QUANTILE GRAPHICAL MODELS: PREDICTION AND CONDITIONAL INDEPENDENCE WITH APPLICATIONS TO FINANCIAL RISK MANAGEMENT

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Abstract. We propose Quantile Graphical Models (QGMs) to characterize predictive and conditional independence relationships within a set of random variables of interest. This framework is intended to quantify the dependence in non-Gaussian settings which are ubiquitous in many econometric applications. We consider two distinct QGMs. First, Condition Independence QGMs characterize conditional independence at each quantile index revealing the distributional dependence structure. Second, Predictive QGMs characterize the best linear predictor under asymmetric loss functions. Under Gaussianity these notions essentially coincide but non-Gaussian settings lead us to different models as prediction and conditional independence are fundamentally different properties. Combined the models complement the methods based on normal and nonparanormal distributions that study mean predictability and use covariance and precision matrices for conditional independence.

We also propose estimators for each QGMs. The estimators are based on high-dimension techniques including (a continuum of) ℓ_1-penalized quantile regressions and low biased equations, which allows us to handle the potentially large number of variables. We build upon recent results to obtain valid choice of the penalty parameters and rates of convergence. These results are derived without any assumptions on the separation from zero and are uniformly valid across a wide-range of models. With the additional assumptions that the coefficients are well-separated from zero, we can consistently estimate the graph associated with the dependence structure by hard thresholding the proposed estimators.

Further we show how QGM can be used to represent the tail interdependence of the variables which plays an important role in application concern with extreme events in opposition to average behavior. We show that the associated tail risk network can be used for measuring systemic risk contributions. We also apply the framework to study financial contagion and the impact of downside movement in the market on the dependence structure of assets' return. Finally, we illustrate the properties of the proposed framework through simulated examples.

Key words: High-dimensional sparse model, tail risk, conditional independence, nonlinear correlation, penalized quantile regression, systemic risk, financial contagion, downside movement

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Date: First version: November 2012, this version July 1, 2016.

We would like to thank Don Andrews, Debopam Bhattacharya, Xu Cheng, Valentina Corradi, Victor de la Pena, Aureo de Paula, Francis Diebold, Ivan Fernandez-Val, Jiaying Gu, Roger Koenker, Arthur Lewbel, Whitney Newey, Dong Hwan Oh, David Pacini, Andrew Patton, Elisabeth Perlman, Stephen Portnoy, Shu Shen, Senay Sokullu, Sami Stouli, Aleksey Tetenov, and Yanos Zylberberg for comments and discussions. We would also like to thank the participants from the Boston University Seminar Series, 2015 Warwick Summer Workshop, 11th World Congress of the Econometric Society, and the University of Bristol Seminar Series.

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