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Bayesian operational modal analysis of offshore rock lighthouses: Close modes, alignment, symmetry and uncertainty

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A R T I C L E I N F O

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Despite use of GPS, lighthouses remain critical infrastructure for preserving safety of mariners and maritime trade, and the most dramatic examples are probably the Victorian era masonry towers located on remote offshore reefs around the British Isles and exposed to extreme weather conditions. Due to their age and likely increasing future loading, dynamic field investigations were undertaken for condition assessment.

The field investigations of a sample of seven lighthouses had focused on experimental modal analysis (EMA) of shaker force and acceleration response data in order to identify sets of modal parameters (MPs) specifically including modal mass, which is useful for linking loading and response. However, the EMA missed significant useful information, which could be recovered from operational modal analysis (OMA) of additional ambient vibration data recorded during the field measurements, as well as from subsequent long-term monitoring of Wolf Rock lighthouse.

Horizontal vibration modes of the towers appear as pairs of modes of similar shape and with close natural frequency due to the quasi-axisymmetric structural form(s), and the lowest frequency pairs are most important to identify since they contribute most to response to breaking wave impact loads. Reliably identifying both the close natural frequencies and the corresponding mode shape orientations was impossible with EMA.

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Bayesian OMA (BAYOMA) provided the most insight into the modal behaviour, while at the same time providing insight into the fundamental limitations for identifying close modes.
Specific conclusions from the OMA described in this paper are:

- Due to varying degree of asymmetry in the 'concave elliptic frustum' lighthouse shapes, mode frequencies in a pair were found to differ by between 0.75% and 3.8%.
- Unlike EMA, OMA was able to identify (or estimate) the horizontal directions of the mode pairs corresponding to the very close natural frequencies.
- Visually apparent structural symmetry may not be strongly linked to mode shape orientations.
- Mode frequency variation over time may exceed -but is not accounted for in- the calculated identification uncertainty of MPs.
- There is a trade-off between mode shape orientation uncertainty and closeness of frequencies in a close-mode pair.

1. Introduction: modal properties of lighthouses and other axisymmetric vertical structures

Lighthouses are iconic structures that provide visual navigational aids to protect mariners and preserve vital maritime trade. Some of the most spectacular examples are the masonry towers constructed in the Victorian era (1837–1901) on rock outcrops or reefs around the British Isles, which have survived decades of extreme weather. A collaborative research project was established in 2016 to examine the performance of a set of these structures, evaluate the wave loads they experience and provide guidance for their future management. A significant component of this project has been evaluation of the lighthouse dynamic characteristics i.e. their modal properties (MPs), since these provide insight into the way the structures resist forces from the waves that break on the reefs, as well as the means to infer the distribution of extreme loads through extended monitoring of dynamic response.

To this end an experimental programme of modal testing spanning June 2016 to October 2017 was organised to provide sufficient information to calibrate finite element simulations and to infer wave loads [1] on a set of seven lighthouses. This programme primarily relied on forced vibration testing (FVT) using a small electrodynamic shaker to produce input-output data for experimental modal analysis, but ambient response measurements were also made, which are the subject of this paper.

The shaker testing identified dynamic curiosities:

1. What appeared on first examination to be single modes (with the same features whichever direction the shaker was aligned) turned out on more detailed examination to be mode pairs whose orientation could not be directly established, nor could their frequencies be clearly distinguished.
2. For five of the lighthouses equipped with a helideck atop the masonry tower, the first and second modes (or mode pairs) displayed opposite mode shape ordinate phase for the helideck.

For each of the shaker tests, force and response measurements were made in orthogonal axes chosen (with respect to convenient internal features) to allow most accurate alignment of accelerometers at the various levels. Using responses in both directions to single direction excitation was not an effective approach to distinguishing mode pairs with close frequencies.

While horizontal structures such as bridges can sometimes exhibit modes with close natural frequencies, the reasons are specific to a structure (sometimes due to fixity at supports), and mode shapes are usually very different. For vertical structures such as tall industrial chimneys [2] and office towers [3,4] modes occur at very close frequencies but with similar mode shapes due to having multiple lines of horizontal symmetry. Office and residential towers with polygonal shapes tending to circular have multiple lines of symmetry tending to axial symmetry, but in all cases there should be no more than two independent modes with the same mode shape, even if (horizontal plane) angles between measured mode shape vectors do not need to be exactly 90°. For perfectly axisymmetric towers there will appear to be a single omni-directional horizontal mode.

In reality perfect axisymmetry does not exist and there is a tendency to align in specific directions, for example with industrial chimneys [2] structural layout of flue ducts and openings in the lower part of the structure controls direction. For lighthouses structural symmetry could be affected by topology of the rock reef at foundation level, fixity, location of doors and windows and internal fitout.

For system identification of tall structures, ambient vibration testing is the usual choice, but few tests are reported for vertical structures likely to have close modes (e.g. of square section towers) including information on mode alignment [3,5] and most studies simply assume alignment with geometric symmetry axes [4]. Published ambient vibration tests on nominally axisymmetric structures such as cylindrical chimneys [2], with mode shapes, are even rarer. Hence the ambient vibration data obtained for these lighthouses provide a useful set to examine the effect of structural symmetry on mode shape alignment and the parallel challenge of resolving very close frequencies with small directional dependence of stiffness or mass properties.

The paper briefly describes the candidate subset of lighthouses relevant to this study in terms of structural features likely to affect modal identification, then provides some details of the experimental modal testing programme. Frequencies, damping ratios and masses obtained using shaker testing are presented for the two (single) modes with opposite helideck phase of the helideck for three examples.

The focus is then on the ambient vibration measurements which, via operational modal analysis (OMA), provided particular insight into the lighthouse behaviour due to ability to discriminate and more fully characterise modes with very close frequencies. Existence of close modes is revealed via singular value decomposition of cross-spectral densities of ambient response, which also indicate initial values and frequency bands for the Bayesian operational modal analysis (BAYOMA) procedure employed. BAYOMA is used in preference to other classical methods because it allows the physical mode shapes (in contrast to operational deflection shapes) of close modes be identified and because it provides both the 'posterior' (i.e., given data) means and variance of all modal parameters, including mode shapes and the power spectral density matrix of modal forces.

2. Construction and structural arrangement of lighthouses

Five lighthouses are studied in this paper and are summarised in Table 1. These structures, operated by Trinity House (TH), Irish Lights (IL) and Northern Lighthouse Board (NLB), are all highly exposed to storm wave loads. All the lighthouse towers are 'concave elliptic frustums' and three have helidecks fixed to the crown of the masonry structure and straddling the lantern. These were retrofitted between 1973 and 1981 to simplify access by lighthouse keepers and later by maintenance crews when the lighthouses became automated. Fastnet and Dubh Artach are accessed by adjacent helipads.

Design of British lighthouses evolved by trial and destruction. Smeaton's Tower on Eddystone Rocks (off Plymouth, completed in 1759) was the prototype shape, with multiple circular masonry courses comprising shaped and dovetailed granite blocks, copied in all subsequent structures. With bolted fixing of the lower courses into foundation rocks, as described in detail for Wolf Rock [6] and Eddystone Lighthouses [7], the lighthouses should act as monolithic granite towers continuous with the foundation rock.

As such, rigid body rocking as was described by many lighthouse keepers should be impossible, and it has since been established at least for Wolf Rock [1] that the sensation observed is an interpretation of the impulsive elastic response of the structure to the impact of waves shaped by the reef bathymetry and breaking on the lighthouse. This means that assumptions of linearity apply and modal analysis procedures that assume linearity can be used to identify modal properties which themselves can be used for forward or backward analysis of loading and response.

Les Hanois, Eddystone and Wolf Rock are chosen because they represent a spectrum of scale for the same type of structure, with retrofitted helidecks. Fastnet and Dubh Artach have no helideck and have axisymmetry disrupted in different ways.

3. Modal testing

Following pilot studies [8] on Eddystone Lighthouse by researchers from the University of Plymouth, a research programme was initiated with aims to

- identify experimentally modal parameters (MPs) for a set of rock lighthouses,
- monitor and process structural response of at least one lighthouse over an extended period,
- develop structural models based on construction information and calibrated through dynamic testing and hydrodynamic experiments,
- investigate worst case hydrodynamic loading and
- formulate guidance on procedures for structural condition assessment and management based on objective structural performance data.

In total, seven lighthouses were studied and the programme of modal testing is reported in [1] and [9]. All the lighthouses are unmanned and the general lighthouse authorities (GLAs) comprising Trinity House, Northern Lighthouse Board and Irish Lights operate a schedule of bi-annual or quarterly maintenance visits planned a year in advance. The relevant GLA provided additional helicopter flights to support the modal testing programme during maintenance visits.

When originally planning these modal tests, there was little to guide the choice between experimental modal analysis (EMA) of forced vibration test (FVT) data which is referred to in this paper as EMA, or operational modal analysis (OMA) of ambient vibration test (AVT) data which is referred to in this paper as OMA. Preliminary response monitoring of Eddystone Lighthouse [8] using geophones had shown a fundamental frequency of 4.4 Hz, and Douglass' paper [7] gives the mass of the granite structure as 4.76×10^6 kg. The modal mass accounting for a linear mode shape unity scaled at lantern level was estimated at 600 tonnes, which is suitable for a small 180 N electrodynamic shaker, depending on the strength of ambient

Table 1
Lighthouse details. TH = Trinity House, IL = Irish Lights, NLB = Northern Lighthouse Board.

Lighthouse	Location	Built	Designer	Tower height	Modal test date
Les Hanois (TH)	Guernsey west coast 49°26′06.2″N 2°42′08.4″W	1860–1862	James Douglass	36 m	2/6/2016
Wolf Rock (TH)	15 km west of Land's End 49°56.72′N 5°48.50′W	1861–1869	James Walker	41 m	18/7/2016
Fastnet (IL)	Southwest Ireland 51°23.358′N 09°36.178′W	1904	William Douglass	54 m	5/12/2016
Dubh Artach (NLB)	Southwest of Mull, Scotland 56°07.946′N 006°38.079′W	1876	Thomas Stevenson	38 m	10/5/2017
Eddystone (TH)	21 km southwest of Plymouth 50°10′48″N 4°15′54″W	1879–1882	James Douglass	49 m	10/10/2017–11/10/2017

response to wind and wave loads. Hence ambient vibration measurements were used as both backup and complement to shaker testing, aiming to provide information about vibration modes beyond the frequency range of shaker excitation.

FVT requires a specific choice of shaker alignment (two directions at 90° to each other) and in each case this was chosen on the basis of lighthouse internal layout and reference features that allowed a common orientation of accelerometers at different levels. The primary aim was logistical simplicity and the pair of shaker and accelerometer directions could not be expected to coincide with actual or expected mode shape directions. Most likely, actual mode shape directions would coincide with weak and strong directions, i.e. 'principal axes', making the assumption that mode shape vectors at each lighthouse level would align neatly in a vertical plane.

Full details of the measurements are given [1] and [9] and details of accelerometer numbers and location are given in Table 2. In all modal tests, the shaker was located at the highest feasible location in the masonry structure, generally the floor of the lantern room. 12 uniaxial accelerometers were used for the majority of tests, 16 in the case of Eddystone, and these were arranged either as two groups (setups or sets) of biaxial pairs or as two sets one with accelerometers in each of the two shaker directions. The former approach was used for the first two tests (Les Hanois and Wolf Rock lighthouses) aiming to achieve a better merging of horizontal mode shape ordinates along a vertical axis using OMA of ambient response, but relocation between levels proved to be logistically inefficient (using up precious limited time on station) and the latter approach was subsequently used.

Example modes identified by EMA are shown in one direction of shaking in Fig. 1 for Les Hanois and Eddystone Lighthouse; the complete set except for Fastnet Lighthouse can be found in [1], the Fastnet test is reported in [9]. For Les Hanois the modes are merged from the two measurement setups, for Eddystone they derive from a single setup. The modes were identified applying the global rational fraction polynomial procedure or GRFP [10] to identify two modes in the frequency band 5–8 Hz or 4–9 Hz. All the helideck-equipped lighthouses demonstrated the type of behaviour shown, with helideck mode shape ordinate switching phase between (what appeared to be) a single low frequency mode and (what appeared to be) a single high frequency mode, with varying ratios of modal ordinates, a result of low mass, low stiffness structure fixed to a high mass, high stiffness structure.

For each of these two EMA modes, detailed inspection of the frequency response functions, e.g. as Nyquist plots [1], shows that there are in fact two modes with close frequencies which could be distinguished using multi-mode identification techniques. GRFP could be set to identify both modes in a pair [1] having e.g. four modes in the band 5–8 Hz for Les Hanois, rather than just two modes (as in Fig. 1), although both modes would be assumed to align in the shaker direction. So for the purposes of inverse load identification based on response, single modes were reported corresponding to and (assumed to be) aligned with each shaker direction and without uncertainty bounds.

EMA was able to deliver useful and repeatable results even with relatively poor signal to noise ratios and despite operational problems with shakers. Standard OMA techniques such as NExT/ERA [11] and stochastic subspace identification (SSI-COV) procedure [12] were subsequently applied to the ambient response but could not provide unique descriptions due to the various implementations, options and interpretations. OMA did, however indicate mode pairs having frequencies sometimes less than 1% different and with directions varying both between different modes (for the same level) and between levels of the lighthouse (for the same mode). Hence it was decided to use Bayesian OMA [13]. The method can identify the physical mode shapes of close modes (in contrast to operational deflection shapes) consistent with conventional structural dynamics assumptions. It also yields identification uncertainty in accordance to Bayes' rule, allowing us to investigate the close modes, their directionality and the uncertainty in identification, attempting to account for the observed variations and establish links with imperfections in the tower symmetry.

4. Foundation arrangements and other factors on symmetry

Various influences on symmetry could include visible features such as openings in the thick masonry walls for windows and doors, particularly at lower level, the shaping of the courses to fit the reef topology, and the internal fitout. Invisible factors could be variation in masonry and grout stiffness and mechanical fixing between courses (by keying and bolts) and to the foundation rock (by bolts). The various factors are presented for the five lighthouses based on available information including archive drawings and photos.

Table 2
Ambient vibration data sets used (*used for Figs. 7, 8 and 12).

Lighthouse	Set	Levels and directions	Duration (s)	Sample rate (Hz)
Les Hanois	1	1,2,5,6,9,10 x + y;	940	204.8
	2	3,4,7,8,9,10 x + y;	340	204.8
Wolf Rock	1	1,2,5,6,8,9 x + y;	960	256
	2	3,4,7,6,8,9 x + y;	64	256
Fastnet	1	1 – 7x, 8x + y, 9x;	31,936 (8.9 h)	128
Eddystone*	S1R6	1 – 6x, 7 – 11x + y;	896	128
Eddystone*	S2R19	1 – 6y, 7 – 11x + y;	896	128
Eddystone	1	1 – 6y, 7 – 11x + y;	35,968 (10 h)	128
Dubh Artach	1	1,2x, 3x + y, 4 – 6x, 7x + y;	960	128
	2	1,2y, 3x + y, 4 – 6y, 7x + y;	896	128

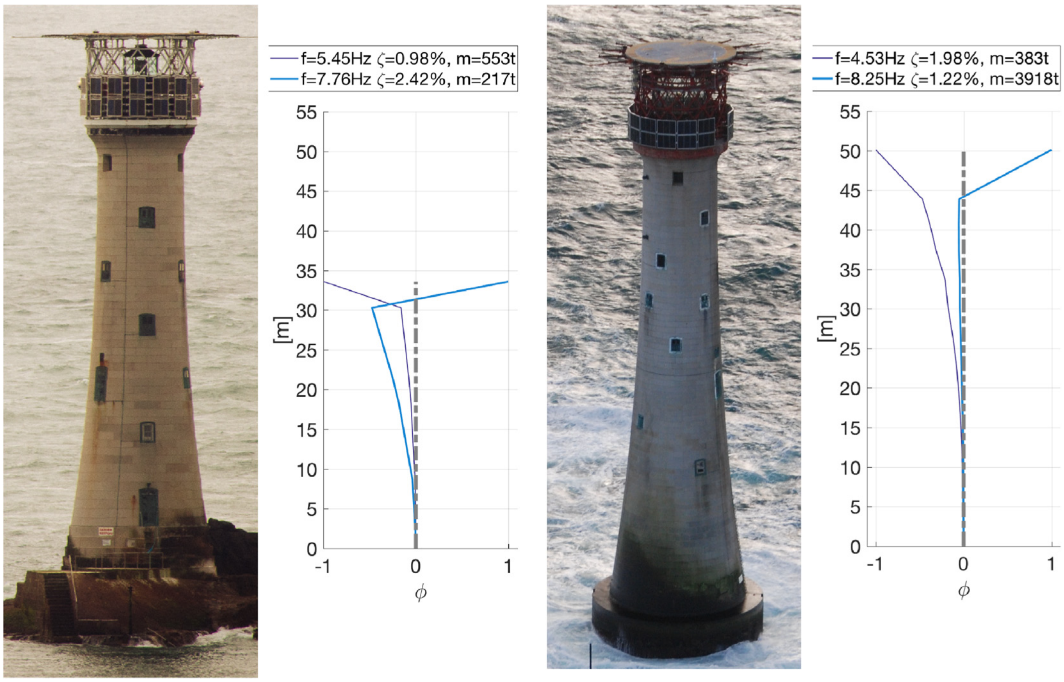


Fig. 1. Les Hanois (left) and Eddystone (right) Lighthouses and first two modes identified from experimental modal analysis of forced vibration test data for nominal x direction. Photos: Emma Hudson (left), Trinity House (right).

Les Hanois Lighthouse (Fig. 1) has no irregularity in alignment of openings and few features strongly disrupting axisymmetry. Shaking provided apparently identical frequencies of 5.45 Hz for the two measurement directions, with positive acceleration measurement directions set to north (0°, x) and west (270°, y).

Eddystone Lighthouse (Fig. 1) is a much larger structure than Les Hanois and also has no obvious disruption to axisymmetry, since openings are at irregular locations on different levels, and the foundation is only mildly asymmetric. Hence only one frequency, 4.53 Hz, is found for the modes in the measurement directions bearing north (0°) for x and west (270°) for y.

Wolf Rock (Fig. 2) is adjoined by a masonry landing that could add mass and/or stiffness, and while the first course is only two masonry blocks on the otherwise level rock foundation, other courses are axisymmetric. The landing area aligns approximately with the entrance (now used as emergency exit) at the tower base which is at a bearing 30°, and the (accelerometer) axes for the measurement were at (compass) bearings 168° (for x) and 78° (for y).

Even with the landing, almost identical frequencies were observed using EMA [1]. The EMA mode shapes show that the first mode is dominated by the helideck; subsequent numerical (finite element) modelling for the entire structure reliably reproduced the modes and revealed that without the helideck the lower mode would be absent and the higher mode frequency increased by ~2%, while the helideck as an independent fixed-base structure would have a (first) mode frequency of 5.1 Hz.

Fastnet Lighthouse (Fig. 3) has no aligning features in the tower itself, but the foundation level courses are strongly asymmetric due to the location perched at the sloping end of Fastnet Rock with courses descending on one side to the sea. Distinct

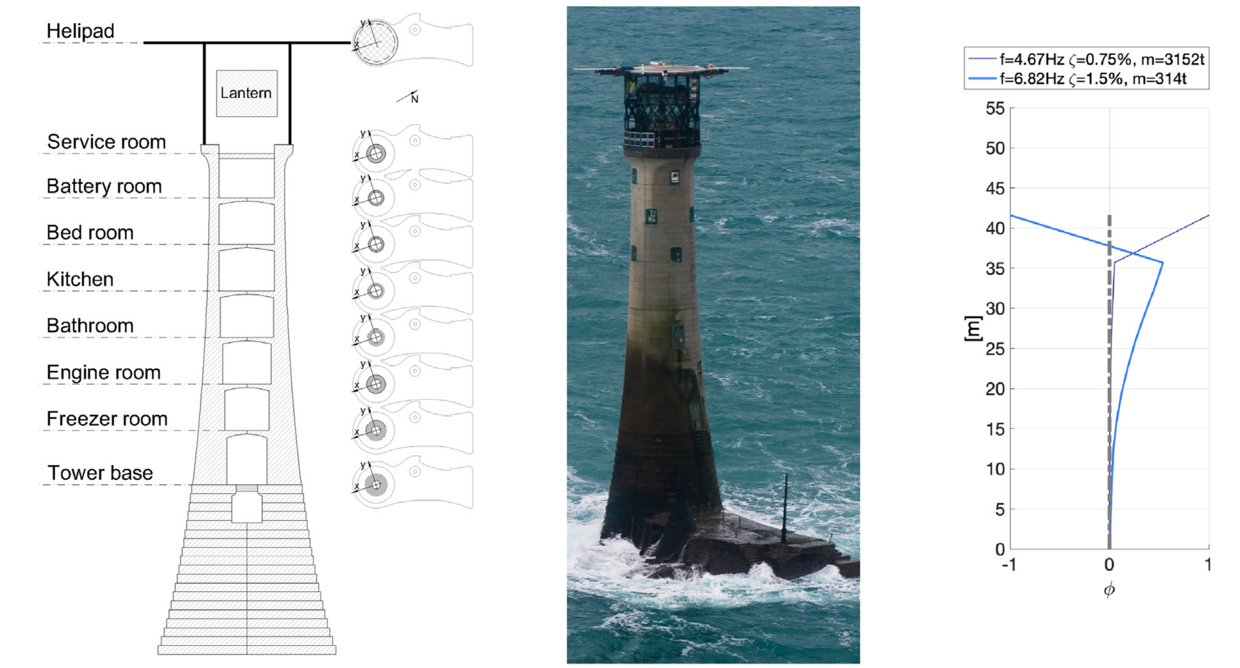


Fig. 2. Wolf Rock Lighthouse. Vertical section through tower, accelerometer angles; aerial view of lighthouse and landing area; EMA modes. Photo: James Bassitt.

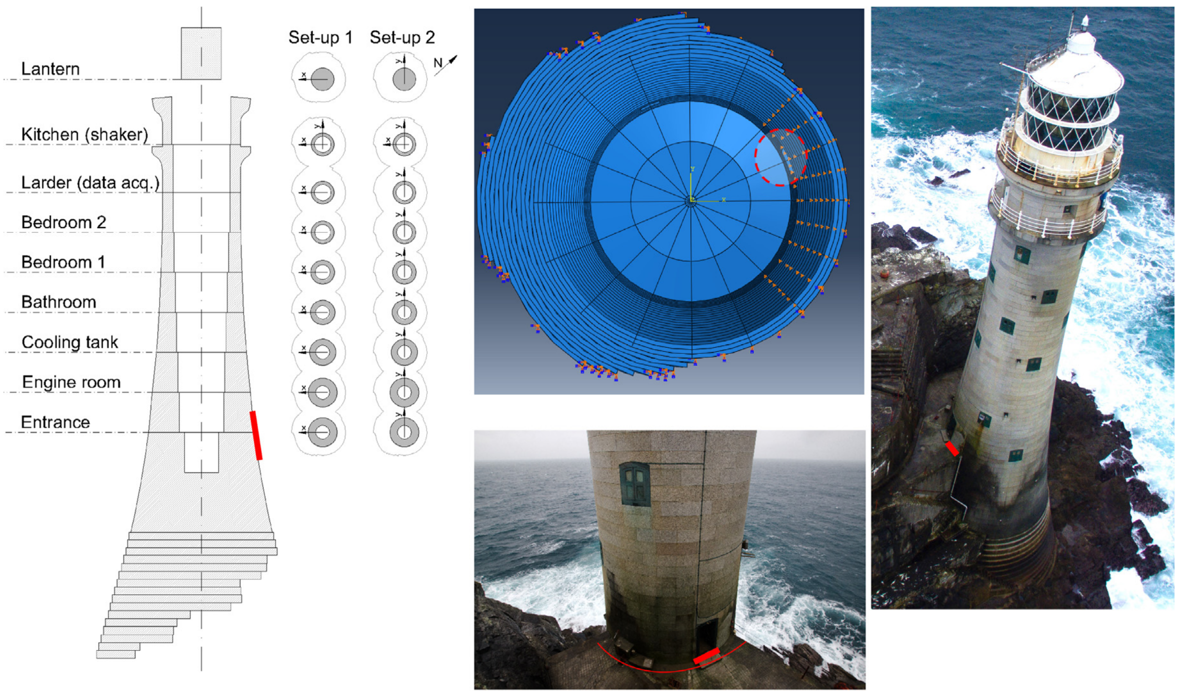


Fig. 3. Fastnet Lighthouse: (left) vertical section through structure and accelerometer arrangements in two sets, (mid upper) FEM showing door, (mid lower) view of rock restraint from stump of old tower, (right) drone view showing extended courses. Measurement x axis is through door, FEM x axis is rotated 22.5° clockwise from door. Photos: James Bassitt and Athanasios Pappas.

frequencies (4.8 Hz appearing strongest in y direction and 5.0 Hz appearing strongest in x direction) for the fundamental mode were identified by EMA in spite of problems with the shaker. The centreline of the deeper (sea-side) courses bears 250°, with measurement axis bearings 312° (for y) and 222° (for x) i.e. not conventional order.

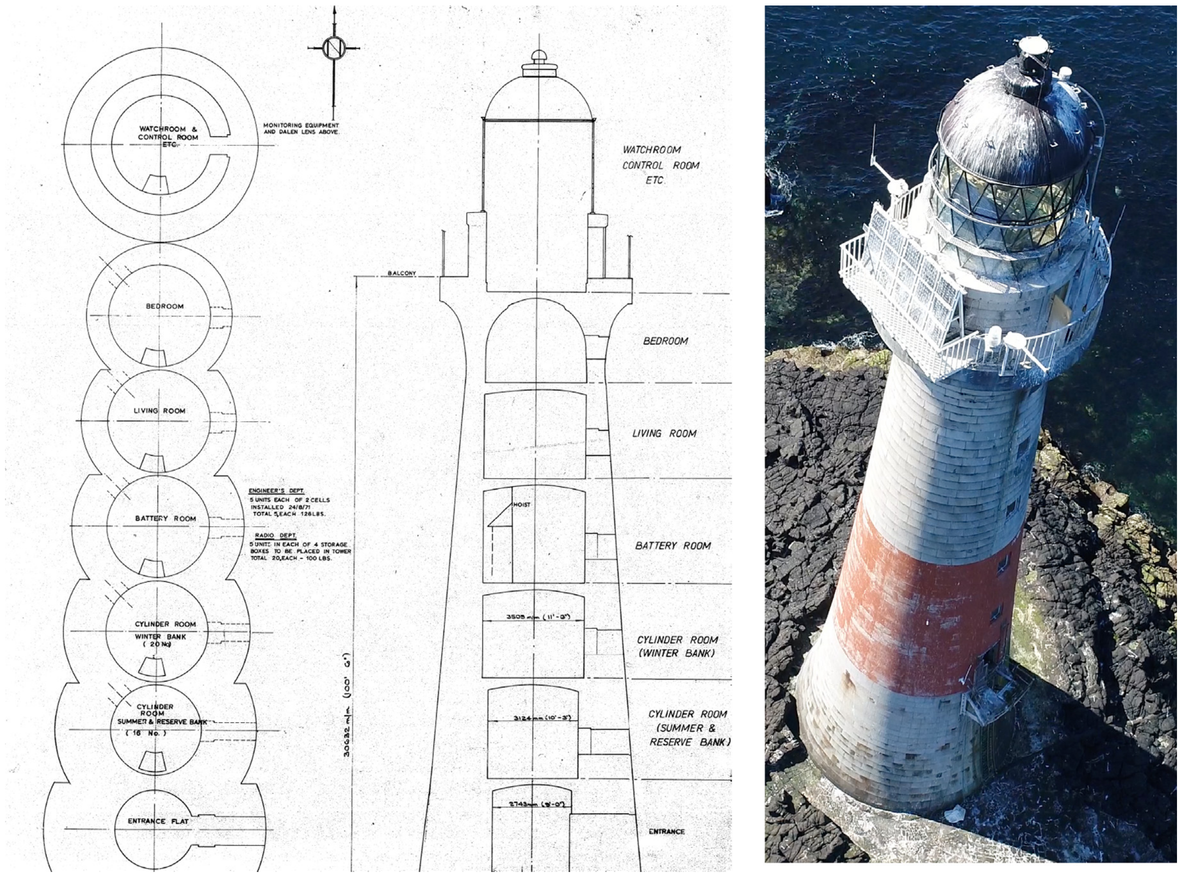


Fig. 4. Dubh Artach Lighthouse horizontal and vertical sections and aerial view, showing alignment of openings. Photo: James Bassitt, drawings courtesy of Trinity House.

Dubh Artach (Fig. 4) sits on a level rock platform and is perfectly axisymmetric at foundation level, but has all the openings aligned on the east face which might lead to distinct directions and frequencies. However, EMA identifies apparently the same mode from both x direction (aligned with openings at bearing 90°) and y direction measurements.

5. Operational modal analysis: power spectral densities and their singular value decomposition

Compared to EMA, operational modal analysis (OMA) is attractive because it does not rely on using a shaker with a particular location and alignment, nor does it depend on its operating bandwidth. Power spectral densities (PSDs) of ambient response are a common starting point for investigating modal properties via OMA, and are obtained as the diagonals of the cross-spectral density (CSD) matrix of all acceleration response measurement channels. Typically, PSDs are obtained by breaking a signal into frames and calculating RMS averages using the Welch procedure. The decreased variance of PSD spectral line amplitudes with increased record length (more averages) is well established [14] and there is a trade-off with frequency resolution that also affects bias in estimates using some older frequency domain OMA techniques. Assuming that there is no modelling error, e.g. MPs are time-invariant and modal forces are stochastic stationary in the data window, the direct relationship of record length with uncertainty in modal parameter estimates has more recently been established [15], hence the longest possible duration recordings permitted by the experimental constraints were used, and during which environmental and operational conditions were stable.

Theoretically, strongly stationary data (where they actually exist in the real world) have no time variation (certainly no trend) in mean, variance or autocorrelation function, and while there are many tests for stationarity in data, they assume no knowledge of the underlying phenomenon and judge stationarity via an arbitrary threshold. Authors prefer the simple approach suggested in [14] that associates real-world stationarity with time-invariance of the physical factors generating the data, and this is in practice judged by inspection of spectrograms (short time Fourier transforms) and band-pass filtered time series. Up to the final section of this paper, all MP estimates are obtained with time series no longer than 16 min, for which the field test teams observed no obvious loading transients (wind, wave or impact) or rapid changes (e.g. fast developing storm). When there *is* clear time-variation of response signals or MPs themselves and an obvious breach in stationarity requirement, the duration for OMA should trade a balance between identification uncertainty and modelling error. This is addressed in the last section of the paper.

The measurement sets used for OMA specified in Table 2 are provided as a dataset linked with the paper, as follows. Measurement levels within a masonry tower are numbered starting from 1 at reef level of entrance/exit, and continuing up into lantern (one or more levels) and helideck (where installed).

PSDs were prepared using the diagonal entries of CSD matrices for each data set, as described above. The example for Les Hanois (Fig. 5) indicates three possible modes (or mode pairs) up to 8 Hz. EMA only identified two modes (Fig. 1) and the peak above 6 Hz has a shape uncharacteristic of a vibration mode and could be due to the generator that operates during maintenance visits. Nevertheless, OMA techniques identified (a pair of) modes around this frequency.

PSD does not indicate the possibility of multiple close modes contributing to a spectral peak, so the type of mode indicator function based on singular values employed in experimental modal analysis of FRF data [16] is adapted. Instead of basing the singular value decomposition (SVD) on the FRF data [16], SVD is applied to the CSD matrices. Near the resonance band of modes, the rank of the CSD matrix at any frequency line is theoretically equal to the dimension of the subspace spanned by the 'partial mode shapes' (i.e. confined to measured DOFs), which in the present case is equal to the number of contributing modes. In practice the rank is usually identified by a sudden reduction of singular values to noise. Hence SVD of the ambient response CSD matrix indicates the number of modes clearly through traces of singular values (SVs) peaking in a frequency band. The SVD is also known as complex mode indicator function (CMIF) [12] and can be used directly for mode identification. SVD plots for the set of lighthouses (using the longest available recordings) are shown in Fig. 6. The modes of greatest interest are those with simple cantilever shape in the masonry tower, such as in Fig. 1, since such lower frequency modes are known to respond most strongly to wave impact [1]. The higher modes are relevant for model updating [9] but have character with greater variation among the lighthouses and are not discussed here.

Les Hanois SVD clearly shows two modes at 5.45 Hz and at least two modes around 7.76 Hz. It also indicates that the 6.2 Hz PSD peak believed to be generator mechanical excitation could be two genuine modes, a theory supported by the absence of such peaks in data for the other lighthouses where generators were also running (at the standard 1500 revolutions per minute, with about 10% variation depending on load).

Wolf Rock SVD indicates two modes each around 4.7 Hz and 6.8 Hz plus a single peak around 5 Hz not identified using EMA.

For the Fastnet Lighthouse modal test only a single level had a biaxial accelerometer pair and the EMA clearly indicated a pair of modes with distinct frequencies around 5 Hz reflecting the structural asymmetry. Hence SVD of the ambient response data is less useful, but confirms the well separated mode frequencies. With no helideck there is no second pair of close modes and the higher frequency peaks seem to be for modes in a single direction. It is possible that the 9 Hz mode is due to the very tall lantern room which for Fastnet appears to behave as a separate structure.

Dubh Artach Lighthouse, which also has no helideck, was tested on a very calm day so the SVD reflects a very low signal to noise ratio compared to the other modal tests. Even so, there is a very clear pair of modes with close frequencies around 5.3 Hz.

Eddystone Lighthouse test benefitted from much better understanding of the modal properties of the structural type, the largest set of (16) accelerometers including many biaxial pairs and an overnight stay that allowed for more comprehensive and long duration measurements. Hence the SVD shows clear mode pairs in bands around 4.3 Hz and 8.2 Hz.

OMA of the ambient response data was initially attempted using NExT/ERA [11] merging CSD matrices from two setups, but did not provide reliable mode shapes, most likely due to poor coherence of weaker signals at lower lighthouse levels.

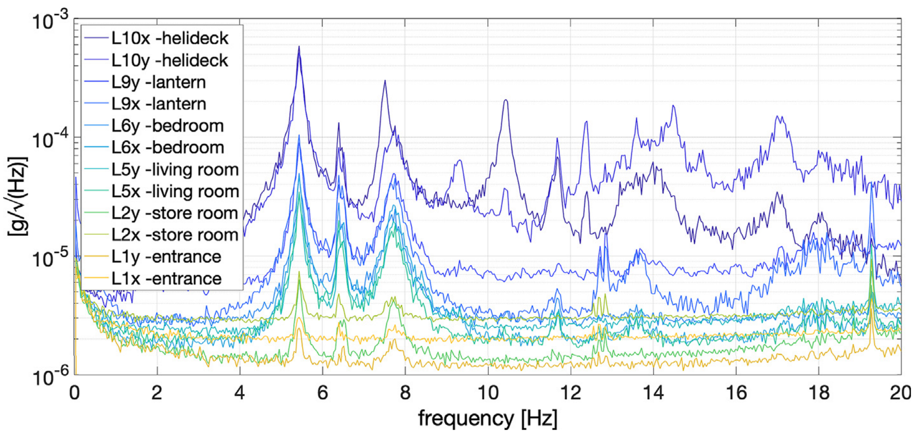


Fig. 5. Les Hanois Lighthouse ambient response power spectral density.

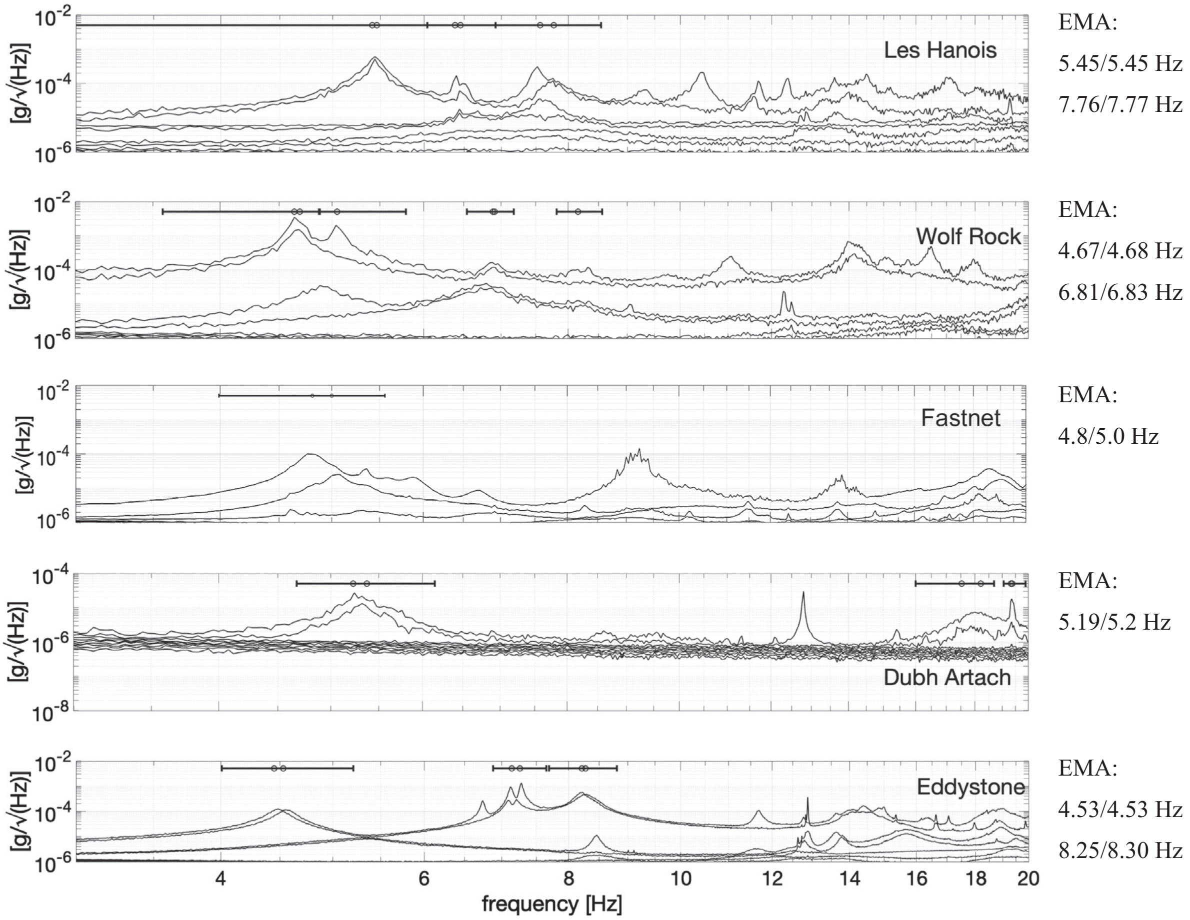


Fig. 6. Singular value decomposition of averaged ambient response data spectra, with corresponding mode frequencies identified by experimental modal analysis (EMA) of shaker test data with arbitrary orthogonal axes. Error bars indicate frequency bands manually chosen for BAYOMA identification and circles indicate BAYOMA-identified mode frequency (most probable values).

The stochastic subspace identification (SSI-COV) procedure [12] was applied to approximately 15 min of time series data from one set for each of the three structures, with 100 lines in a covariance matrix obtained from time series resampled at 50.2 Hz (for Les Hanois), 64 Hz (Wolf Rock) and 32 Hz (Eddystone) and system order up to 30 poles. SSI estimates depend on the sample rate, the number of samples and the algorithm used to determine a 'stable' mode e.g. where the mode repeats a minimum number of times at the same frequency and reasonable damping ratio as system order is increased. The stabilisation plots, superimposed on SVD of the data used (Fig. 7) show variable performance in estimating both the EMA-identified modes and the 'extra' modes observed in the singular value spectra.

For Les Hanois only the first mode pair (Fig. 1) with higher modal mass (normalised to unity at lantern level) is stable in SSI for a large range of poles. For Wolf Rock, the second mode (~6.8 Hz as in Fig. 2, with low modal mass) is not identified while a mode around 5 Hz is picked up. For Eddystone, SSI finds several modes additional to the EMA (pairs) at ~4.5 Hz and ~8.25 Hz shown in Fig. 1. The 'extra modes' appear to involve significant helideck movement and would not have been picked up in shaker testing, with poor signal to noise ratio; while the granite towers responded mainly to wave, helidecks were noticeably sensitive to wind.

Rather than rely on interpretation of stability plots, BAYOMA was used since it relies only on SVD to define modes, providing the corresponding 'most probable' modal parameters with corresponding variance errors. It proved to be very useful in characterising the very close modes and what appears to be ill-conditioned directionality of mode shapes.

6. Bayesian operational modal analysis: data and procedure

BAYOMA [13,17] operates on the Fast Fourier Transform (FFT) of ambient vibration time series, in a selected frequency band around the subject modes. The FFT is used directly without windowing or averaging. In a Bayesian perspective of modal identification, given the data the uncertain modal parameter has approximately a Gaussian distribution with a mean equal to

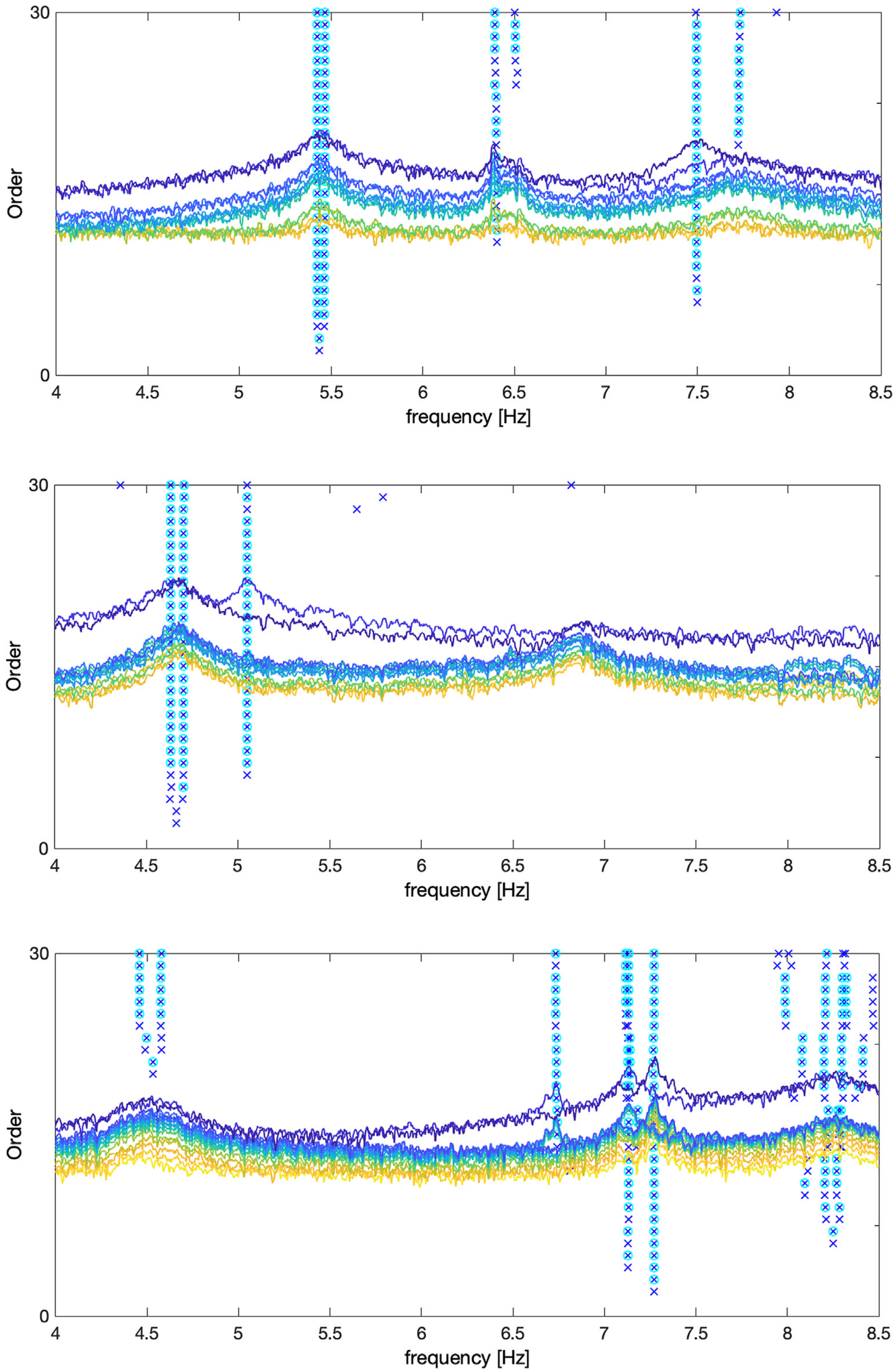


Fig. 7. SSI-COV applied to Les Hanois (upper), Wolf Rock (middle) and Eddystone (lower) data.

the most probable value (MPV) and standard deviation equal to σ, the square root of variance (σ^2). This is a fundamentally different view from the 'frequentist' approach where MPV might be derived from hypothetically repeated experiments. Assuming uniform prior distribution (as is typical), the MPV of the set of modal parameters Θ is found by minimising the negative log of the likelihood function (NLLF) for the given data. The identification uncertainty of modal parameters can be quantified by the covariance matrix of Θ associated with the Gaussian distribution, which is equal to the inverse of the Hessian matrix of the NLLF at the MPV. The coefficient of variation (COV) is σ normalised by the MPV and expressed as a percentage, and is reported with the MPV.

The mathematics of BAYOMA are challenging so efficient algorithms are used to calculate the values. The computational time is generally slightly greater than using NExT/ERA or SSI; being a procedure that finds the minimum of a function (NLLF), the computational time is typically a few seconds for well-separated modes and a few seconds to minutes for close modes, depending on how close the modes are, how well they are excited, etc. Unlike NExT/ERA or SSI, model order is pre-determined (so far manually) via plots such as Fig. 6.

Ideally a broad frequency band around a single mode frequency is chosen, but for close modes (as in this case) the analyst can choose to model the modes together with a broad band (the better choice) or separately with narrow bands. The method is robust/immune to coloured activities outside the resonance band of subject modes.

The identification results are consistent with conventional structural dynamics assumptions and so are directly transferrable to analysis and design. In BAYOMA, the modes are assumed to be classically damped while the noise and modal force (excitation) PSDs are assumed to be independent and to have constant PSDs (only) within the selected frequency bands, which are indicated (for each lighthouse) in Fig. 6. If response is not stationary, the FFT represents an average over the different scales of vibration and identified modal properties represent an aggregate over the different vibration levels experienced. This is especially relevant for the modal force PSD and damping ratio that is often perceived to be amplitude-dependent.

BAYOMA identifies mode shapes in contrast to operational deflection shapes which include contributions of modes away from their resonant peaks and are necessarily orthogonal as a result of matrix decomposition. Identified mode shapes need not be orthogonal as they are confined to the measured DOFs only. Some applications of BAYOMA are presented in [18–20]. See, e.g., [21–23] for OMA identification uncertainty quantification in a non-Bayesian 'frequentist' sense.

Like other OMA methods assuming time-invariant and stationary models, BAYOMA applied to a single time window does not quantify the variation of modal properties over an extended duration. Currently a simple empirical way to track variation is to apply BAYOMA to different time windows and track the results accordingly.

Once the real-valued mode shapes in each set are identified (in terms of their MPV), they can be glued using a global least square method [24] that minimises (under norm constraint of glued mode shape) a quadratic measure-of-fit function accounting for the discrepancies in all sets. Such procedure can be shown to give the same MPV as in a Bayesian method when the signal-to-noise ratio is high. For challenging cases where the signal to noise ratio is low in some sets leading to erroneous results from least square methods, a multi-set Bayesian algorithm may be adopted as it is found to give more robust results, although currently it can only be done efficiently when the modes are well-separated [25,26]. MPVs and particularly COVs may differ between sets, particularly when set durations differ significantly (e.g. Wolf Rock).

7. Bayesian operational modal analysis: results

Based on the SVD plots shown in Fig. 6, bands containing one or more modes were chosen for identification, and are indicated by tolerance bars in the figure.

Results are presented as mode shape plots for Les Hanois, Wolf Rock and Eddystone lighthouses in Fig. 8, combining sets for Les Hanois and Eddystone to obtain the full mode shapes in elevation and plan. For the plan views the accelerometer x axis is at 0° degrees, and for the elevation views the best fit vertical plane of each mode is identified and the mode is rotated around the vertical to provide a common view. Mode shapes for Les Hanois and Eddystone appear to be smooth shape whereas those for Wolf Rock, which are from a single measurement set (not merged ambient response) are, unexpectedly, not smooth. This is due to neither an alignment nor calibration error, and elevation plots for the other two lighthouses tested but not described here (Bishop Rock and Longships) did not provide expected clarity of mode shapes in elevation.

Three closely spaced mode pairs are identified for Les Hanois. The first and last pair correspond to the EMA modes and the two modes of the first pair have the same scale in the masonry tower, so overlay in the elevation view. For the second pair, which was not clear enough for identification by EMA, the tower/helideck phase is the same as the first pair. All damping estimates are believable and the modes in each pair are approximately orthogonal, although there is significant rotation between the pairs, suggesting no strong symmetry axis for strength or mass distribution.

Two mode pairs are identified for Wolf Rock, around the frequencies identified by EMA, and there are two additional single modes. As for Les Hanois, the two modes in the first pair have identical scales and overlay in the elevation view. Only the second mode pair is (self) orthogonal, but there is no pattern for the other modes. The third mode (one of the singles, at 5.05 Hz) is almost entirely of the helideck, with negligible mode shape in the tower, while the sixth mode, at 8.16 Hz (also a single), which has significant mode shape in the tower, was not identified by EMA due to very poor signal to noise ratio.

Eddystone data are good quality i.e. clear of transients (spikes, shifts) so all were usable for OMA, and like Les Hanois there are three mode pairs, with the second unclear in and missed by EMA, and all three pairs are (self) orthogonal. The first pair of modes overlay, as do the second and third pairs.

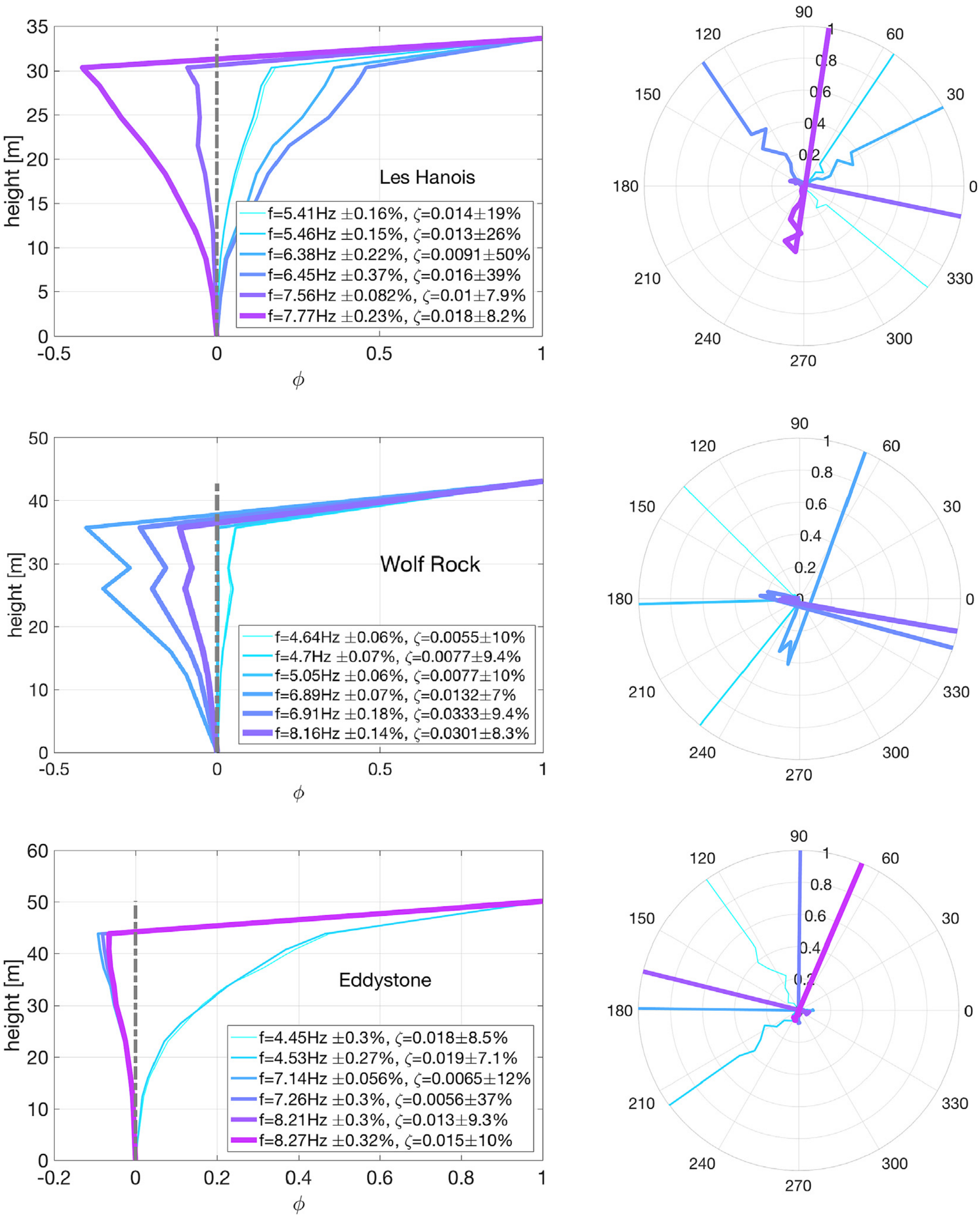


Fig. 8. BAYOMA modes. Left elevation, rotated to same plane; right in plan.

8. Mode directionality

Usually modal tests for identifying sway and torsional modes of buildings are planned with a specific orientation and aligned with e.g. core walls, rectangular columns or building envelope [27]. For buildings with square or rectangular section, modes are expected to align with the two symmetry axes. Even with clear symmetry this is not guaranteed and directions need not align precisely with geometric symmetry axes, neither do the directions need to be at exact right angles at any level. Misalignment is particularly evident in higher modes [28].

For structures like lighthouses with circular section there is not even any expected alignment which begs the question of whether modes have predominant directions and how they reflect variation of structural stiffness and mass. Few full-scale tests on supposedly axisymmetric structures have been reported, the closest analog to the studied lighthouse being

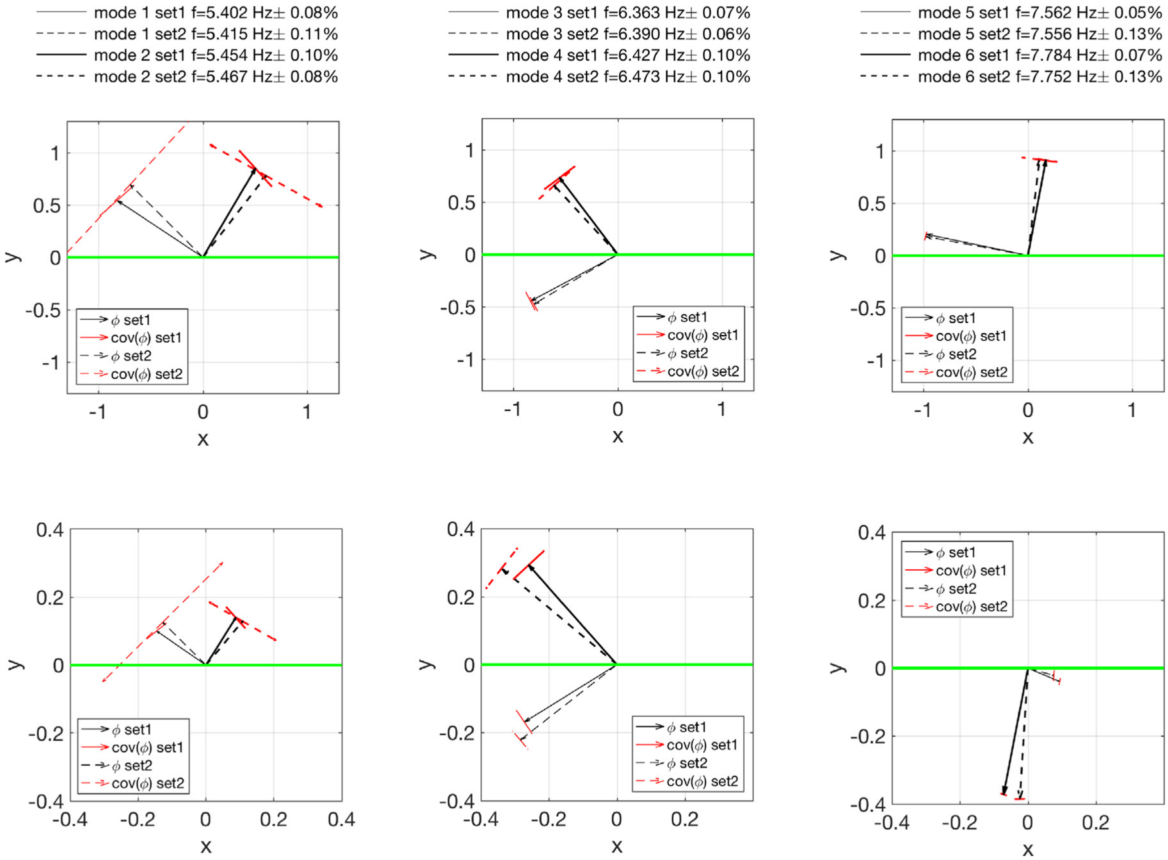


Fig. 9. Les Hanois first three modes at L10 helideck (upper row) and L9 (Lantern), normalised to unity at L10 (helideck).

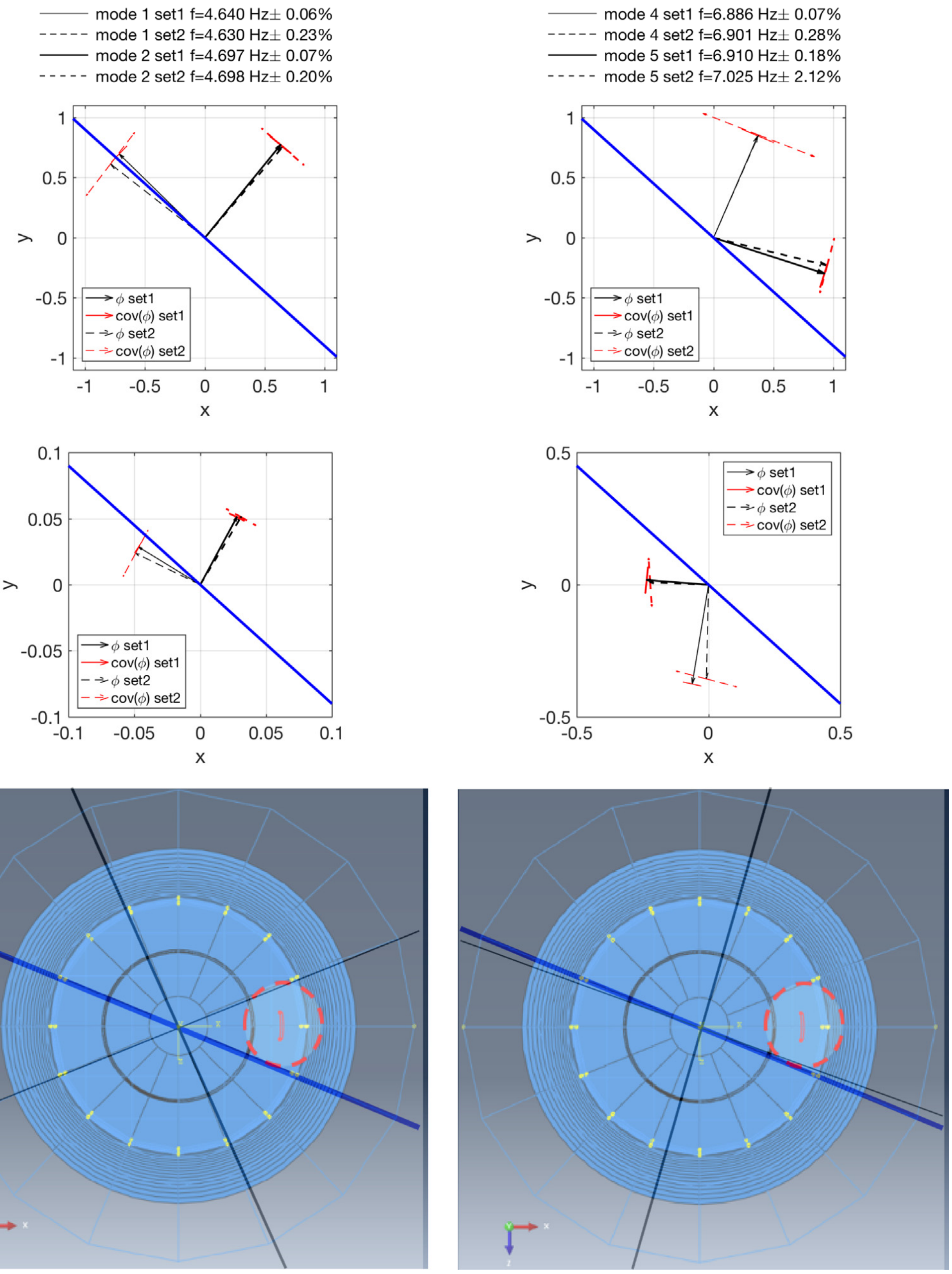
industrial chimneys. Studies invariably neither report full-height mode shapes nor their alignment, although some papers e.g. [27] do report on the very close modal frequencies.

The BAYOMA mode shapes are presented in Fig. 9 to Fig. 12 beginning with Les Hanois (Fig. 9). The MPV of mode shapes are illustrated as (black) vectors originating from the vertical axis. Being vector-valued the dominant uncertainty of each mode shape is obtained from the eigenvector (maximum eigenvalue) of its posterior covariance matrix multiplied by the square root of the eigenvalue. This $\pm 1\sigma$ uncertainty is shown as (red) lines which are roughly perpendicular to the vector tip. Thicker lines are shown for MPV and $\pm 1\sigma$ uncertainty for second mode of the pair. Estimates from both sets are shown, for helideck and lantern levels. Les Hanois set 2 duration was approximately one third of set 1 duration and this is reflected in the larger variance for the first mode pair. While mode pairs are self-orthogonal, directions of the three mode pairs do not correlate. The phase reversal for third pair is as per the EMA result although the different vector lengths at level 9 are surprising, i.e. mode shapes in elevation are not the same for this pair. There being no features of the foundation or structure suggesting a specific alignment, measurement x axis (also North-South axis) is indicated as a green line.

For Wolf Rock (Fig. 10) the first (slightly lower frequency) mode of pair seems to align with brickwork landing and almost with the FEM mode axis, that is rotated a little anticlockwise. The second mode pair aligns with neither the landing nor the FEM mode axis. The FEM mode frequencies are a good match with the experimental modes and a curious result is that the 5.1 Hz first FEM mode of the helideck (by itself on hypothetical fixed base) matches quite well the additional mode at 5.05 Hz identified by both BAYOMA (Fig. 8) and SSI (Fig. 7).

Fig. 11 (left) shows Dubh Artach first mode pair (there is no helideck so the pair stands alone) where the lower mode aligns closer to the line of openings. Unlike all other lighthouses all openings align exactly for Dubh Artach so it is possible that these weaken the structure on that side. The two modes are not self-orthogonal at either level. While this seems strange, orthogonality should be considered over all DOFs weighted with mass distribution rather than a measured subset of DOFs without weighting.

Fastnet experimental modes (upper right) obtained from a ten-minute segment of the dataset are well separated compared to the other six lighthouses and the tower has no helideck, so the mode pair stands alone. The second mode being along the FEM axis (and agreeing with the FEM result, lower right) suggests that the lighthouse is stiffer in that axis, possibly



mode 1 4.69 Hz; mode 2 4.72 Hz   mode 3 6.78 Hz; mode 4 6.85 Hz

Fig. 10. Wolf Rock first (left columns) and second (right columns) mode pairs. Level 9 (upper) and 8 (middle). FEM modes (lower), red curve locates upper door. (Thick) blue lines are in direction of landing platform (also the entrance). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

since it is constrained against the rock on that side. Angles for first mode pair obtained from SSI (dashed lines) match BAYOMA reasonably well.

Fig. 12, shows the plan view of first mode pair for Eddystone, combining ordinates at the four upper levels. There is no FEM for comparison and no clear symmetry axis. i.e. it is almost perfectly axisymmetric. There is distinct twisting of the vectors descending the tower.

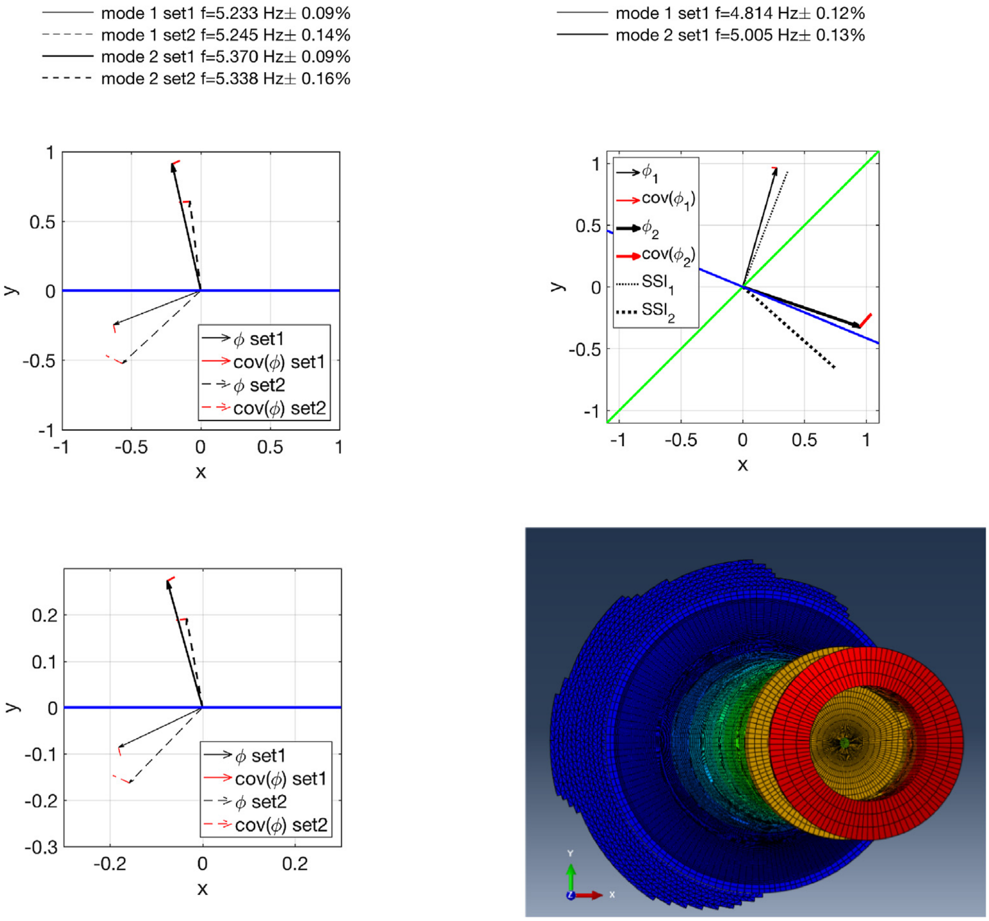


Fig. 11. Left column Dubh Artach level 7 (upper) and level 3 (lower); x axis (blue line) is aligned with openings (windows, doors). Right column Fastnet; x axis aligns with entrance, green line is North-South axis, blue line is x axis of FEM. FEM second mode, along the model axes shown bottom right. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

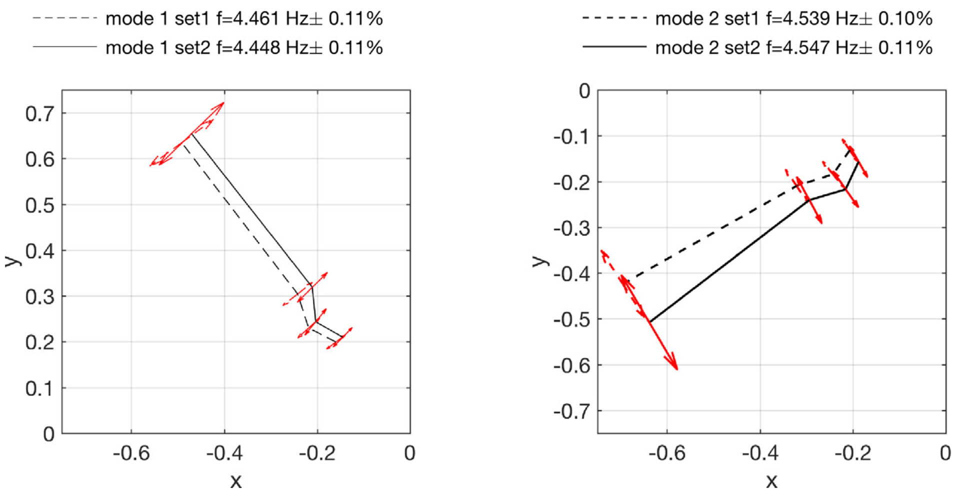


Fig. 12. Eddystone first pair of modes for upper level ordinates.

9. Time varying modal parameter identification

Variation of wind speed, wave loading and sea level (sea state) and internal live loading will change the spectrum of dynamic loading (the modal force), while other factors such as temperature and moisture content can lead to variation of MPs and collectively result in non-stationary response. Often MP and load variation are correlated and this is sometimes very clear [29,30], but sometimes the correlation may be buried in MP estimation uncertainty. This is a particular problem for the significant bodies of research concerned with model updating [31] and diagnosing structural condition via MP changes [32–34].

Response data used to generate the preceding (BAYOMA) results are too short to reveal non-stationarity (including MP variation) and data that are strongly non-stationary are best obtained during long term monitoring, during storms, when lighthouses are unmanned. Fig. 13 shows response of Wolf Rock on 16th October 2017, during passage of ex-Hurricane Ophelia, first during the whole day, second around 7.30 AM during strong winds and third at 3 PM. Monitoring over the 2017/2018 UK winter storms period shows that strong responses coincide with high tide and waves presumably reaching high up the lighthouse. Tidal range can be up to 6 m and during low tide the response is much smaller and largely wind-driven. The 7.30 AM data are typical of the signals used for the BAYOMA analyses in the preceding sections.

As well as the Wolf Rock monitoring, extended overnight recordings are available for Fastnet and Eddystone. Eddystone is the best example due to having good quality biaxial signals for five levels with 10 min mean wind speeds gradually increasing over 10 h from 7 m/s to 11 m/s. Natural frequency estimates shown in Fig. 14 are from both 5 min (left) and 20 min (right) segments, with error bars representing $\pm 1\sigma$ identification uncertainty about MPVs. Using the same frequency range among the plots highlights the differing 'closeness' as well as the inherent MP variation. The same slow variations (but no clear and consistent trends) are shown for the two segment lengths and clearly the longer segments have reduced variance (which goes approximately with square root of data duration). There appears to be no greater uncertainty where the frequencies (or rather their MPVs) are changing faster with time. The identification uncertainty associated with an MP estimate from longer duration data 20 min of merged data would not reveal such variation since it assumes time-invariance over the duration. The segment length is therefore a trade-off between controlling identification uncertainty (the longer the better) and modelling error arising from potential MP variation (the shorter the better).

Variation of corresponding mode shape angle MPV is shown in Fig. 15. Longer data clearly benefit the identification, and there seems to be a strong link between closeness of frequencies in mode pairs (Fig. 13) and inherent mode shape variation (Fig. 14), the strongest effect being evident for the 3rd mode pair (top rows, modes 5, 6). Even though it is an intuitive result, the evidence is surprising.

Finally, identification of Wolf Rock first mode pair frequencies from the data of Fig. 13 is shown in Fig. 16. The first 10 h of data could be considered stationary, the data for the rest of the day are anything but. Practically all requirements for BAYOMA application would be violated with the strongly impulsive (wave driven) response, nevertheless data were analysed in 10-minute segments, often failing to converge to MP estimates yet suggesting natural frequency reduction and damping increase with response often observed in civil structures.

10. Observations and conclusions

From a practical perspective the published results [1,9] of the experimental modal analysis (EMA) of the set of seven rock lighthouses are adequate for the purpose of validating finite element models and forward and backward identification between wave loading and response. However, the measurements left a number of unanswered questions prompting an investigation using operational modal analysis. The questions mainly related to the nature of the mode pairs apparently behaving as one for EMA, specifically their directions and relationships with structural features, and the uncertainties in the modal parameters including natural frequency, damping ratio and mode shape.

Specific outcomes and conclusions are:

- The varying extent of imperfection in axisymmetry i.e. degree of asymmetry, leads to pairs of modes with close frequencies that EMA (as used) was not intended to discriminate. Only OMA was able to reliably quantify the mode frequencies, and the precision of the estimates.
- As was found by EMA for all helideck-equipped lighthouses, OMA shows pairs of close modes having similar mode shape in masonry structure but opposite phase in the helideck ordinate(s). Some spectral peaks observed in EMA were assumed to be effectively helideck-only modes and OMA, which does not depend on location of a shaker, identifies them more readily.
- OMA generally produced mode shapes in elevation with much less clarity than from forced vibration testing, depending on details of experimental setup such as number of biaxial accelerometer pairs in, and duration of, a measurement setup.
- Quantification of mode shape orientation/alignment and its uncertainty was only possible using OMA and showed that mode alignment did not necessarily relate to visually apparent structural symmetry.
- Bayesian OMA provides statistical quantification of MP values which are particularly important for structures having close frequencies for modes with almost identical horizontal mode shape.

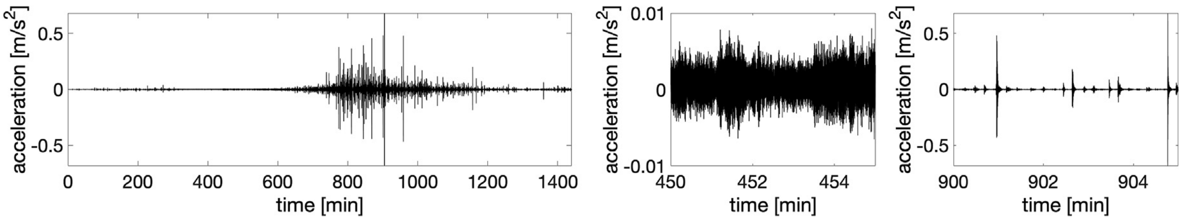


Fig. 13. Wolf Rock acceleration response during ex-Hurricane Ophelia, 16th October 2017. Left, for the whole day, middle for 7.30 AM (low tide), right for 3 PM.

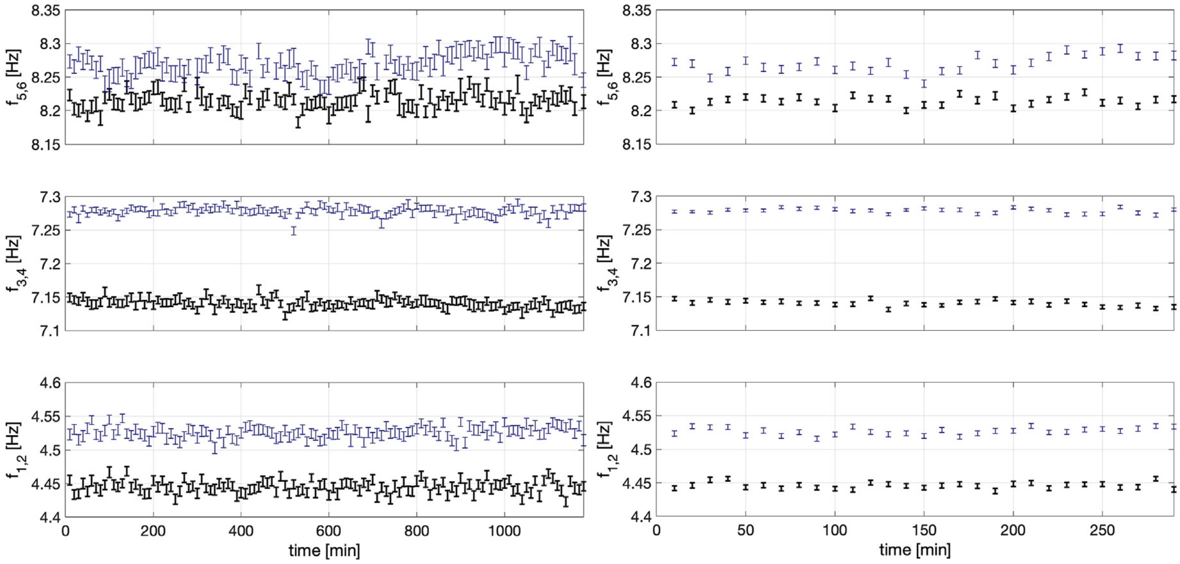


Fig. 14. Mode pair frequency variation for overnight recording at Eddystone Lighthouse.

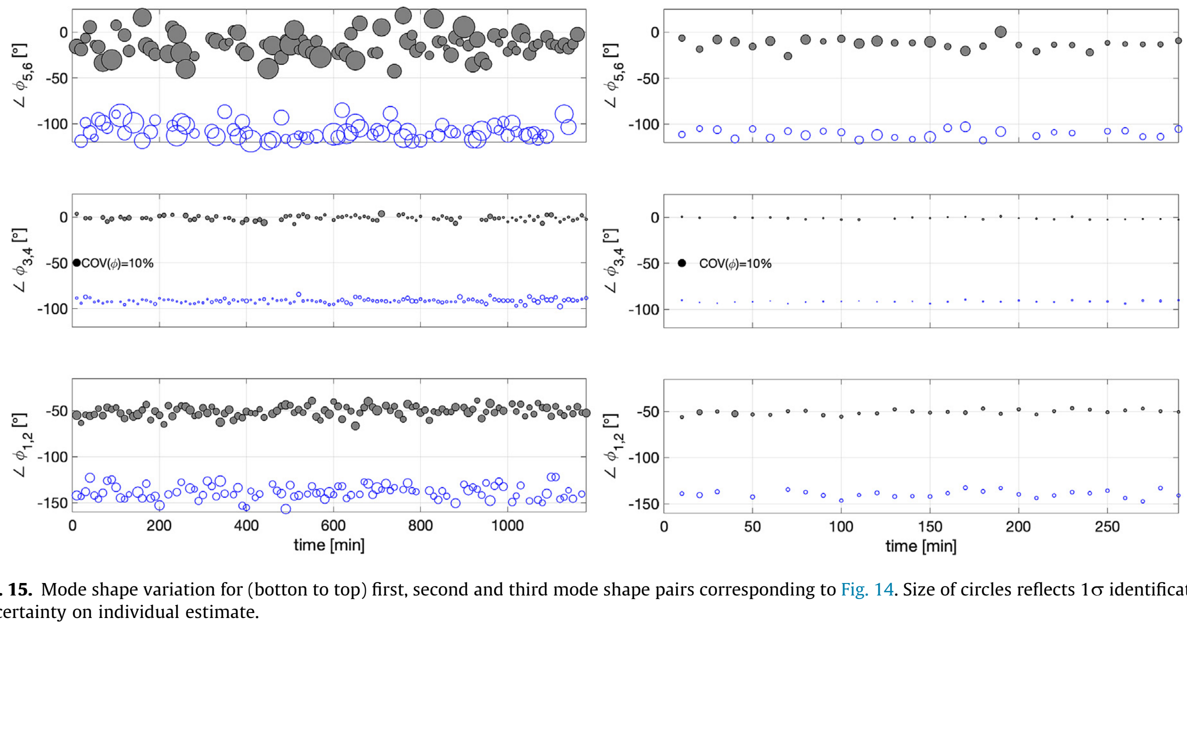


Fig. 15. Mode shape variation for (botton to top) first, second and third mode shape pairs corresponding to Fig. 14. Size of circles reflects 1σ identification uncertainty on individual estimate.

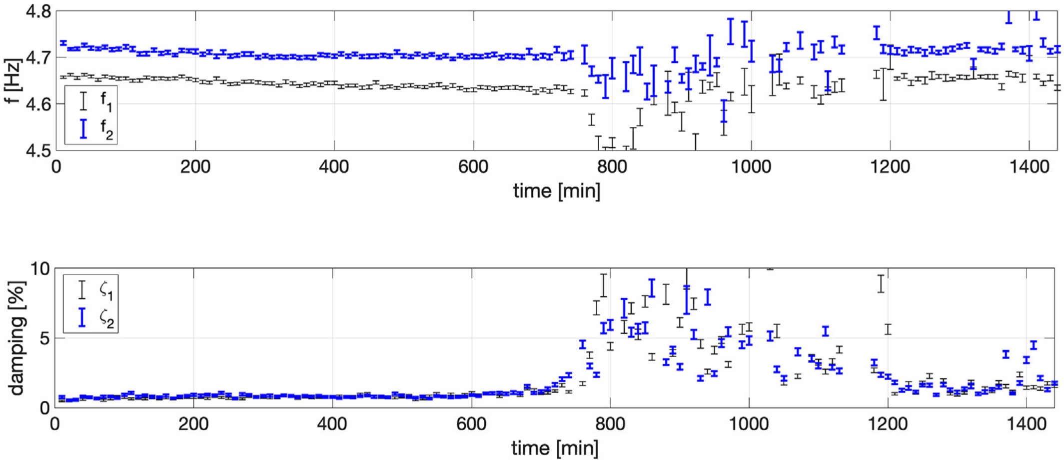


Fig. 16. Wolf Rock monitoring. Frequency estimation (upper) and damping estimation (lower) from application of BAYOMA to data of Fig. 16.

- There appears to be a strong link between closeness of mode pair frequencies and uncertainty in mode shape orientation: Closer frequencies come with greater mode shape uncertainty.
- For modal parameters tracked over extended periods of time while loading and response vary weakly, identification uncertainty appears not to be affected by underlying MP variation, in other words MP variance over four consecutive periods with different mean MP values is still always approximately twice that when analysing a single long period.
- While this last observation may not necessarily be true for strongly non-stationary data, BAYOMA appeared to be surprising robust to the nonstationary high amplitude response of Wolf Rock that contained numerous strong impulse-like response events.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.24378/exe.1043.

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