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CRABTREE'S THEOREM

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1986

Mingling reverentially over the years amongst all the awe-inspiring depth of Crabtree scholarship in its literary, linguistic, artistic, historical, legal, medical, pharmaceutical, electromagnetic and divers other modes, a mere mathematician has felt the impossibility of ever making a contribution that could stand alongside the imposing monuments to our poet raised by the giants of the discipline. Ineluctably, one of your Foundation's members was conscious, beside those massive erections, of an unaccustomed sense of impotence.

With Joseph Crabtree, of course, as with that comparably polymathic genius Francis Bacon, it could be seen as tempting to go far beyond the legitimate bounds of historicoliterary speculation and attribute hastily, without proper scrutiny of evidence to the contrary, works of genius of hitherto accepted provenance to an age's most widely-ranging mind. But no! Crabtree scholarship has always firmly set its face against any such trifling with the inexorable dictates of our precious source material in all its majestic albeit tantalising sparseness. Mere speculation could never find a place alongside the established works of rigorously analytical definition and exegesis of the Crabtree oeuvre. And so over the years I reconciled myself to my inactive posture as a naïf imbiber of all the scholarly nectar decanted from deep reservoirs of wisdom in those whom I felt fortunate to call colleagues but whom I could never aspire to emulate.

Then the blind workings of stochastic serendipity, for which no individual scholar can claim any credit at all, brought me gradually amongst the company of those dedicated to identifying that part within the historical development of the human consciousness which had been played by our polymath's hitherto unrecognized insights. There was no one moment encapsulating this change of direction of my scholarly activity. I had rather been led towards a reluctant acknowledgment of the inevitability of gradualness in the progress of research as I contemplated the extraordinary depth of the dilemmas facing me in my work as would-be historian of the *Decline and Fall of Classical Algebra*.

But here I must seek if at all possible to shun what Smith, quoting Mark Twain, called 'that gravity, that profundity and that impressive incomprehensibility which are so proper to works of solemn scientific inquiry'. Being most firmly of the opinion that higher algebra is accessible to everyone, I will just remind you of some facts which may no longer be in the forefront of all your minds but have long been known without exception to the proverbial 'every schoolboy', including, without doubt, the precocious Joseph Crabtree as he browsed in the school library during his boyhood years in the Cotswold village of Chipping Sodbury.

I make no apology for venturing to set forth from what some may feel to be the all-too-familiar jumping-off ground of QUADRATIC EQUATIONS, called quadratic because of the presence of the x squared term in an equation such as:

$$ax^2 + bc + c = 0.$$

The young Joseph was of course fully familiar with this elementary equation and with its standard solution, requiring us to extract the square root of $b^2 – 4ac$. All this and more had been needed for the brilliant thirteen-year-old to carry out that devastatingly enlightening critical juxtaposition of some of the more extreme concepts propagated by his namesake Joseph Priestley in an otherwise sound *History of Electricity* with the conservative doctrines of Newton's *Principia*, upon which Jones has riveted our collective attention. The algebra text which will have found its place alongside these last-named works of physics in the Chipping Sodbury school library must have been the standard *De Algebra Tractatus* of John Wallis, reprinted in so many editions in the eighteenth century; indeed, the most arduous Sodburian researches pursued with rigour and devotion have never even suggested that this indispensable tome was absent from the school's copiously stocked shelves. We can be certain that its early sections on the theory of the quadratic equation did indeed imprint themselves on the receptive though critical mind of Joseph Crabtree because, later on, he was to need that material in order to comprehend in its totality Thomas Young's wave theory, which he was able to subject to such penetratingly perceptive criticism.

Nothing, of course, could have been more natural than for our inquisitive young scholar to be simultaneously tasting at two separate branches of the Pierian spring, wooed not only by the muse of amorous poetry, Erato, but also by that austere Pythagorean Queen of the universe, Mathematics. Profound insight was shown by the great nineteenth-century mathematician Karl Weierstrass, when he wrote, 'A mathematician who is not also something of a poet will never be a complete mathematician'; and so it was with Joseph Crabtree. Tay demonstrated how Erasmus Darwin's lectures led Crabtree into evolutionary questions, while at the same time generating in him a healthy scepticism towards unproven assumptions in this as in many other areas; which must be a lesson to us all. No doubt the Swan of Sodbury had been drawn to those lectures by an instinctive sympathy towards a fellow poet-scientist – one, indeed, whose later works such as *The Loves of the Plants* have been shown by Freeman to stem from Crabtreaceous roots.

And this brings us back to the extraction of roots which, with regard to quadratic equations, plays so fundamental a role in that general solution (with its emphasis on the square root of $b^2 - 4ac$) which had fascinated Crabtree. From his dearly loved *De Algebra Tractatus* he must have known how in an exactly similar way the extraction of a cube root comes into the general solution of a cubic equation (that is an equation including not only x squared but also x cubed) published in 1539 by Cardano. This will have drawn him on to the case of a general quartic equation (including x to the power of four) with its solution also disclosed by Cardano in his profound treatise the *Ars Magna* — an equation whose general solution requires, not perhaps surprisingly, the extraction of fourth roots.

Now in all likelihood there may be some in the audience who are expecting me to reveal that the young Crabtree (for, indeed, mathematics has been truthfully designated by the late G.H. Hardy as 'a young man's game') may have simply gone on from cubics and quartics to the case of a quintic equation (one bringing in x to the power of five) and found a solution in the shape of a formula involving a finite sequence of operations of addition, multiplication, division and extraction of roots. Such a relatively pedestrian discovery, while not perhaps worthy of stigmatization in the words of the Royal Society's Standing Order No. 42 as 'mere accumulation of detail' and therefore unworthy of publication by the Society, would nevertheless have lacked the characteristics that make it suitable for concentrated historical analysis in front of a great learned Foundation two centuries later. It is indeed even probable that Crabtree, like another precocious genius Niels Henrik Abel studying in Oslo four decades later, may have temporarily believed he had cracked this problem which had been specified in Wallis's *Tractatus* as unsolved. Abel was to submit his 'solution' for publication, but soon afterwards to withdraw the paper after finding a serious error in it and before a referee had reported. Crabtree, noted for what a predecessor characterized as 'the studied deliberation with which he refused to publish anything but an occasional gem under his own name', will have refrained from exposing himself in so puerile a fashion. Yet on finding the mistake, he may have been spurred, unlike Abel, to ask the much more fundamental question whether it is indeed possible to express the solution of a general quintic in the shape of a formula involving a finite sequence of operations of addition, multiplication, division and extraction of roots.

As every mathematician now knows, the answer is in the negative and the proof that no such formula is possible was only able to be achieved by abandoning as insufficient all the weapons of Classical Algebra and putting on instead the whole armour of Groups, Rings and Fields — that armour with which our modern algebraists sally forth to win their Fields Medals! If, indeed, all this panoply of new ideas came from the fertile mind of the young Crabtree, he certainly did not allow any of them to filter out into the literature for many decades; indeed, not until 1832. This was the year when, as Thomas demonstrated, Crabtree chose to abandon the legal practice he had long pursued in London; yet it may have been more than simply an old man's disgust with the Reform Bill that led him to shake off those shackles.

He may rather, in 1832, have felt drawn to make yet another visit to his beloved France now that the revolutionary excesses of 1830 were over; that France with which our *savant* interacted continuously from, as Tancock was to put it, the age of Louis Quinze up to the beginning of the Second Empire. But he could have had a rather special motive for making *this* visit. Within the England (and even in the Scotland) of the first third of the nineteenth century, there was absolutely no pure mathematician of standing. The days of Wallis and Newton and Maclaurin were over and the centre of gravity of mathematical development had moved to France and Germany. If Joseph Crabtree had at last decided to reveal epoch-making new ideas in algebra to the world, there was no chance of their being taken seriously as coming from the British Isles, 'Das Land ohne Mathematik' as it seemed then to Carl Friedrich Gauss. In fact every paper sent to Germany was simply suppressed by Gauss, who took the view that any interesting discovery in mathematics must merely duplicate one of the numerous existing works that he had carried out as a young man but never had time to publish.

In these circumstances, what could have been more natural than for Joseph Crabtree to turn to France and to attempt to father his own boyhood insights on some young Frenchman who might then die in romantic circumstances, which would appeal to the French and cause them to take seriously the ideas of their deceased compatriot, determined as they still were in 1832 that Paris was the intellectual centre of the world, so that revolutionary discoveries in mathematics were clearly ten times as likely to have been made in Paris as in all other locations put together. These are the conclusions to which decades of study of the entire primary source material on the birth of Modern Algebra — a birth leading, as I put it earlier, to the *Decline and Fall of Classical Algebra* — have inexorably led me.

Nevertheless, I am obliged to convince my audience, and to this end I must first give a true summary of that primary source material which is mainly to be found in an article by Auguste Chevalier in the *Revue Encydopédique* for 1832. This purports to be an Obituary of a friend of M. Chevalier named Evariste Galois, who, at the age of 20, was killed in a duel employing pistols at 25 paces. The object of the article is to insinuate that Galois was the inventor of all the ideas and discoveries in Modern Algebra to which I have alluded; and, indeed, this is the view which the world of mathematics has generally accepted, utilising phrases such as Galois Fields and Galois Extensions and supposing that such a fundamental definition as that of a simple group is due to the youthful Galois.

Yet Chevalier's own account is riddled with inconsistencies. He states that there is no record of mathematical talent on either side of Galois' family. (How different from that of Crabtree, whose ancestor Henry Krabtree has been singled out by Smith for his scientific curiosity and authority!) Again, the young Evariste Galois received no teaching except from his mother till he was 12 years of age. Then he entered the Lycée Louis-le-Grand in Paris. There his work was reported as equally mediocre in Latin, Greek and mathematics, while his conduct is stated to have been contemptibly dissipated. Later he is said to have browsed superficially in the works of Adrien-Marie Legendre and our old friend Niels Henrik Abel, and to have put on some airs affecting ambition and originality. We all know those students! Both in 1827 and in 1829 he failed the examination for entry to the Ecole Polytechnique, being so frustrated during the oral on the latter occasion that he threw an india rubber at his examiner's face. He affected ambition to the extent of submitting a paper to the great Augustin-Louis Cauchy but whatever that paper was, it was rejected by Cauchy, who has been regarded by history as a fair-minded man. Chevalier goes on to strain our credulity with a story of another brilliant manuscript submitted to the Académie des Sciences for the Grand Prize in Mathematics, and of a sequel in which the Secretary to the Academy dies suddenly 'without having had time to look at' a submission of which no trace is subsequently found among his papers! Later, the distinguished mathematician Siméon Denis Poisson, also known as a fair judge, is said to have rejected contemptuously a Galois paper. By then Galois was spending most of his time participating in the 1830 revolution as an artilleryman of the Garde Nationale. Later, the artillery was *disbanded* and Evariste was brought to trial for drinking Louis Philippe's health with an open pocket-knife in his hand. He was acquitted after pleading that he had been cutting up his dinner with the knife; but then he was rearrested and imprisoned for six months for contumaciously continuing to wear his artilleryman's uniform. On his emergence from prison, he is stated to have been amorously entrapped and indeed 'initiated' by an individual described as 'quelque coquette de bas étage'. What a contrast to those far more civilised initiations described by Freeman in *Crabtree and Country Matters*! The Galois liaison led directly to his fatal duel, about which he wrote, 'I die the victim of an infamous coquette'. Finally, Chevalier asks us to believe that the last twenty-four hours of Galois' life were devoted to writing out a complete prospectus for all of the subsequent developments in modern algebra, a process during which he kept breaking off to scribble in the margin the words 'I have no time! I have no time!'

It is impossible to stress too strongly how unlikely the story by Chevalier may appear to anyone who knows the complexity of the new ideas expounded in that so-called 'scientific last will and testament'. Time and again, I have studied Chevalier's implausible account and mused about what the truth must have been. Evidently Galois was ideal material for use by a genius of exceptional modesty who had spent a lifetime perfecting original ideas which he (the genius) had fashioned first of all in the brilliance of his youth and now sought to launch on a sceptical world in his old age in a manner that would ensure their acceptance. Galois was ambitious (nay, desperate!) for mathematical recognition. Here was a chance, with his pathetic young life just about to reach its end, for him to win that posthumously. Half understanding the theory described to him, he will have begun feverishly copying from his mentor's manuscripts, conscious, as the moment of the duel approached, that there was no time, no time to complete the copying process.

Yet I have long been aware that none of these conjectures revising the history of algebra could have any chance of acceptance, unless an identity for that hidden mentor could be discovered and unless some substantiation could be given for some of the ideas having existed long before Galois was born. Here my historical researches must have been influenced, unconsciously at least, by all I had imbibed from the scholarly activities of this Foundation. Nevertheless, it was hard indeed to complete the puzzle by identifying reliably who might have been the 'onlie begetter' of what we traditionally call the Galois theory, with its power to transform an entire branch of human knowledge.

And then, surprisingly, it was none of my colleagues in this Foundation who gave me the ultimate clue which I needed; rather, it was John Keats. In pondering how I could identify someone capable of changing the face of algebra, I suddenly thought of Keats's sonnet which begins, 'Great spirits now on earth are sojourning'. Keats knew Crabtree of course; and Tattersall has traced the powerful influence of our poet's *Ode to Claret* on Keats's uncharacteristically derivative passage beginning:

> O for a draught of vintage that hath been
> Cooled a long age in the deep-delved earth.

Yet in the octet of the 'Great Spirits' sonnet he chooses to allude first to Wordsworth and then to Leigh Hunt. Then the association of ideas perhaps, with Leigh Hunt and Joseph Crabtree having both served spells in prison, seems to lead him to make an unmistakable choice within the sestet, where he writes:

> And other spirits there are standing apart
> Upon the forehead of the age to come;
> These, these will give the world another heart,
> And other pulses. Hear ye not the hum
> Of mighty workings from wise Joseph's art?
> Listen awhile, ye nations, and be dumb.

Later of course, probably in response to Crabtree's protests regarding the confidentiality of their conversations in the field of Algebra, he was to suppress the conclusion of the line beginning 'Of mighty workings ...'. It was impossible to match the rhyming felicity of the phrase '... from wise Joseph's art', although one of Keats's notebooks shows that he tried the rather weak '... in some distant Mart', from which he was discouraged by Benjamin Robert Haydon. Finally, he was to adopt the aposiopesis which we know today:

> ... Hear ye not the hum
> Of mighty workings? —
> Listen awhile, ye nations, and be dumb.

Yet to us in the Crabtree Foundation, the omitted words shout just as loud as if they had been printed.

Crabtree appreciated being bracketed by his young friend Keats with Wordsworth and Leigh Hunt, and he seems to have seen Byron too as a kindred spirit, not only because of a common devotion to the muse Erato, but also because of his penchant for swimming. Visiting that small Channel Island Sark, I found ancient records in the vile Sark *patois*, so much rougher than its kindred Guernsey dialect, which seemed to refer to a poet-mathematician 'qui nageoit atour de lisle' and the local suggestion that it was Byron (who was practically innumerate) *must* be rejected in favour of the far more plausible Crabtree.

Often our poet must have meditated on the quincunx of seminal poets Byron, Crabtree, Keats, Leigh Hunt and Wordsworth, to give them in alphabetical order; and then he will have permuted the orders in accordance with various different kinds of criteria, generating several different permutations.

Suddenly, in the course of these meditations, he must have come across the fundamental fact which, in the language of group theory, prevents the quintic equation being soluble in terms of extractions of roots. It is the fact that the group of even permutations of any five individuals is a simple group without normal subgroups. Immediately, the last splendid link in the great chain of deductive inference leapt into place and he abandoned his legal practice and set off for Paris, where he found Galois all ready to be tempted by the offer of posthumous glory. Crabtree, with that partial medical background, which he shared with Keats, was no doubt successful in offering his medical services for the duel at 25 paces in order to ensure that the ambitious young man should not be barred by any unwanted physical survival from the scientific immortality which his acquiescence had earned him. Crabtree afterwards could retire to Ashburton in Devon and peacefully enjoy the consciousness of having transformed an entire branch of mathematical knowledge.

As Larratt indicated in another context, 'final proof eludes us' – always a distressing matter to a mathematician. Nevertheless, I have to confess that I for one do now feel fully justified in abandoning, within all of my future writings in the field of algebra, misleading designations such as Galois Fields and Galois Extensions in favour of the much more roundly sounding, and historically based, Crabtree Fields and Crabtree Extensions; and above all I shall always refer to the theorem that the solutions of a quintic equation cannot be expressed through a formula involving a finite number of additions, multiplications, divisions and extractions of roots as CRABTREE'S THEOREM.