CENTRE FOR ADVANCED SPATIAL ANALYSIS

Working Paper Series

Paper 4

LOCAL MOVEMENT: AGENT-BASED MODELS OF PEDESTRIAN FLOW

Michael Batty
Bin Jiang
Mark Thurstain-Goodwin

Centre for Advanced Spatial Analysis
University College London
1-19 Torrington Place
Gower Street
London WC1E 6BT

Tel: +44 (0) 171 391 1782
Fax: +44 (0) 171 813 2843
Email: casa@ucl.ac.uk
http://www.casa.ucl.ac.uk

http://www.casa.ucl.ac.uk/local_movement.doc

Date: June 1998

ISSN: 1467-1298

ABSTRACT

Modelling movement within the built environment has hitherto been focused on rather coarse spatial scales where the emphasis has been upon simulating flows of traffic between origins and destinations. Models of pedestrian movement have been sporadic, based largely on finding statistical relationships between volumes and the accessibility of streets, with no sustained efforts at improving such theories. The development of object-orientated computing and agent-based models which have followed in this wake, promise to change this picture radically. It is now possible to develop models simulating the geometric motion of individual agents in small-scale environments using theories of traffic flow to underpin their logic. In this paper, we outline such a model which we adapt to simulate flows of pedestrians between fixed points of entry - gateways - into complex environments such as city centres, and points of attraction based on the location of retail and leisure facilities which represent the focus of such movements.

The model simulates the movement of each individual in terms of five components; these are based on motion in the direction of the most attractive locations, forward movement, the avoidance of local geometric obstacles, thresholds which constrain congestion, and movement which is influenced by those already moving towards various locations. The model has elements which enable walkers to self-organise as well as learn from their geometric experiences so far. We first outline the structure of the model, present a computable form, and illustrate how it can be programmed as a variant of cellular automata. We illustrate it using three examples: its application to an idealised mall where we show how two key components - local navigation of obstacles and movement towards points of global locational attraction - can be parameterised, an application to the more complex town centre of Wolverhampton (in the UK West Midlands) where the paths of individual walkers are used to explore the veracity of the model, and finally it application to the Tate Gallery complex in central London where the focus is on calibrating the model by letting individual agents learn from their experience of walking within the environment.

Local Movement in the Urban System

Below a certain scale particularly where spatial representation is more appropriate in terms of the built form rather than territorial subdivision, aggregative approaches to explaining and predicting urban phenomena begin to lose their meaning. For example, in town centres, in shopping malls and housing estates, location and movement is best described in terms of the individuals who use such spaces rather than in terms of their aggregation by social or any other attribute. Predicting local movements, patterns of crime, deprivation, and the individual usage of urban facilities must thus be simulated by treating the populations involved as distinct 'objects' or 'agents'. Such approaches although still novel are not inconsistent with the more aggregative approaches which have dominated urban simulation hitherto, but at scales where buildings, public spaces, and streets must be represented as distinct objects, the associated behaviour patterns of individual users must be directly simulated if impacts of changes to the geometry of the local environment are to be understood.

To date, there have been few long term efforts to simulate the patterns of pedestrian flow within small scale environments. Despite the fact that walking in western cities constitutes up to 30 percent of all journeys made (GSS, 1997), and in city centres such as the West End of London up to 40 percent (Whiteley, 1997), this mode of travel has been almost entirely neglected in mainstream transportation modelling which still emphasises the automobile. Early attempts at modelling pedestrian flows followed three rather different approaches. Descriptions of how pedestrians move and congregate have provided statistical data for the distributions of queues, and these have been used to predict volumes on street segments from linear relationships estimated by regression (Older, 1968; Stilitz, 1969; Sandahl and Percivall, 1972). More formal approaches based on loose analogies with fluids, gas kinetics, and other physical flow systems have been proposed and tested (Henderson, 1971, 1974; Helbing 1995) but by far the most usual approach has been based on adaptations of spatial interaction models, often in their discrete choice rather than gravitational form.

Attempts to simulate pedestrian flows (Borgers and Timmermans, 1986a, 1986b) from individual trip probabilities by mode, often incorporating trip chaining have been developed, but the predictions from such models must then be scaled back to more aggregate units if they are to be generalised spatially. Models of movement patterns in the urban system based on microsimulation appear promising but notwithstanding the effort that has gone into such simulation in recent years (Clarke, 1996), there have been very few applications at a fine spatial scale. Such approaches concentrate on simulating generic patterns or profiles within a population based on 'typical' or 'prototypical' individuals but ways of mapping these profiles onto detailed spatial representations are poorly developed.

There has been more activity of late in associating movement patterns and the location of pedestrian volumes with measures of spatial accessibility, proximity, or connectedness. Since the 1940s, measures of accessibility based on evaluating the potential of any location with respect to how near and how attractive it might be to all other locations have been used to provide indices which show the actual or potential density of use at any location within the city or regional system. For example, Hansen's (1959) residential land use model was based on forecasting growth or change at any location in proportion to the value of its accessibility, based on a variant of Stewart and Warntz's (1958) measure of population potential. In terms of networks, the density of traffic flowing across any node has been correlated with degrees of nodal connectivity within such networks (see Haggett and Chorley, 1969) while a more recent theory of natural movement due to Hillier et al. (1993) has sought to correlate movement patterns at the local scale with the relative accessibility of streets to one another. This theory, referred to as space syntax, is a first order generalisation of accessibility or connectivity of streets where streets are treated as nodes and network measures computed if streets relate to one another in various physical ways such as through their intersection. However, these accessibility measures although providing indices associated with forecasting trip volumes, are not based on models which simulate processes of movement and thus do not provide methods for predicting the impact of locational changes on patterns of pedestrian flow. In short although these indices can show changes in flow due to changes in the geometry and location of entire streets, they are unable to account for comprehensive movement patterns which link facilities at different locations to one another. Although space syntax deals with movement economies, it has not yet been possible to link such indices to the socio-economic structure of the city at the local scale (Hillier, 1997).

The need for a much richer theory of local movement accounting for individual behaviours which determine pedestrian flow suggests that all aspects of the environment within which such behaviour takes place as well as the individuals generating such behaviour must be represented explicitly, as distinct objects. Recently object-oriented approaches to simulation have become popular due to developments in programming technology as well as due to the increasing perception that local behaviour is fundamental in explaining global pattern. To develop models of such local behaviour, individuals must be represented explicitly and from this comes the idea of 'agent-based' modelling (Axelrod, 1997a). This is entirely consistent with recent developments in complexity theory where the complexity of the system emerges in global and structural terms from actions, each of which are simple in themselves, of relatively autonomous agents, acting with their own self-interest in mind, without appeal to any grand design or response to any overall global rationality or utility. Models for simulating artificial life are the best exemplars (Langton, 1995; Adami, 1998). Of course many systems cannot be characterised in this way but local movement patterns and behaviours in small-scale built environments appear to fit the approach rather well.

Small-scale environments capture the global properties of the urban system in such a way that local responses are usually consistent with macro properties. For example, local geometries such as the juxtaposition and type of buildings, the scale and direction of streets, and the location of transport facilities all imply a global order to the city. Thus when individuals respond to what is in their immediate neighborhood, this will reflect a more global order. The relative attraction of facilities at different locations is usually mirrored locally and local responses are thus consistent with behaviours which reinforce the global order. For example, if an individual is searching for an attractive retail location and is in a relatively unattractive location, then the local situation is likely to contain clues as to how to move towards a more attractive location. However, local movements are also heavily influenced by much more idiosyncratic factors such as physical obstacles around which to navigate, localised congestion, and serendipitous decision-making with respect to what is immediately attractive. Local movements must also account for different varieties of behaviour ranging from movements which are well-defined and completely purposive to those which are more random and exploratory, based on walkers who know the environment completely to those who do not know the local geometry and attractions of the environment at all. An agent-based approach is the only way in which we might account for such diversity and design models which can be tuned to quite different walker situations.

A number of agent-based models are being developed which have direct relevance to smallscale urban environments. The *TRANSIMS* model under construction at Los Alamos (which is associated with the elaborate microsimulation system called *SWARM*) has been developed to model individual trip movements at the level of the automobile (see *http://www.lanl.gov/*). The model system is noteworthy because it can be simplified in various ways to show its consistency with various major themes in complexity theory such as local movement based on cellular automata ideas as well as self-organised criticality (Nagel and Paczuski, 1995). It is perhaps the most developed such system to date and lies one step beyond the ideas we will develop here. As such it represents a potential next stage in our own research. A much more focused set of approaches is being developed by Helbing and his co-workers at Stuttgart who have developed several variants of pedestrian model, particularly based on analogies between social and physical forces (Helbing, 1991; Helbing and Molnar, 1995). In these models, the patterns of walking are reinforced by the very activity of walking, such that the system and its movement patterns self-organise according to certain interactive behaviour (Helbing and Molnar, 1997). However although *TRANSIMS* and these active walker models incorporate many relevant features of local movement, their purpose is less geared to predicting the importance of locations to which pedestrians move than the approach that we will develop here.

Other models of local movement in which location is prominent are those in which actors or agents respond to others in their vicinity. For example, the range of models developed onwards from the work of Schelling (1978) in which actors move or migrate to be closer to those to whom they perceive some affinity, thus creating polarised clusters or ghettos, might be adapted to deal with local movements. Their focus however is on longer term spatial migration rather than the more routine and frequent kinds of movement implicit here. Axelrod's (1997b) model of regional polarisation based on cultural convergence and Epstein and Axtel's (1996) '*Sugarscape*' model of an artificial society are more recent variants on this theme that provide exemplars of why agent-based approaches are essential. Finally, the models which emanate from the simple manipulation of local geometry as embodied in the multiple *Logo* approach developed by Resnick (1994) amongst others, are well adapted to dealing with local movement as illustrated in a variety of examples such as insect movement, trail formation, and percolation through sparse and dense media. These are all rooted in the logic of cellular automata to which we will return.

There are more pragmatic reasons for developing agent-based models of small-scale urban environments. First there is a strong commercial imperative for predictive models which are able to simulate how attractive different locations are to consumers. The location of points at which such consumers are 'discharged' into spatial markets, the characteristics of streets which enable such consumers to reach the market sites in question, and the relative attraction of other adjacent sites all directly affect profit margins through patronage. The same issues pertain to sites in the leisure industries which rely upon visitors. More focussed social concerns such as the spatial incidence of crime, and the need for secure environments are intrinsically bound up with the location and type of facilities and the extent to which these are patronised. In one sense, the problem emphasised here is little different from the more general problem of urban location at larger scales except for the interaction between local geometry and purposive and exploratory behaviours which seek to optimise locational attraction. The fact that such modelling is now possible also relates to the availability of new data sets, many of them collected by the various commercial interests which have greatest need for good predictions involving their retail trade. Good digital data on built form is now available at 1-2 meter resolution while pedestrian flows are being increasingly monitored automatically and remotely using closed circuit TV. Questionnaire surveys of origin and destination movement patterns and person-following techniques for precise walking inventories are increasingly available. Data from electronic point-of-sale (EPOS) is being associated with spatial and locational data at a variety of scales, from movement within the store at one extreme to movement within the metropolis at the other.

There are many types of environment and problem context at these small scales which require different variants of this approach and at the outset, we should be clear as to the issues that we consider important and those that we will ignore. Our model will apply to relatively closed geometric systems such as malls and town centres, galleries and theme parks within which visitors or walkers engage in purposive activities such as shopping or leisure for fixed periods of time. We will make the assumption that all the individuals who move within such systems enter and leave the system at the same points (although we can easily relax this) which we will call 'gateways'. In each case, an individual's trip can thus be divided into active and passive stages where we assume that an individual is active for most of the trip but once a decision is made to return to the gateway, the trip becomes passive in that the goal changes from visiting the most attractive locations to that of returning to the gateway. In this sense, the variants of the model that we illustrate here will not apply to movements in residential neighborhoods or in work trip contexts. Furthermore, all our experiments will be of an exploratory nature. These kinds of pedestrian model are in their infancy and it is likely that very different versions of them will emerge once the research momentum builds up. We consider that all such models so far are 'proofs of concept' rather than fully operational predictive structures. However this does not imply that we cannot gain insights from such applications.

In the next section, we will outline the key issues that characterise the kinds of local movement we will simulate, illustrating the rudiments of the model and the way these can be articulated. We then develop the mathematics of our generic model, showing how a typical walker moves through the geometric system and interacts with other walkers within the same space. We implement the model as a cellular automata, using highly visual software which enables us to examine movements and patterns as they take place in the geometric space which we visualise through a variety of map layers and agent movements. We then develop three related applications or experiments, Our first experiment involves the simplest model - an idealised retail mall based on a symmetric geometry with a simple unimodal retail attraction surface. We show how the pedestrian movement can be varied as various parameters controlling the interaction of the local geometry with the surface attraction can be manipulated, thus illustrating how we can 'calibrate' the model. Our second example, involves the same model applied to a real town centre, that of Wolverhampton which is a medium sized town (population circa 244K) with a well-defined and self-contained centre. We attempt to calibrate this model in the conventional way by searching over the parameter space for parameter values which generate patterns of realistic-looking movement. Our last experiment involves introducing considerably more complexity into the model through very complex local geometry that characterises the space as well as the more complex locational attraction posed by a structure of closed rooms linked by galleries and corridors. We show how we can model pedestrian visitor movements in London's Tate Gallery and then outline how the model can be calibrated to real data which provides realistic simulations of movement, by letting the agents 'learn' the best parameter values as they move through the space. Finally we draw these various threads together, speculating on next steps in the research programme and arguing the need for taking this research to a more sophisticated level of simulation.

Representing Pedestrian Flows

The generic framework we develop will be flexible enough to take account of several different problem characterisations, all of which involve relatively self-contained local movement. However, this framework is based on the requirement that pedestrian movements are important to predicting how many individuals are attracted to different sites within the local system of interest. The flow volumes on the various routes that relate these sites are of interest but the detail of the flow is not relevant. In essence, the model allocates walkers from fixed origins to various destinations and in doing so enables their assignment to the various streets, sidewalks, squares and precincts which link origins and destinations together. The elements of the model loosely reflect ideas of attraction and deterrence in more aggregative traffic models that build on traditions in spatial interaction modelling (Willumsen and Ortuzar, 1990). Other aspects of pedestrian flow are of lesser interest;

for example, the streaming of pedestrians, their movement in groups, their behaviours at street intersections, gates and doors, and their velocity are of little relevance as the focus here is upon the location of origins and destination, the geometry of their linking, and the flows which bind various locations together. Lastly as we have already implied, trips in this kind of system begin and end at the same origin and exist over a fixed interval of time such as a shopping trip period - hours or parts of days - which might be aggregated to 'weekly' or 'monthly' behaviour. The interest is upon how many visits are made in total to various destinations, notwithstanding the fact that those controlling or managing such destinations might be interested in more detailed temporal movements.

In any model, it is possible to relax assumptions but we will begin by specifying the model in the strictest form possible. All **origins** are referred to as **gateways**. These are nodes in the system such as car parks or bus stations from which walkers are 'discharged' and from which they begin their walk through the system. We will assume that at each gateway, individuals change their transport mode - in car parks from car to walk, in bus or rail stations and at bus or rail stops from bus or rail to walk. The same kinds of transition can be assumed for linear gateways such as on-street parking lines; for those who walk directly into the system from outside, we assume a line of gateways surrounding the system from which these walkers assume their pedestrian mode. Related to this assumption is the fact that all pedestrians return to the gateways from which they enter the system. This is not so severe an assumption as it might appear and it has only been introduced to simplify the simulation; if necessary it can be relaxed.

While gateways are always discrete points, we represent **destinations** somewhat differently, as continuous **attraction surfaces** which cover the entire extent of the local geometry. This is primarily due to the fact that such surfaces can be easily computed from very detailed point location data using various spatial averaging techniques. These enable each location in the system to be considered relative to any other, and thus to be independent of the local geometry of buildings and streets. In short, this is a way of aggregating and smoothing out the effects of individual buildings and stores and also of adding together various data to produce composite indices of attraction. For example, such attractions might be built up from data on retail turnover, floorspace, rents and so on, most of which are not available for individual shops but are available for very fine scale geometries such as postal delivery codes. This implies that a large store in the centre of town is likely to have a level of attraction per unit of its space rather similar to an adjacent store which is much smaller, selling different goods, and attractive to different customers. This concept of the attraction surface can be varied to relate to different geometries; for example, in the Tate Gallery example, the attraction surface is based on rooms which are disjoint from one another but with equal levels of attraction within each room. Finally, it is quite possible to have more than one surface of attraction, each picking up different features of attraction at various locations, and being either combined or used sequentially through the walking process.

The third element of representation involves the local geometry of buildings and streets - the built form. The kind of resolutions that we are concerned with here range down to 1-2 meters. Fairly precise movements can be modelled at this level but to model very detailed interaction of pedestrians with one another in the manner which the Stuttgart group have employed (Helbing and Molnar, 1995), it would be necessary to represent the system down to 0.1 meter resolution so that crowding could be thoroughly examined. Our limit is consistent therefore with our interest in aggregate flows and flows attracted to different locations. Our characterisation does not pick up the kind of detail where one walker might relate to another except in terms of total counts of crowding. For example, we do not simulate how two walkers might collide, nor do we simulate how they might behave at intersections or at doorways. However, our model should be able to detect crowding which occurs as pathways and streets narrow. It should also reflect the intrinsic attraction of the sidewalks along buildings relating to the fact that streets never go directly to the points of highest locational attraction (which are within buildings) although walkers attempt to do so. Lastly, we will not simulate the behaviour of pedestrians who might act in groups other than through the usual mechanism of increasing attraction as more pedestrians visit a place or reducing attraction as congestion thresholds are passed.

We can now outline the key principles for movement which we will build into the model and which we believe are borne out through current observation and our causal knowledge of how walkers behave in small-scale urban environments. Our walkers move one step in each time period (if they are able) and this is in a given direction which in turn is measured through an angular heading or through xy coordinates. We define five components which determine direction in each time period of the simulation:

1. The key heading is always in the direction associated with the surface of locational attraction. At any location and at a given time, a walker who is still engaged in active movement in the system (that is, not returning to their origin or gateway), sets a heading in the direction of the gradient of the attraction at that point. In the case where there is a unimodal surface of attraction with the highest point at the centre of the town say or at the prime retail pitch, the walker will

continually readjust this heading in an effort to climb to the top of the surface: the hill-climbing analogy is relevant. There are many factors that might obstruct this process; where there might be multiple local optima or several surfaces of attraction evaluated in sequence, say. But the most likely changes in direction which distort this process are due to the interaction of the surface with the local geometry as we will indicate below.

2. The default direction in the model is for any walker to move in the direction that they are already travelling, that is forward. In each situation where the walker is moving forward, this heading is perturbed by a random change in direction whose probability of occurrence declines proportionately and nonlinearly with the size of the deviation from the forward direction.

3. Obstacles to forward movement occur through the local geometry of streets and buildings. At each stage of the walk, a running count of progress in terms of distance travelled is kept. If a walker hits an obstacle such as the edge of a street, the walker continues to advance with their heading perturbed randomly. If after a number of tries, no progress has been made, the walker reverses direction and several new directions are tried until one which initiates progress is found.

4. At a given time period, the level of congestion at each location in the system is evaluated. If there are more walkers at or around a point than a predetermined threshold, then walkers are perturbed in terms of their subsequent directional movement until congestion falls below the threshold.

5. In different areas of the spatial system - rooms for example in a gallery, or squares and street segments within a town centre, change in the number of walkers and their totals are measured in each time period. These changes and totals can be related directly to the attraction of the place in question, attraction increasing up to the point when congestion sets in and attraction declines. In this way, the number of walkers in any area can also be controlled.

Walkers are thus connected to one another through locational attraction and through congestion. In this way, various positive feedbacks are reinforced and the system can be self-organising in a similar manner to that used in the models developed by the Stuttgart group (Helbing and Molnar, 1997).

Movement is calculated in each time period t in the direction r for each walker k, where k = 1, 2, ..., K, there being K walkers in the system. K will vary with time t but to introduce the model, we do not need to detail the way K changes yet. Each location in the system is given by coordinates xy (which pertain to pixels in the computable model developed below), and thus the direction of each walker is defined as $r_k(x,y,t)$. In the model, we will compute direction from the heading θ at xy where $0<\theta<2\pi,\theta$ being measured in radians. However we will first state the model in general terms in this section before we develop its computable form in the next. In general, the direction for any walker k at time t + 1 is defined from

$$r_k(x,y,t)=\sum_i \tau_i f_i \qquad (1)$$

where f_i is a function of one of the five components affecting the heading, and τ_i is a temporal switch which activates the function or force on an appropriate time cycle. If for example, this switch applies to every time period and every function, then $\tau_i=1,\forall i,t$ but it is usually different from this as we will explain below. We can write equation (1) explicitly for the five components as

$$r_k(x,y,t)=\tau_g f_g+\tau_d f_d+\tau_b f_b+\tau_c f_c+\tau_a f_a \qquad (2)$$

where f_g is the function associated with evaluating the gradient of the attraction surface at xy, f_d is the function involving movement in the forward direction which is the default, f_b is the function that controls the perturbations needed to navigate a physical barrier, f_c is the congestion function that involves perturbing the direction of any walkers who are exceeding the threshold, and f_a is the function that enables the attraction surface to be updated with respect to the number of walkers in that area of the system in question.

Each of these functions should be further defined so that the elements involved in each can be specified prior to the full model being stated. The direction $r_k(x,y,t)$ is defined as a sum of each of five components which in turn are evaluations of the heading required to make independent progress in each of the five directions. Which are relevant depends of course on the temporal switches $\{\tau_i\}$ but the computable model ensures that these components are added to form a composite directional vector. The first component f_g is a function of the attraction surface ϑ_{xyt} in terms of its gradient, and this can be stated as

$$f_g = f_g\left[\nabla \vartheta_{xyt}\right] \qquad , \tag{3}$$

while the second function f_d, the default forward direction with random perturbation $\varepsilon_k(x,y,t)$ is defined as

$$f_d = f_d\left[r_k(x,y,t), \varepsilon_k(x,y,t)\right] \qquad . \tag{4}$$

The third component involves the perturbations required to avoid locations within the barrier set $\{B(x,y)\}$ - edges of streets or walls of rooms - and is of the same form as the second

$$f_b = f_b\left[r_k(x,y,t), \{B(x,y)\}\right] \qquad . \tag{5}$$

The fourth and fifth components are those which lead to direct positive feedbacks between walkers. Congestion is computed as the sum of walkers *k* in the congestion neighborhood of point *xy*, called Z_{xy}, with appropriate perturbations $\omega_k(x,y,t)$ in direction to ensure that congestion is reduced below the threshold χ, that is

$$f_c = f_c\left[\{\sum k \in Z_{xy}\}, r_k(x,y,t), \omega_k(x,y,t), \chi\right] \qquad , \tag{6}$$

while attraction ϑ_{xyt} is altered according to the number of walkers in the attraction area Ω_{xy} within which the point *xy* exists

$$f_a = f_a\left[\{\sum k \in \Omega_{xy}\}, \vartheta_{xy}\right] \qquad . \tag{7}$$

The precise functional forms for these five components will be specified below. The simultaneity which is implied in equations (1) to (7) is resolved in the computable model once these forms are made discrete and the sequencing implied through $\{\tau_i\}$ in equations (1) and (2) is specified.

A Computable Form for the Generic Model

We will first examine the five components of movement and then show how these are assembled into the integrated model before dealing with detailed algorithms and programming in the next section. All movement in the model is driven from setting a new heading θ in each time period t for each walker k henceforth referred to as walker w_k at location xy. As we have indicated, we assume that each walker k makes a unit step forward or remains stationary in each time period and this implies that changes in the x and y coordinate directions given by Δx and Δy respectively are chosen so that $r_k(x,y,t) = 1, \forall kxyt$. In the following outline, we will suppress the indices *k, x, y,* and t wherever it is obvious from the context. Changes in direction are computed from

$$\Delta x = r_k(x,y,t)\cos\theta \quad \text{and} \quad \Delta y = r_k(x,y,t)\sin\theta$$

where using the assumption of unit length in walking, the new coordinates become

$$x_{t+1} = x_t + \Delta x = x_t + \cos\theta, \quad \text{and} \tag{8}$$

$$y_{t+1} = y_t + \Delta y = y_t + \sin\theta \quad . \tag{9}$$

Equations (8) and (9) apply to the evaluation of any motion, whether or not the walker is in the active or passive (returning) phase of their trip and whether or not this motion actually takes place.

The first and most basic component which affects each walker's heading is the function involving the gradient of the attraction surface $f_g = f_g\left[\nabla \vartheta_{xyt}\right]$. This gradient is the total differential

$$\nabla \vartheta_{xyt} = \frac{\partial \vartheta_{xyt}}{\partial x} + \frac{\partial \vartheta_{xyt}}{\partial y} \tag{10}$$

from which changes in direction can be computed in proportion to

$$\Delta x' = \frac{\partial \vartheta_{xyt}}{\partial x} dx \qquad \text{and} \tag{11}$$

$$\Delta y' = \frac{\partial \vartheta_{xyt}}{\partial y} dy \qquad . \tag{12}$$

A notional single step distance of dx = 1 and dy = 1 implies that progress is directly proportional to the partial derivatives of the surface in their respective directions. In fact the angular variation can be directly computed from

$$\theta = \tan^{-1}\left[\frac{\partial \vartheta_{xyt}}{\partial x} / \frac{\partial \vartheta_{xyt}}{\partial y}\right] \tag{13}$$

where θ is used to update the heading and to define the increments Δx and Δx in equations (8) and (9).

The second component is based on a function $f_d\left[r_k(x,y,t), \varepsilon_k(x,y,t)\right]$ which updates the heading by applying a random perturbation to the existing heading. This is made on the assumption that walkers continue in the direction that they are going but with some probability that they will adjust their heading marginally. The adjustment, an increment to the heading $\Delta\theta$ is defined from

$$\Delta\theta = \pm\{\pi / random(100)\} \tag{14}$$

where it is clear that the absolute value of $\Delta\theta$ varies inversely with the random occurrence of any number in the uniform interval [1:100]. 90 percent of the time, this value will be less than 18 degrees or 0.314 radians. This perturbation to direction is applied to all headings, including those which are computed from the other four components.

The third component involves the function $f_b\left[r_k(x,y,t), \{B(x,y)\}\right]$ which tests whether or not the forward motion computed from any function hits a geometric barrier or obstacle in the set $\{B(x,y)\}$. If the new coordinate pair $\left[x_{t+1}, y_{t+1}\right] \in \{B(x,y)\}$, then no movement takes place because the location is not part of the system where walking is allowed. In such cases, a progress variable λ_{kt}, measured in terms of distance travelled, is set as 0, and the walk continues; each subsequent time period, this variable is accumulated and tested to see how much progress has been made. If progress is less than a certain threshold Λ, then it is assumed that the obstacle cannot be circumnavigated without the walker reassessing its position and heading. Formally

$$if\ \lambda_{kt} < \Lambda, then\ x'_{t+1} = x_t - \Delta x \quad and \quad y'_{t+1} = y_t - \Delta y \tag{15}$$

and the walker attempts to reverse. If $\left[x'_{t+1}, y'_{t+1}\right] \in \{B(x,y)\}$, the heading is reassessed as

$$\Delta\theta = random\ \{2\pi\} \tag{16}$$

and the algorithm implied by equations (15) and (16) is repeated a preset number of times *m*. *m* is 6 in the experiments reported here but like the progress threshold Λ, it is a parameter of the system and can be calibrated if required.

The last two functions - the fourth and fifth components - involve introducing interactions between walkers. The component $f_c\left[\{\sum k \in Z_{xy}\}, r_k(x,y,t), \omega_k(x,y,t), \chi\right]$ counts the level of congestion in the vicinity Z_{xy} of location *xy* in terms of the number of walkers N_{xyt} defined as

$$N_{xyt} = \sum_{k \in Z_{xy}} w_k \quad . \tag{17}$$

Then if $N_{xyt} < \chi$, the headings of the relevant walkers in Z_{xy} are perturbed by a directional component $[\omega_k(x,y,t)]$ although in practice this is achieved by setting the heading to $\Delta\theta = random\ \{2\pi\}$ as in equation (16) above. There is no guarantee that this will reduce the level of congestion *per se* although combined with other processes, it appears to work effectively. So far, we have set Z_{xy} as single point locations because the scale of resolution for the examples we have developed is such that this is appropriate. However, this set can be modified in terms of area covered if required.

Finally, the function $f_a\left[\{\sum k \in \Omega_{xy}\}, \vartheta_{xy}\right]$ relates the locational attraction to the number of walkers in the area Ω_{xy} where this might be a square, a segment of street, a shop or a room within the system in question. We first count the number of walkers M_{xyt} in the set as

$$M_{xyt} = \sum_{k \in \Omega_{xy}} w_k \tag{18}$$

arguing that the attraction ϑ_{xyt} is proportional to the number of walkers visiting the place. Of course, we preset the locational attraction surface in most cases but we are able to modify this preset attraction to take account of economies and diseconomies of scale not built into the simulation through prior data. For example, we can set the attraction at time $t + 1$, ϑ_{xyt+1} as

$$\vartheta_{xyt+1} = \vartheta_{xyt} + \beta_1 M_{xyt} - \beta_s M_{xyt}^2 \tag{19}$$

where equation (18) is clearly parabolic, implying that for small values of M_{xyt}, ϑ_{xyt+1} is an increasing function of the number of walkers in Ω_{xy}, while for larger values, this function is decreasing. The precise form will depend upon the parameters β_1 and β_2 while the baseline value of the function is set to the prior value of locational attraction ϑ_{xy} which is input data. So far in our simulations, we have not conducted exhaustive tests of this device although we consider the function important in pushing the model towards greater realism. It will be of particular use in later applications where we divide walkers into those who already know the space in which they are moving and those who do not. The latter group will move in response to those already in the system, being influenced by prior movement which is captured through changes to the attraction surface that they respond to and which will be computed in the manner of equation (19).

All the components of movement have now been assembled but as equation (2) implies, these five functions can act differentially in time. As this is important to the logic of the process, it is necessary to demonstrate how these functions are nested and sequenced within the model. The order in which these five components are evaluated as well as the points at which they are switched on and off combine to produce quite complex modulations. The temporal switches $\{\tau_i\}$ differ in form. The congestion switch τ_c is either on or off; if off, the function f_c is never evaluated while if on, it is evaluated at each time period. When this switch is on, and if the congestion level is exceeded at any location, then walkers are moved within a time period, that is they are moved immediately, and thus this factor does not affect any of the other functions. It can be operated anywhere within a time period, and in the current version of the model, it is evaluated first.

Three of the functions - the gradient f_g, the attraction level f_a, and the barrier function f_b are switched regularly but occasionally in the current implementation, usually on different sequences but the switches τ_g, τ_a, and τ_{ib} can be set to any time series, the default being their operation at every time. The current model evaluates gradient before attraction before barriers but in the case of the barriers function, a barrier must have been encountered and less progress made than the given threshold, for the function to be activated. However, an order of precedence is established when the time switches coincide; the barrier takes precedence over the attraction level which in turn takes precedence over the use of the gradient to fix the new heading of the walker. At this point, it is worth noting that only the congestion and barrier functions apply to walkers who are in the passive state, returning to their point of entry or their gateway. When these functions are not being activated - that is, when there is no congestion or no infringement of a barrier - then the heading of a returning walker is fixed to ensure that progress is made back to the gateway. This heading is fixed before the final function - perturbation in the directional heading - is invoked. This function f_d controlled by the switch τ_d is always switched on in this model, that is, it operates in every time period. It is evaluated last and this means that the dominant heading determined by any of the functions already evaluated or the heading on the previous time period if none of the functions just discussed have been activated in that time period, is perturbed in the usual manner. This incorporates local fluctuations in movement which always occur in reality.

The logic of these operations is illustrated in the flow chart in Figure 1 where it is immediately clear that what operates in each time period depends upon the way in which the switches are configured and the order in which the five components are evaluated. In the default case where all functions operate in each time period, then the congestion function always applies and is self-contained while the barrier function if invoked will always take precedence over the attraction function which in turn dominates the gradient function. Note that when the attraction function is evaluated, then this involves the computation of a new gradient. The directional heading function of course applies in every time period. The critical sequencing then depends on the frequency at which the gradient, attraction and barrier functions are evaluated. The precedence order will only come into play however when these sequences coincide. For example, if the gradient function is evaluated in every 5th time period, the attraction in every 7th and the barrier in every 4th, then only in the 140th period will the barrier take precedence if operative. However, various combinations of these frequencies would mean that the following sequence would apply: when $t = 4$, f_b; $t = 5$, f_g; $t = 7$, f_a; $t = 8$,

f_b; $t = 10$, f_g; $t = 14$, f_a; $t = 15$, f_b; $t = 16$, f_b; $t = 20$, f_b; $t = 21$, f_a; $t = 24$, f_b; $t = 25$, f_g; $t =$ 28, f_b; $t = 30$, f_g; $t = 32$, f_b; $t = 35$, f_g; ....; $t = 140$, f_b; and so on ....

As well as the structural logic that we have developed which determines the precedence of certain operations, the model has several parameters which need to be specified and calibrated. As it stands, it represents a subtle combination of local and global factors and functions, based on the way local geometry combines with global attraction. Some might argue that such models should be based on local factors entirely and that local geometry should determine the way walkers respond and use various attractions. This may be the case for situations where visitors have no knowledge of the local situation but in systems where some idea of the global properties of the environment are known from previous contact which is often, in fact usually the case in town centres and shopping malls, then some global functions are essential. Moreover these models are ones which simulate routine movements; they are not meant to enable trails to be formed or paths and streets in cities to be developed. They are much more akin to traffic distribution and assignment models than to evolutionary models. Although we have added a fifth component dealing with the modification of attraction based on movement volume, we consider this to be a factor which enables congestion to be handled, rather than mirroring any real processes which govern the way locational attraction surfaces might evolve.

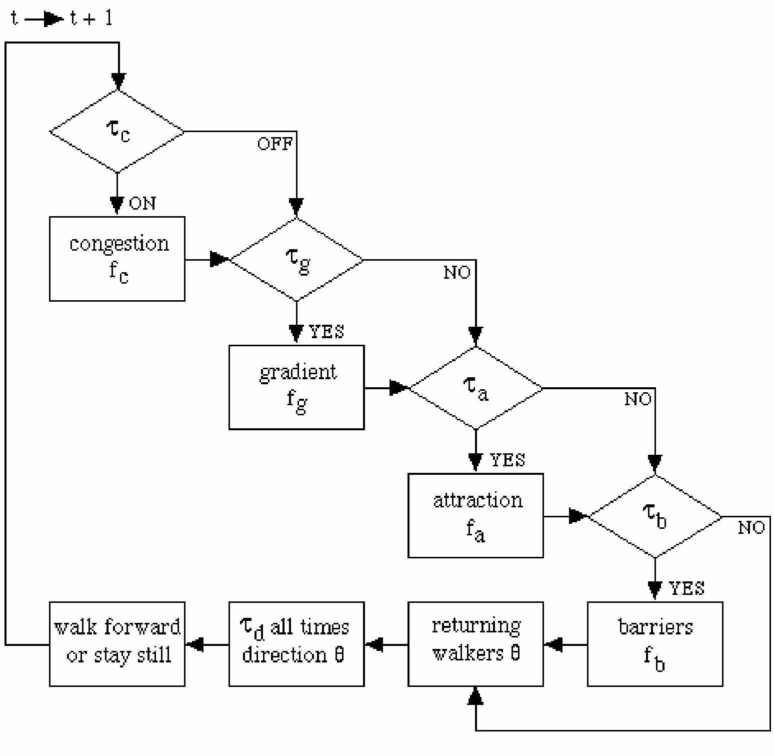


Figure 1: Components forming the Walking Algorithm

Implementing the Model as a Cellular Automata

The way pedestrians use town centres and malls has been conceived here as a bottom-up process, in which walking is a subtle interplay between the effects of local geometry, attractions which are 'discovered' as walkers move, and more global attractions which provide an overall rationale for such trip making in the first place. As we implied earlier, there are several ways in which such simulation might be implemented based on the rationale of spatial interaction modelling or microsimulation of event and agent profiles, although the emphasis we have given here to local action makes a cellular automata (CA) approach attractive. CA however does provide quite strict limits on the extent to which global factors can be treated in that most functions in such frameworks deal with entirely local neighborhoods, thus making it hard to examine any phenomena of interest at a distance from any given location. However with judicious use of neighborhood data within which global features are contained, such simulations can be very effective.

We make an initial distinction between agents - in this case pedestrians or walkers - and the environment in which motion takes place. The environment is always conceived as a 2-dimensional space based on a homogeneous grid of pixels which represent *xy* locations, upon which various characteristics or attributes of the environment can be stored as spatial layers. Agents also have characteristics and attributes, and motion depends upon the interaction between agents and their environment as well as interactions between agents, both types of interaction being effected through comparisons of attributes. The local geometry and the global attraction of locations are represented as part of the environment while agents move by making comparisons between these characteristics of the environment and their own motivations for motion which are represented by their personal characteristics. For example, a walker, who is actively shopping, will respond to the environment in terms of attractions and geometry in a different way from a walker who is returning from such a trip to their point of entry or gateway into the system.

Agents are virtually 'blind' in such a system in that the CA principle restricts their viewshed to their immediate neighbourhood. This is useful for evaluating immediate obstacles but is quite limited for evaluating progress which depends on lines of sight and memory of destinations. Because streets and even squares and precincts tend to be directional for the most part, once a walker is set on a heading which makes progress, then this is reinforced by the model with walkers following lines of sight from the interplay of the local street geometry and the behavioural need to maintain direction. More global factors must therefore be encoded locally through the attributes of layers. Where there is a global optima to location - represented as a unimodal attraction surface for example, then the local gradient is sufficient for walkers to make progress. More complex surfaces, however, must be disaggregated into constituent and simpler parts which are handled by walkers separately. A spatial interaction model, for example, would consider all destinations simultaneously and thus account for very complex attraction directly whereas in CA modelling, only the immediately adjacent locational attractions can be examined. If these are to be used to influence direction, then these must embody some local element which reflects the global attraction; this is the case where the surface is particularly simple and the local gradient is the key to where the walker is on the overall surface (Batty, 1998). It is possible in CA to decompose the surface into different trends, each implying some significant destination but the number of such decompositions is limited.

In short, only for certain problems is the CA approach relevant. Problems where there are many local optima or where trip making is dominated by very specific destinations are likely to be unsuitable. Here the Tate Gallery exemplar is instructive for visitors to the gallery are likely to be newcomers, most being first time visitors to the particular exhibition and thus movement is dominated by much exploratory walking. As there is only one gateway or entrance, then there needs to be a global function relating to drawing walkers into the gallery and this is set up as an orientation surface. The local attraction of different rooms provides another surface and the way the model works is by interweaving these two surfaces in such a way that combined with local geometry, realistic walking patterns are simulated. Nevertheless, such ideas are fairly experimental as yet and all we can say so far is that this approach appears promising.

CA is a very effective approach for processing local information rapidly and efficiently (Toffoli and Margolus, 1987). The version we use here makes a distinction between agents and environment in the manner we noted above and is based on Resnick's (1994) version of *Logo* (called *StarLogo*) where multiple agents (called turtles) interact with a spatial environment whose layers (called patches) contain the geometric and other characteristics of the spatial system. This system can handle up to 16K agents, spatial environments up to 200 x 200 cells in size (arranged in grid/pixel fashion) whose attributes can be encoded in up to 64 layers. The software provides a means for rapid prototyping of these kinds of model. The visualisation capabilities of the system are good, with a user-friendly interface for modelling operations, fast animation of dynamic change, in this case motion, and the ability to visualise layers at will. The agents can be individually interrogated as can the cells which comprise the environment and this enables the numerical state of any agent or cell to be examined whenever this is required. Attributes can also be plotted and written to file although as yet, we have not invoked these functions. A feature of the software is that parallel processing is simulated in that all updating of agent behaviour is made simultaneously. The program can be stopped and started at any time and parameters can be changed on-the-fly. Although we have structured the walk routine quite formally as we illustrated in Figure 1, it would be possible to set up each of the five components of motion as independent modules, which would run separately, the parallel processing compiler taking care of the order in which these operations were effected. Although this is attractive and we will consider it in later applications, the current logic follows the sequential mode of operations shown in Figure 1.

The structure of the program is shown in Figure 2. It is built around the walk routines with local geometry and global attraction providing the key inputs. These layers provide data which control the walk routines, but they also provide the backcloth on which various inputs, outputs and the animation itself can be visualised. The open circles labelled **A**, **V**, **D**, and **P** represent program modules which are called to animate the walks (**A**), visualise the geometry and surfaces (**V**), display numeric data (**D**), and plot such data (**P**). As in many visual programs, the data concerning the built form, the gateways - entry and exit points to and from the system, and the locational attraction are encoded in layers which can be displayed visually. The number of agents N and the total time periods T (at which time each walker will be in the state of returning to their gateway) are input first, while the local geometry captured through the barrier set $\{B(x,y)\}$ and the locational attraction surface $\{\vartheta_{xy}\}$ are read in as layers. The number of walkers at each gateway whose coordinates are given as $\{X_e, Y_e\}$ are specified as $N(X_e, Y_e)$ where $N = \sum_e N(X_e, Y_e)$.

The key output from the model is the density of movement at each point xy in the system. The position of each walker k at each time period t is counted by the system as w_{kxyt} and the total density at each place xy up to time t is thus computed as

$$P_{xyt} = \sum_{k\tau} w_{kxy\tau}, \quad (\tau = 1, 2, \ldots, t) \qquad . \tag{20}$$

Total distance travelled in the system during the simulation is computed as

$$D(t) = \sum_{kxy\tau} r_k(x, y, \tau), \quad (\tau = 1, 2, \ldots, t) \tag{21}$$

and the average $\overline{D}(t)$ follows directly as

$$\overline{D}(t) = D(t) / N \quad . \tag{22}$$

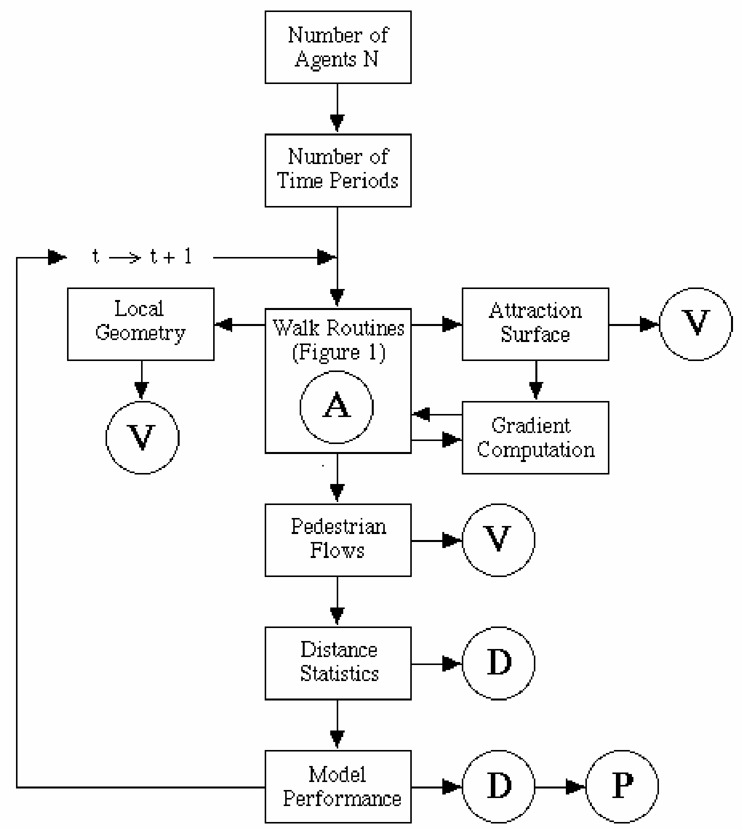


Figure 2: Key Elements in the Simulation

Another output from the model used mainly for diagnostic purposes are the actual paths taken by any walker *k*. These can be switched on and off at will but are only useful when small numbers of walkers are in the system. The level of resolution of the system and the inability to store paths as output data makes the analysis of path behaviour only relevant if individual walks are to be examined; this is important in examining what happens when walkers confront different geometric obstacles such as the narrowing of streets, doorways and so forth. Finally, as each walker in the system returns ultimately to their entrance point or gateway, we make the cavalier assumption that over the period of time *T* for the simulation, a constant proportion of these walkers shift from active to passive mode. At any time t, the probability of a walker being in the passive, returning mode is computed as

$$p_k(t) = t / T \tag{23}$$

which implies that when $t = T$, all walkers will be returning to their gateway. Of course as the number of walkers returning increases linearly with time and as their headings are fixed on their return gateway, then once walkers reach their gateway, they disappear from the system and thus *N(t)* falls with time. We consider this to be a reasonably realistic assumption which we invoke for these simulations although we could easily use a non-uniform probability function, based on the normal density for example, if this were felt to be more realistic.

Finally, we will collect together and comment on the various parameters which we have introduced before we use these to explore and calibrate the examples in later sections. Table 1 lists all the parameters we have defined and notes whether these are 'hard-wired' into our structure, that is, preset in the program or capable of being varied, that is, under the control of the user. There is enormous scope for varying the values of these parameters, the forms of the various functions used to fix headings, as well as the algorithms used to move walkers who get congested or need to circumnavigate obstacles. In developing the model structure to date, we have tested many different variants but in Table 1, apart from our ability to change the number of walkers N and time periods T, there are only three key parameters which we need vary: whether the gradient is on or off and if on, at what frequency τ_g is evaluated to fix a walker's heading; at what frequency the barrier function τ_b is evaluated; and whether the congestion function τ_c is off or on, and if it is on, at what level χ it is set. Nevertheless the parameter space that these three variables set up is big enough for a first test of the model. Our interest, however, is not exclusively on model performance but in the qualitative behaviour of walkers and this can only be assessed by watching the progress of the walks through the animations which illustrate how the simulation is proceeding.

Table 1: Preset and Variable Model Parameters

Parameter Type	Variable	Preset but can be varied if re-programmed
Number of Agents	*N (1, ..., 16K)*	
Number of Time Periods	*T (1, ..., 16K)*	$p_k(t) = t / T$
Gradient	[on $\tau_g =$ [period 1 to 99 [off	
Direction		$\tau_d =$ [on $\Delta\theta = \pm\{\pi / random(100)\}$
Barrier	$\tau_b =$ [period 1 to 99	$\Lambda = 0.2$
Congestion	[on $\tau_c =$ [[off level χ	Z_{xy} set as single points *x-y* in the runs reported here
Attraction	[on $\tau_a =$ [period 1 to 99 [off polynomial coefficients β_1, β_2	Ω_{xy} the attraction switch τ_a is set as off in the runs reported here

Experiment 1: Movement in an Idealised Shopping Mall

Our first example is characteristic of a planned shopping mall of the kind that developers are locating on the edges of urban areas, particularly in North America. The mall is square and symmetric; at each corner, there is a car park or gateway from which shoppers enter. These gateways are connected by outer pedestrian ways which in turn connect to the central cross routes which converge on the centre of the mall assumed to be the prime retail pitch. Shops and other facilities are located along the routeways. The level of spatial resolution is rather coarse based on a 51 x 51 square grid of pixels but this was chosen to optimise speed of running the model in its development stage. Just over half the mall is given over to pedestrian routeways: of the 2601 pixels comprising the mall, 1305 are routeways while 1296 are retail areas and car parks (gateways). This relatively simple geometry is shown in Figure 3(a) although the CA software like most, treats the screen as though it is mapped onto a torus; this means that walkers who walk off the screen in any direction appear on the opposite side and are never lost to the system. The screen wraps and in terms of this example, there is, strictly speaking, only one pedestrian gateway for the four corners of the screen join as one on the torus. We can remove this wrap-around feature quite easily but we have not yet done so.

The local geometry in Figure 3(a) is complemented by a global attraction surface $\{\vartheta_{xy}\}$ based on the relative linear accessibility to the central cell $\{x_0 y_0\}$ measured by crow-fly distance. Then

$$\vartheta_{xy} = \Phi - \sqrt{[(x - x_0)^2 + (y - y_0)^2]} \tag{24}$$

where Φ is a constant ensuring that the central location $\{x_0 y_0\}$ is a positive value and the most attractive position in the system. This is shown in Figure 3(b) where it is immediately clear that the gradient $\{\nabla \vartheta_{xy}\}$ is directly proportional to this unimodal surface. Figures 3(a) and 3(b) are the two key inputs to the model; the number of pedestrians for all these experiments has been set at 300 which is appropriate given the scale of the example and these are randomly allocated to the four gateways prior to the model being run. It is hard in a text to provide a real sense of the animation which occurs as the model runs but this is important as we shall see for the paths that walkers take are an important diagnostic to developing the model as well as important to the realism that we seek to generate. In Figures 3(c) and (d), we show two stages in a typical simulation; in Figure 3(c), we show a picture of walkers leaving their gateways just after a run of the model has begun (at time t = 20) while in Figure 3(d), we show walkers converging on the central focus which is the kind of steady state that is typical when the gradient exercises a stronger effect than the local geometry. Note also that in Figure 3(d), the background geometry has been switched off. The program is so configured that either the local geometry, the attraction surface, outputs such as the density of walking so far, and the paths of walkers can be switched on or off, or the screen cleared while the animation takes place.

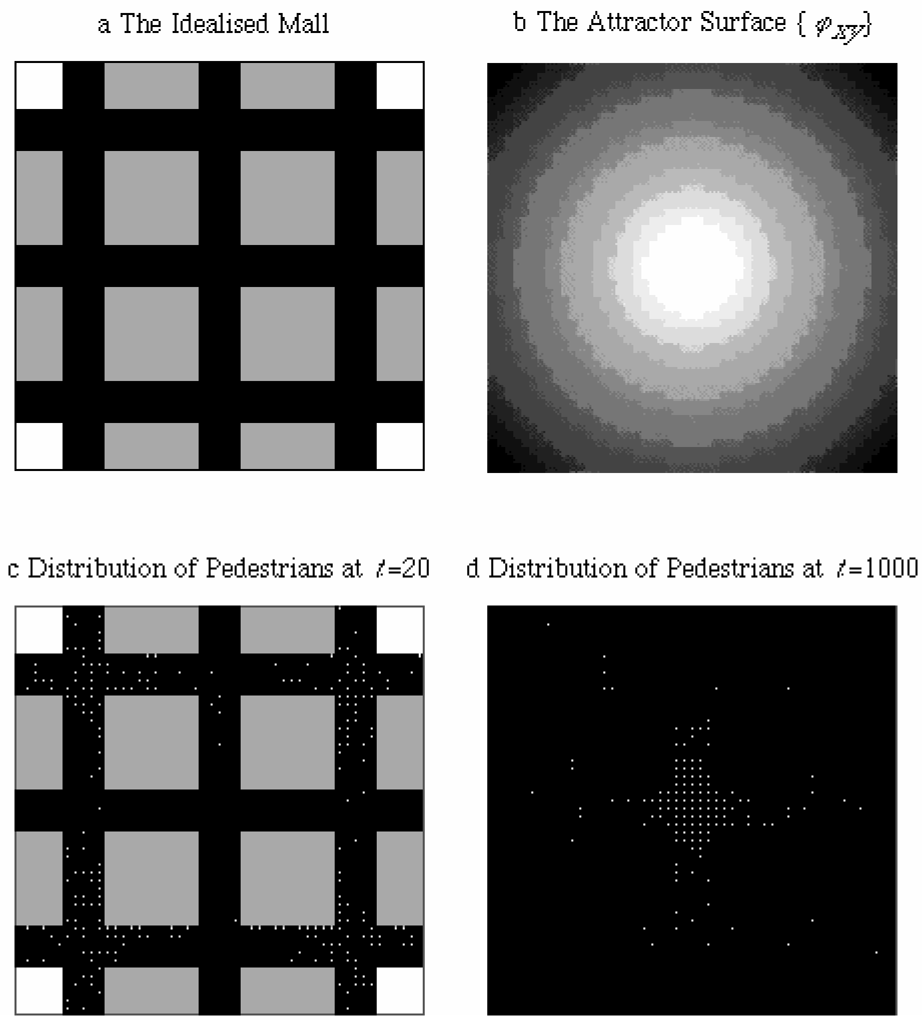
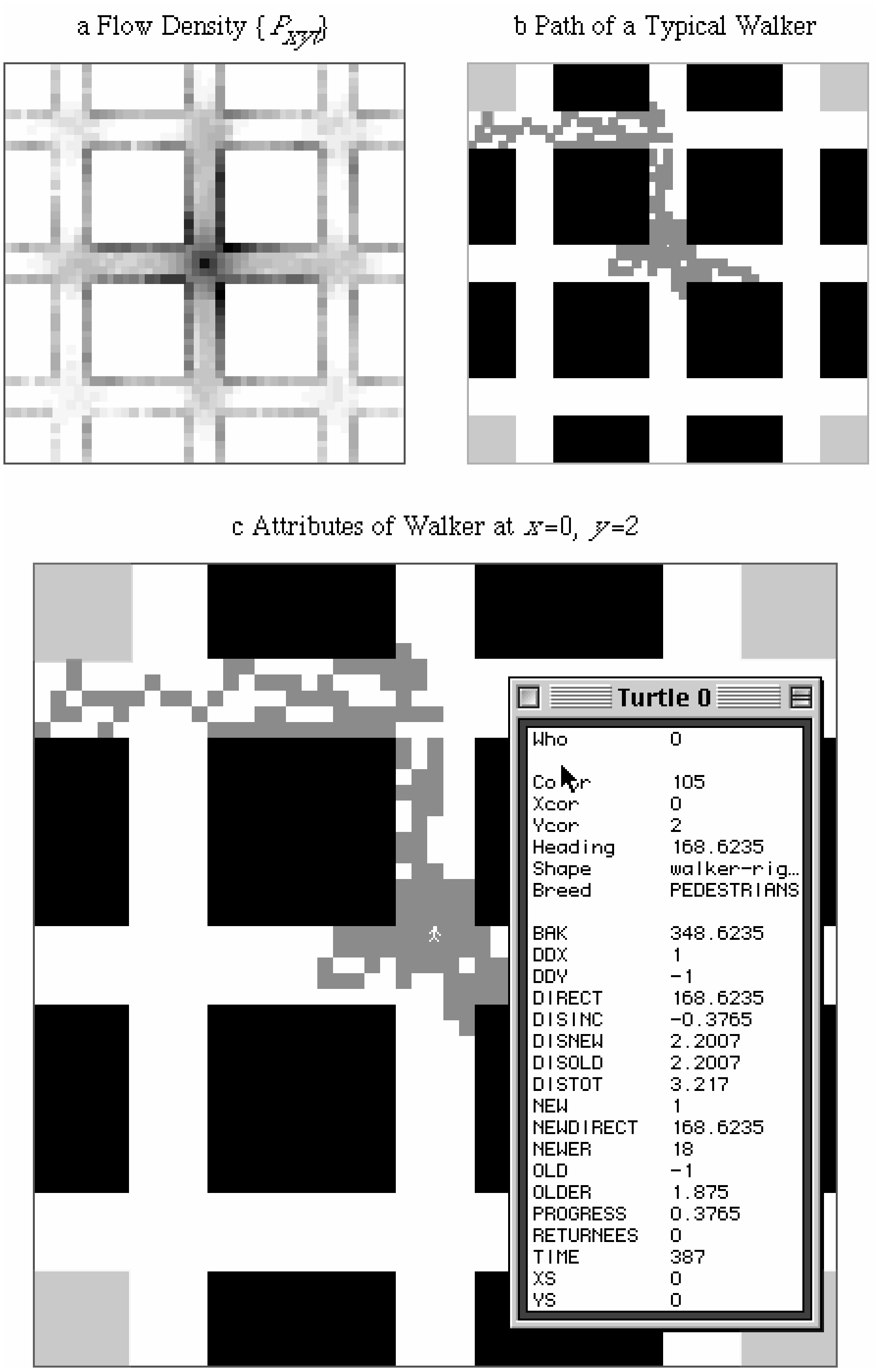


Figure 3: Local Geometry, Global Attraction and Walking in the Idealised Mall

We defined various outputs from the model in the previous section where we noted paths, densities and distance variables. In Figure 4(a), we show an example of the flow density surface based on $\{ P_{xyt} \}$ in equation (20) where the lighter tones show higher densities and the darker lower. This is in effect a key output from the model as it records the pattern of movement so far. In Figure 4(b), we show the path of a typical walker in the system where the parameters have been set to make the walker move towards the prime retail pitch which is clearly sensitive to local movement in terms of the local geometry. At this level of resolution, we can only show a single walker but once again, this is a useful diagnostic in showing the effect of various parameters on the behaviour of typical walkers. In Figures 4(c) and (d) we also show another feature of the software. Attributes associated with any agent or any location in the environment can be examined at any point in the simulation. In Figure 4(c), we show the key characteristics of the point $x = 0$, $y = 2$ while in 4(d) we show the attributes of the single agent (turtle 0) in the system. This can be done for any agent, any location, and at any time in the simulation. It is a useful way of checking that variables are within bounds.



d Attributes of Point x=0, y=2

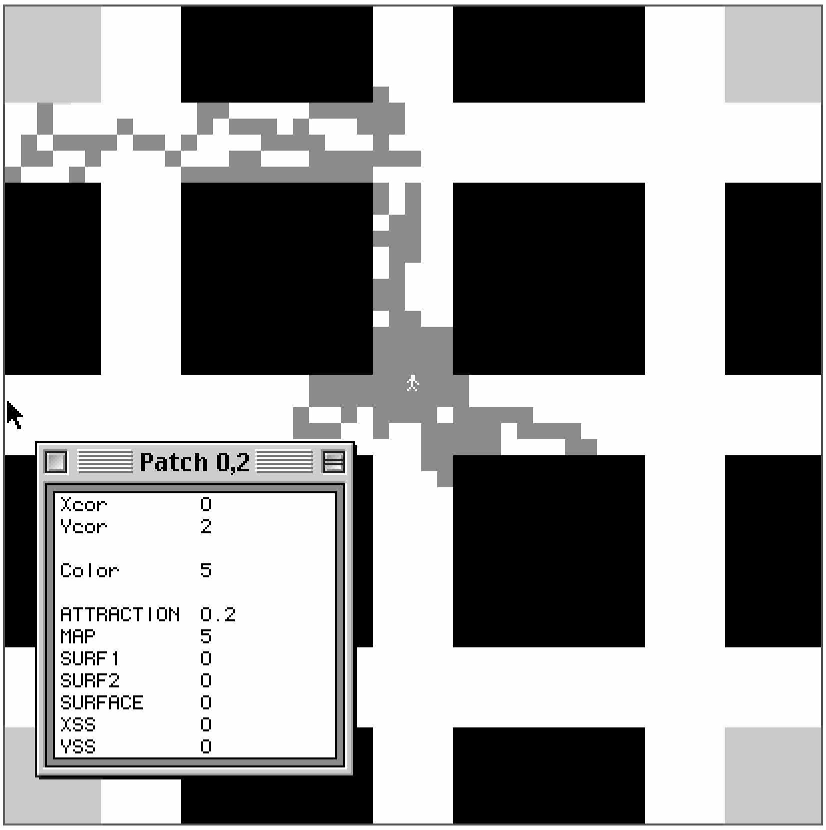


Figure 4: Typical Paths and Flow Densities

Our first tests of the model involve assessing the key feature of this framework - the extent to which local geometry and the global attraction interact in producing reasonable and realistic walking behaviour. To explore this, we vary the two parameters controlling these effects - the barrier and the gradient switches τ_b and τ_g, with the congestion parameter τ_c switched off. When we examine the behaviour of the model, we must always generate the steady state associated with each pair of parameters τ_b and τ_g and this means we must negate the returning walker effect and any constraints on time spent in the system. In each case, we therefore examine the patterns of movement for 300 walkers who have spent 1000 time periods in the system. This is enough time for the steady state associated with any set of parameter values to have emerged. The range of parameter values is shown in Figure 5 where the combination of values used in runs of the model are indicated by the bold dots. These values are the times when any local geometric obstacles or barriers are dealt with and when the gradient surface is evaluated. These two effects do not cancel each other out when applied at the same time period as the barrier effect only applies when no progress has been made in previous time intervals. In 1000 time periods, with the highest value for τ_b= 100, the barrier function is only invoked 10 times while with the largest value for τ_g= 250, the gradient is evaluated only 4 times; at these extremes, these values have hardly any effect on the simulation.

Figure 5 shows two sets of nested designs, the first involving the area bounded by $1 \le \tau_b \le 100$ and $1 \le \tau_g \le 250$, the second by $1 \le \tau_b \le 20$ and $1 \le \tau_g \le 20$. We have crudely explored the larger area first. In fact, it is instructive to start with the values ($\tau_b = 1$, $\tau_g = 1$), to increment τ_b to 100, then τ_g to 250, then to decrement τ_b back to 1, and then τ_g back to 1; this is a trace around the parameter space from the bottom left-hand corner of the grid in Figure 5 in clockwise direction. To an extent, we can guess what is likely to happen to the walkers as we do this. At the point ($\tau_b = 1$, $\tau_g = 1$), walkers are always perturbed as soon as they get stuck, and they continuously set their heading, assuming they are not stuck, in the direction of the global attraction. As expected, walkers leave their gateways, quickly moving into the outer routeways, and thence focusing on the central cross mall. There is, however, sufficient perturbation in this model to make walkers who are always converging on the central point of the mall, to move off this point. Thus although the centralising effect of these parameters is strong, there is some circling movement around the point of central attraction. As we increase the value of τ_b, obstacles posed by the local geometry become more and more significant. What happens is that walkers leave their gateways but then in their quest to move to the centre, cling to the walls. The barrier effect is not evaluated frequently enough to move walkers fast towards the centre. When τ_b = 100, walkers literally creep around the walls on their way to the centre but all reach the centre within 1000 time periods. In effect once the walkers reach the centre, they have no incentive to move away from this and the polarisation is extremely strong. This contains an interesting almost 'literal' demonstration of the idea of path dependence (Arthur, 1988) in that although the walkers always reach the centre of the mall in the steady state, the particular position of walkers prior to this state being reached is very sensitive to the paths they have already taken which are determined almost entirely by local randomness combining with local geometry.

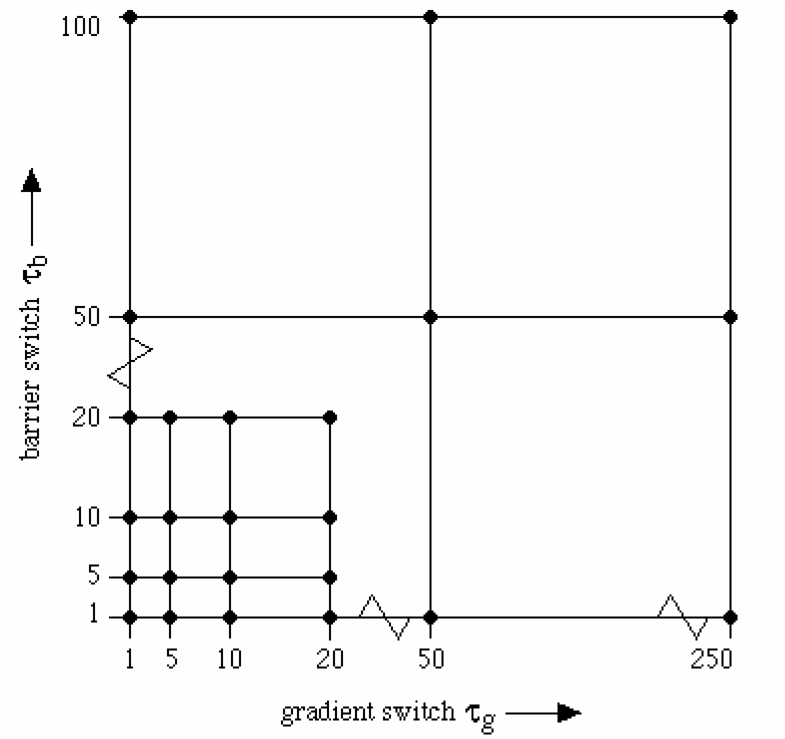


Figure 5: The Nested Experimental Design in Parameter Space

With the effect of local geometry effectively ignored, and as we loosen the effect of global attraction by increasing τ_g, then less numbers of walkers focus on the centre and many simply cling to the walls. When τ_g is completely relaxed and there is no effect of local geometry (τ_b= 100, τ_g= 250), walkers move randomly in the system although there is a noticeable effect here of walking along walls within a generally random pattern of movement. As the effect of local geometry is increased, this emphasis on walking along walls or edges decreases and a much more random pattern emerges as $\tau_b \to 1$. Finally as we reduce $\tau_g \to 1$, the randomness decreases as the central focus begins to reassert itself. What is interesting and useful is that this kind of variation can be achieved in one model run by adjusting the parameters and letting the steady state emerge before making further adjustments in similar fashion.

Quantitative and visual outputs from these runs support these conclusions. Overall the average density of occupance of each pixel does not change much as the parameters are varied. Walkers have to be located somewhere and with 300 walkers given 1305 pixels to walk in over 1000 time periods, each pixel would, on average, be visited around 230 times. In fact, because walkers are not counted if they remain stationary, the average density of pixels varies from around 205 where local geometry effects are discounted entirely to around 222 where movement is largely random.

However very significant changes are seen if the flow patterns $\{ P_{xy1000} \}$ are examined. The patterns over the range $1 \le \tau_b \le 100$ with $\tau_g = 1$ are very highly polarised around the point $x_0 y_0$. As the gradient effect is relaxed, then the effect of walking along edges and walls becomes much more significant with the patterns around $\tau_b = 100$ and $\tau_g = 50$ being dominated by the edges of the blocks defining the routeways. As $\tau_b \to 1$ with $\tau_g = 50$, then the pattern becomes much more random with the local geometry effect being almost non-existent and the centralising effect beginning to take over. We will show some of these patterns below when we have concluded this analysis. The last feature worth noting is that the average distance travelled by walkers in the system dramatically decreases as the effect of local geometry becomes more and more significant. When barriers to movement are continually assessed, then the average distance travelled by a walker is nearly four times that when barriers are rarely if ever assessed. Finally average distance increases a little when there is a mild centralisation effect, when $\tau_g \sim 50$. These statistics and those for the finer scale experimental design are presented in Table 2.

Table 2: Average Distance and Densities under Varying Parameter Regimes

	τ_g					
τ_b	*1*	*5*	*10*	*20*	*50*	*250*
100	**191**	**-**	**-**	**-**	**282**	**219**
	206	-	-	-	222	221
50	**201**	**-**	**-**	**-**	**284**	**228**
	206	-	-	-	223	223
20	**216**	**659**	**681**	**404**	**-**	**-**
	204	226	255	229	-	-
10	**281**	**700**	**625**	**442**	**-**	**-**
	221	231	230	229	-	-
5	**353**	**725**	**604**	**522**	**-**	**-**
	225	232	223	227	-	-
1	**786**	**825**	**806**	**804**	**789**	**785**
	223	221	222	224	220	222

The **bold** numbers are the average distance travelled $\overline{D}(1000)$;
The numbers below are the average flow density per pixel/location

The finer scale exploration based on the ranges $1 \le \tau_b \le 20$ and $1 \le \tau_g \le 20$ mirrors to a large extent the coarser scale effects. With the gradient evaluated at each time period, that is $\tau_g = 1$, the effect of increasing the impact of the local geometry (which is the same as increasing the difficulty of walkers circumnavigating local obstacles) by increasing τ_b, is to tighten the focus on the central point of attraction but make it more difficult for walkers to get there quickly. This means that walkers hit the edges of the routeways more frequently for there is less chance of their being bounced off. Walkers begin to crawl along the edges on their way to the centre. As these barriers are assessed less frequently, whenever they are assessed, it is easy to spot significant pulses of activity as walkers spin off only to correct their headings back to the centre immediately after. This pattern of pulsing with a degree of cycling around back onto the main heading, is maintained even as the effect of the gradient is relaxed, when τ_g increases.

When τ_g = 10, then there is still a strong focus on the central point of attraction but there is more dispersion around this point in the immediately adjacent pixels and there is even movement as far away as the outer routeways when the steady state has been reached. However, this is a little deceptive in that although there is quite a spread of walkers, most walkers are still focussed on the blocks of pixels within a few of units of distance from the central focus. When we relax τ_g to 20, there is much more randomness notwithstanding the fact that the central focus is still evident. In fact with these parameters set at $\tau_b = \tau_g = 20$, there is fairly random movement everywhere. Whenever the barrier is evaluated, there is wave of activity which is most evident in the central mall and the effect is almost to throw walkers out of the random pattern against the walls of the routeways and thence to bounce them back into the random pattern until the next major perturbation takes place 20 units of time later. The trace of these effects is best seen in the density flow maps. In Figure 6, we show four very different pictures of what the model can generate. First in Figure 6(a), we show a strong central focus where the flows are all concentrated on the central cross at the centre of attraction under conditions where barriers are evaluated frequently τ_b = 5, and the gradient is still strong τ_g = 10. This is perhaps as realistic a pattern as we might hope for in this kind of idealised geometry. At an extreme, when the gradient is evaluated every time period and when the barrier effect only operates every 100'th period (τ_g = 1, τ_b = 100), then the pattern is one in which there is rapid movement away from the gateways to the edge of the outer routeways and thence a slow crawl to the central focus which repeatedly gets reinforced in the steady state. This gives a somewhat stylistic density flow pattern reminiscent of a carpet design which we show in Figure 6(b). At another extreme, when we keep the effect of the local geometry very tight but relax the gradient (τ_g = 250, τ_b = 100), we get the pattern illustrated in Figure 6(c) which is quite random but with edge effects strongly emphasised. Finally in Figure 6(d), when we consider the barrier effect every time period $\tau_b = 1$ but relax the gradient effect to $\tau_g = 250$, we get as random a pattern as we are likely to get. Even then, there is a mild lowering of densities in the central routeways between the crossing points which have slightly higher densities.

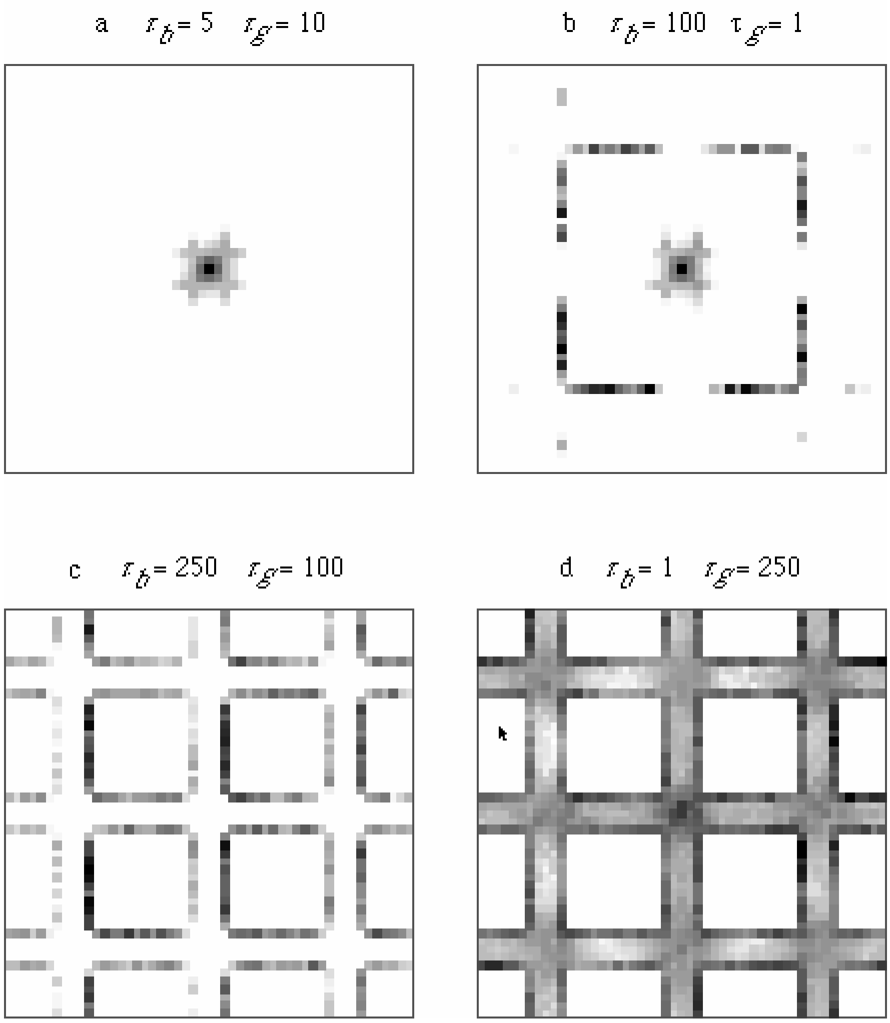


Figure 6: Variation in Flow Densities

The result of all these experiments is to show that the model is extremely sensitive to these parameter values which is most encouraging. It is quite likely that in realistic situations based on this kind of idealised mall, then the gradient switch τ_g should be between 10 and 50, with the barrier switch τ_b being evaluated much more frequently, at something like a frequency between 1 and 5. These are good portents for the further development of this approach to simulating local movement. It might be argued that if the model is designed to be sensitive to such parameters, then it is somewhat trivial to find that it is but those who know the problems of designing complex simulation models will be well aware that effects such as these cannot be assured in advance. The effect of local geometry, the level of resolution, the way time periods are structured and in this case the parallelism that the software imposes can all act together in an uncertain fashion. The experiments of this section however give us considerable confidence that the model will be useful for more realistic situations, and it is to these that we now turn.

Experiment 2: Walking Through a Medium-Sized Town Centre

In this paper, we will not develop any fully-fledged simulations of real systems. Our purpose is to develop the generic model and to explore its implications for spatial systems which imply very different kinds of local movement. Yet to do this, we must use examples which are realistic and accordingly our first foray into the real world is based on simulating walking behaviour in a medium-sized town centre. We have developed the model for the centre of Wolverhampton, a very well-defined town centre in the British Midlands which is loosely bounded by a complete ring road, and has a partly pedestrianised central shopping area focused on a covered mall. The urban area which this centre serves has around 244K population, and the total employment within the centre bounded by the ring road is 24659 of whom 4766 work in retailing. The retail turnover of the centre is around £229 million per annum which in UK terms suggests a lower than average turnover per retail employee, notwithstanding the fact that Wolverhampton has thriving entertainment and leisure facilities within its centre drawing population from a wide hinterland. The centre is basically circular, about 1 km in diameter and is largely non-residential with less than 30 people living there. Most movement is generated by persons visiting the centre for work, shopping or entertainment and thus the gateways which 'discharge' pedestrians into the centre from cars, buses and trains are especially important as the origins of local movement. The number of on-street parking spaces by comparison is very small.

We have excellent data for this town centre including a comprehensive set of surveys on pedestrian movement. However all we will be concerned with here is demonstrating the problems that emerge when the model is applied to the kinds of convoluted local geometries that characterise many town centres of which Wolverhampton is typical. We will in fact develop a fully-fledged simulation of pedestrian movement in Wolverhampton's town centre in a later paper. The Ordnance Survey 1:1250 digital streetline and building plot data has been used to produce a generalised local geometry which is consistent with developing this application at a scale with enough streets and pedestrian ways for detecting the significant pattern of local movement. This geometry is shown in Figure 7(a) where the 20 gateways from which all walkers emanate, are also identified. These gateways are largely car parks but the bus station and rail station gateways are included. To date, we have not taken account of on-street parking, nor have we addressed the problem of those who walk into the town centre across the ring road. These are easy enough to handle using various kinds of dummy gateway but at present, such additions are too refined for these applications.

The global attraction surface is particularly well-developed for this example. As part of a wider project involving the definition of town centre boundaries for statistical purposes (DETR, 1998), we have computed a composite retail attraction surface based on several layers of employment, turnover, floorspace, and rental value data which is available at around 100 meter resolution level. This data has been smoothed using the kernel density estimating function within the *Spatial Analyst* module of the GIS package *ArcView*. The surface for Wolverhampton town centre is shown as a contour map in Figure 7(b) as an overlay to the digital map data. This surface is fairly simple in that it is unimodal and has similar properties to that used in the idealised shopping mall example in the previous section. We have positioned the local geometry in such a way that it is centred at the point of maximum retail attraction in the centre (the Mander centre). At this stage however, we have not used the surface shown in Figure 7(b) but have computed our own circular surface based on the same centre but using the logic previously introduced and based on equation (24); we show this in Figure 7(c). At present, we consider that we should change as few parameters as possible as we make the model more realistic and thus we will not use the real surface until we complete the fully-fledged test that we are currently working on and which we will report in a later paper.

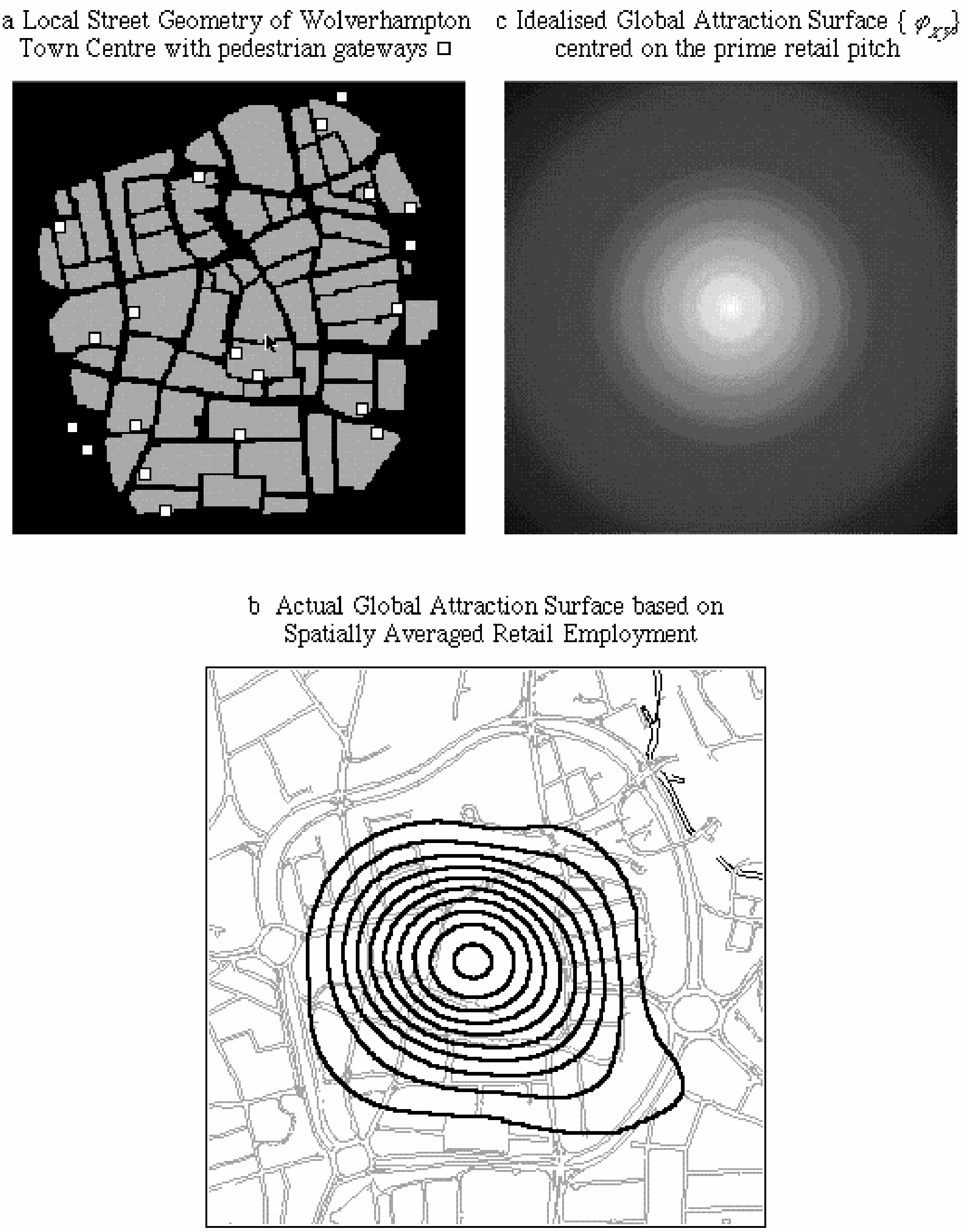


Figure 7: Wolverhampton Town Centre: Local Geometry and Retail Attraction

The critical issue that we will address in these experiments involves the extent to which the behaviour of the model developed for an idealised geometry is transferable to the more complex, realistic geometry of Wolverhampton. The practice of building models that blend local and global factors in various ways is not well developed and from our casual explorations of these models, we know that the effect of local geometry can be very problematic. Walkers can get stuck in the most unlikely places where local and global forces act to keep agents "in stasis" so to speak. We will thus begin to test this model by examining the paths traced out in the system by single walkers, so that we can quickly gauge the extent to which realistic behaviour can be simulated. In all our tests on Wolverhampton, we have used the same variant on the generic model that we applied to the idealised mall, that is we have not considered congestion, switching off the parameter τ_c. We begin by exploring what happens when we let single walkers wander through the system with the barrier and gradient parameters set at $\tau_b = 1$ and $\tau_g = 10$, those that gave us the most realistic walker behaviour in the idealised mall.

In Figure 8(a), we show the streets and other pathways within the town centre which act as the containers for the local movements emanating from the 20 gateways which are also shown. The resolution of the system is based on a 187 x 187 grid and of these 34969 pixels, some 6390 or 18 percent are taken up as streets and thus available for walking. This ratio is far smaller than the previous idealised mall example where the ratio was close to 50 percent. Thus we might expect the local geometry to exercise a much greater constraining effect on walk behaviour in Wolverhampton, making it more difficult to move around the entire centre. We let each walker roam the system for 10000 time periods. If a walker made a positive increment of distance each step, the distance travelled would be 10000 units and thus when we compare this with the actual distance travelled, this gives us an idea of how much time the walker is stationary. In the idealised mall examples, although this ratio varied as the parameters varied, then for the optimal parameters set at $\tau_b = 1$ and $\tau_g = 10$, this was around 80 percent (see Table 2). In short, in the idealised mall example, most of time was spent walking, revealing that the geometry had little effect on keeping walkers still.

In Figure 8(b), using these parameter values which were optimal for the idealised mall, we show the path of a typical walker who, in this case leaves, a gateway in the north of the centre and begins to walk. Every time period, progress is assessed and if stuck, the walker is bounced in the manner described above. Every 10'th time period, the walker's heading is set to the gradient. This we might imagine would move the walker fairly rapidly to the most attractive point in the town centre which is the physical centre of the map. Not so. The walker takes a circuitous walk around the centre, entering the prime retail pitch at one point but then leaving it. The path traced after 10000 time periods is shown in Figure 8(b) and the average distance travelled $\overline{D}(10000)$ - the total distance too for one walker - is 3292. This means that

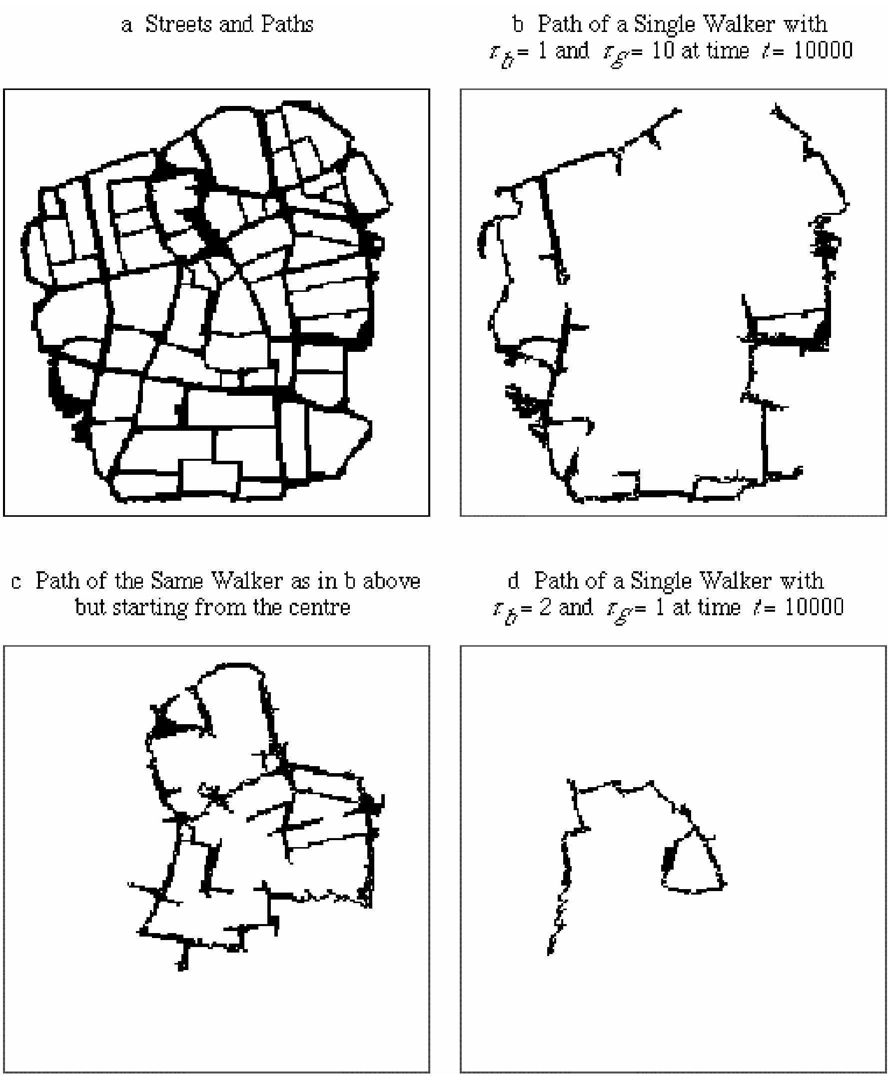


Figure 8: Exploring Paths of Single Walkers

nearly 70 percent of the time the walker is stationary, which is simply another illustration of the highly constraining effect of the local geometry. It also means that the walker has walked the length and breadth of the system nearly 18 times in comparison with the 54 times that could have been achieved had the walker not been obstructed. We have experimented with placing the walker at different points in the system and the same kind of circuitous paths result. To demonstrate this finally, we have placed the walker, very close to the central point of attraction and even in this position, the local geometry combined with the effect of frequent bouncing pushes the walker away from the most attractive point, thus generating in this case almost a spiral walk around the centre. We might consider this kind of behaviour to be one in which the walker is forever trying to get back to the centre but the local geometry continually frustrates this quest. This walk is shown in Figure 8(c) and these experiments suggest to us that the gradient is not being reinforced strongly enough. Reluctantly because the model takes quite while to run in this mode, we must explore further variations in the parameter values which hopefully might generate somewhat better behaved walks, enabling walkers to travel further and faster in search of the most attractive locations.

What we have done is to start the walker at the central point of attraction with parameters $\tau_b = 1$ and $\tau_g = 10$, systematically increasing the frequency of the gradient switch τ_g (i.e. decreasing its value to 1) and examining the walk. We do not reduce the circuity of the walk much this way. It appears that the barrier parameter acts to keep the walker in stasis much of the time in that if the walker is in a street whose direction is against the attraction gradient, frequent bouncing which will occur in this context will simply stop the walker from walking in the 'wrong' direction to the point where they can 'correct' their direction. Thus somewhat ironically, frequent evaluation of the barrier factor stops rather than enhances movement. However when we increase τ_b to 2, this problem begins to disappear. We therefore suggest that a 'better' set of parameter values which generate more realistic local movement are $\tau_b = 2$ and $\tau_g = 1$. Using these values, we show the path generated by a walker (placed in the system in the south west corner) in Figure 8(d). It takes the walker some 200 time periods to reach the central point of attraction, but from then on, there is not much further change in general location although considerable movement occurs in and around the central point. In fact, 48 percent of the time [$\overline{D}(10000) = 4852$], the walker is moving and this is much more akin to the kinds of levels that were observed for realistic movement in the idealised mall. It is also clear that when $\tau_g > \tau_b$, walkers find it increasingly hard to maintain their focus on the point of global attraction and thus it appears that this application is highly sensitive, too sensitive in fact, to the parameter values. Future experiments with the model will explore this further.

Using these parameter values, we can now simulate the full system. We randomly allocated 2500 walkers to the 20 gateways. By time period 1000, it becomes clear that walkers are beginning to converge on the central point but it takes many iterations for most walkers to reach the locations in the neighbourhood of this centre of global attraction. By t = 10000, 95 percent of all walkers are in this position; at the actual centre, there are 1096 walkers. To the east of this pixel, there are barriers for this is the side of a street and the pixel below the centre is also barred. Immediately to the west there are 268 walkers while north and south of this there are 51 and 52 walkers respectively. These five pixels account for half the walkers in the system with most of the others in neighbouring locations. In Figures 9(a) and (b), we show the animation at t = 1000 and t = 10000 respectively while in Figure 9(c) we provide a trace of all the paths made in the system which shows that although most walkers end up at the centre, most of the available walking space in the system is traversed at one time or another during this simulation. Finally in Figure 9(d), we show the final cumulative density at t = 500 with the flows represented by their logarithm. In this example, we must clearly invoke the congestion threshold for it is quite unreasonable to let over 1000 walkers occupy one location, one pixel which is some 5 meters x 5 meters. In the fully-fledged application to Wolverhampton which is under construction, these kinds of details will obviously be resolved.

There are two analogies which are helpful in thinking about the model. First, if the available space is considered to be a liquid where the current is proportional to gradient of global attraction, then the path of a walker is like placing a drop of die in the stream and watching it diffuse through the system. In this sense, we can examine the sensitivity of the system to changes in the behavioural rules. In the same way, if we think of the algorithm for bouncing walkers at regular frequencies if they get stuck in the local geometry as a method for shaking the system locally, then you can see quite clearly that if we shake the system differentially at different times and in different places, then the global repercussions are hard to disentangle. For example, imagine a walker gets stuck in a narrow street. If we keep shaking the walker every time period, then we never give the walker a chance to move in any direction for long enough to move away from the local problem and re-evaluate its position. If we shook the walker less, then we have more chance of it finding a good direction. In the same way if we fail to shake the walker enough, then it may never find the best position for it never gets enough opportunities to explore its local environment. The problem is that under a given set of parameter values, dependent upon where the walker is, both situations can exist. This makes it impossible to find a single set of parameter values which resolve both difficulties. We clearly need to explore this problem further. We have not yet considered walks which start and then end at the same gateway for it is possible that the time spent in the system makes an important difference to the flow patterns that are generated. This we will consider in future papers.

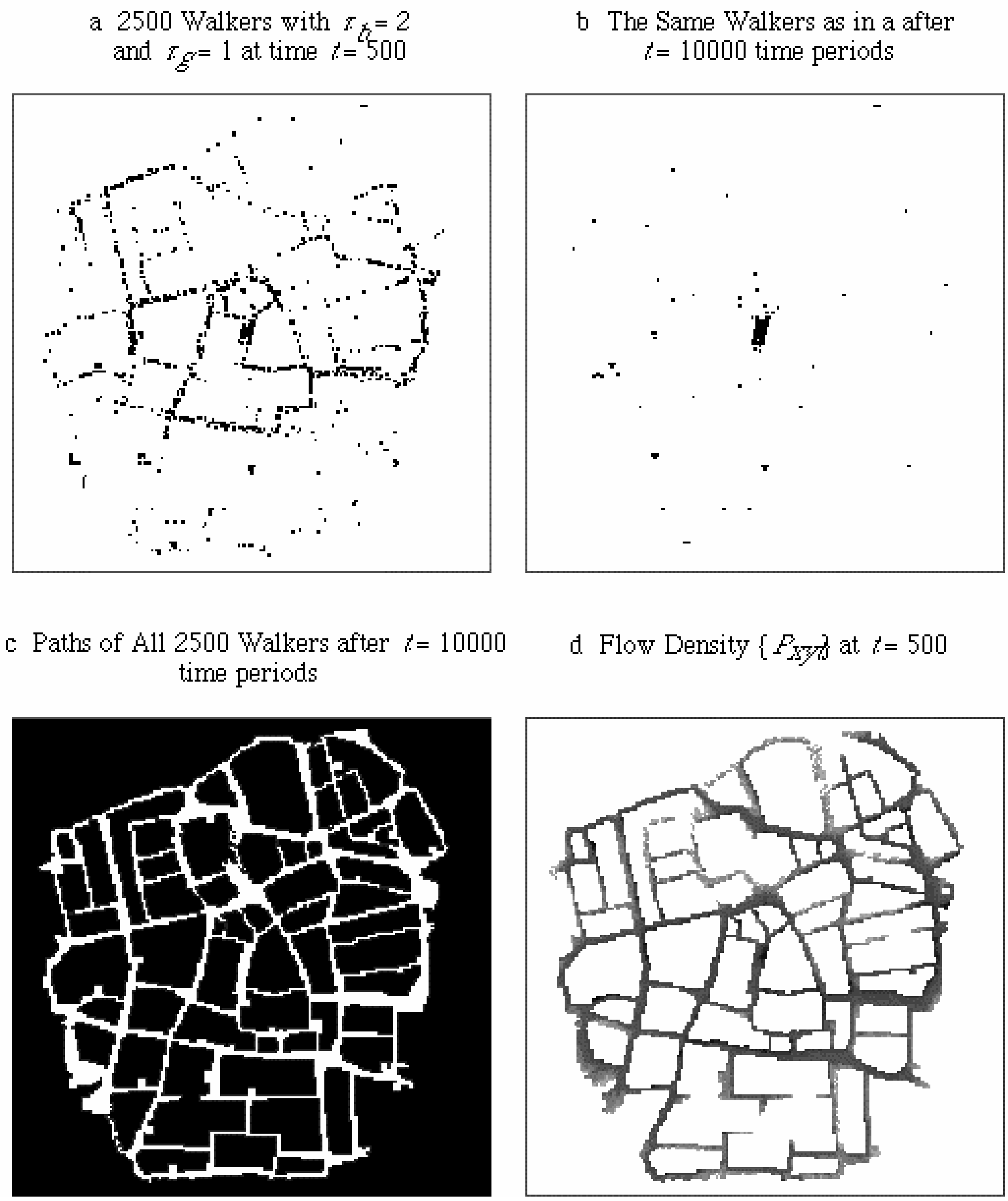


Figure 9: Paths and Flow Densities for 2500 Walkers in Wolverhampton

Experiment 3: Viewing Exhibits in a Gallery Complex

Our last application is very different from the previous two, in that it is essentially a closed building complex whose linear features are connectors between a series of 41 rooms which act as the foci of movement. This is the Tate Gallery which is an internationally prominent art museum in Central London. We have excellent data concerning pedestrian movement, originally collected by the Unit for Architectural Studies in August, 1995 (UAS, 1996) and this enables us to model the main part of the Gallery, excluding the smaller Clore annex. Without the Clore, there is one gateway, the main entrance through which 97 percent of visitors enter the Gallery. The 41 rooms had an average occupancy of some 13.4 persons per hour during the 12 hours over which visitors were observed. The geometry of the main gallery is shown in Figure 10(a) and the frequency of room visits in Figure 10(b) where the actual rates of occupancy per hour are indicated. From the survey, we deduce that during any hour there are around 550 visitors in the gallery and this is consistent with viewing rates at the peak period of the year when the survey was carried out. 93 pedestrians were also tracked individually in this survey although we have not made use of this data as yet.

It is important to establish the geometric dimensions of the problem to get some idea of the potential densities and distances which are likely to be encountered in these experiments. The modelled system is based on a 187 x 187 grid of pixels giving some 34969 cells in all of which 17831 constitute the gallery itself. Of this subset, some 8239 pixels are occupied by the geometric construction of the complex, mainly walls and associated abutments but also rooms and other areas closed to the public. The remaining space within which visitors might walk thus comprises 9592 pixels which is some 54 percent of the entire space, a little higher than the idealised mall in experiment 1 which was 50 percent but much higher than Wolverhampton town centre in experiment 2 which was only 18 percent. It is obvious that a typical visitor would not walk on most of the space. Each pixel is about 1 meter x 1 meter in size and if it were assumed that a walker were to 'hop, step and jump' across the gallery attempting to land on each square and that each adjacent move took 2 seconds, then it would take over 5 hours to accomplish. In fact, this notion of walking in the gallery is not very useful; visitors pause, sit, reflect, browse, rather than walk *per se*. The notion of walking density which in the previous examples was computed for each pixel is not relevant here but rather density per room is the measure that should be examined. The other feature which did arise in the previous two experiments but was not dealt with there involves congestion. With the size of pixel used, no more than one pixel can be occupied at any one time and in fact, for comfortable viewing which embraces the idea of a personal space around each person into which another would not trespass, the density should be at least one person for every 4 pixels. In the very first runs, we have not invoked this congestion threshold but in all subsequent runs, we will set the size of the space Z_{xy} as a single pixel and the congestion threshold as $\chi = 1$.

In Figure 10(a), all 550 visitors enter from the gateway which is located at the centre of the building at the bottom of the plan which is clearly marked. As in all our previous experiments, we will launch all 550 visitors at once for our purpose at this stage is to examine where a body of walkers end up in the steady state. Most of our experiments have not invoked the returnee stage of the walk for it is important to let the walkers flood into the system and then to track where they end up. We have not yet examined what occurs when we launch these walkers randomly - teleporting them into various rooms say - and letting them begin their walks, but in later experiments this will be necessary because undoubtedly there is considerable path dependence in terms of the ultimate steady states which result when walkers are launched from only one gateway. A comment must be made on the observed patterns of room occupancy which we show in Figure 10(b). There is a clear bias to more frequent movements within the left-hand side of the gallery and the work which has been developed by UAS (1996) suggests that this might be due to the fact that this side of the gallery is more intelligible, more connected to the main focus than the right-hand side, However we must be careful. If we first normalise the observed occupancies by area of each room, the asymmetry is less marked. If we then consider the number of rooms on each side of the gallery, then it is clear that it takes longer to penetrate the complex on the right-hand side as these rooms are smaller, there are more of them, and they are more remote from the main axis of the gallery, hence they are probably harder to get to. Then there is the question of the left-handedness of the signage and various customs associated with moving in the gallery. Symmetry might be broken by visitors having to enter on the left-hand side, to avoid others by using the British custom of walking on the left.

In developing the model, we have begun with the same form as that developed in the previous section for Wolverhampton. In principle, what we require is an attraction surface $\{\vartheta_{xy}\}$ which reflects the actual attraction of each room to visitors, likely to be based on the quality of the pictures and other exhibits. However, when the survey was taken, this information was not explicitly recorded and thus we are left with having to use the observed occupancy rates as attractions, in the same way for example retail modellers often have to use some measure of existing patronage in models which are designed to predict this same patronage. Defining the occupancy P of each room J as

$$P_J = \sum_{xy \in J} P_{xy} \qquad (25)$$

where P_{xy} is the hourly average number of walkers that frequent pixel *xy*, then we have computed the attraction surface $\{\vartheta_{xy}\}$ as a spatially averaged function of P_J where the averaging assigns the room value for *J* back to each cell $xy \in J$. $\{\vartheta_{xy}\}$ is computed from

$$\vartheta_{xy} = \text{spatial average } \{P_J | xy \in J\} \quad . \tag{26}$$

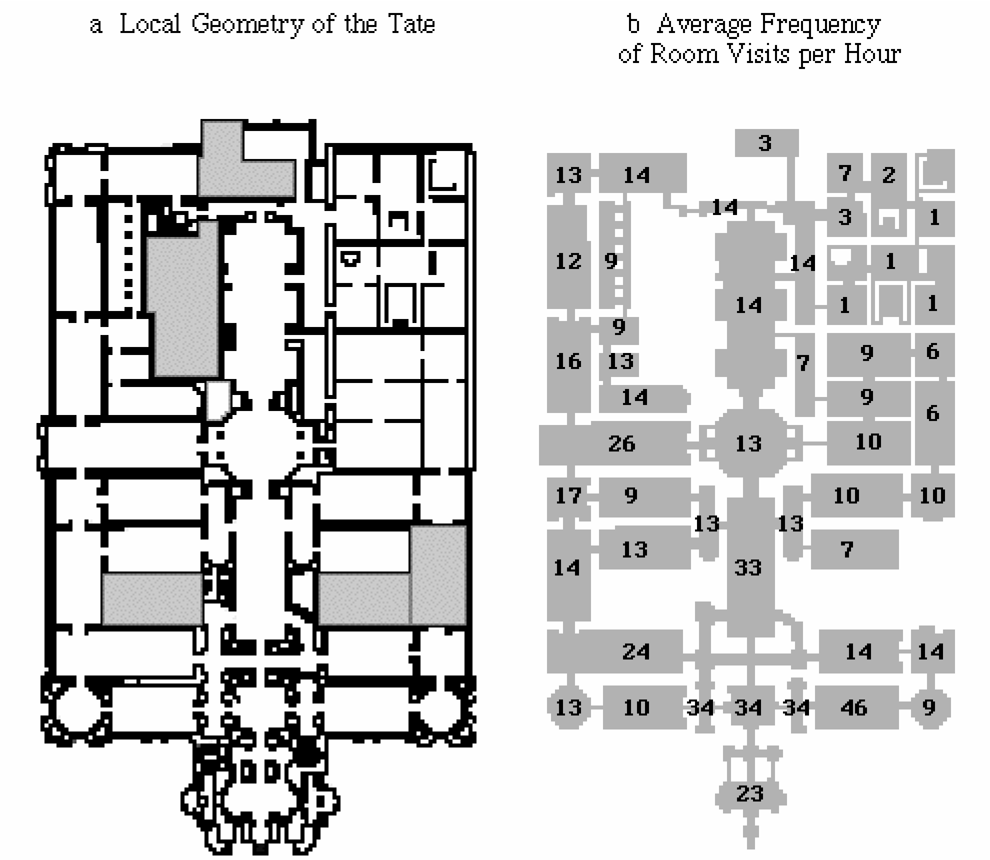


Figure 10: Room Geometry and Occupancy in the Tate Gallery, Millbank, London

This surface is shown in Figure 11. It clearly reflects the recorded pattern of room occupancy in Figure 10(b) with this pattern having been put through a filter based on diffusing 0.75 of the level of P_J to adjacent cells each of 8 times, thus establishing a sharp gradient between rooms. With the congestion factor τ_c switched off, we have run the model until a steady state emerges using the best parameter values of τ_b and τ_g for the idealised mall ($\tau_b = 1$ and $\tau_g = 10$) and town centre examples ($\tau_b = 2$ and $\tau_g = 1$) respectively. What these effectively show is that the model in its current form is quite unsuitable for simulating walking in a gallery complex such as this. The basic problem is twofold. First, the walkers take many time periods to move out of the gateway area into the rest of the complex. This is because, there is no incentive other than through random movement to move anywhere other than the locally optimal attraction - the room they are within - and although if the model is run long enough a gradual diffusion takes place, it is clear that the absence of any function to spread walkers around the mall is a major limitation. Second, there are problems over local obstacles. Walkers easily get trapped and continual bouncing simple moves them around within a local area. This is even more problematic when the returnee function is switched on. When a walker decides it is time to return to the origin - the single gateway - the local geometry is so convoluted, that walkers make little progress in that they continually bounce around, searching blindly for narrow entrances and exits to rooms that might speed their way towards the origin. Even if they do manage to find such exits, these are often in the wrong direction in terms of the gateway and this then frustrates such progress instead of aiding it. To address this issue, we need much more structured paths through the gallery for returnees. This needs to be incorporated into the model as well.

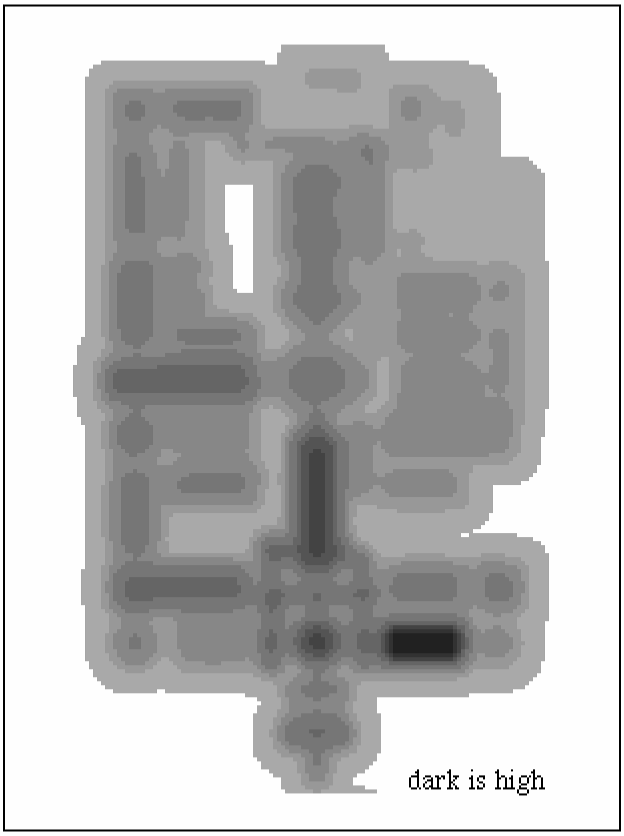


Figure 11: The Local (Room) Attraction Surface $\{ \vartheta_{xy} \}$

To extend the model to meet these problems, we must first consider breaking the attraction surface into two components, one dealing with room attraction as we have specified it already through equations (25) and (26), the other with a more global basis for attraction. In short we must partition the function $\tau_g f_g$ into

$$\tau_g f_g = \tau_{local} f_{local} + \tau_{global} f_{global} \qquad . \tag{27}$$

Now we set $\tau_{local} f_{local}$ as illustrated in Figure 11 but we define a new function for $\tau_{global} f_{global}$ based on a surface γ_{xy} which enables walkers to be drawn to the extremities of the building once they enter. As the gateway is at the bottom centre of the building, we define three points of attraction at the mid-west, mid-east and mid-north points of the rectangular complex, measuring a composite accessibility to these three points from the gateway defined as $x_g y_g$. The new surface is

$$\begin{aligned}\gamma_{xy} = \Gamma &- \sqrt{[(x_g - x_0)^2 + (y_g - y_{86})^2} - \sqrt{[(x_g - x_{-86})^2 + (y_g - y_0)^2} \\ &- \sqrt{[(x_g - x_{86})^2 + (y_g - y_0)^2}\end{aligned} \tag{28}$$

where Γ is a constant which ensures that all values of γ_{xy} are positive, and the subscripts xy give the grid reference points of the system, the central screen pixel being $x_0 y_0$. This surface is shown in Figure 12(a). Now to ensure that walkers return to the gateway from any point in the system, we note the gallery can be represented as a graph with the gateway as the root node. It is easy to construct a maximal spanning tree to all the rooms, each room being given nodes, usually at entrances and doorways which connect it directly to its neighbouring rooms. The maximal spanning tree from such a graph of connections between these nodes gives direct connections between rooms which provide the shortest ways of returning to the gateway. This graph is shown in Figure 12(b). When a walker is put into return mode, then the walker heading is always taken with respect to the node next nearest the root node, and the walker heads in this direction. Once it reaches this node, the next nearest node is taken and in this way the walker gets back to the gateway in the quickest way. The way this is handled is obvious from Figure 12(b).

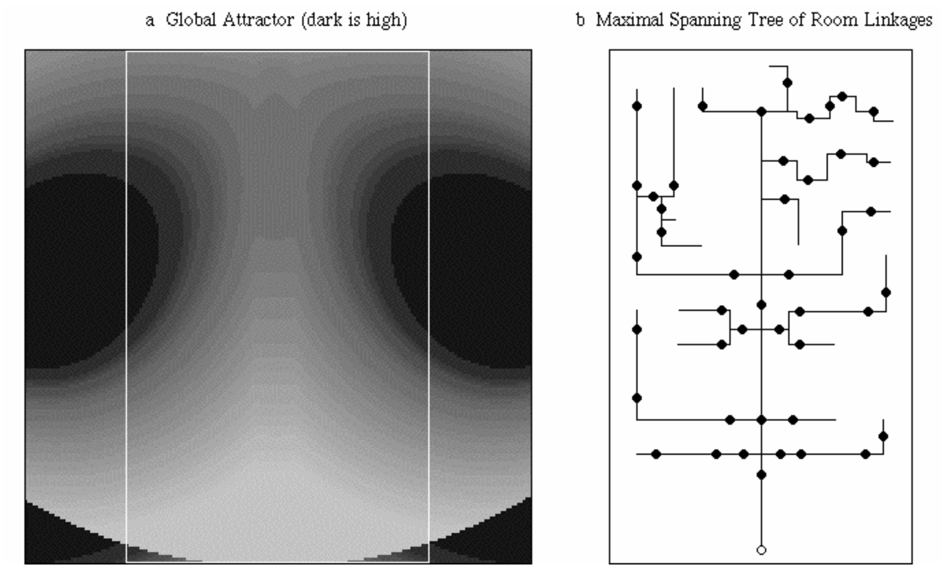


Figure 12: The Global (Orientation) Attraction Surface {γ_{xy}} and the Depth Tree

Our first set of experiments involve calibrating the key model parameters τ_b, τ_{local}, and τ_{global} with the congestion factor $\tau_c = 1$ and its level set at $\chi = 1$. Note that there is no attraction force, that is $f_a = 0$ and that the forward movement parameter is evaluated at every time period, that is $\tau_d = 1$. With 550 walkers launched simultaneously, each simulation is run for 10000 time periods by which time we consider the steady state for the frequency of room visits has been reached. At the end of each run, apart from the various visual outputs such as the distribution walkers, flow densities, and walker paths, we have computed two statistics, namely the average distance travelled $\overline{D}(10000)$ defined above in equation (22), and the predicted occupancy P'_J for each room *J*. This is defined as

$$P'_J = \sum_{xy \in J} \overline{P}_{xy10000} \tag{29}$$

where $\overline{P}_{xy10000}$ is the total flow density at point *xy* less the number of walkers at *xy* who move there to avoid obstacles. Note the difference between $\overline{P}_{xy10000}$ and $P_{xy10000}$ in equation (20) is accounted for by those walkers who visit a location purely to avoid obstacles in their path rather than to make forward progress. $\overline{D}(t)$ is also computed in such a way that movements back and forth around the same point to avoid obstacles cancel one another out and this variable thus measures direct progress.

The distribution of walkers is extremely sensitive to the variation in the values of the parameters τ_b, τ_{local}, and τ_{global} which we henceforth will refer to as the barrier, local (room) gradient attractor, and global (orientation) attractor. Simple exploration of the model reveals certain key effects of these parameters which we can easily anticipate. *Ceteris paribus*, when τ_b is evaluated every time period, that is when $\tau_b = 1$, walkers are continuously subjected to being bounced around to avoid obstacles where and when these occur. As this switch is applied less frequently (as τ_b increases in value), it becomes ever more difficult for walkers to move away from barriers. They begin to cluster and cling to walls, the pattern that is then etched out beginning to define the edges of the rooms themselves. In contrast when $\tau_{local} = 1$, the local attractor forces walkers to orient themselves to the central points of rooms which reflect the highest points in the local surface shown in Figure 11. As τ_{local} increases and the local gradient is evaluated less frequently, much more random movement takes place. In the case where $\tau_{global} = 1$, walkers move rapidly to the edges of the gallery on the west (left-hand) and east (right-hand) walls as well as to the saddle point of the global attractor surface which is located at the top of the axis hall of the gallery (see Figures (10(a) and 12(a)). As τ_{global} increases, the global attractor exercises less and less effect and eventually the walks dissolve into random motion.

Although we cannot demonstrate this here, one very interesting feature of the model is its ability to provide possibilities for exploration of the sensitivity of parameter values on-the-fly. If the model is run with $\tau_b = 1$, $\tau_{local} = 100$, and $\tau_{global} = 100$, then a fairly random pattern results, in fact perhaps the most random pattern that can be generated by the model where randomness through avoiding obstacles is the only force operating. With a distribution of walkers in the steady state posed by this combination of parameter values, we can alter each parameter in turn to illustrate the effects we have described above. Decreasing the local gradient parameter $\tau_{local} \to 0$ leads to a concentration of walkers on the central point in each space although there is still rapid circling around such points due to the fact that the barrier function is still effective. When $\tau_{global} \to 0$, the global attractor asserts itself and walkers will move to the extreme points of attraction; as they are randomly distributed within the various rooms, they will drift to the edge of each room trying continually to escape north, east, or west. When $\tau_b \to 100$, randomness associated with the model decreases; if walkers are clustering around the local attractors, these points become focal while if walkers are escaping to the nether regions of global attraction, then they become strongly clustered along the room edges. Further concentration of these patterns occurs when the congestion switch is switched off, thus negating the last randomising effect in the model. This ability to explore the model on-the-fly is invaluable in that it enables users to invoke evolutionary strategies for calibrating the model, which are akin to evolutionary strategies for 'learning' about the model. In fact, this has proved necessary here for the search space is so large that exploring its limits and then homing in on the best areas by trial and error has been the only feasible method for calibrating the model to date.

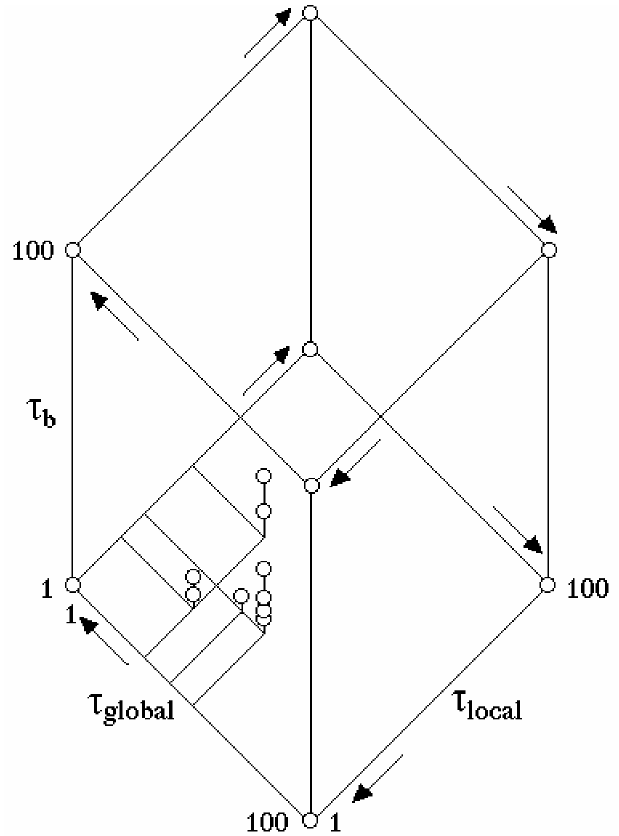


Figure 13: The Parameter Space Defined by the Barrier, Local and Global Attractors

We will however begin more conventionally by setting the limits to the parameter space as $\tau_b \leq 1$, $1 \leq \tau_{local} \leq 100$, and $1 \leq \tau_{global} \leq 100$,. The experimental design we have used is shown in Figure 13 where it is clear that having evaluated the model at the limits of the parameter space, we have homed in on a finer area of the space wherein we consider the parameter values giving best model fits must lie. As in the case of the idealised mall model, we can begin by tracing around the limits of the parameter space for each horizontal plane. Starting with $\tau_b = 1$, $\tau_{local} = 1$, and $\tau_{global} = 1$, we then increase τ_{local} to 100, then τ_{global} to 100, and thence τ_{local} back to 1. This traces around the perimeter of the space as shown by the arrows in the clockwise direction in Figure 13. When $\tau_b = 1$,

$\tau_{local} = 1$, and $\tau_{global} = 1$, the barrier, local and global attractors affect movement in every time period. The pattern of walkers that results in the steady state is fairly clustered on the central points of each room but the distribution is not untenable and there is a very slight asymmetric effect which emphasises the left-hand side of the gallery (as observed in the UAS study). As we reduce the local attractor effect, then the global attractor becomes important and walkers are shifted to the west and to the north east of the gallery. In fact combined with the effects of local geometry, a very strange pattern emerges with all the walkers either bouncing around in the rooms to the left of the gateway, or in the far north east corner, emphasising the edges of the rooms in those vicinities. Keeping the local attractor at minimal effect and then reducing the impact of the global attractor, we reach a situation as near as random as we might envisage with $\tau_b = 1$, $\tau_{local} = 100$, and $\tau_{global} = 100$. In this state, the density of walkers is very uniform but with a slight fall off in density with increasing distance into the gallery from the gateway. The only effects are of local geometry with walkers simply responding randomly to obstacles in rooms and entrances to various spaces. Finally as we increase the local gradient effect back to $\tau_{local} = 1$, then a pattern not dissimilar from the starting point of this exploration emerges but with more focus on the gateway and with the global attractor entirely discounted.

The upper circuit around the parameter space is navigated in the same fashion by increasing the barrier switch to $\tau_b = 100$ and moving around in clockwise direction as is also illustrated by the upper set of arrows in Figure 13. With $\tau_b = 100$, $\tau_{local} = 1$, and $\tau_{global} = 1$, the only randomness that occurs is due to the congestion switch which is on and which moves walkers when more than 1 occupy a single pixel. Walkers find it hard to escape the gateway area and there is considerable clustering there. As the local gradient is relaxed, the global attractor becomes predominant and walkers all move up to the saddle point area of the global surface. Because the congestion effect is still operative, walkers begin to spill out to the right of this area, and fall through an entrance into an adjacent room and then find it hard to gravitate back to the saddle point focus. This means that a kind of sandpile builds up there; this does not occur if the congestion is switched off and then all the walkers cluster around the top of the gallery in the dead centre at the point of balance between the twin east-west maximum attraction points of the global surface (see Figure 12(a)). When we reduce the effect of this global attraction to the point where $\tau_b = 100$, $\tau_{local} = 100$, and $\tau_{global} = 100$, we get a fairly random distribution but with the walkers clinging to walls because there is no mechanism to bounce them away from such obstacles. What emerges is a random spread but with the walls etched out within this. Finally when we increase the local gradient effect back to $\tau_{local} = 1$, walkers concentrate even more in the vicinity of the gateway for there is no process to bounce them out and the only effective heading is towards the points of maximal attraction in the rooms around the gateway. There is a cornucopia of different patterns resulting from this model, and in Figure 14, we illustrate four of these which show the very different distributions that might result from the extreme points which bound this parameter space.

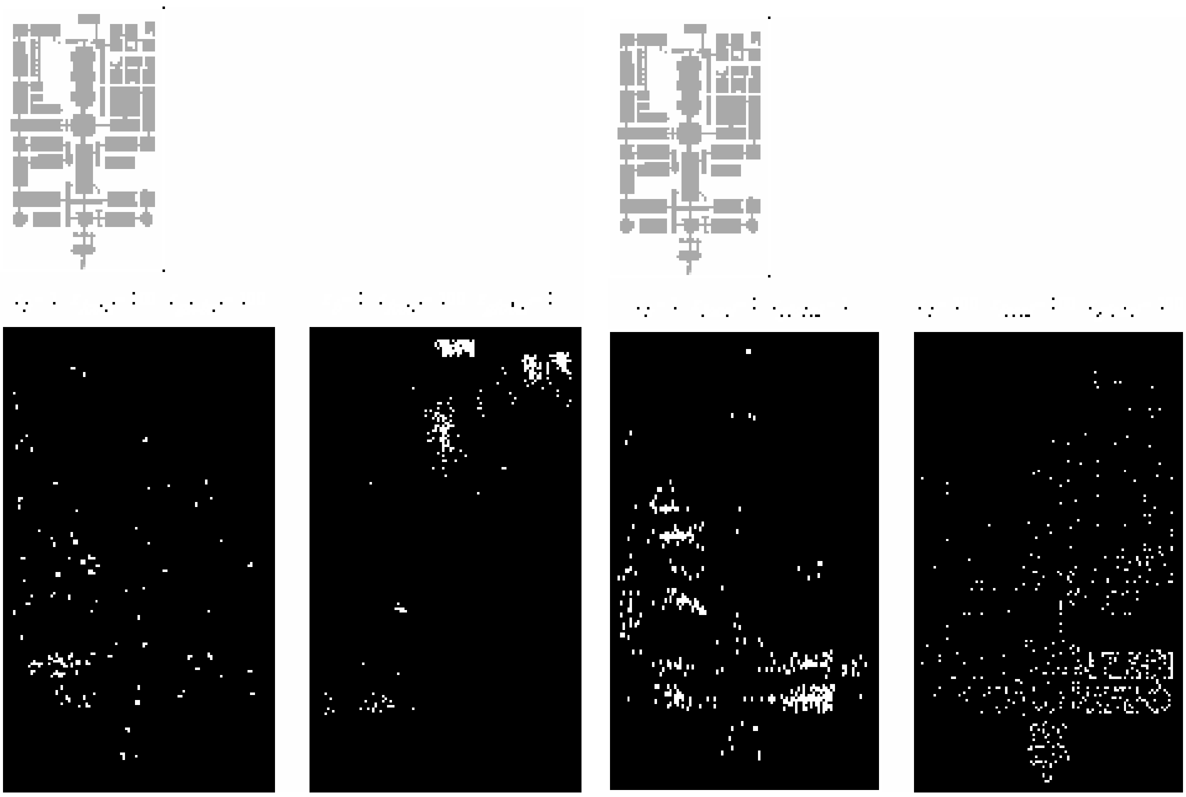


Figure 14: Steady State Walker Simulations Associated with Extremes in the Parameter Space

Through this process of judicious search, we have explored the performance of the model in detail in the areas where τ_b is nearer 1 while τ_{local} and τ_{global} are an order of magnitude greater, between 30 and 50. These give much more realistic distributions although it is clear when τ_{local} is evaluated more frequently than τ_{global}, better distributions are simulated. When τ_{global} is more frequent, there is a distinct tendency for walkers to move to the edges of rooms, marking out these edges. Although the model is quite sensitive to these parameters and because our explorations are in no way complete as yet, then it is entirely possible that there are other combinations of values in this general area that lead to better performance and contradict these tentative conclusions. We have computed a simple measure of model performance Θ based on the squared differences between observations P_J and predictions P'_J which is defined as

$$\Theta = \sum_J (P_J - P'_J)^2 \quad . \tag{30}$$

This measure is computed for each run of the model shown in Figure 13 and is presented in terms of the parameter space in Figure 15. Within the extreme values defined by this space, it is clear that $\tau_b << \tau_{local} << \tau_{global}$ with the best fit occurring at $\tau_b = 5$, $\tau_{local} = 20$, and $\tau_{global} = 50$. We have explored the sensitivity of the parameter values around this optimum and conclude that this is the approximate area where the best fit lies. As we have not explored the surface in complete detail however, we do not know if this surface is unimodal with a global optimum and this must await further work on this and similar applications.

However these best parameters do produce a well fitting model which we can illustrate in various ways. In Figure 16(a), we show the distribution of pedestrians in the steady state and this reveals that there is little geometric bias - walkers do not cling to walls, walkers do not remain in the gateway area, and walkers disperse throughout the gallery. There is a mild asymmetry between the left- and right-hand sides of the gallery, and the bookshop is the most frequently visited room; these bear out the observations made by UAS (1996). In Figure 16(b), we show the flow density diagram associated with 16(a) and this too confirms these interpretations. A more complete test of the model is given when observed and predicted pedestrian flows as measured by the number of visitors per room, are correlated. Figure 17 presents a scatter graph of P'_J versus P_J where it is clear that there are two major outliers - the room immediately above the bookshop and the top gallery hall along the main axis. In one sense, it is clear why these two rooms have attracted many more visitors than are observed. The room adjacent to the bookshop may live in its shadow to an extent and such an effect is hard to build into such a model. Second, the upper hall on the axis is at the focal point of the global attraction surface - at its saddle point between the right- and left-hand foci of the global attractor and we must conclude that this is given undue importance in the model. The correlation between observations and predictions is 0.736 which yields an r^2 of 0.541; if the two outliers are removed, then this correlation increases to 0.844 with a consequent r^2 of 0.712, which is really rather good. Combining these measures with the visual realism generated by the model suggests that this approach is more promising than we originally anticipated.

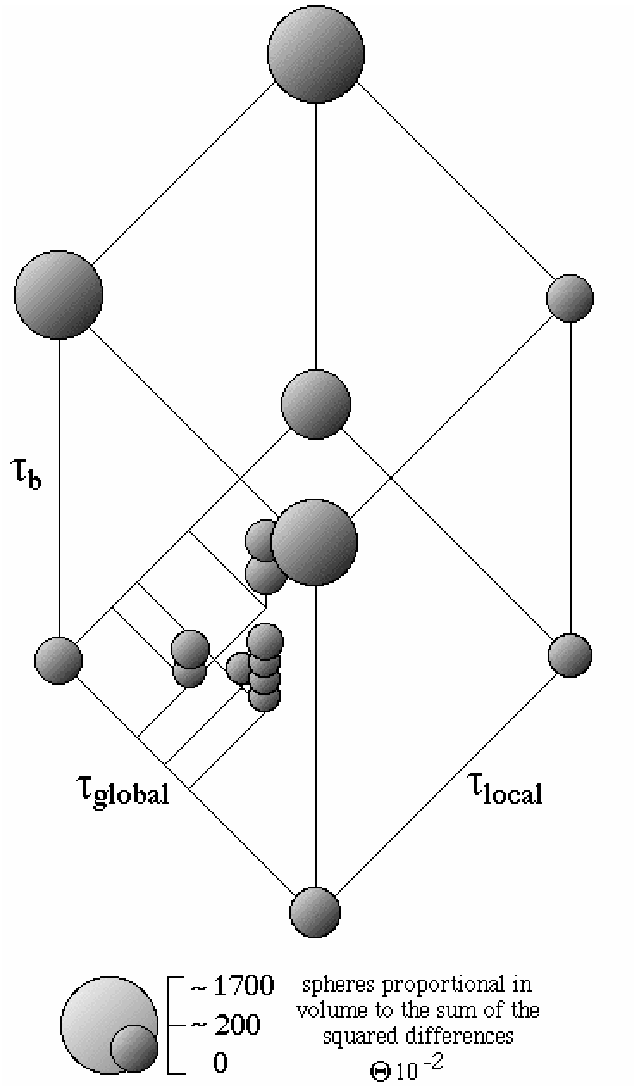


Figure 15: Performance of Various Model Runs in the Parameter Space

Our last foray into pedestrian modelling involves developing rudimentary learning capability within the model. The Tate example is the best of all our applications on which to develop this; we have greater flexibility in developing movement in this example than any other and thus we are able to explore in visual terms the extent to which agents might learn their way around the building complex. The most obvious metaphor for pedestrian systems of this kind involves comparing the way pedestrians might learn to move in a complex geometry with the way animals learn to navigate within a maze. In such situations, there must be some objective function which is successively improved through the changed behaviour which is associated with learning. It is immediately clear that such a function cannot be the same as the global performance function which we used earlier to find parameter values which optimised the motion of the collectivity of agents. Learning must take place for each individual within the model and this suggests that parameters must be set for each individual agent. As this would involve a major extension to the model, all we will do here is sketch the way such parameters might be specified and optimised as a prelude to future work.

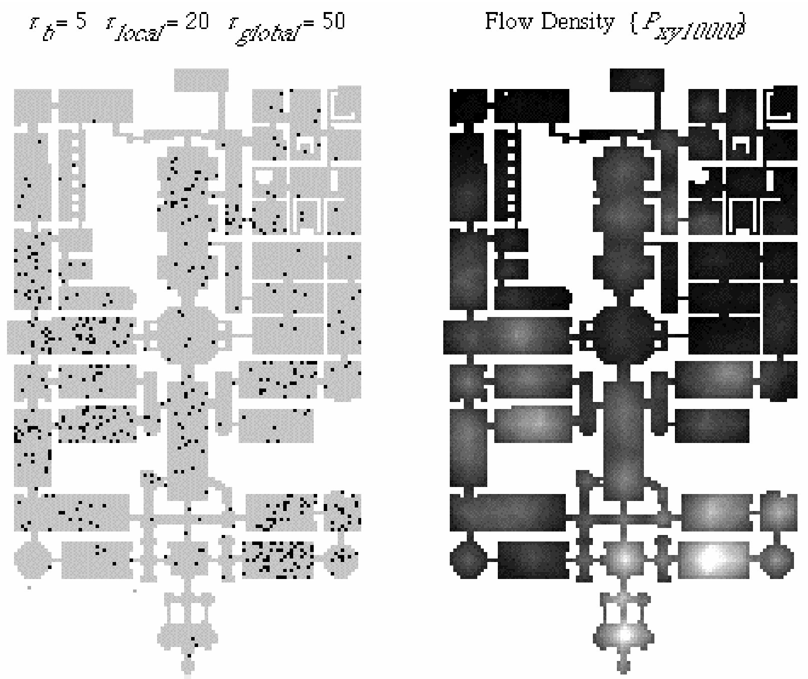


Figure 16: The Best Fitting Distribution of Walkers and their Flow Density

With the model in its current form, we are able to simulate any number of walkers (up to 16K) and thus we can use the model to explore the behaviour of a single walker as indicated in previous sections. With one walker, the parameters at τ_b, τ_{local}, and τ_{global} are associated with a single individual and we can use these and the other parameters concerning motion, which are predetermined, as a basis for changing behaviour due to learning. One way of proceeding would be to simply let a single agent continually evaluate his/her performance in terms of the number of times each room in the gallery is visited, and to adjust the values of these three parameters in such a way that eventually the walk traced out is closest to that observed for many walkers. This substitution of the external calibration process for a process whereby the agent makes the changes instead of the model user is not very realistic. Model performance based on room visits is associated with many walkers, not one. In practice, a single walker would not visit each room in proportion to the number of actual visits by all walkers. A single walker would only visit a limited set of rooms for only when the number of walkers was increased to the number observed would their collective room visits be comparable to those observed.

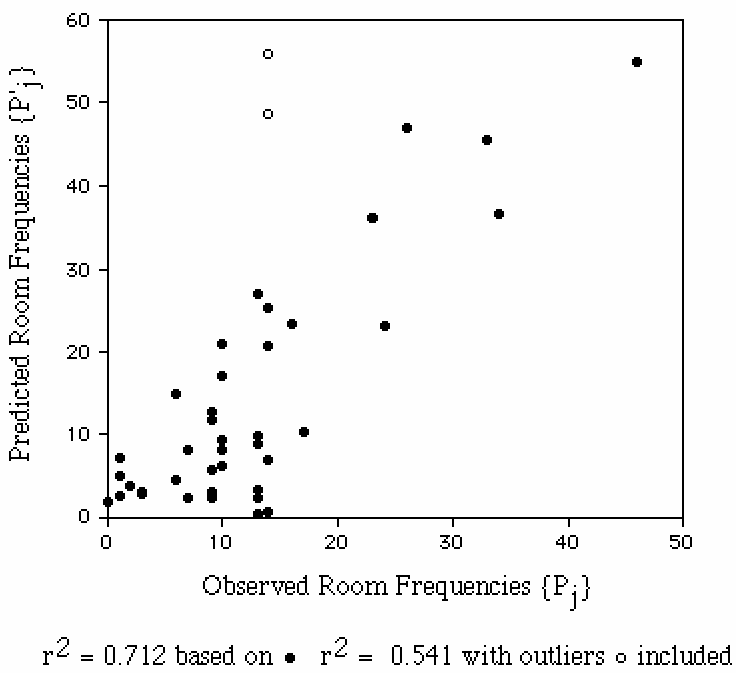


Figure 17: Predicted and Observed Room Occupancies based Frequencies of Visits

Secondly, it is unlikely that walkers would adjust their behaviour to optimise so abstract a function as the number of rooms visited; it is much more likely that behaviour would be modified through learning about how to navigate the local geometry successfully. It is this that we consider to be the most promising feature for endowing such learning capabilities within the model. At present, every time t an obstacle is encountered, a progress function λ_{kt} is evaluated and if this is below some threshold Λ, then a procedure is invoked which moves the walker *k* around the obstacle. Currently this procedure consists of: moving the walker back to the previous position where the obstacle was not encountered; incrementing the heading by $\pi / 2$, moving one step forward, and if encountering an obstacle, moving one step back. This sequence is repeated six times. The progress threshold Λ is fixed in advance and in all the runs to date, this has been set at 0.2 units of distance. If the goal of the walker is to make progress in walking as rapidly as possible, to circumnavigate obstacles as easily as possible, spending as little time as possible being bounced in this fashion, then as the agent experiences obstacles on the walk, changes can be made to Λ, to the number of times the sequence is activated, and to the headings that are used when the walker gets stuck. This is easy enough to implement but as in any scheme which involves optimising an objective whose form in the parameter space is unknown, there may be multiple optima and there is no certainty that the learning will be robust.

In principle, it is possible to set the agent walking from the gateway and after a fixed number of time periods, *T*, to have the agent check the amount of time spent in circumventing obstacles, and making progress, and to test the sensitivity of these values by making an adjustment to one or several of the features of the obstacle circumnavigation routine itemised above. If the adjustments reduce time spent and increase progress, then they are accepted and the agent continues to walk around the system until at time *2T*, the same sensitivity testing is invoked. Such a process is similar to the way search techniques operate but the nice feature of this model, is that once an adjustment is made, agents simply resume their walk. As they can walk indefinitely in the gallery and as the model takes no more than 30 seconds of real time to simulate 100,000 time periods (on a late 1995 Macintosh PowerBook 5300), then many, many possibilities can be tried out in this way. The big problem is that we do not know the extent to which the local geometry of particular places determines the speed of walking and the rate of progress. In other words, every time the model is run, a single walker will take a different path through the system and the extent to which learning behaviour is robust between different runs is unknown. Possibly by letting the agent walk around the system for 24 hours or more which would constitute billions of time periods, then learning may become robust. Moreover, learning may not simply be a matter of altering parameters values but of selecting different strategies for encountering different types of obstacle situation. For example, with one type of obstacle, one might first move back, then forward with a different heading and so on; with another one might move forward first with a different heading and so on. It is possible in this model to encode this kind of strategic learning into the simulation although such detailed conditions tax the power of the current software and can only be developed in detail once the programming effort moves to a more purpose-built software environment.

The last issue we will broach involves the problem of moving from a population of many agents to individual agents. As we have noted, when we examine the behaviour of one agent over many different runs, the spatial patterns traced out by this agent will differ from run to run. To illustrate this, in Figure 18, we show four walks traced out by a single agent starting from the gateway, walking for 20000 time periods, using parameter $\tau_b = 5$, $\tau_{local} = 30$, and $\tau_{global} = 50$ for the best fitting simulation. Each of the four walks is different in terms of location as one might expect but each is similar in terms of macro parameters such as distance travelled, and the amount of time spent in circumnavigating obstacles. Over a very large number of runs, these patterns are likely to follow a distribution similar to that simulated earlier when many agents walk in the system simultaneously. However, this is not always the case for agents interact through the congestion and attraction functions f_c and f_a and this means that exploration of the behaviour of one agent cannot be easily generalised to many. Our comment in an earlier section that exploring the behaviour of a single agent is instructive for what it might tell one about the entire system is still valid in part for in our models here, the two forces f_c and f_a have not be widely used. Nevertheless, although the idea that one might be able to get a single agent to learn and then transfer this experience to a population of agents is attractive, the interactions between agents are likely to be such that transfer will be limited in fully-fledged, more realistic simulations.

Conclusions: Developing Agent-Based Models

The microsimulation model we have developed is composed of the individual dynamics - the motion - of agents, the behaviour of each agent being largely conditioned by autonomous responses to highly local spatial conditions. We have assumed that the model is calibrated in a traditional manner by scaling up the actions of individual agents into some global pattern which is then matched against some set of observed data. Our assumption is that such global behaviour can be predicted and possibly controlled through interventions that change the spatial geometry of the system with consequent impacts on individual behaviours. Our approach to date, however, largely ignores the measurement and analysis of individual dynamics using appropriate statistics. We might be more interested in fine-tuning the actual behaviour of each agent rather than in any macro tuning of the agent population. There is even an argument for assuming that the population will take care of itself as long as individual behaviours are modelled correctly. There remains a huge area of investigation only touched upon in the last section, concerning the distributions of different spatial behaviours within the larger population. As yet we have little idea how the statistics of the global patterns we have predicted decompose into individual behaviours but this is an important area for continued exploration in the current set of applications.

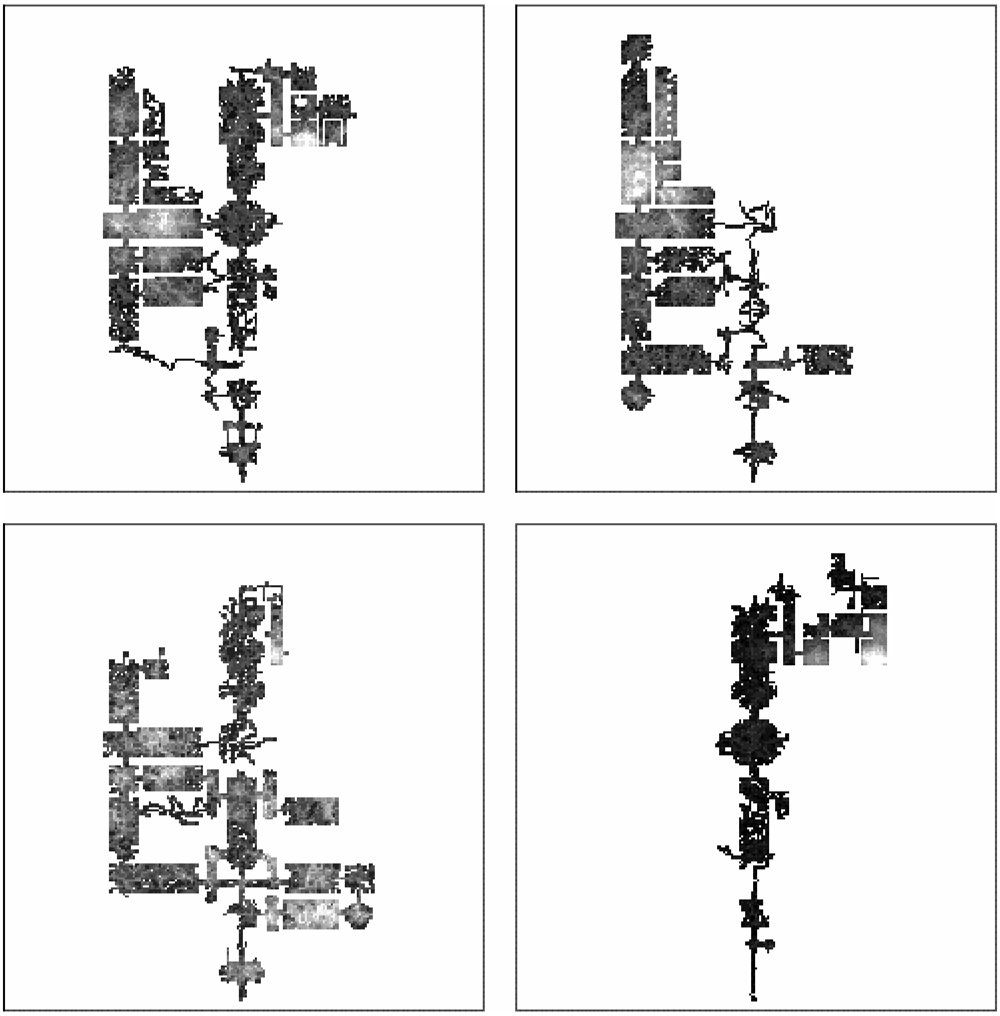


Figure 18: Flow Densities Associated with Different Walks of a Single Agent

The aggregation problem for these types of agent-based model involves representing many different classes of agent. So far, all our agents, our walkers, obey the same rules and respond to the same spatial events and patterns. Making such models more realistic must therefore involve defining key classes and types. For example, there is distinct difference between walkers who are already familiar with local situations and those who are not. Moreover walkers may have different degrees of familiarity with different parts of the same system. Some will be familiar with different neighbourhoods while others will be familiar with different activities such as varieties of retailing and entertainment. Walkers whose goal is shopping or entertainment have different demographic profiles which condition how and what attractions they respond to and this implies that different attractor surfaces must be developed for various categories of walker. This kind of disaggregation suggests that more emphasis be given to different classes of behaviour, something which is limited within the CA software used here where the emphasis is mainly on the way local actions generate global pattern.

The existing model does not contain much agent interaction. In the Tate Gallery example, we switched on the congestion threshold which meant that walkers tend to avoid each other when a congestion threshold is reached. However the positive and negative feedbacks between the number of walkers in a place and the level of spatial attraction was not implemented. Interaction between agents only comes into its own when different classes of agent are introduced. For example, it is well-known that crowds form when individuals see other individuals congregate. When we have walkers who are familiar with a place and know where they are going, other walkers begin to follow. This kind of self-reinforcement also induces a kind of learning in that walkers who do not know a place will find out about it more quickly when they follow others who know it. In fact every time a walker visits a place, something new is learnt even if they are thoroughly familiar with a place. The randomness of motion and the fact that there are differences in the composition of other agents each time a place is visited mean that agents are continually exposed to new information, and this conditions future behaviour. In a sense, this kind of learning can only be embodied in the model if there is a shift to more individually specific modelling software where agents are treated as individuals and where massive amounts of variety can be built into the way agents respond to even quite simple situations. Again a move to more purpose-built software is necessary.

If we develop a framework in which each individual is tracked in detail, then it is likely that the local geometry through which the agents move, must be represented by something other than continuous attraction surfaces. In fact, locations should be represented by parcels and like objects which in turn can then be endowed with different attributes. The notion of moving through the space can then be articulated as the search for objects of attraction whose attributes match the individual profiles of agents. For example, an agent with a certain set of preferences for a bundle of goods might only find those goods in a small subset of places which in turn are attractive enough to draw the agent. The outputs of the model then become profiles of realised trips which are characterised by the match between what the agent requires - demands - and the way those requirements are met - supplied. The model would thus take on a more explicit economic flavour incorporating elements of choice theory in the manner of disaggregate travel demand modelling (Willumsen and Ortuzar, 1990). In this way, we might begin to build in multi-purpose trips. Finally, if we so disaggregate, we are likely to be able to incorporate the more detailed dynamics of walker behaviour similar to that in the pedestrian models designed by Helbing and Molnar (1995).

This requires us to move to purpose-built software and to relinquish, for a time at least, the powerful graphics and animation capabilities that the current models have. But it also assumes that we begin to build in more detailed profiles of how walkers respond to their local environment and this in turn will require detailed data on their demographic and related behavioural characteristics. This forces us back to ideas in microsimulation but this time in a spatial context where the agents are modelled explicitly and individually according to the behavioural profiles which drive such simulations. These are all elements that we will consider in moving the research to the next level, while at the same time continuing to develop the existing models for Wolverhampton and the Tate Gallery. All these developments will be reported in future papers.

References

Adami, C. (1998) ***Introduction to Artificial Life***, Springer-Verlag, New York

Arthur, W. B. (1988) Urban Systems and Historical Path Dependence, in J. H. Ausubel and R. Herman (Editors) ***Cities and Their Vital Systems: Infrastructure Past, Present and Future***, National Academy Press, Washington DC, pp. 85-97

Axelrod, R. (1997a) ***The Complexity of Cooperation: Agent-Based Models of Competition and Collaboration***, Princeton University Press, Princeton, NJ

Axelrod, R. (1997b) The Dissemination of Culture: A Model with Local Convergence and Global Polarization, ***Journal of Conflict Resolution, 41***, 203-226

Batty, M. (1998) Urban Evolution on the Desktop: Simulation Using Extended Cellular Automata, ***Environment and Planning A, 30***, 1943-1967

Borgers, A., and Timmermans, H. J. P. (1986a) City Centre Entry Points, Store Location Patterns, and Pedestrian Route Choice Behaviour: A Microlevel Simulation Model, ***SocioEconomic Planning Sciences, 20,*** 25-31

Borgers, A., and Timmermans, H. (1986b) A Model of Pedestrian Route Choice and Demand for Retail Facilities within Inner-City Shopping Areas, ***Geographical Analysis, 18***, 115128

Clarke, G. (Editor) (1996) ***Microsimulation for Urban and Regional Policy Analysis***, Pion Press, London

DETR (1998) ***Town Centres: Defining Boundaries for Statistical Monitoring***, Department of the Environment, Transport and the Regions, The Stationery Office, London

Epstein, J. M. and Axtel, R. (1996) ***Growing Artificial Societies: Social Science from the Bottom Up***, MIT Press, Cambridge, MA

GSS (1997) ***Transport Statistics Report: National Travel Survey 1994-1996***, Government Statistical Service, The Stationery Office, London

Haggett, P. and Chorley, R. (1969) ***Network Analysis in Geography***, Edward Arnold, London

Hansen, W. G. (1959) How Accessibility Shapes Land Use*,* ***Journal of the American Institute of Planners, 25***, 73-76

Helbing, D. (1991) A Mathematical Model for the Behavior of Pedestrians, ***Behavioral Science, 36***, 299-310

Helbing, D. (1995) Improved Fluid-Dynamic Model for Vehicular Traffic, ***Physical Review E, 51***, 3164-3169

Helbing, D., and Molnar, P. (1995) Social Force Model for Pedestrian Dynamics*,* ***Physical Review E, 51***, 4282-4286

Helbing, D. and Molnar, P. (1997) Self-Organization Phenomena in Pedestrian Crowds, in F. Schweitzer (Editor) ***Self-Organization of Complex Structures: From Individual to Collective Dynamics***, Gordon and Breach Science Publishers, Amsterdam, The Netherlands, pp. 569-577

Henderson, L. F. (1971) The Statistics of Crowd Fluids, ***Nature, 229***, 381-383

Henderson, L. F., (1974) On the Fluid Mechanics of Human Crowd Motion, ***Transportation Research, 8***, 509-515

Hillier, B. (1997) Cities as Movement Economies, in P. Droege (Editor*)* ***Intelligent Environments: Spatial Aspects of the Information Revolution***, North Holland, Amsterdam, The Netherlands, pp. 295-342

Hillier, B. Penn, A., Hanson, J., Grajewski, T., and Xu, J. (1993) Natural Movement: Configuration and Attraction in Urban Pedestrian Movement, ***Environment and Planning B, 20***, 29-66

Langton, C. G. (Editor) (1995) ***Artificial Life: An Overview***, MIT Press, Cambridge, MA

Nagel, K., and Paczuski, M. (1995) Emergent Traffic Jams, ***Physical Review E, 51***, 2909-2918

Older, S. J. (1968) Movement of Pedestrians on Footways in Shopping Streets, ***Traffic Engineering and Control, 10***, 160-163

Resnick, M. (1994) ***Turtles, Termites and Traffic Jams: Explorations in Massively Parallel Microworlds***, MIT Press, Cambridge, MA

Sandahl, J., and Percivall, M. (1972) A Pedestrian Traffic Model for Town Centers, ***Traffic Quarterly, 26***, 359-372

Schelling, T. S. (1978) ***Micromotives and Macrobehavior***, W. W. Norton and Company, New York.

Stewart, J. Q. and Warntz, W. (1958) Physics of Population Distribution, ***Journal of Regional Science, 1***, 99-123.

Stilitz, I. B. (1969) The Role of Static Pedestrian Groups in Crowded Spaces, ***Ergonomics, 12***, 821-839

Toffoli, T., and Margolus, N. (1987) ***Cellular Automata Machines: A New Environment for Modeling***, MIT Press, Cambridge, MA

UAS (1996) ***Tate Gallery, Millbank: A Study of the Existing Layout and New Masterplan Proposal***, Unit for Architectural Studies, Bartlett School, University College, London

Whiteley, B. (1997) Putting London Back on its Feet, ***London Calling*** *(Journal of the RTPI London Branch),* ***2***, 2-5

Willumsen, L. G. and Ortuzar, J. de D. (1990) ***Modelling Transport***, John Wiley and Sons, Chichester, UK

Acknowledgements

This project was funded by grants from the Joint Information Systems Committee (JTAP Grant 45), and the Office of Science and Technology's Foresight Challenge VR Centre for the Built Environment (Grant EPSRC GR/L54950).