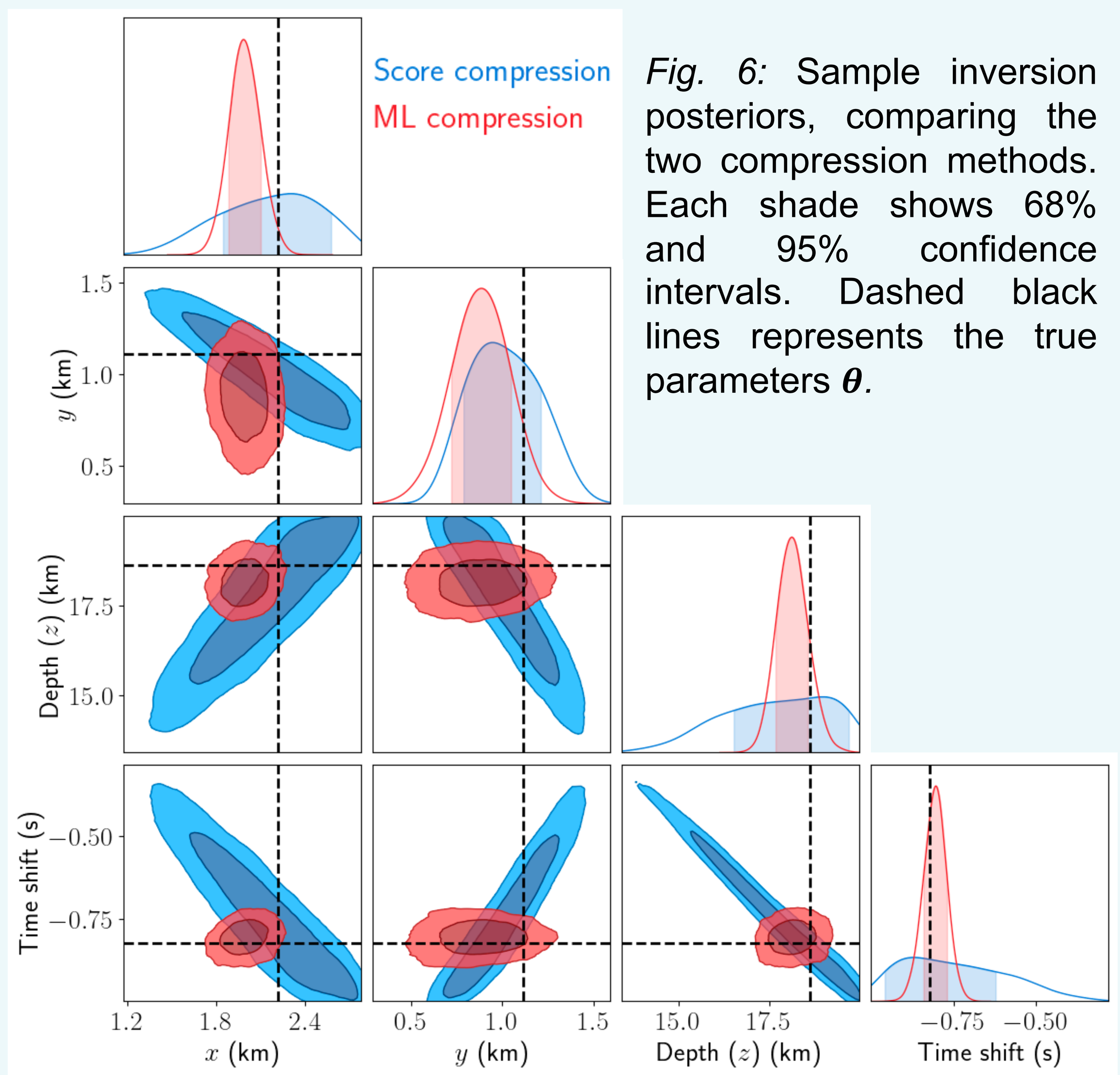



*Fig. 6:* Sample inversion posteriors, comparing the two compression methods. Each shade shows 68% and 95% confidence intervals. Dashed black lines represents the true parameters $\boldsymbol{\theta}$.

## 7. Conclusions

- SBI can incorporate full-waveform information and account for non-Gaussian noise effects in the posterior distribution. Future work will address and quantify these advantages.
- Samples are generated in the compressed space $\{t, \theta\}$, foregoing the forward model and giving $\sim 60\times$ speed-up over MCMC using *Instaseis.*
- The choice and tuning of the compression technique is important.
- More work is needed to avoid failure in the presence of modelling errors.

**References:**
**[1]** Germain et al. (2015), *CoRR* - **[2]** Alsing & Wandelt (2018), *MNRAS* - **[3]** Münchmeyer et al. (2021), *GJI* - **[4]** UPFLOW project, *upflow-eu.github.io*
**[5]** Dziewonski & Anderson (1981), *Phys. Earth Planet. Inter.* - **[6]** IPMA, *www.ipma.pt* - **[7]** van Driel et al. (2015), *Solid Earth*.

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