Theory of Standard and Non-Standard Neutrino Oscillations

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- Hamiltonian of system:

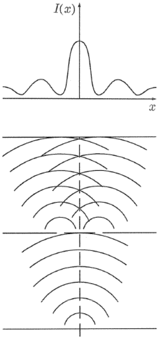
$$H_{system} = H_{propag} + H_{int}$$

- H_{int} diagonal in $\alpha = e, \mu, \tau \Rightarrow$ flavour basis $|\nu_\alpha\rangle$
- H_{propag} diagonal in $k = 1, 2, 3 \Rightarrow$ mass basis $|\nu_k\rangle$
- Related by a rotation:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha k} |\nu_k\rangle$$

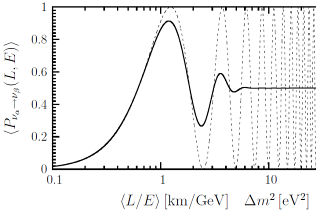
where $U_{\alpha k}$ is the PMNS mixing matrix

- Analogous to Young's double slit experiment:
  - Slits $\Leftrightarrow$ Neutrinos
  - Intensity pattern $\Leftrightarrow$ Detector spectrum
  - Coherence of light source $\Leftrightarrow$ Coherence of neutrinos
  - Observation of slit $\Leftrightarrow$ Precise energy measurement



[Peacock 1855]

[Feynman, Hibbs 1965]



[Giunti, Kim 2007]

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Localisation – $|\nu_k\rangle$ with equal 3-momenta $\mathbf{q}_k$ interfere. But plane wave $|\nu_k\rangle$ have different momenta $|\mathbf{q}_k| = \sqrt{E_\mathbf{q}^2 - m_k^2}$. Therefore no interference unless each $|\nu_k\rangle$ has a finite σ_p. From

$$\sigma_x \sigma_p \geqslant \frac{1}{2}$$

each $|\nu_k\rangle$ must be localised in space as a wave packet:

$$|\nu_k\rangle = \int \frac{d^3 q_k}{(2\pi)^3} \frac{1}{\sqrt{2E_\mathbf{q}}} f(\mathbf{q}_k)\, |\mathbf{q}_k\rangle$$

Use Schrödinger equation

$$i \frac{d}{dt} |\nu_k\rangle = H_{propag} |\nu_k\rangle \quad \Rightarrow \quad |\nu_k(t)\rangle = e^{-iE_\mathbf{q} t} |\nu_k(t_0)\rangle$$

to construct an amplitude for $|\nu_\alpha^P\rangle \to |\nu_\beta^D\rangle$ over a baseline L:

$$A_{\nu_\alpha \to \nu_\beta} = \langle \nu_\beta^D | \nu_\alpha^P(T)\rangle = \sum_i U_{\alpha k}^* U_{\beta k} \int \frac{d^3 q_k}{(2\pi)^3} f^P(\mathbf{q}_k) f^D(\mathbf{q}_k)\, e^{-iE_\mathbf{q} T + i\mathbf{q}_k L}$$

[Beuthe, 2003]

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Standard oscillation probability

$$P_{\nu_\alpha \to \nu_\beta}(L, E_{\mathbf{q}}) \equiv |A_{\nu_\alpha \to \nu_\beta}|^2 \approx \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j} \, e^{-i \frac{\Delta m^2_{kj}}{2E_{\mathbf{q}}} L}, \qquad \Delta m^2_{kj} = m^2_k - m^2_j$$

$$\Rightarrow \quad P^{(2\nu)}_{\nu_\alpha \to \nu_\beta} = \sin^2 2\theta \sin^2 \left(\frac{\Delta m^2}{2E_{\mathbf{q}}} L \right), \qquad \alpha \neq \beta$$

- 3-ν mixing with $\Delta m^2_{12} \ll |\Delta m^2_{23}|$ an excellent fit to experiment [Esteban at al., 2017]
- Reactor and accelerator anomalies (GALLEX, LSND, MiniBooNE) examined in the context of (3+s)-ν models. Tensions with KARMEN, OPERA, CMB + LSS... [Dentler at al., 2018]

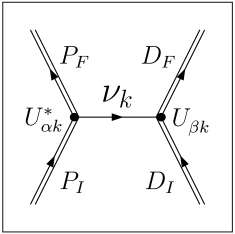
Problems with QM derivation:

- Ad-hoc wave packets
- No consideration of production and detection processes

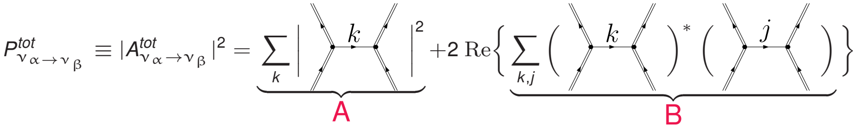
In Quantum Field Theory (QFT) I

QFT formalism – Entire process is a macroscopic Feynman diagram

- Incoming/outgoing particles $\Rightarrow$ Wave packets
- Neutrinos $\Rightarrow$ Virtual intermediate particles



Schematically the total probability is:



A – Incoherent mixing ($|U_{\alpha k}|^2$ and $|U_{\beta k}|^2$ factors from production and detection)

B – Interference of different ν_k – suppressed if production/detection are not sufficiently localised

[Akhmedov Kopp, 2010]

In Quantum Field Theory (QFT) II

Macroscopic propagation distance

In the limit $L \to \infty$ the virtual neutrinos, which can be off-shell

$$E_{\mathbf{q}} \neq \sqrt{\mathbf{q}_k^2 + m_k^2}$$

are forced to be real and on-shell with $E_{\mathbf{q}} = \sqrt{\mathbf{q}_k^2 + m_k^2} \quad \Rightarrow \quad \mathrm{B} \propto e^{i(\mathbf{q}_k - \mathbf{q}_j)L} \approx e^{-i\frac{\Delta m_{kj}^2}{2E_{\mathbf{q}}}L}$

But how to define a probability of *oscillation*? First use:

$$\Gamma^{tot}_{\nu_\alpha \to \nu_\beta} \propto P^{tot}_{\nu_\alpha \to \nu_\beta}$$

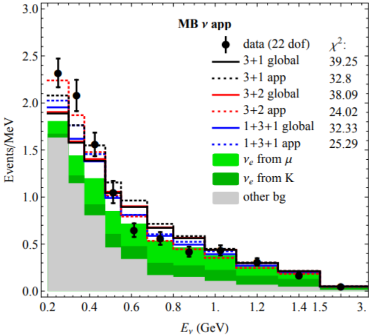
Factorisable – $\Gamma^{tot}_{\nu_\alpha \to \nu_\beta}$ must be separable into a production rate, oscillation probability, and detection cross section:

$$\Gamma^{tot}_{\nu_\alpha \to \nu_\beta} \propto P^{prod}_{\nu_\alpha} \cdot P^{osc}_{\nu_\alpha \to \nu_\beta} \cdot P^{det}_{\nu_\beta} \propto \frac{1}{4\pi L^2} \boxed{\Gamma^{prod}_{\nu_\alpha}} \cdot \boxed{P^{osc}_{\nu_\alpha \to \nu_\beta}} \cdot \boxed{\sigma^{det}_{\nu_\beta}}$$

$\Rightarrow$ Valid for relativistic and quasi-degerate neutrinos $\Rightarrow$ In those cases rearrange for $P^{osc}_{\nu_\alpha \to \nu_\beta}$

An oscillation experiment might be observing an 'anomaly' for a number of reasons:

$$\underbrace{\frac{d\Gamma^{tot}_{\nu_\alpha\to\nu_\beta}}{dE_{\mathbf{q}}} \propto}_{A} \underbrace{\frac{1}{4\pi L^2}\frac{d\Gamma^{prod}_{\nu_\alpha}}{dE_{\mathbf{q}}}}_{B} \cdot \underbrace{P^{osc}_{\nu_\alpha\to\nu_\beta}}_{C} \cdot \underbrace{\sigma^{det}_{\nu_\beta}}_{D}$$



[Dentler at al., 2018]

A – Eq. (7) is not valid because the total rate cannot be factorised

B – Systematics or NP in the production spectrum (e.g. reactor/beam fluxes)

C – Non-standard vacuum or matter oscillations (e.g. $\Delta m^2 \sim 1$ eV^2)

D – Systematics or NP in the detection cross section (e.g. MINERνA, SciBooNE)

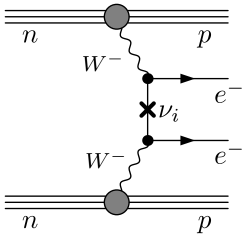
Correlated B, D systematics removed with near + far detector (e.g. LBL experiments, DANSS, NEOS)

[Ankowski Mariani, 2017]

Nature of neutrinos still an open question

- No dependence of $P^{osc}_{\nu_\alpha \to \nu_\beta}$ on Majorana CP phases

$\therefore$ $\nu_\alpha \to \nu_\beta$ unable to distinguish Majorana from Dirac neutrinos



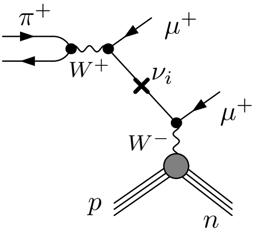
LNV Process – $0\nu\beta\beta$ is possible if neutrinos are Majorana

$\Rightarrow \Delta L = 2$ also possible over a macroscopic distance: $\nu_\alpha \to \bar{\nu}_\beta$

- **Suppressed** by

$$\left(\frac{m_k}{E_{\mathbf{q}}}\right)^2 \lesssim \left(\frac{0.05\text{ eV}}{1\text{ GeV}}\right)^2 \sim 10^{-21}$$

- Charged lepton ID at MINOS far detector $\Rightarrow$ Bound on $|\langle m_{\mu\mu}\rangle|$



[Xing, 2013]

Non-Standard Model Interactions

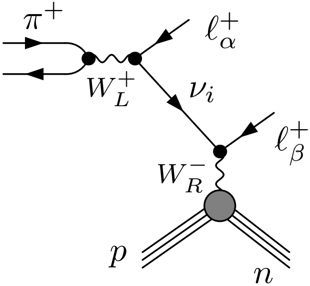
Positive signal of $0\nu\beta\beta \Leftrightarrow \begin{cases} \text{Majorana neutrinos?} \\ \text{Some mechanism from LNV extension of the SM?} \end{cases}$

$\Rightarrow$ Same applies for the $\nu_\alpha \to \bar{\nu}_\beta$ process

For example, a right-handed ($V + A$) current at detection would replace

$$\left(\frac{m_k}{E_\mathbf{q}}\right)^2 \to |\varepsilon^R|^2$$

$\therefore$ Obtain bound on the relative strength of the RH current to SM



[Deppisch et al., 2012]

Other phenomenological models:

- NSIs of neutrinos in matter
- Neutrino oscillations through DM
- Effect of neutrino EM properties on scattering (e.g. coherent, quasi-elastic)

Conclusions and Outlook

QM vs. QFT

- Neutrino flavour oscillations can be described in QM or QFT
- QM simple but suffers from consistency issues
- QFT more rigorous
- Necessary conditions for oscillation clearer in QFT

Non-Standard Oscillations

- A number of unexplained anomalies
- May arise from misunderstood (or new) production, propagation or detection effects
- QFT a consistent framework to study NP at production and detection

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JHEP **01**, 087 (2017)

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Updated Global Analysis of Neutrino Oscillations in the Presence of eV-Scale Sterile Neutrinos
[https://arxiv.org/abs/1803.10661]

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Systematic uncertainties in long-baseline neutrino oscillation experiments
J. Phys **G44** 5, 054001 (2017)

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Neutrino oscillations: Quantum mechanics vs. quantum field theory
JHEP **04**, 008 (2010)

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Phys. Rev. **D87**, 053019 (2013)

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Neutrinoless Double Beta Decay and Physics Beyond the Standard Model
J. Phys **G39**, 124007 (2012)

Backup

In Quantum Field Theory (QFT) III

Two cases:

- **Relativistic** ν – Production and detection vertices must be chiral ($V - A$ current). In the limit $m_k \to 0$ only negative helicity neutrinos contribute and

$$P^{tot}_{\nu_\alpha \to \nu_\beta} \propto \sum_{\text{spins}} A^{tot}_k A^{tot*}_j \propto \left(\sum_{\text{spins}} |A_P|^2 \right) \left(\sum_{\text{spins}} |A_D|^2 \right) \propto P^{prod}_{\nu_\alpha} P^{det}_{\nu_\beta} \quad \checkmark$$

- **Quasi-degenerate masses** – As above, the spin structure simplifies for $m_k \simeq m_j \equiv m$
  $\Rightarrow$ Meson and heavy neutrino oscillations $\checkmark$

$\therefore$ Both produce $\quad P^{osc}_{\nu_\alpha \to \nu_\beta} = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\alpha j} U^*_{\beta j}\, e^{-i \frac{\Delta m^2_{kj}}{2E_{\mathbf{q}}} L}$

$\Rightarrow$ If $P^{tot}_{\nu_\alpha \to \nu_\beta}$ is not factorisable, $P^{osc}_{\nu_\alpha \to \nu_\beta}$ cannot be disentangled from production and detection