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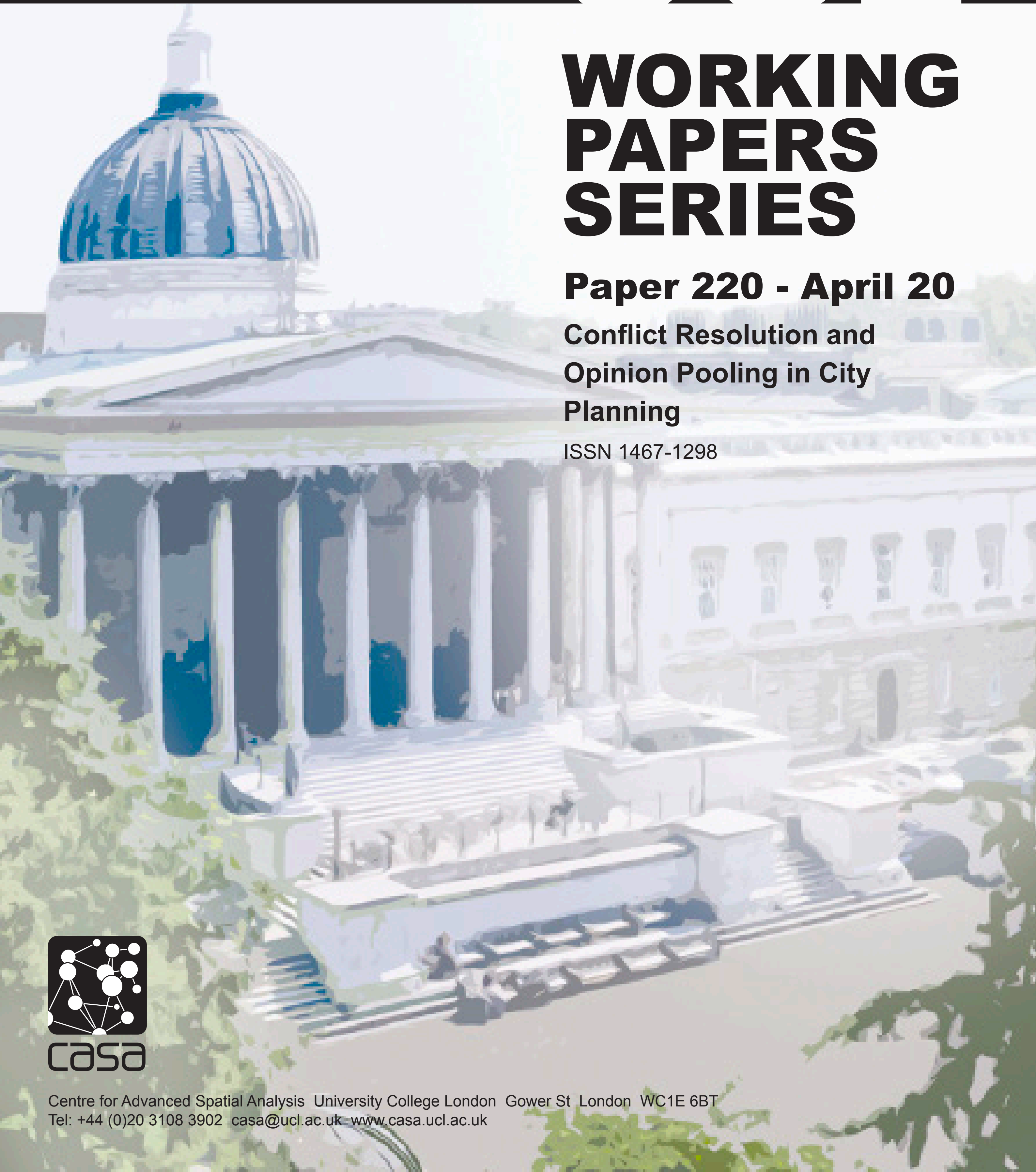

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Conflict Resolution and Opinion Pooling in City Planning

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Conflict Resolution and Opinion Pooling in City Planning

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Abstract

Opinion pooling is a method whereby actors who hold different opinions from one another are able to change their opinions according to the opinions of other actors to whom they are related. This process of change is based on the assumption that actors are rational, respect the individual integrity of each other's opinion, and are prepared to make a genuine compromise between the opinions of others to whom they are linked. A true consensus where all the actors are in agreement can only occur however if the actors are linked to one another, either directly or indirectly. We first describe this process and we then present the standard linear model of opinion pooling, illustrating how the dynamics of opinion change takes place and how an equilibrium occurs. We illustrate this using numerical values given to the opinion of each actor which at the beginning of the process are a set of 'best guesses'. This sequence converges to an agreed number which is the 'agreed guess' at the end of the process. We then illustrate the process using a set of opinions based on the desirability of different locations defined as a 'map'. We show how a unique equilibrium emerges and then we change the focus and assume the desirability indices are coordinates of places where actors are located. When rationally averaged, these coordinates change and the process leads to actors moving to the same location. We finally illustrate this process using a highway location problem, and then conclude by suggesting how we can predict the form of the network rather than simply focussing on the process of pooling.

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Introduction

There are multiple theoretical, academic and professional perspectives associated with the study of cities. A key distinction is between urban science that reflects the city itself in terms of its physical form and function in contrast to city planning where the main focus is on reconciling conflicting views about how the quality of life and the sustainability of cities might be improved. The development of networks in each of these domains is very different. In urban science, the concern is for the way the city functions through a multitude of networks that embody urban processes that distribute energy, materials, ideas and social interactions between its many parts. In city planning, however, the concern is largely with ways in which different views about the future of cities can be represented and then reconciled where the network of connections between various factors important in such a process of design often remains implicit. In short, networks in urban science are those that define the city and its component parts, whereas networks in city planning define ways in which ideas about the future are related to one another. Most of the chapters in this book are about the former – about urban processes that determine how cities function and evolve– whereas in this chapter, the emphasis will be on how networks can be used to represent processes associated with the actual planning of the city. These two rather different approaches to networks are usually developed by different constituencies of researchers and professionals and there are few attempts at reconciling them (Batty, 2013).

In city planning, there are many varieties of process that are used to explore the future; from intuitively inspired design to formal policy-making organized through different actors and stakeholders to mathematical models that seek to find the best locations that optimize the goals and objectives that define more sustainable, equitable and efficient futures. Here we will articulate these processes quite formally defining various objectives that either support or conflict with one other, and which must be resolved to produce some consensus or compromise. The way in which our model of the planning process works is by defining the key relationships between actors or agents who have a stake in the outcome (the future plan) with this set of relationships represented as a network. The network provides the basis for examining conflicts and concurrences and then resolving these if this is possible. To demonstrate its use, we will define different variants of planning problem to illustrate the process of resolution that the model attempts to achieve, and in this case, we will assume that the different objectives can be represented by different physical locations which imply where the best locations are for the future city. As these locations differ, the model enables a rational process of conflict resolution which homes in on the overall best location. This is achieved by altering the location or by resolving the differences in this location as actors in the process become aware of the conflicts involved and the need to compromise between them.

The model we will outline is generic in that it can be applied to many different kinds of planning problem. In fact, our ability to resolve conflict in such a model is a very special case. There are many representations of the problem where the network of relationships is highly fractured and where it is not possible to reach a consensus. Arguably many planning problems are of this nature and there is a wide class of problems that involve designing the network rather than using it to resolve a problem. In short, we can think of planning problems as involving different types of optimization and in this context, we use the network model to achieve such an optimization. Here we will assume the network of relationships is akin to a 'social network' and that actors who identify with specific objectives resolve their differences according to the objectives held by those to whom they are directly linked. If this process is achieved systematically and in an ordered way, we will see that a consensus akin in this context to an optimization, is realized. We might begin with a fractured network where no consensus can be realized but in such a case one might also repair the network so that ultimately every actor is related to every other either directly or indirectly thus ensuring a consensus. There are many possibilities but here our goal is to demonstrate the importance of thinking about city planning in this relational way. The model is thus more like a method for planning than anything that actually exists as a formal network in the city itself. Clearly the networks we will work with here are part of the city for planning is often seen as simply another function in the city – planning being often said to be part of the more generic problem – but this is another perspective and we will not develop it further here.

In the sequel, we will first outline the model using the simplest possible example which serves to introduce the kind of network we will work with, the objectives represented hypothetically in numerical terms, the method of compromise, and the convergence towards a solution. This is sometimes called 'opinion pooling' and it has a formal interpretation as a well-known process of forming weighted averages which we can represent as a Markov chain. We explore this in a mathematical interlude in the section that follows. There are multiple applications of this linear model. The problem can be set up the problem as one of numerical weighting of all locations to achieve a single location which is clearly best, one of binary reclassification, and one involving the identification of a single best location by moving different partial 'best' locations towards each other. We will catalogue all these in our third part. We then move on to show an application finding the best location for a highway, a problem we have reworked before (Batty, 1971; Batty, 1974). This enables us to extend the formalization of the problem to one in which we can predict' or rather 'generate' the network from two different bi-partite graphs. This enriches the problem and provides us with a basis for using the model to resolve more complex problems where we formally link location – the problem of the city – with actor preferences – the problem of planning.

Linear Opinion Pooling by Weighted Averaging

Let us begin with the simplest possible model which consists of four stakeholders (which we refer to interchangeably as agents or actors). Each actor has an opinion about the solution to the problem with respect to their own objectives and these opinions differ. What is required is a pooling of opinions so that these differences might be resolved and to this end, we need to define an appropriate network which shows how these actors might be related. We define such a network in Figure 1(a) where the communication channels between each actor are two-way. In fact we need to show this as a directed graph (digraph) which we illustrate in Figure 1(b) noting that the self-arrow means that when communication takes place, the actor remembers its own opinion and transmits this to those actors to whom it is connected. To best illustrate the problem, we will assign numerical values to each actor and in Figure 1(c), the range of these values is [10, 90, 60 and 30]. To fix ideas, it is best to think of these as first 'best guesses' to a number which lies somewhere in the range from 10 to 90.

Now opinion pooling consists of each of these actors sending its best guess to those actors to whom it is connected. In Figure 2(a) we show these for the links from actors 1, 2 and 4 to actor 3 with the assumption that when actor 3 receives these three values which are 10, 90 and 60, it will act rationally by taking this information and pooling it – averaging it if you like -with its own best guess which is 30. We assume that the actor is 'rational' and believes the guesses of those whom communicate their guesses to it are just as good as its own and thus it forms the average which is 10+90+60 together with its own which adds to 190. Dividing this by 4 which is the number of actors communicating with it including itself, its new guess becomes (10+90+30+60)/4= 47.5. Now all these actors do the same on the first round of swapping their best guesses and the opinions change to those that are shown in Figure 2 (b). This process of course generates the new guesses which move from the first best guesses at the start of the process [10, 90, 60, 30] to [33.33, 75, 47.5. 33.33]; and then the process begins again with these new guesses. You can see immediately that these new guesses lie between the range of 10 to 90 at the start to 33.33 to 75 and if you reflect a little about what the process will do in subsequent rounds, it is intuitively clear that these differences will reduce. Ultimately if the process of transmission continues, the guesses will move to some sort of average between all the initial opinions. In short the initial differences will disappear and a consensus will be reached. This of course is a weighted average where the average takes place according to the network. As the network varies in its links, then the consensus will also differ. If every actor is connected to every other actor, the consensus takes place in one round and becomes a true average of the initial set of guesses.

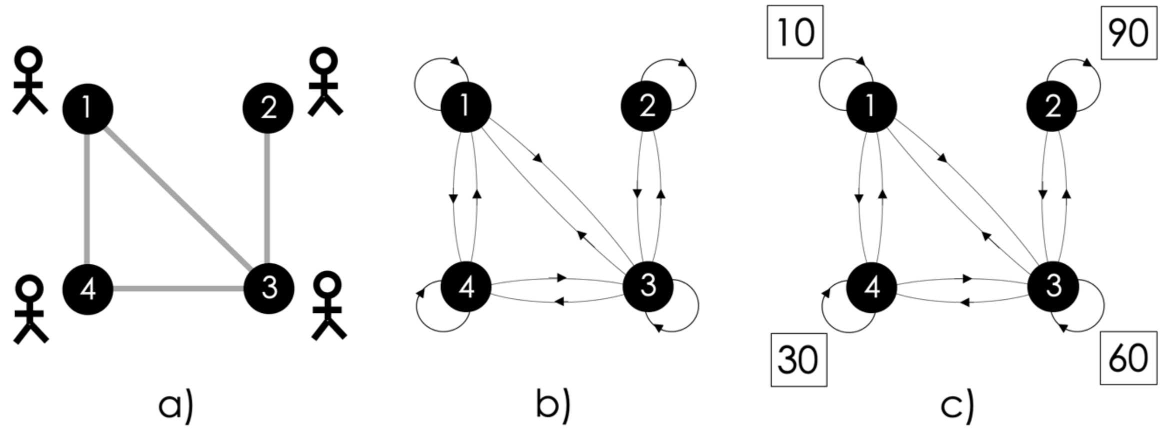


Figure 1: (a) A Typical Network of Four Actors, b) as a Digraph, and c) Their Opinions/Guesses Scored as Numbers

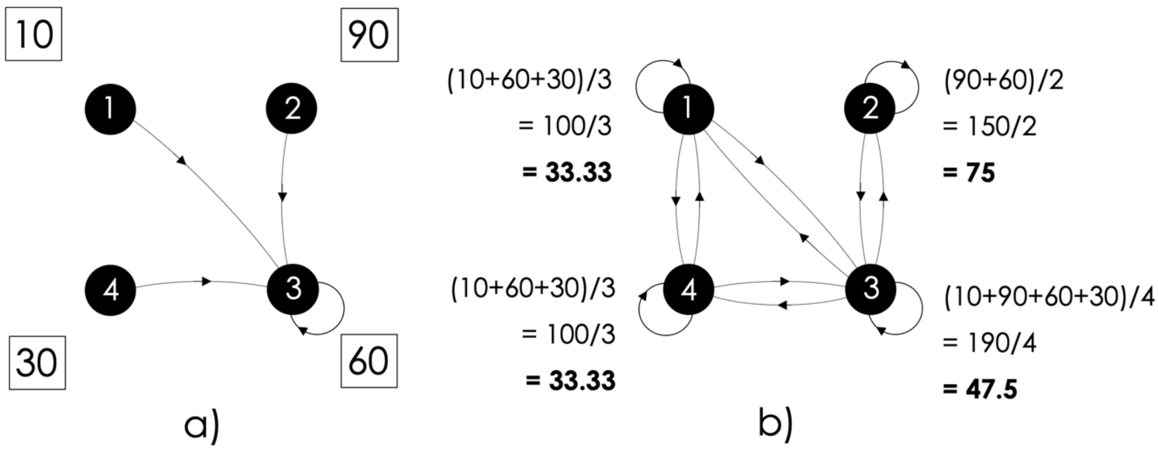


Figure 2: Transmission: a) Averaging for Those Communicating with Actor 3, and b) Averaging in the First Round of Communication Between All Actors

We show the succession of guesses for each actor over several iterations of the process in Table 1. What is clear although we do not demonstrate it formally, is that the convergence is very rapid at first and then moves towards an asymptote where all the best guesses are identical.

Arguably this process mirrors the kind of estimate that you might make if you were surrounded by a crowd who were pitching their best guesses. In fact it is easy to see that the best guess must lie between the lowest and largest in the range, between 10 and 90 and if we were simply to average these two opinions, we would get 50 which is close to the ultimate value which from Table 1 is about 45. Of course it is the network that makes a difference. If the network were very big and sprawling with many densely clustered subgraphs and various directional obstacles, assuming it was strongly connected in that every actor could reach every other either directly or indirectly, then the average would be much harder to guess. The really interesting feature however is when no consensus takes place.

Iteration No.	Actor 1	Actor 2	Actor 3	Actor 4
Best Guess→	**10.00**	**90.00**	**60.00**	**30.00**
1.00	33.33	75.00	47.50	33.33
2.00	38.06	61.25	47.29	38.06
3.00	41.13	54.27	46.16	41.13
4.00	42.81	50.22	45.68	42.81
5.00	43.77	47.95	45.38	43.77
6.00	44.30	46.66	45.21	44.30
7.00	44.61	45.94	45.12	44.61
8.00	44.78	45.53	45.07	44.78
9.00	44.87	45.30	45.04	44.87
10.00	**44.93**	**45.17**	**45.02**	**44.93**

Table 1: Convergence of the 'Best Guesses' to a Stable Equilibrium

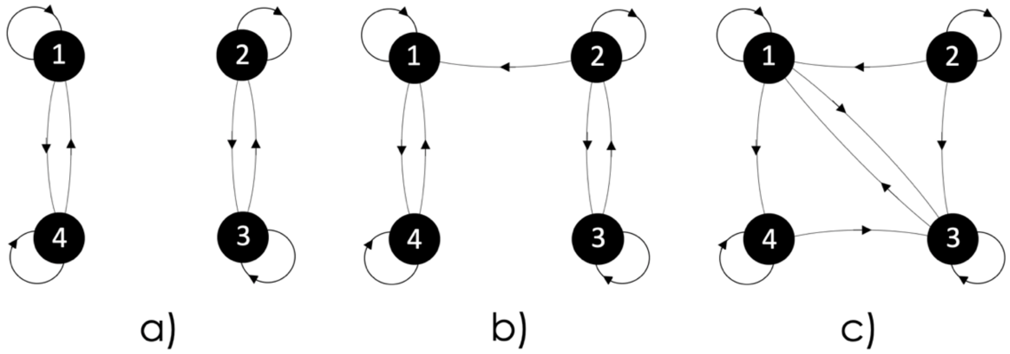


Figure 3: Networks with a) Divided Opinions b) Where Actors 2 and 3 Dominate, and c) Where Actor 2 Dominates

In Figure 3 we show three variants of this very simple 4 node network; first in Figure 3(a), the network is separated into two subgraphs where 1 and 4, and 2 and 3, form their own guesses which in this case are the simple averages of 1 and 4, and of 2 and 3, giving 20 and 75 respectively. This is a little like the system dividing into two groups where each person in the same group completely agree with one another to form a simple compromise but never talk to other actors in the other group. We all have experience of such a stand-off. Figure 3(b) mirrors a situation where 2 and 3 agree to compromise but together through Actor 2, this subgroup relentlessly pushes their opinion on Actors 1 and 4 with the ultimate result that Actors 1 and 4 take the same opinion as the average of 2 and 3. This is a situation where Actor 3 gets 2 to do its bidding but Actor 2 then continues by insisting that 1 and 4 take its view. Ultimately 2 (and of course 3) prevail. Figure 3(c) is yet another situation where Actor 2 by itself prevails and the other 3 relent to its opinion. In these last two cases, Actors 2 and 3 form an average which is 60 while in the last case, Actor 2 prevails and everyone is converted to its initial unwavering opinion of 90. Only when we build bigger networks can we generate many different cases. However even this simple network produces quite a lot of variety.

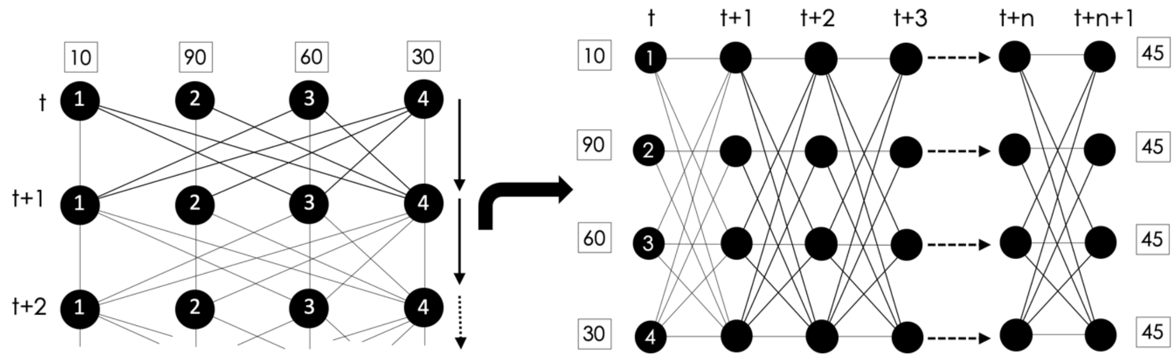


Figure 4: Successive Averaging: a) The Network in Layers b) The Network Splayed Out in Neural-Net Like Fashion

There is one last thing to present with respect to this simple model and that involves showing specifically the pattern of influence through the successive generations. In Figure 4, we show the sequence of iterations by organizing the network first from top to bottom (a) and then we twist it in the horizontal direction to show it in the manner of a neural net (b), where each layer is connected to another at each subsequent iteration based on the network. This is a succession of the original network with identical weights used for the set of links at each iteration. Strictly this is one of the simplest neural nets where the weights are compounded as the succession of opinion changes take place. To show the power of the method, we might change the weights or we might reconfigure the network at each iteration. As we start with a series of guesses at iteration 1, we then aim to reach a conclusion – a stable guess – by the end of the process choosing weights that make that particular guess feasible. In this sense, this is how a neural network actually works. You feed it some guesses and iterate and adjust the weights by a method of forward and backward propagation until the desired result is achieved. In fact the guesses you feed it are data and if we had millions of guesses, the net could be trained to produce the required guesstimate (Gurney, 1997).

This relationships to neural nets is not something we will take any further here but it does represent another dimension to this discussion and one that is worth exploring. However, what we need to do now is to ground the method in a more formal context. This will enable us to say something about the process and the equilibrium that results, and to generalize the method in applying it to different types of planning problem.

A Mathematical Interlude

If we first examine how a single actor responds to the transmission of the best guesses from its neighbours who are directly connected to it, we can begin to generalize the process to all actors.

In Figure 2(a), we show the communication of the opinions of Actors 1, 2, and 4 with respect to how Actor 3 averages these opinions with its own and we can write these in vector form as

$$\left.\begin{aligned}[47.5] &= \left[\tfrac{1}{4}\right][10] + \left[\tfrac{1}{4}\right][90] + \left[\tfrac{1}{4}\right][30] + \left[\tfrac{1}{4}\right][60] \\ &= \begin{bmatrix}\frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}\end{bmatrix}\begin{bmatrix}10\\90\\60\\30\end{bmatrix}\end{aligned}\right\} \quad . \tag{1}$$

Now we can generalize equation (1) to all four actors in our simplest example writing the matrix-vector equation out in full for the first iteration using the values from Figure 2(b) as

$$\begin{bmatrix}33.33\\75.00\\47.50\\33.33\end{bmatrix} = \begin{bmatrix}\frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3}\\ 0 & \frac{1}{2} & \frac{1}{2} & 0\\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4}\\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3}\end{bmatrix}\begin{bmatrix}10\\90\\60\\10\end{bmatrix} \quad . \tag{2}$$

Defining the 4x4 weighting matrix as $\boldsymbol{P} = [P_{ij}]$, the initial guesses as the vector $\boldsymbol{w}(t) = [w_j(t)]$ and the new vector of averaged guesses as $\boldsymbol{w}(t+1) = [w_i(t+1)]$, we can write the generalized version of equation (2) as

$$w_i(t+1) = \textstyle\sum_j P_{ij} w_j(t) \ , \qquad \sum_j P_{ij} = 1 \ , \tag{3}$$

which in matrix-vector notation is

$$\boldsymbol{w}(t+1) = \boldsymbol{P}\boldsymbol{w}(t) \qquad . \tag{4}$$

Note that $t = 1$ is the beginning of the process.

We are now in a position to demonstrate how the model is able to simulate a steady state where all the actors are reconciled with one another. Equation (4) is a recursive relation and if we work this through for a succession of averaging steps as portrayed in Figure 4, we can write it as

$$\boldsymbol{w}(t+n) = \boldsymbol{P}\boldsymbol{w}(t+n-1) = \boldsymbol{P}^n\boldsymbol{w}(t) \qquad . \tag{5}$$

Now it is easy to demonstrate that equation (5) converges to a vector of opinions that are all identical, that is $\boldsymbol{w} = \boldsymbol{w}(t+n)$ as $n \to \infty$. As equation (3) shows, $\boldsymbol{P}$ is a stochastic matrix with row sums equal to 1. A property of such a matrix is that when it is multiplied by itself, it remains a stochastic matrix and its rows begin to converge to stochastic vectors which get closer and closer to a steady state vector $\boldsymbol{z^T}$, that is $\boldsymbol{Z} = \boldsymbol{P}^n$ as $n \to \infty$ where each row of $\boldsymbol{Z}$ is this steady state vector. Then in the limit, equation (5) can be written as

$$\boldsymbol{w} = \boldsymbol{Z}\boldsymbol{w}(1) \quad . \tag{6}$$

which generalizes to

$$\left.\begin{aligned} \boldsymbol{w} &= \boldsymbol{z}^{\boldsymbol{T}}\boldsymbol{w}(1) = \boldsymbol{z}^{\boldsymbol{T}}\boldsymbol{w} \\ \boldsymbol{z}^{\boldsymbol{T}} &= \boldsymbol{z}^{\boldsymbol{T}}\boldsymbol{P} \end{aligned}\right\} \quad . \tag{7}$$

Note that the vectors are column vectors in this presentation and we transform them in the usual way to row vectors by taking their transpose. Also note that the final steady state opinion vector $\boldsymbol{w}$ can be computed directly from steady state vector $\boldsymbol{z}^{\boldsymbol{T}}$ and the initial opinions $\boldsymbol{w}(1)$ in equation (7) and this steady state vector in turn can be generated by solving the simultaneous equations in the second row of equation (7). All these results are standard in linear algebra and a full presentation is in the author's book (Batty, 2013).

There are a couple of comments that need to be made about this process. First, the model is based on a well-known discrete Markov process (or chain). Second it can be manipulated in many ways. We have assumed here that the Markov or probability matrix $\boldsymbol{P}$ is strongly connected. This means that it is possible for every actor to reach every other actor either directly or indirectly. In short, the matrix cannot be partitioned into separate components for then there would be no equilibrium solution. There are several variants such as those we showed in Figure 3 where various actors might dominate and these can be accommodated within the framework. The probability matrix that we have used here is based on simply converting a binary matrix based on a connected and symmetric network into a matrix where each link is given equal weight at the level of the connectivity. But the differential averaging and the probabilities are entirely dependent on a link being present or not within the original network. In fact we could weight these links quite differently. We could, for example, assume a completely connected matrix where every actor interacts with every other actor as for example in spatial interaction matrix but that the links are based on some measure of cost or distance. In this sense, actors would be swapping opinions based on their relative distance from one another. Whether the final equilibrium would be meaningful would depend on how the problem was formulated but this indicates that the same logic as developed here could be developed for networks in city systems which do not necessarily depend on the kinds of social consensus assumed here but on functional relationships between different parts of a city.

Applications of the Markovian Framework

We will outline three different applications of this model to problems of city planning. We begin with the simple opinion pooling model and then introduce an application to highway design originally developed by Alexander and Manheim (1962). This is in the spirit of design with nature developed by McHarg (1969) and reworked by Batty (1974). We have illustrated the opinion pooling model for a numerical example but where each actor holds an opinion which is quantified as a guess. As an abstract concept, this simply illustrates the logic of the process but in any application, such quantification has to have some meaning. In fact there is an old story about the 'wisdom of crowds' told by Surowiecki (2004) which illustrates this process. In 1906, Francis Galton, the man who developed biometrics and a man of many parts, visited a livestock fair in England's West Country. There, an ox was the centre of attention and the villagers were asked to guess the animal's weight after it had been slaughtered. Some 800 people hazarded a guess but no one got near to the actual weight of 1198 pounds. When all the estimates were averaged, these gave 1197 pounds which was a much closer estimate than any by the 800 individuals, thus illustrating the possibility that large groups of people can converge on something which is a correct value simply by reflecting the diversity of views in a decentralised, independent group of people with no prior coordination. This is a classic example of crowdsourcing to some obvious purpose. Some planning problems of great complexity are of this nature and in a professional group, averaging this diversity might be a good strategy for producing an optimum plan. Our model could be used in this way for appropriate problems.

However to give this some physical structure, let us assume that we have a set of n actors that each have a view about the desirability for locating a facility in a city which is divided up into a series of m locations. We can define a desirability map for each actor j and each location on the map k as D_{jk}. We also have a weighting matrix in probability form as in the previous section called $\boldsymbol{P} = [P_{ij}]$ and the process that we now initiate is where the desirability scores are averaged assuming complete rationality on the part of the actors until they all converge on an agreed set of scores. This is no more or less than applying the model in equations (1) to (4) above to a set of opinions which each actor holds rather than a single opinion. We show this in algebraic form as

$$D_{ik}(t+1) = \sum_j P_{ij} D_{jk}(t) \ , \qquad (8)$$

where it is clear that the scores for each location are independent of one another. Once the process has converged, we can select the most desirable location from

$$D_i = \max_k D_{ik} = D_{ik}(t+n) \ as \ n \to \infty \qquad . \qquad (9)$$

It is possible to introduce some local averaging of scores if the locations are adjacent in some way. Assuming that we identify the closest location as $D_{j\ell \in k}$, we form a local average of this which is articulated as

$$D_{ik}(t+1) = \sum_j P_{ij} \{[(D_{jk}(t) + D_{j\ell \in k}(t)]/2]\} \qquad . \qquad (10)$$

There are many such schemes that we might devise but one which we have applied to identify all or nothing locations for urban development involves scoring each location as being possible or not for development. If we revert to our simple example which we show in equation (2), then we can assume that each of the four actors has a map of the city divided into say 16 zones, arranged on a 4x4 grid as we show in Figure 5. Noting that we need to scale the values by 100 so that we can represent the rounded desirability scores as integers, we can write the averaging equation for the first iteration as

$$\begin{bmatrix} 67\ 67\ 67\ 33\ 33\ 67\ 33\ 67\ 67\ 0\ 67\ 33\ 33\ 100\ 33\ 33 \\ 50\ 50\ 100\ 50\ 50\ 50\ 50\ 50\ 50\ 50\ 50\ 50\ 0\ 50\ \ 0\ \ 50 \\ 50\ 75\ 75\ 50\ 25\ 50\ 50\ 75\ 50\ 25\ 50\ 25\ 50\ 75\ 25\ 50 \\ 67\ 67\ 67\ 33\ 33\ 67\ 33\ 67\ 67\ 0\ 67\ 33\ 33\ 100\ 33\ 33 \end{bmatrix} = 100 \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 1\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 1\ 1\ 0 \\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0\ 1 \\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 0\ 0 \\ 0\ 1\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 1\ 0\ 1 \end{bmatrix} \qquad (11)$$

The four maps, one for each actor in Figure 5 represent cells in the space where each actor suggests that development can take place $D_{jk} = 1$ and where it cannot $D_{jk} = 0$. In fact there is trick in this simplified system for computing the final maps when all the actors agree with one another. Rather than iterate the equation in (11) according to the recursion in (3), we note that the steady state vector is proportional to the indegrees (or out degrees) of the initial digraph from which the probability matrix is derived. These indegrees, in this case for the four actors, are [3 2 4 3] from which we compute the steady state vector as

$$z = [1/4, 1/6,\ 1/3,\ 1/4] \quad . \tag{12}$$

This vector adds to 1 as it is the set of probabilities of the actors contributing to the final solution. Applying this to the set of maps $[D_{jk}(t = 1)]$ from first line of equation in (7), we can produce the solution directly as

$$[58\ 67\ 75\ 42\ 33\ 58\ 42\ 67\ 58\ 17\ 58\ 25\ 42\ 83\ 25\ 42] = 100 \begin{bmatrix} \frac{1}{4} & \frac{1}{6} & \frac{1}{3} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 1\,1\,1\,0\,0\,1\,0\,1\,0\,0\,1\,0\,1\,1\,1\,0 \\ 0\,1\,1\,1\,0\,0\,1\,1\,0\,1\,0\,0\,1\,0\,0\,1 \\ 1\,0\,1\,0\,1\,1\,0\,0\,1\,0\,1\,0\,0\,1\,0\,0 \\ 0\,1\,0\,1\,0\,0\,1\,1\,1\,0\,0\,1\,0\,1\,0\,1 \end{bmatrix} \tag{13}$$

which we can also represent as the converged scores for all actors as

$$\begin{aligned} D_k &= [\ 58\ 67\ 75\ 42\ 33\ 58\ 42\ 67\ 58\ 17\ 58\ 25\ 42\ 83\ 25\ 42\ ] \\ &= \max_j D_{jk} \begin{bmatrix} 58\ 67\ 75\ 42\ 33\ 58\ 42\ 67\ 58\ 17\ 58\ 25\ 42\ 83\ 25\ 42 \\ 58\ 67\ 75\ 42\ 33\ 58\ 42\ 67\ 58\ 17\ 58\ 25\ 42\ 83\ 25\ 42 \\ 58\ 67\ 75\ 42\ 33\ 58\ 42\ 67\ 58\ 17\ 58\ 25\ 42\ 83\ 25\ 42 \\ 58\ 67\ 75\ 42\ 33\ 58\ 42\ 67\ 58\ 17\ 58\ 25\ 42\ 83\ 25\ 42 \end{bmatrix} \quad . \end{aligned} \tag{14}$$

In the vector in equation (14), these are the scores that the actors finally agree upon. We find the highest score by finding the maximum value of this vector which is location 14 which has a score of $83 = \max_k D_k$.

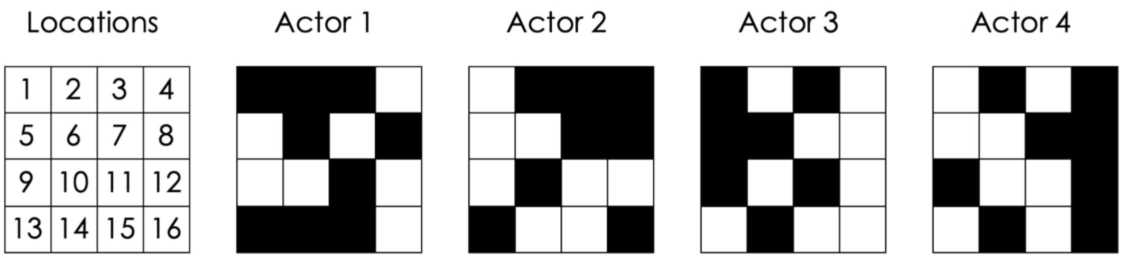


Figure 5: Actor Desirability Scores for Locating Urban Land Uses in a 16 Zone Spatial System
(black is desirable, white is undesirable)

The second application involves defining the position of each actor in some 2-dimensional space. In fact we could consider the application in equations (11) to (14) above as being coordinates in a 16-dimensional space with the convergent solution being the finally agreed coordinates of the actors once they have reached a consensus about the agreed position. We can best illustrate this in 2-dimensional space where the actors have an initial position and wish to move to some location where all of them are able to meet. This might be a problem where all the actors agree that there is need for a central location rather than where they are located at the start of the process, while this is a little bit like Hotelling's (1929) spatial problem of agglomeration. But the power structure is such that the solution is not likely to be a true centroid but one which is weighted according to the importance of each actor. In a situation where the actors are all able to compromise immediately which implies that in the 4x4 actor matrix of probabilities, the links are of equal weight, the final solution which is achieved in one iteration is the true centroid of the initial positions. However for the matrix of the weighted digraph in Figure 2(b), we define the first iteration of the process with the matrix of coordinates $[x_j \; y_j]$ shown in Figure 6 as

$$\begin{bmatrix} 4.33 & 4.33 \\ 7.50 & 5.50 \\ 5.00 & 5.50 \\ 4.33 & 4.33 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 2 & 8 \\ 7 & 9 \\ 8 & 2 \\ 3 & 3 \end{bmatrix} \qquad , \qquad (15)$$

and using the fast solution used for the full problem, we get the weighted centroid expressed in terms of the four set of coordinates as follows

$$\begin{bmatrix} 5.08 & 4.92 \\ 5.08 & 4.92 \\ 5.08 & 4.92 \\ 5.08 & 4.92 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{6} & \frac{1}{3} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 2 & 8 \\ 7 & 9 \\ 8 & 2 \\ 3 & 3 \end{bmatrix} \qquad . \qquad (16)$$

We show this convergence in Figure 6 where we have joined the four points to define the space in which the solution exists. In this simple problem, the convergence is quite fast but in a very big ramified network with many clusters, weak ties and asymmetric links, the process might take many iterations.

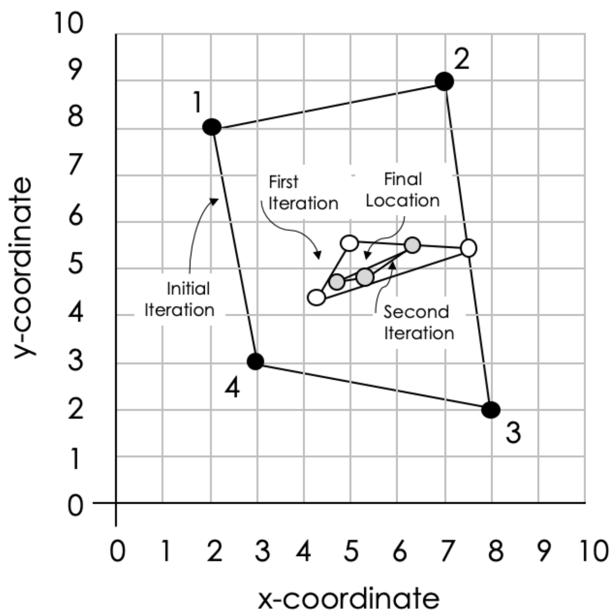
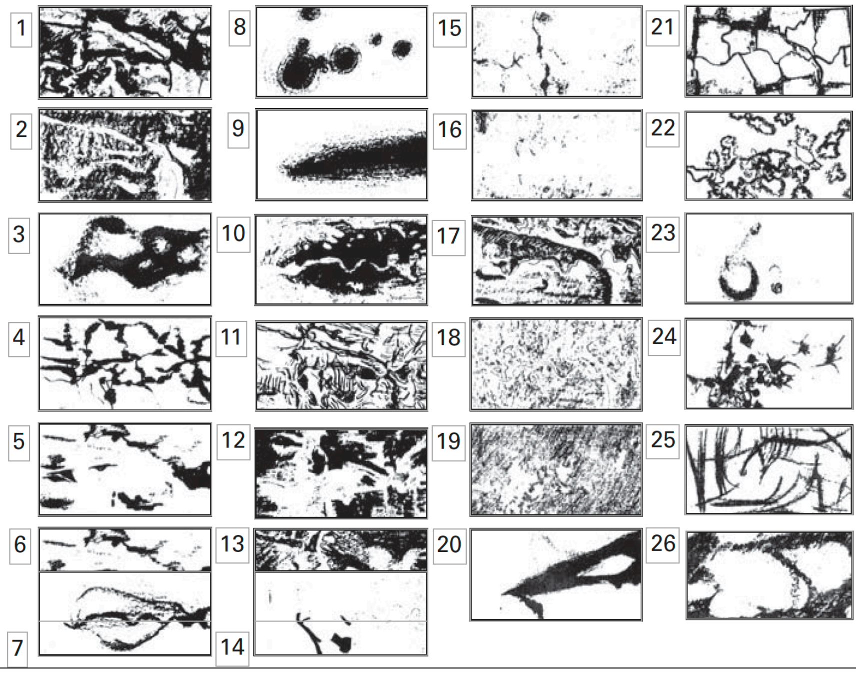


Figure 6: Convergence to a Stable Location
Where Opinions Are Positions (Coordinates) in a Map Space

This application might be seen as an optimization problem in location theory but where the network is buried beneath the spatial solution. The 2-dimensional location problem where the actors pool their opinions so that they might agree on a central location somewhere within the wider space that their preferred locations define, appears in many contexts particularly in areas such as robotics, games, and problems of movement dynamics (Mesbahi and Egerstedt, 2010; Ren, Beard, and Atkins, 2007). In these domains, there is a strong focus on how the dynamics of the way actors or objects move reflect other theories of how animals and humans flock and herd, usually in highly stylized ways that are characterised by well-defined physical situations (Vicsek et al., 1995). There are also many applications that have emerged in computer science where the focus on agents transmitting information by pooling has been articulated using the linear Markov model (Blondel, Hendrickx, Olshevsky, A. and Tsitsiklis, 2005; Blondel, Hendrickx, and Tsitsiklis, 2009). But the origins of these ideas really lie in studies of social power, the model here being developed originally by French (1956), and then formalised by Harary (1959) and Harary and Lipstein (1962) as part of the vibrant field of group dynamics which flourished in the 1950s and 1960s. These models were then picked up in statistical and mathematical applications in sociology and political science (see de Groot, 1974) and then in a succession of developments by Friedkin and Johnsen (1999), Hegselmann (2004), and Kelly (1981) amongst several others working with consensus and opinion pooling.



1	Earthwork Costs	10	Pavement & Subgrade Costs	19	Public Financial Losses
2	Comfort and Safety	11	Drainage Patterns	20	Major Current Traffic Desires
3	Regional Development	12	Bridge Costs	21	Catchment Areas
4	Local Land Development	13	Land Costs	22	Local Accessibility and Integrity
5	Obsolescence	14	Eyesores	23	Future Transportation Systems
6	Interference in Construction	15	Noise	24	Existing Transportation Systems
7	User Costs	16	Air Pollution	25	Duplication of Facilities
8	Services	17	Weather Effects	26	Self-Induced Congestion.
9	Travel Time	18	Public and Private Losses		

Figure 7: Factor Diagrams for the Alexander-Manheim Highway Location Problem.

To conclude these applications, we will briefly present a more practical problem of highway location first introduced by Alexander and Manheim (1962). The problem involved the location of a highway between Springfield and Northampton in Western Massachusetts where a series of 26 factors were identified as implying different routes for the highway. The factors were mapped as desirability surfaces and all could thus be represented spatially pertaining largely to physical constraints on the location of a highway. Many of the factors pertained to landscape amenity, topography, weather, drainage and so on but they included a mix of socio-economic factors such as local accessibility, urban development, and transportation. The factors in question are shown in Figure 7 and the usual way of finding the best location (where in Figure 7 the darker areas show more desirable locations for the highway) is to overlay these surfaces and compute an overall index of desirability for each location which would then guide the choice of the best line for the highway. If one were to do this directly, it would assume that each factor had the same weight as any other and the result would simply be a 'sieving' or simple averaging. This is akin to methods used in professional planning and landscape design through much of the 20th century referred to as overlay analysis (McHarg, 1969; Steinitz, Parker, and Jordan, 1976) and the layer model is still writ large in GIS. It is akin to a situation where each factor is associated with a single actor and actors then pool the factors or 'opinions' as reflected in their desirability surfaces. It might be a stretch to consider factors as the opinions of actors but with judicious choice of problem and context, one can consider this to be the case.

What Alexander and Manheim (1962) did was to form a matrix of differences between the factors (actors) and to decompose this matrix into clusters which then produced an order for the way in which the factors could be combined. This order produced an implicit set of weights with the most clustered based on the greatest differences being combined first and those on the edge of the network being the least different from the majority and thus combined last. As the order is based on differences, it gets progressively easier to merge the factors into one another as the order progresses up the hierarchy of relations. Strictly Alexander and Manheim should have recomputed this matrix as it is progressively reduced in size – that is as factors are merged – and there are many variants which could improve the process but most have never been tested. As the factors are merged, they are strengthened in their form for the focus is on extracting a linear form – a highway – and reinforcing this as factors are successively merged.

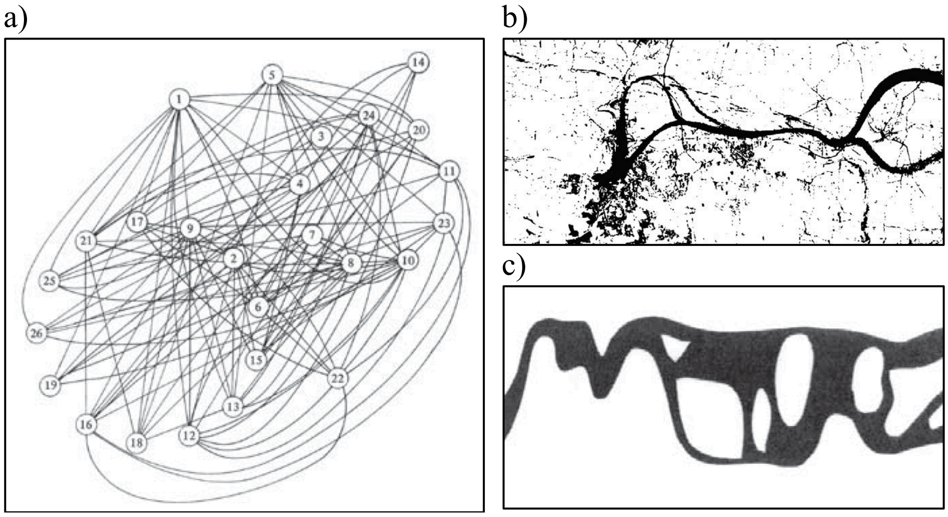


Figure 8: Solutions to the Highway Location Problem

a) The Relationship Graph-Network used for the Cluster and Markov Averaging Methods
b) The Alexander-Manheim Cluster Weighted Solution
c) The Solution Space from the Markov Averaging

The final solutions are shown in Figure 8. What we have done here is take the matrix of relations used in the original problem and then use Markov averaging to achieve a comparable solution. In terms of equation (11), the $\boldsymbol{D}$ matrix is the set of maps where i is the factor (or actor) and k is each cell of the map. In short each cell k in the i'th map in Figure 7 is defined as $D_{ik}(t = 0)$. The relationships matrix P_{ij} is defined from the graph in Figure 8(a) which was constructed by the author from the account given by Alexander and Manheim (1962). This led to the solution shown in Figure 8(b) while the Markov averaging methods associated with the examples earlier in this chapter produce a solution space in Figure 8(c) in which many different but similar locations for the highway might be chosen. There are many other issues involved in the Markov model and these are illustrated in Batty (2013) while the wider literature that we noted above particularly in sociology, shows how we might begin to ground these ideas in empirical applications.

These methods do not produce definitive solutions in the sense of an optimization for they are more like tools for defining the solution space within which the designer or policy maker or planner might make an informed choice. In this sense, they act to support plan design just as participatory tools are being used to inform similar kinds of geodesign which involve pitting actors against one another in the search for a best plan (Steinitz, 2012). There are many variations that we can continue to make with respect to models of design processes which reflect opinion pooling and what is clearly required is a sustained attack on these variants. To conclude, we will begin to explore how we can take these forward by focussing on the networks of relations rather than on the processes of opinion pooling *per se*. To this end, we will sketch how we can begin to predict the network of interaction with respect to the way it can be formed.

The Challenge: Deriving Networks of Actor Interaction

We have assumed that the network of actor relations is observed in some way but very often these networks are not easy to define and sometimes it is possible to see the network as a result of more indirect processes. In the case of the highway location problem, we examined the relationships between the maps with respect to how close each actor was to another in terms of the strength of each relationship. If we take each element of the map k with respect to each actor i, D_{ik}, where for the moment we drop the iteration index t, we can form an $n \, x \, n$ matrix of similarities $[X_{ij}] = \boldsymbol{X}$ like correlations between the actors/factors from

$$X_{ij} = \sum_k D_{ik} D_{jk} \text{ which in matrix terms is } \boldsymbol{X} = \boldsymbol{D}\boldsymbol{D}^{\boldsymbol{T}} \qquad . \qquad (17)$$

Each cell of the similarity matrix is based on a sum of how close an actor's desirability scores are to another actor's. We then form the averaging matrix by simply summing the row elements of $\boldsymbol{X}$ and normalising it as

$$P_{ij} = X_{ij} / \sum_j X_{ij} \, , \qquad \sum_j P_{ij} = 1 \qquad . \qquad (18)$$

Now it is possible to compute the steady state vector $\boldsymbol{z}^{\boldsymbol{T}}$ which gives the relative power of each actor once a consensus has been reached. The final map could be derived in the manner we illustrated in our first example above where we defined changes in the map from successive averaging as

$$D_{ik}(t+1) = \sum_j P_{ij} D_{jk}(t) \qquad , \qquad (19)$$

which in matrix terms is

$$\boldsymbol{D}(t+1) = \boldsymbol{P}\boldsymbol{D}(t) \qquad . \qquad (20)$$

From the previous results ,the final distribution of elements in the map $\boldsymbol{D}(t \to \infty)$ are computed as

$$\overline{\boldsymbol{D}} = \boldsymbol{z}^{\boldsymbol{T}}\boldsymbol{D}(t=1) \qquad . \qquad (21)$$

There is an important point to note here. The original transition matrix $\boldsymbol{P}$ is computed from $\boldsymbol{D}\boldsymbol{D}^{\boldsymbol{T}}$ and thus we are compounding the map into the process of averaging notwithstanding that the average itself depends on the original map. There is thus an element of 'double counting' in the process of constructing the averaging.

The key insight into expressing the averaging matrix through correlation and similarity is that there is a very obvious *dual* problem, noting that the set of equations from (18) to (21) define a *primal*. In the main method here, the whole process involves how n actors interact with one another but as soon as we introduce similarities, we need to consider the similarity between any pair of actors in terms of some other dimension, in this case factors expressed as m maps. Instead of measuring these similarities between actors, we can also compute them between the maps themselves by first forming the $m \ x \ m$ matrix

$$Y_{k\ell} = \textstyle\sum_j D_{jk}D_{j\ell} \text{ which in matrix terms is } \boldsymbol{Y} = \boldsymbol{D}^{\boldsymbol{T}}\boldsymbol{D} \qquad . \qquad (22)$$

This is clearly a dual problem where instead of finding the relative power of the actors in generating an average map, the problem is finding the relative power of each map cell/location from which we are able to compute a sort of average actor. This problem is less meaningful in terms of map factors but it does have more relevance to situations where one set of actors is related to another but different set of actors. As in all the applications here, the choice of problem is critical in assessing the relevance of the method and as we have already made clear, this is a set of tools that gives us some flexibility in handling problems such as these.

To complete the dual process, we form the $m \ x \ m$ probability matrix

$$Q_{k\ell} = Q_{k\ell}/\textstyle\sum_\ell Q_{k\ell} \ , \quad \sum_\ell Q_{k\ell} = 1 \qquad . \qquad (23)$$

with the process continuing as

$$D_{jk}(t+1) = \textstyle\sum_\ell Q_{k\ell}D_{j\ell}(t) \qquad , \qquad (24)$$

which in matrix terms is

$$\boldsymbol{D}^{\boldsymbol{T}}(t+1) = \boldsymbol{Q}\boldsymbol{D}^{\boldsymbol{T}}(t) \qquad . \qquad (25)$$

If we call the steady state vector $\boldsymbol{q}$ where $\boldsymbol{q} = \boldsymbol{q}\boldsymbol{Q}$, then we can compute the averaged actors best map location as

$$\widehat{\boldsymbol{D}}^{\boldsymbol{T}} = \boldsymbol{q}\boldsymbol{D}^{\boldsymbol{T}}(t=1) \qquad . \qquad (26)$$

We will conclude by taking our example based on the simple network in Figure 1 with the 4 maps in Figure 5 and we can simply state the primal and dual results from the steady state equations directly as

$$
\begin{aligned}
&[50\ 79\ 75\ 50\ 21\ 50\ 50\ 79\ 46\ 25\ 50\ 25\ 54\ 75\ 29\ 50] \\
&= 100[0.29\ 0.25\ 0.21\ 0.25]\begin{bmatrix} 1\,1\,1\,0\,0\,1\,0\,1\,0\,0\,1\,0\,1\,1\,1\,0 \\ 0\,1\,1\,1\,0\,0\,1\,1\,0\,1\,0\,0\,1\,0\,0\,1 \\ 1\,0\,1\,0\,1\,1\,0\,0\,1\,0\,1\,0\,0\,1\,0\,0 \\ 0\,1\,0\,1\,0\,0\,1\,1\,1\,0\,0\,1\,0\,1\,0\,1 \end{bmatrix} \text{, and}
\end{aligned} \tag{27}
$$

$$
[66\ 56\ 47\ 56] = 100 \begin{bmatrix} 0.06 \\ 0.09 \\ 0.09 \\ 0.06 \\ 0.03 \\ 0.06 \\ 0.06 \\ 0.09 \\ 0.06 \\ 0.03 \\ 0.06 \\ 0.03 \\ 0.06 \\ 0.09 \\ 0.03 \\ 0.06 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1. & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0. & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1. & 0 \\ 0 & 0 & 0. & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1. & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} . \tag{28}
$$

We have applied the method to a problem involving key building sites and actors involved in the development of a small high profile neighbourhood in the City of London where the model is able to show the importance of the actors and the significance of the sites in question (Batty, 2016). But the real power of the method consists in starting with a problem involving two sets – of objects, actors, locations, whatever. The notion of devising bipartite networks or graphs as in this case for the relations between actors and locations, always leads to primal and dual problems which involve twisting the problem one way or the other, thus gaining real insights into the nature of the networks involved and the power relations if the problem is formulated in social terms. There is in fact a much more detailed development of this primal-dual problem where two different bipartite matrices/graphs are combined as in equations (17) and (22) respectively. If these matrices are already stochastic, the interpretations are somewhat easier in terms of the steady state distributions. Th is variant of the model was first suggested by Coleman (1973) and its wider foundations are presented in his book (Coleman, 1998). This has inspired many applications in social exchange theory and theories of social power but there is still a dearth of observational data and most of these applications to date simply elaborate the theoretical framework. It awaits widespread empirical testing.

Conclusions: Next Steps

The message of this chapter is simple. Network models not only exist for the physical and spatial functions determining the way cities transit information, ideas, people, materials and energy, they pertain to the ways in which we communicate information about the future form of the city itself within the wider context of design. We might think that this implies that the model in this chapter could be widely used to simulate actual flows between agents that make up the entire city but here the set of agents is very specific. As designers and planners, the networks they use are to communicate ideas about the future city, rather than determining and supporting our understanding about the actual functioning of the city. These models must be taken at their face value. They are not necessarily meant to be empirically testable statements of how expert groups of designers get to grips with the problems they need to resolve while also resolving differences in opinions about the importance of ideas and factors. Notwithstanding that they are hard to validate largely because the process of formulating a plan is highly convoluted and is much more than solving a simple problem in group dynamics, they are in fact empirically applicable although the processes of application are not well developed or understood. There are thus some important challenges in developing such applications because the conceptual ideas are widely applicable to many different kinds of problem.

The problems in this chapter are highly simplified and extremely small with respect to their networks of interaction. At times we have hinted that the power of the method only comes into its own when the problem has many more actors. However there is a limit to this for the networks are strongly purposive in that they are appropriate only for situations where the actors have a common mission in mind. The sorts of network that one now finds in social media are clearly not relevant here and thus networks with more than 100 actors are not likely to be found in these types of design and planning problem. In fact it is more likely that the model needs to be extended to deal with problems that have many more actors by developing ways in which actors can enter or leave the process as the problem morphs. In this sense, planning problems involve an explicit process of learning and even if one can conceive of planning problems that do involve thousands of stakeholders, the kind of models that might explain and inform the way such a large group might move to a consensus are likely to be very different from those introduced here. The focus here is still on small group dynamics although the literature on opinion pooling in groups like this has not got any closer to empirical applications in the last decade. It has become ever more theoretical concerned with the dynamics of processes moving to consensus, particularly with a focus on disruptions and disturbances from equilibrium (Dietrich and List, 2014; Parsegov et al. 2017).

The most promising focus of this research is in areas associated with participatory planning and design, and the clearest statements of this have been in geodesign. Geodesign is about how one uses informal but structured problem solving amongst groups who articulate planning problems as problems of merging map layers in the manner shown here. The paradigm of GIS is based on such map layers in that it originally emerged from the development of spatial overlay analysis in landscape architecture and planning. Steinitz (2012) argues that geodesign brings professional stakeholders and analysts together to solve a common problem such as the search for development sites in a city or region but with each stakeholder or team of stakeholders interacting with other groups who are pursuing their own designs independently. In the general, the groups know the opinions about the emerging solutions which other groups hold as the process proceeds. In this sense, it is very close to the model developed here and although we have considered its use as part of the International Geodesign Collaboration IGC (2020), there is much to do in first articulating the process of geodesign more explicitly, notwithstanding that there is increasing experience in its use (Pettit et al., 2019; Steinitz, 2020).

To an extent this kind of spatial averaging has come out of design methods where the group dynamics is minimal but as we demonstrated in terms of Alexander and Manheim's (1962) highway location problem, it is entirely appropriate to interpret 'factors influencing development' defined as maps of desirability associated with individual actors or stakeholders. Just as the work on small groups in sociology and social psychology has become less significant, formal work on design methods has fallen out of fashion. To an extent, formal methods for design have become more participatory, like geodesign. In many contexts, however, networks related to who communicates to who are being measured and represented in ways that enable detailed analysis of their structure to take place. But formal processes operating on these networks are not being developed, other than the model that has been described in this chapter. As crowdsourcing is rapidly developing however, there are clearly opportunities for developing more formal processes connected to the ways in which ideas appear in this context and a number of developments in this direction look promising (Lim, Quercia, and Finkelstein, 2010).

Ways of extending this approach to the design of networks is another avenue to be researched (Batty, 2013) but other network methods are also worth exploring. The development of the neural net model sketched briefly earlier in this chapter suggests that there are new ways in which we can train the network to generate different solutions and in approaching an ideal outcome, the neural net process encounters many less than ideal outcomes which nonetheless might be feasible. In fact if the neural net does not converge, then less than optimal solutions are useful and to an extent, the sensitivity of the actual outcomes (plans) can also be exploited by following this route. As we have emphasised, very few network models of problem-solving processes like this exist and although this particular model represents a baseline, we have identified many potential developments which, if pursued, will provide much richer and more relevant ways we might generate and evaluate plans for the future city.

References

Alexander, C., and Manheim, M. L. (1962) The Use of Diagrams in Highway Route Location, Civil Engineering Systems Laboratory, **Research Report R62-3**, MIT, Cambridge, MA.

Batty, M. (1971) An Approach to Rational Design: Part 1: The Structure of Design Problems, Part 2: Design Problems as Markov Chains **Architectural Design, 41,** 436-439, 498-501.

Batty, M. (1974) A Theory of Markovian Design Machines, **Environment and Planning B**, **1**, 125-146.

Batty, M. (2013) **The New Science of Cities**, The MIT Press, Cambridge, MA.

Batty, M. (2016) Evolving a Plan: Design and Planning with Complexity, in J. Portugali and E. Stolk (Editors) **Complexity, Cognition, Urban Planning and Design**, Springer Proceedings in Complexity, Springer International Publishing Switzerland, 21-42.

Blondel, V. D., Hendrickx, J. M., and Tsitsiklis, J. N. (2009) On Krause's Multi-Agent Consensus Model With State-Dependent Connectivity, **IEEE Transactions on Automatic Control**, **54**, 11, 2586-2597.

Blondel, V. D., Hendrickx, J. M., Olshevsky, A. and Tsitsiklis, J. N. (2005) Convergence in Multiagent Coordination, Consensus, and Flocking, **Proceedings of the Joint 44th IEEE Conference on Decision and Control, European Control Conference**. Seville, Spain, December 12–15.

Coleman, J. S. (1973) **The Mathematics of Collective Action**, Heinemann Educational Books, London.

Coleman, J. S. (1998) **Foundations of Social Theory**, Harvard University Press, Cambridge, MA.

de Groot, M. H. (1974) Reaching a Consensus, **Journal of the American Statistical Association, 69**, 345, 118-121.

Dietrich, F., and List, C (2014) Probabilistic Opinion Pooling, in Hitchcock, C. and Hajek. A. (Editors) **Oxford Handbook in Probability and Philosophy**, Oxford University Press, Oxford, UK, at https://mpra.ub.uni-muenchen.de/54806/

French, J. R. P. (1956) A Formal Theory of Social Power, **Psychological Review**, **63**, 181-194.

Friedkin, N. E., and Johnsen, E. C. (1999) Social Influence Networks and Opinion Change, **Advances in Group Processes**, **16**, 1-20.

Gurney, K. (1997) **An Introduction to Neural Networks**, UCL Press Ltd, London.

Harary, F. (1959) A Criterion for Unanimity, in D. Cartwright (Editor) **Studies in Social Power**, Institute for Social Research, Ann Arbor, MA.

Harary, F. and Lipstein, B. (1962) The Dynamics of Brand Loyalty: A Markovian Approach, **Operations Research**, **10**, 19-40.

Hegselmann, R. (2004) Opinion Dynamics – Insights by Radically Simplifying Models, in D. Gillies (Editor) **Laws and Models in Science**, King's College Publications, London, 19– 46.

Hotelling, H. (1929) Stability in Competition, **The Economic Journal**, **39** (153), 41–57.

IGC (2020) International Geodesign Collaboration: Changing Geography by Design, at https://www.igc-geodesign.org

Kelly, F. P. (1981) How a Group Reaches an Agreement: A Stochastic Model, **Mathematical Social Sciences**, **2**, 1–8.

Lim, S.L., Quercia, D., and Finkelstein, A. (2010) StakeNet: Using Social Networks to Analyse The Stakeholders of Large-Scale Software Projects, in **Proceedings of the 32nd ACM/IEEE International Conference on Software Engineering**: ICSE'10 Volume 1, Association for Computing Machinery, New York, 295-304.

McHarg, I. L. (1969) **Design with Nature**, Doubleday/Natural History Press, Garden City, NY.

Mesbahi, M., and Egerstedt, M. (2010) **Graph Theoretic Methods in Multiagent Networks**, Princeton University Press, Princeton, NJ.

Parsegov, S. E., Proskurnikov, A. V., Tempo, R., and Friedkin, N. E. (2017) Novel, Multidimensional Models of Opinion Dynamics in Social Networks, **IEEE Transactions on Automatic Control**, **62** (6), 2270-2285.

Pettit, C. J., Hawken, S., Ticzon, C., Leao, S. Z., and Afrooz, A. E., Lieske, S. N., Canfield, T., Ballal, H., and Steinitz, C. (2019) Breaking Down the Silos through Geodesign – Envisioning Sydney's Urban Future, **Environment and Planning B: Urban Analytics and City Science**, **46**(8) 1387–1404.

Ren, W., Beard, R. W., and Atkins, E. M. (2007) Information Consensus in Multivehicle Cooperative Control, **IEEE Control Systems Magazine**, April, 71-82.

Steinitz, C. (2012) **A Framework for Geodesign: Changing Geography by Design**. ESRI Press, Redlands, CA.

Steinitz, C. (2020) Re-thinking the Growth of London: An Interactive Multi-Stakeholder Perspective Derived in Geodesign and Negotiation, **Land Use Policy**, forthcoming

Steinitz, C., Parker, P., and Jordan, L. (1976) Hand-Drawn Overlays: Their History and Prospective Uses, **Landscape Architecture**, **66**, 444–455.

Surowiecki, J. (2004) **The Wisdom of Crowds**, Doubleday & Co, New York.

Vicsek, T., Czirók, A., Ben-Jacob, E., Cohen, I., and Shochet, O. (1995) Novel Type of Phase Transition in a System of Self-Driven Particles, **Physical Review Letters**, **75**, 1226-1229.

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