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WEIGHTED AND METRIC SURFACE NETWORKS - NEW INSIGHTS AND AN INTERACTIVE APPLICATION FOR THEIR GENERALISATION IN TCL/TK

Sanjay Rana
*Jo Wood

Centre for Advanced Spatial Analysis
University College London
1-19 Torrington Place
Gower Street
London WC1E 6BT

Tel: +44 (0) 171 391 1782
Fax: +44 (0) 171 813 2843
Email: casa@ucl.ac.uk
http://www.casa.ucl.ac.uk

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*Jo Wood, Department of Information Science, City University, Northampton Square, London EC1V 0HB.

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ABSTRACT

The idea of characterising the different forms of natural topographic surfaces by a topological model based on their fundamental surface features has attracted many proposals. In this paper, a detailed discussion and new proposals on various issues related to the concept, generation, and visualisation of two graph theoretic based surface topology data structures – Weighted Surface Networks and their improved version, Metric Surface Networks - are presented. Also presented is an interactive Tcl/Tk application called *Surface Topology Toolkit*, which has been developed to support the discussion on aspects of their generalisation and visualisation. The highlight of the *Surface Topology Toolkit* is the utility to allow arbitrary contraction unlike the usual vertex importance based criterion. This paper proposes that effective automated surface topology modelling based on these surface networks requires (a) further research in the development of "computing" algorithms that will accurately locate critical surface points, be able to establish topological links, and also check topological consistency, (b) transforming their 2D straight line graph like appearance to 3D to improve visualisation and contraction, and (c) assessment and user-awareness about the effects of each type of contraction criterion on the topography.

INTRODUCTION

From the early days of spatial information science, attempts have been made to parameterise the continuously varying topographic surface, into a set of its fundamental surface features and then describe the topography as a framework woven around the geometrical and topological relationships between these features. Prominent examples of such surface topology data structures are Contour Tree (Morse, 1968; Mark, 1977), Weighted Surface Network (Pfaltz, 1976; Wolf, 1988), Symbolic Data Framework (Palmer, 1984), Interlocking Ridge and Channel Network (Werner, 1988), and Drainage and Divide Network (Bennett, Armstrong, 1996). For a review on different surface topology data structures refer to Wolf (1993) and Rana (1998). The present work addresses the graph theoretic based data structures - Weighted Surface Network (WSN) and its improved version Metric Surface Network (MSN), which have proved to be suitable for both characterising and generalising the surface topology (Pfaltz, 1976; Mark, 1977; Wolf, 1988, 1989, 1990, 1991, 1993). For simplicity, WSN and MSN (both also called Pfaltz-Graphs after the original work by Pfaltz, 1976) will be referred to as surface networks throughout. This paper presents a detailed analysis on these surface networks and an interactive application, which is simple and more powerful than the currently available program for their generalisation. A brief description of surface networks taken (with minor modifications) from texts of Pfaltz (1976) and Wolf (1991) as a background for this paper will be repeated in the next section. Interested readers are strongly recommended to read the original works.

WEIGHTED AND METRIC SURFACE NETWORKS - BACKGROUND

It is essential to understand that the term topographic surface here means a “twice continuously differentiable function *f (x, y)* associating with each point *(x, y)* its respective altitude and being defined over a domain, which is simply connected and bounded by a closed contour line”. It is assumed that all critical points of *f (x, y)* are non-degenerate, therefore its relative minima, saddle points, and relative maxima (respectively representing the pits, passes and peaks of the corresponding surface) are isolated**.** These critical points (pits, passes and peaks) also termed as ‘surface-specific points’ contain significantly more information than any other point on the surface and thus are exceptionally qualified for characterisation of topographic surfaces. Other ideas also exist on the classification and terminology of fundamental features in topographic surfaces, alternatively called landform elements (Speight, 1974), surface specific features (Fowler, Little, 1979), symbolic surface features (Palmer, 1984), surface patches (Feuchtwanger, Peucker, 1987), critical surface features (Wolf, 1992), and specific geomorphological elements (Tang, 1992) amongst others. The closed contour line represents either the surrounding pit or peak. Fig. 1A shows a topographic surface containing pits, passes, peaks, ridges (lines leading from a pass to a peak) and channels (lines leading from a pit to a pass). The topological structure of any topographic surface can now be modelled as an edge weighted directed graph with the vertices representing the pits, passes, and peaks, the edges being the channels and ridges and the edge-weights specifying the differences in elevations (Fig. 1B).

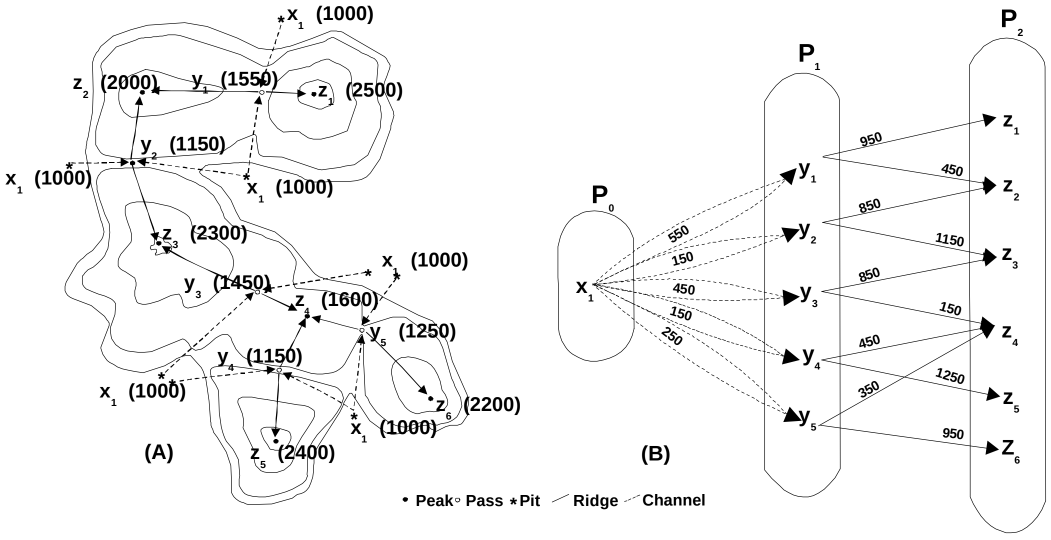


Fig. 1 (A) Ridges and channel lines of a topographic surface (number in parentheses indicate altitude of the point) with its (B) Tripartite graph representing the topological structure of the surface; edge weights are indicated on the edges (From Wolf, 1991). Note the straightline like appearence of ridges and channels and the single surrounding pit x_1 *of the surface.*

However, not all such graphs can be regarded as abstract models of topographic surfaces but only those, which satisfy the following properties:

A weighted, directed, tripartite graph $W = (P_0, P_1, P_2; E)$, where P_0, P_1, P_2 are the three vertex sets representing the sets of all pits, passes and peaks, respectively, while E is the set of all edges, is termed a (weighted) surface network if

P0: W is planar.

This means that an intersection of edges, for instance an intersection of ridges and channels is not allowed.

P1: The subgraphs $[P_0, P_1]$ and $[P1, P2]$ are connected.

This means that channels connect pits and passes, and ridges connect peaks and passes.

P2: $|P_0| - |P_1| + |P_2| = 2$.

This means that the number of pits minus the number of pass points, plus the number of peaks must always be two.

P3: *For all* $y \in P_1,$ $id(y) = od(y) = 2$.

This means that from all passes exactly two channels and exactly two ridges emanate, thus excluding the existence of degenerate passes.

P4: $val(x,y_i) = val(y_i,z) = 1$ *implies that there exists* $y_j \neq y_i$ *such that* (x,y_j) , $(y_j,z) \in E$.

This guarantees that if there exists a path from pit x via pass y_i to peak z, which consists only of edges with valency one, then there exists another path from pit x to peak z via a distinct saddle y_j.

P5a: (x,y) *is an edge of a circuit in the bipartite graph* $[P_0,P_1]$ *if and only if* $val(y,z) \neq 2$ *for all* $z \in P_2$

P5b: (y,z) *is an edge of a circuit in the bipartite graph* $[P_1,P_2]$ *if and only if* $val(x,y) \neq 2$ *for all* $x \in P_0$

This property asserts that a configuration, as shown in Fig. 2, is not possible.

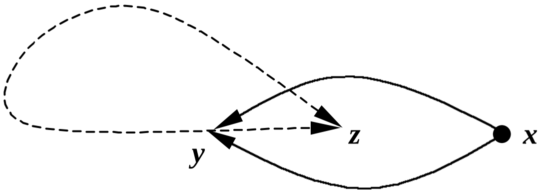


Fig. 2 P5a and P5b assert that this configuration is impossible (From Wolf, 1993).

P6: $w(e_i) > 0$ *for all* $e_i \in E$
This means that all edge weights must be greater than zero. For instance if $h(x_0)$, $h(y_0)$ and $h(z_0)$ represent the elevations of a pit, pass and peak, respectively, then the weight of a channel is $h(y_0) - h(x_0)$ and the weight of a ridge is $h(z_0) - h(y_0)$.

P7: *For all* $x \in P_0, y_i, y_j \in P_1, z \in P_2$ *and* $(x,y_i), (x,y_j), (y_i,z), (y_j, z) \in E$ *holds*
$w(x,y_i) + w(y_i,z) = w(x,y_j) + w(y_j,z)$
This means that for all paths from pit *x* to peak *z* the difference in elevation is the same, regardless of which saddle point is passed.

P8a: *If* $val(x,y) = 2$ *with* $e_{i1} = (x,y)$ *and* $e_{i2} = (x,y)$ *then* $w(e_{i1}) = w(e_{i2})$
P8b: *If* $val(y,z) = 2$ *with* $e_{i1} = (y,z)$ *and* $e_{i2} = (y,z)$ *then* $w(e_{i1}) = w(e_{i2})$
This means that all channels from a pit to a pass have the same difference in altitude; the same holds for ridges, too.

In order to add geometrical characteristics to a WSN thus being able to represent river junctions and ridge bifurcations, Wolf (1990) proposed the addition of geographical co-ordinates to the surface-specific points in WSN and termed the resulting network Metric Surface Network (MSN). Stream junctions and ridge bifurcations are represented by channels and ridges with infinitesimally close pits and passes, and peaks and passes, respectively; in practical examples this can be achieved by assigning very small edge weights (Wolf used a value of 2).

The cartographic importance of surface networks results from the fact that they may be condensed by two homomorphic contractions called (x_0, y_0)-*w*-contraction and (y_0, z_0)-*w*-contraction which reduce the number of edges and vertices, but preserve the topological structure of the corresponding topographic surface. In simple terms, the (x_0, y_0)-*w*- and (y_0, z_0)-*w*-contractions cause the removal of an "internal" (the surrounding pit is not allowed to be selected for contraction) pit x_0 or an "internal" (the surrounding peak is not allowed to be selected for contraction) peak z_0 with their highest adjacent pass i.e. the subgraphs $[P_0,P_1]$ and $[P_1,P_2]$, together with all surface-specific lines incident with at least one of these critical points. The pass connected to the contracted x_0 (or z_0), which becomes free, is then connected to the pit (or peak) originally linked to the contracted y_0 thus restoring topological consistency. The complete mathematical proof of the contractions is provided in Pfaltz (1976) and Wolf (1989). The selection of a pit or peak is based on an importance criterion, which depends upon the particular problem and the topography. Based on the edge weights associated with vertices, Wolf (1988) proposed three importance criteria for the selection of pits and peaks. Rana (1998) proposed a User Defined Contraction (UDC), which gives intelligent user added flexibility over the generalisation process. These four different homomorphic contractions are described below:

(i) The maximum of the differences in elevation between a peak/pit and all its adjacent passes serves as a measure.
Of all edges incident with a peak or a pit, the one which is maximum weighted is selected and its weight is assigned as a vertex-weight to the respective surface specific point. In a similar way the weights of the maximum weighted edges of all interior pits and peaks of the surface network are assigned as weights to the respective vertices; then the vertex-weights are put into a list and sorted in ascending order. The vertex with minimum weight - it is always the first in the sorted list - is then contracted.

(ii) The minimum of the differences in elevation between a peak/pit and all its adjacent passes serves as a measure.
This is similar to the first contraction type except that the weight of the minimum weighted edge is assigned as a vertex-weight.

(iii) The sum of the differences in elevation between a peak/pit and all its adjacent passes serves as a measure.
The weights of all edges incident with a peak and pit are added and the sum is assigned as a vertex-weight. The peak or pit with the smallest vertex-weight is contracted. It is expected that this contraction would remove peaks or pits with a degree of one or two. Higher degree peaks or pits represent "crossings" of different ridge and channel lines and are therefore of great importance for the topography of the given area (Wolf, personal communication).

(iv) User Defined Contraction (UDC).
A user can arbitrarily select an internal peak or pit for contraction and the least weighted edge incident on the selected peak or pit is contracted. This type of operation provides extreme power to a learned user to experiment with the contraction sequence. Section 4 gives an example of the potential uses of UDC.

Incidentally, Wolf (1991) was right in envisaging that there could be several ways of calculating importance criteria, for example the slope of an edge can also be taken as a weight of a linked pit (or peak). In addition, giving the flexibility of contracting pits (or peaks) with a user-specified drop in height over its linked edge (simple elevation differences) or slope can extend UDC functionality.

REVIEW ON METHODS FOR GENERATION OF SURFACE NETWORKS

Generation of surface networks broadly involves two steps, firstly the identification of critical points and lines and then assigning topological relationships between them, taking into consideration the topological rules of surface networks, as mentioned earlier. There have only been two reported attempts (Wolf, 1989; Wood, 1998) to create surface networks for natural topographic surfaces.

Wolf (1989) created the surface network for an area in the Latschur Mountains, western Carinthia region of Austria. He did this manually by digitising the critical points and lines on paper maps and then establishing the topological relationships between them. It was only by doing this that he could successfully build and use the surface network in a variety of applications. This is because the generation of surface networks from Digital Elevation Models (DEM) with automated routines is yet to be satisfactorily implemented. This will be discussed in detail.

Wood (1998) proposed a prototype methodology for automated generation of surface networks from DEM (which is a part of the terrain analysis and visualisation software - *LandSerf*). It must be noted that although Wood (1998) did not propose this methodology as the definitive solution for automated generation of surface network from DEM, the methodology has been discussed here to propose suggestions and expectations for its next generation algorithms. The procedure for creation of surface networks (Wood, 1998) is to (i) identify the passes using the feature identification algorithm included in *LandSerf;* (ii) move upwards in the direction of any ridge axes that fall within a circle of radius r until a new grid cell is reached; (iii) recursively repeat step (ii) until no new higher cell is found and (iv) repeat steps (i) – (iii) but moving downwards along channel axes. It is very successful in visualisation of surface networks as it showed a more “natural” look for surface networks (Compare Fig.1 with Fig. 3A). However, its feature extraction and topology building has some limitations (Wood, 1998) as follows:

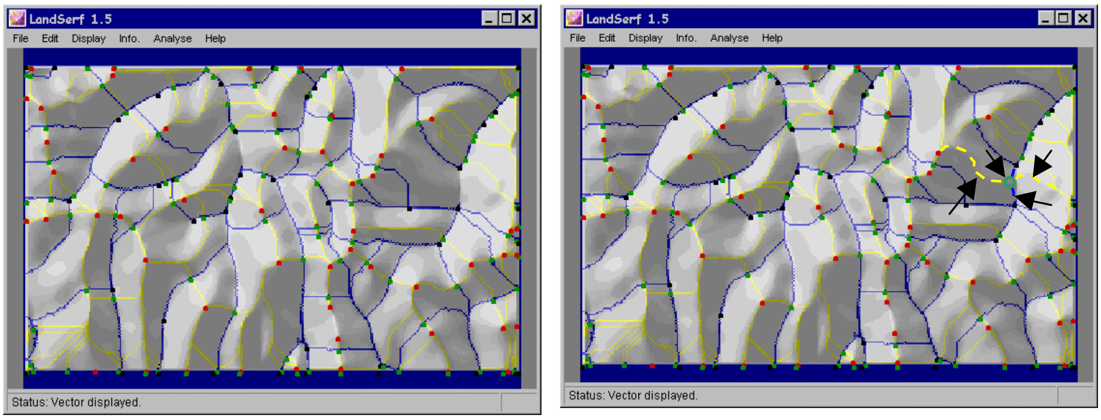
(i) The surface network is not fully connected i.e., there are unconnected critical points giving an impression of missing edges. In Fig. 3B the arrows show the position of the missing channels and ridges,

(ii) Third topological property for the relation between the number of pits, passes and peaks is not attained,

(iii) Channel and ridge edges begining at a pass started from a pass can stop further tracing at channel and ridge junction i.e., if it respectively meet another channel or ridge. This condition was only intended to stop backtracking of the edge and is similar to the surface specific line tracing used by Fowler, Little (1979). However, it is a serious drawback (but can be easily fixed) because surface network topology requires the edges to exist between passes and pits, and passes and peaks,

(iv) Channel and ridge junctions are not stored but can possibly be identified. However, this methodology would neither identify nor store channel and ridge bifurcation as delineation of ridge or channel from a pass is only allowed to lead to a single pit or peak.

Wood (1998) explained the two main reasons for the limitations of surface network creation in *LandSerf*. Firstly, some features are topologically defined showing an obvious morphometric expression. In particular, ridges and channels may show a gentle longitudinal curvature at a much broader scale than that implied by their cross-sectional curvature. Secondly, passes are used as the starting point for all ridges and channels, therefore if a ridge and channel crossing is not expressed morphometrically at the point of intersection, it will not be identified by the algorithm. For example in Fig. 3B the missing pass (shown by the hollow green circle) can actually be the reason for missing edges.



Status: Vector displayed.

Status: Vector displayed.

(A) (B)

Fig. 3 (A) The topological network of ridges and channels derived from the Lake District DEM. Linear features are bounded by pits, peaks and passes (From Wood, 1998). Red dots – Peaks, Green dots – Passes, Black dots – Pits, Yellow lines – Ridges and Blue lines – Channels. (B) The missing topological links in the surface network produced by LandSerf shown on the left. Hollow green circle – Missing Pass, Dashed yellow lines – Missing Ridges, Dashed blue lines – Missing Channels.

Both of these problems relate to issues of scale dependency in DEM based analyses, which is widely discussed in the field of geocomputation (See Wood 1996, 1998 for latest thoughts on scale dependency in DEM processing). One simple reason for missing a critical point can also be a wrong basis of classifying cells into different types of critical points. The missing bifurcation limitation can have two possible solutions (a) to adopt ideas behind multi-directional flow algorithms (Costa-Cabral, Burges, 1994) or (b) to simply allow second tracing, this time from pits to passes and peaks to passes and deciding the final topology based on a combination of both.

SURFACE TOPOLOGY TOOLKIT

Wolf (1991) developed FORTRAN routines that could generalise surface networks and produce simplified contour maps. These routines are capable of efficient processing of WSN but provide limited interactivity and visualisation, both of which form an essential component of cartographic techniques. In this work, a prototype application *Surface Topology Toolkit* (*STT*) coded in Tcl/Tk v8.0 is presented, that performs interactive generalisation and exploratory visualisation of surface network. The high level Tcl/Tk language, developed by Sun Microsystems, is becoming a popular programming language for creating applications on spatial problems in GIS (for example cartographic data visualiser by Jason Dykes [1]).

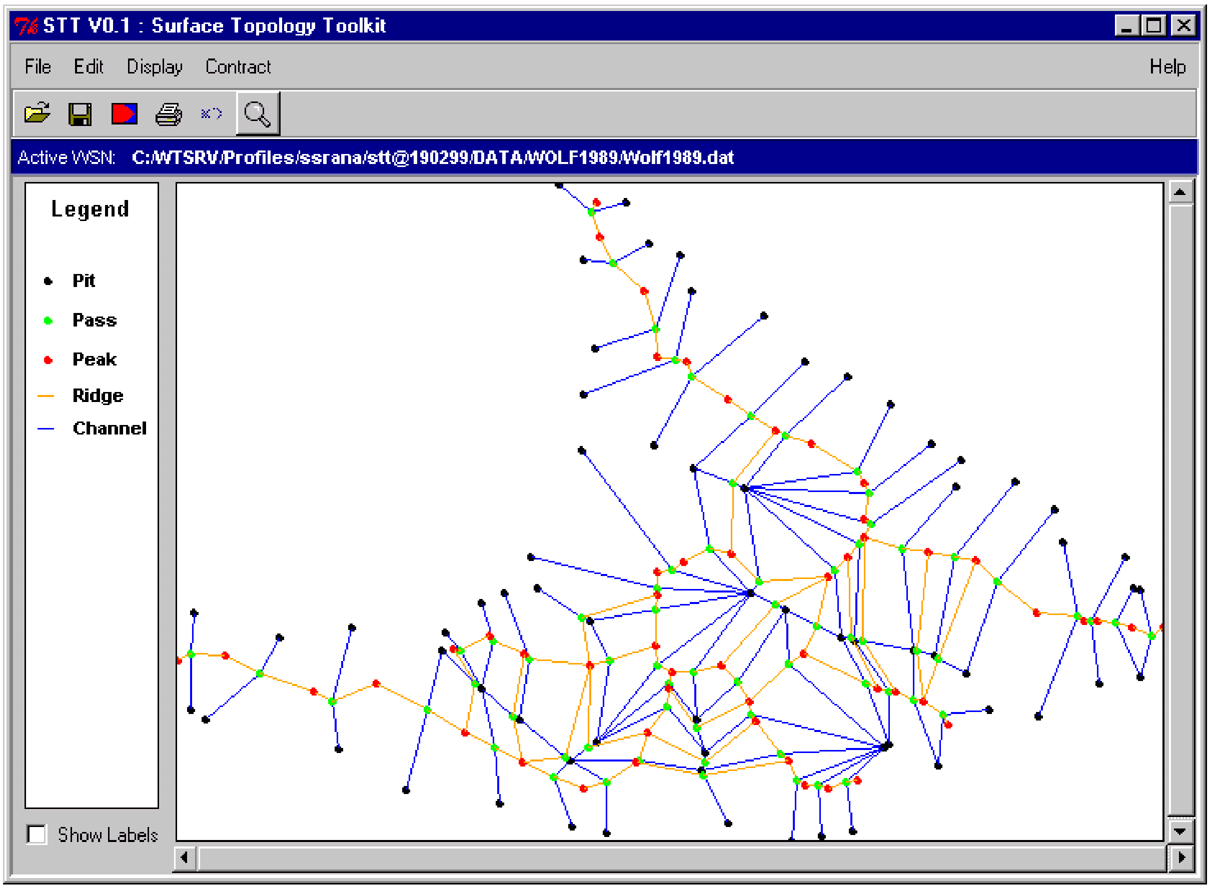
STT has a simple point and click Graphical User Interface (GUI) (Fig. 4) which makes the process of contraction interactive and informative. The versatile, fast and easy commands for programming GUIs and array processing in Tcl/Tk are the building blocks for contractions used in *STT*. Owing to the GUI and UDC present in Surface Topology Toolkit a user is able to achieve a considerable improvement over earlier methods for the generalisation and visualisation of surface networks. The significant achievements of *STT* are as follows:

(i) User is made aware of every contraction (except for continuous contractions) so that a selection can be made more intuitively,

(ii) User can shift from one type of importance measure to a different one in subsequent contractions and can arbitrarily select a pit/peak (if permissible by surface network topological properties) for contraction,

(iii) Surface network can be exported as an ASCII data file that can be used in programs such as SURFER and UNIMAP as an input to create contour maps and 2.5-D surfaces.

(iv) User has the flexibility to undo a contraction to observe the changes in results for better generalisation.



Active WSN: C:/WTSRV/Profiles/ssrana/stt@190299/DATA/WOLF1989/Wolf1989.dat

Fig. 4 Graphical User Interface of STT with Latschur surface network displayed in the plot area.

A pseudo code describing the main steps in the operation of *STT* is as follows:

```
(i)   Load surface network dataset
        - Read information on (x,y)-co-ordinates and elevations of pits, passes
          and peaks.
        - Read connectivity information of channels and ridges.
(ii)  Calculate edge weights
        - For each pit and peak fetch the adjacent passes
          {
              - Find the maximum weighted and minimum weighted incident edge
                and assign their edge-weights as well as the sum of all of
                these weights as weight to the respective vertex, this vertex-
                weight is termed “Importance”.
```

```
            - Insert the three vertex-weights obtained in this way together
              with the unique ID of the respective peak and pit into three
              lists (one for each importance measure) for the entire surface
              network.
        }
    -   Sort the three lists in ascending order so that the least important
        is always the first in the list.

(iii)   Perform Contractions
    -   If asked to do vertex importance based contraction
        {
              Contract the first pit/peak in one of the three lists (based on
              type of importance measure selected).
        } else if asked to do UDC
              Contract the least weighted ridge or channel connected
              respectively to the selected internal peak and pit.
        }
    -   Adjust the topological connections.

(iv)    Repeat step (ii) and (iii) till the desired degree of generalisation has
        been achieved or surface network has reached elementary stage.
```

Tests on functioning of *STT* were performed by carrying out homomorphic contractions on the surface network for the area in Latschur Mountains of Western Carinthia region in Austria, used by Wolf (1989), and then comparing the contraction sequence. The original format of surface network data was simplified and reformatted into the format compatible with *STT*. Appendix 1 contains the list of reformatted surface network data used in this analysis. The following experiments were carried out with *STT*:

(i) Correlation with Wolf's (1988, 1989) observation of the first intersection of edges at the 33rd contraction based on maximum of elevation differences measure,

(ii) Observing effects of generalisation on surface network using different vertex importance based homomorphic contractions, and

(iii) Use of UDC to avoid edge intersections and to improve the quality of condensed contour maps (Based on suggestions of Wolf, 1989).

Fig. 5 shows the surface network of an area in Latschur Mountains in western Carinthia, Austria draped on contour map of the same, produced from SURFER v5.03 using the Triangulation interpolation. Interesting results appeared from the experimentation:

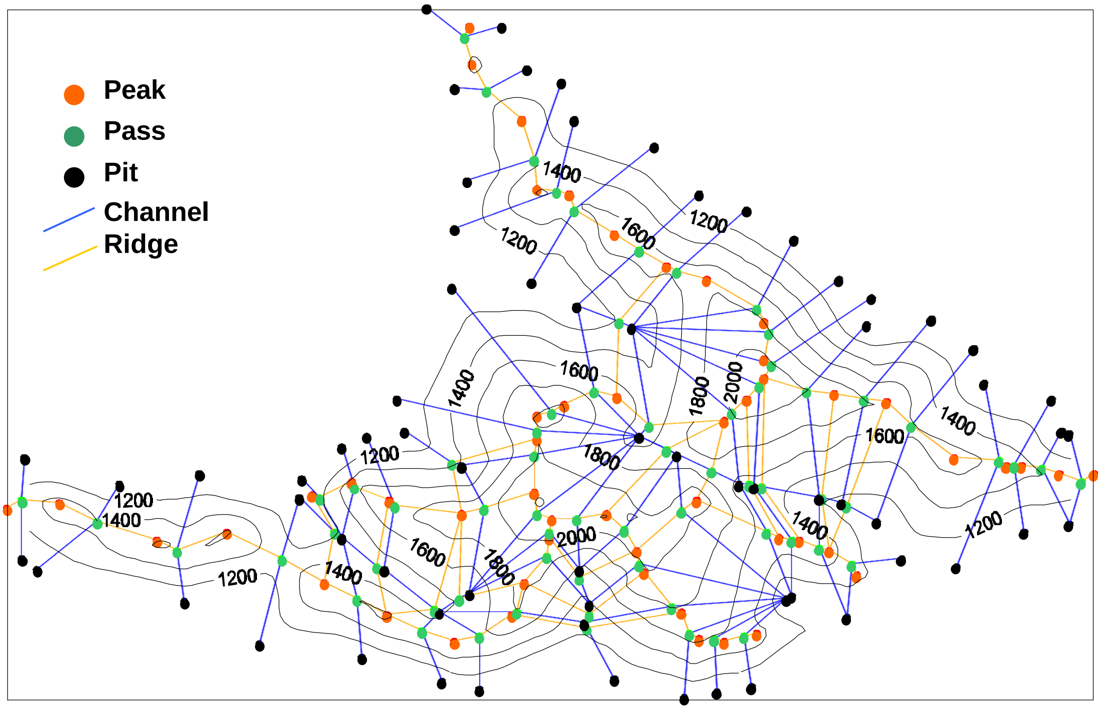


Fig. 5 The topography around Latschur Mountains, Austria as a Surface Network, overlain on a contour map (200m interval).

Case: 1 Correlation with Wolf (1988, 1989)'s observation about the first intersection of edges at the 33rd contraction based on Maximum of elevation differences measure.

Wolf (1998, 1989) reported that elimination of the peak 131 with it adjacent pass 81 in the 33rd step would produce the first intersection of surface-specific lines in the Latschur surface network. However, this work found that the first intersection takes place in the 34th step with the elimination of the peak 130 and pass 78 (Table 1 and Fig. 6). Actually disparity starts at the 27th contraction where Wolf (1988, 1989) contraction routine selects the ridge [Pass 103, Peak 160] while *STT* selects the ridge [Pass 97, Peak 138], which basically cumulates later to show a disparity in the stage of intersection of edges.

In fact after the 26th contraction, ridge [Pass 103, Peak 160] is second to ridge [Pass 97, Peak 138] in line for contraction. The reason for this difference is perhaps an improper selection of the points by the algorithm used by Wolf (1988, 1989). A manual check of the edge weights after the 26th contraction revealed that the maximum weight associated with peak 138 is 291 while the weight associated with peak 160 is 296. Therefore, a minimum weighted edge associated peak 138 would have to be contracted in the 27th contraction step, which is the ridge [Peak 138, Pass 97]. Of the other two contraction sequences only the one based on the minimum of elevation differences measure (Table 2) is same as produced by *STT* while the contraction sequence based on sum of elevation differences measure (Table 3) again show dissimilarities after the 37th contraction step.

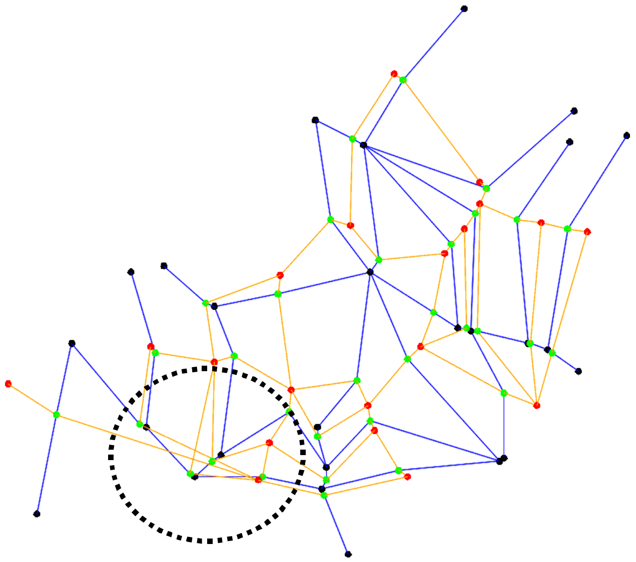


Fig. 6 Latschur surface network at the stage of first intersection of edges (inside the dotted black circle) after the 34th step of contraction based on maximum of elevation differences measure.

Case 2: Observing effects of generalisation on surface network using vertex importance based homomorphic contractions.

In metric surface networks a channel junction/bifurcation is stored as an edge between a pit and a pass and a ridge junction/bifurcation as an edge between a peak and a pass. In both cases the incident vertices lie very close together and the edge-weights are very small - in the following example a value of 2 is used. A comparsion between the sequence of contractions based on the maximum of elevation differences measure (Table 1) and the data (Appendix 1) reveals that ridge junctions (for example Ridge [Peak 196, Pass 195]) are the first to be selected for contractions. This could also be true for channel junctions, which could be a serious drawback, as the deletion of a channel and ridge junction can potentially delete an entire ridge or channel due to the surface network contraction rules. Therefore, junctions and bifurcations need more conceptual attention as they have more importance than simply being artefacts (Wolf, 1990).

Very different results were obtained by performing the three different vertex based importance measures. WSN was exported to an ASCII file after each 15 steps of contraction and contour maps were collected to compare the generalisation of the corresponding topographic surface (Fig. 7 A-L). The following observations were noted:

(i) Contraction based on minimum of edge weights measure started producing intersections between edges very early on in the generalisation process but it was the only one to maintain the continuity of the ridge structure through the entire area. Even after 60 contractions, the ridge structure could be observed in the surface network (Fig. 7H). This could perhaps be because the edge weights on channels would usually be lower than edge weights on ridges. As surrounding pits could not be contracted, it's only the internal pits that are contracted and hence the ridge structure was not affected. This indicates that minimum of edge weights measure would be the most suitable of the three measures for generalisation of topography based on a surrounding pit, if the major structure is to be retained. It could be an interesting experiment to observe the same sets of steps in the case of an area bounded by a surrounding peak.

(ii) Contractions based on sum of edge weights result into the maximum and fastest contraction of the topographic structure (meaning loss of topography). This is most likely due to the large number of degree one edges especially near the peripheries of the surface network.

Fig. 7 Surface Networks and respective contour maps (200m interval) of Latschur surface network after each 15 contractions based on maximum of elevation differences criterion (A-D), minimum of elevation differences criterion (E-H) and sum of elevation differences criterion (I-L).

Contraction No.	Pit ID	Peak ID	Pass ID
1	None	196	195
2	None	179	122
3	None	178	121
4	None	149	92
5	None	148	91
6	None	177	120
7	None	154	124
8	None	157	127
9	None	162	107
10	None	155	126
11	None	142	101
12	None	147	90
13	None	133	80
14	None	170	115
15	None	163	108
16	None	164	109
17	None	165	110
18	None	176	119
19	None	159	104
20	None	193	192
21	None	128	76
22	None	129	77
23	None	137	85
24	None	166	111
25	None	167	112
26	None	169	114
27	None	138 (160)	97 (103)
28	None (238)	160	103 (240)
29	238	None (143)	240 (97)
30	None	143 (132)	84 (79)
31	None (62)	132	79 180)
32	62	None (141)	180 (100)
33	None	141 (131)	100 (81)
34	None	131 (130)	81 (78)
35	None	130 (156)	78 (125)
36	None	156 (171)	125 (116)
37	None (61)	171	116 (182)
38	61	None (168)	182 (113)
39	None (54)	168	113 (185)
40	54	None (144)	185 (87)
41	None	144 (136)	87 (84)
42	None (63)	136	102 (123)
43	63	None (152)	123 (95)
44	None (64)	152	95 (188)
45	64	None (153)	188 (96)
46	None (65)	153	96 (189)
47	65 (237)	None	189 (239)
48	237	None (140)	239 (99)
49	None	140 (135)	99 (83)
50	None	135 (134)	83 (82)
51	None	134 (161)	82 (105)
52	72 (49)	None	186 (183)
53	None	161 (146)	105 (89)
54	49 (72)	None	183 (186)
55	None (60)	146	89 (181)
56	60	None (139)	181 (98)
57	None (70)	139	98 (191)
58	70	None (175)	191 (118)
59	None	175 (145)	118 (88)
60	None (52)	145	88 (184)
61	52	None (158)	184 (102)
62	71	None	187
63	None	158 (172)	86 (117)
64	None (69)	172	117 (190)
65	69	None (173)	190 (94)

	Elementary WSN
Pit ID	13,39
Pass ID	94 (86)
Peak ID	151,173 (138,151)

Table 1 Contraction sequence of Latschur surface network based on maximum of elevation differences criterion. Number in parenthesis is the ID of the point selected by Wolf (1989) for contraction.

Contraction No.	Pit ID	Peak ID	Pass ID
1	52	None	184
2	60	None	181
3	61	None	182
4	62	None	180
5	63	None	123
6	64	None	188
7	65	None	189
8	70	None	191
9	72	None	186
10	237	None	239
11	238	None	240
12	None	144	87
13	None	179	122
14	None	196	195
15	49	None	183
16	54	None	185
17	None	178	121
18	None	134	82
19	None	140	99
20	None	154	124
21	None	158	102
22	None	155	126
23	None	131	79
24	None	149	92
25	71	None	187
26	None	141	100
27	None	171	116
28	None	173	117
29	None	148	91
30	None	166	110
31	None	175	118
32	69	None	190
33	None	164	109
34	None	177	120
35	None	142	101
36	None	169	114
37	None	170	115
38	None	152	95
39	None	156	125
40	None	157	127
41	None	162	107
42	None	146	89
43	None	137	85
44	None	135	83
45	None	168	112
46	None	147	90
47	None	153	96
48	None	161	105
49	None	139	98
50	None	129	77
51	None	128	76
52	None	160	104
53	None	133	80
54	None	136	84
55	None	143	97
56	None	145	88
57	None	163	108
58	None	132	81
59	None	176	119
60	None	159	103
61	None	193	192
62	None	167	113
63	None	151	86
64	None	165	111
65	None	130	78

	Elementary WSN
Pit ID	13,39
Pass ID	94
Peak ID	138,172

Table 2 Contraction sequence of Latschur surface network based on minimum of elevation differences criterion. Number in parenthesis is the ID of the point selected by Wolf (1989) for contraction.

Contraction No.	Pit ID	Peak ID	Pass ID
1	None	196	195
2	None	179	122
3	None	178	121
4	None	149	92
5	None	148	91
6	None	177	120
7	None	154	124
8	None	157	127
9	None	162	107
10	None	147	90
11	None	155	126
12	None	142	101
13	None	163	108
14	None	164	109
15	None	165	110
16	None	170	115
17	None	176	119
18	None	193	192
19	None	128	76
20	None	129	77
21	None	133	80
22	None	166	111
23	None	167	112
24	None	169	114
25	None	159	104
26	None	137	85
27	None	160	103
28	None	130	78
29	None	141	100
30	None	171	116
31	None	131	79
32	62	None	180
33	None	140	99
34	60	None	181
35	None	132	81
36	61	None	182
37	None	134	82
38	None	138 (168)	97 (113)
39	None (65)	168	113 (189)
40	65	None (146)	189 (89)
41	None (49)	146	89 (183)
42	49 (238)	None	183 (240)
43	238	None (143)	240 (97)
44	None	175	118
45	70	None	191
46	None	156	125
47	64	None	188
48	None	153	96
49	None	145	88
50	52	None	184
51	None	135	83
52	None	144	87
53	None	173	117
54	None	143 (152)	84 (95)
55	None (71)	185	54 (187)
56	237	None (161)	239 (105)
57	None (69)	136	102 (190)
58	72 (54)	None	186 (185)
59	None	139 (151)	98 (86)
60	None (63)	158	86 (123)
61	63	None (172)	123 (94)
62	None	152 (158)	95 (102)
63	69 (72)	None	190 (186)
64	None	161 (139)	105 (98)
65	71 (237)	None	187 (239)

	Elementary WSN
Pit ID	13,39
Pass ID	94 (84)
Peak ID	151,172 (136,138)

Table 3 Contraction sequence of Latschur surface network based on sum of elevation differences criterion. Number in parenthesis is the ID of the point selected by Wolf (1989) for contraction.

Case 3: Use of User Defined Contraction to avoid edge intersections and to improve the quality of condensed contour maps (Based on suggestions of Wolf, 1989).

Wolf (1989) observed that the quality of condensed contour maps could be improved substantially if the elimination of the peak 155 and its adjacent pass 126 were shifted from the 10th step to a subsequent one. It was observed that the application of an UDC on nearby peak 156 and its adjacent pass 126 could postpone the contraction of peak 155 to the 35th step. However, since Wolf (1989) did not provide an exact description of the intersection and contour map mentioned in his paper, a comparison on the quality of contour maps with and without the application of UDC could not be done. However, this experiment has been clearly successful in its basic aim of manipulating the contraction sequence.

REVIEW ON SURFACE NETWORK CONCEPTS

Surface networks have great promise as a unified global description of a surface. However, the use of surface networks can become very restrictive, even almost impossible to implement (Pfaltz, personal communication). Some of the limitations in surface networks noticed in this work are described below.

(i) Wolf (1990) regarded a river junction as a pit-pass combination with the distance and elevation difference between pass and pit being infinitesimal (spatial auto-correlation). A ridge junction is similarly a peak-pass-combination. However, as discussed in the last section, this approach has the potential limitation that it makes these junctions prone to homomorphic contractions. These will be regarded as least important edges in a vertex importance based contraction as described in the last section.

(ii) In its current form, surface networks cannot be used to represent realistic drainage networks as information on source nodes is missing i.e., Dry channels or exterior links (as called in “Ridge and Channel networks” terminology) or leaves are not represented. Although some researchers will favour this as they regard exterior channels or leaves not to be significantly important [2]. However, from the cartographic point of view, it creates unrealistic visualisation of a topographic surface, and under-estimation of hydrologic parameters such as catchment area. Therefore, until surface networks do so, it would not be possible for surface networks to become a complete alternative to traditional non-topological drainage/ridge network structures for hydrological calculations. Wolf (personal communication) suggests that the links of the graph can be labeled additionally, for example according to the amount of water in a special river segment with dry channels, obtaining a value of zero.

(iii) Surface networks may only be suitable to model very well developed and dissected fluvial and hilly topography, where clear, distinct point and linear surface features can be identified. Surface networks have not yet been used to model areas with low relief or biased relief such as flat alluvial plains, glaciated surfaces, mainly with pits, so its applicability as a universal model remains yet to proved.

(iv) As WSN is shown as a 2-D Graph based structure, the visualisation for topography by means of a limited set of straight lines is not very appealing. In addition edges may overlap each other (Fig. 6) when their links are updated after a contraction (Wolf, 1989). Wolf (personal communication) suggests that perhaps the only solution is to approximate the (x,y)-projection of a ridge or channel lines by additional points though they will not be surface specific points. The authors feel that the ideal way of viewing WSN would be in the form of a 3D figure in which the user would be able to visualise the differences in elevation far more realistically. The 3D figure would also remove the obstructed views of the edges in cases of intersections. With the current state of programming languages and the ease with which 3D diagrams can be constructed, this aim does not appear to be difficult.

(v) At the time of ordering the pits and peaks, based on their importance measure, two or more pits/peaks may have the same edge weights. Wolf (personal communication) suggests that either of the nodes can be selected arbitrarily as the other node(s) will be contracted in the next step(s) or perhaps an additional criterion for the order in which the nodes can be specified. However it is not sure, and it will be interesting to examine, whether the choice of selection of nodes has any implications in the final surface topology.

(vi) Wolf's proposal that homomorphic contraction can be useful for making simplified contour maps (cartographic generalisation) appears misdirected and needs to be revised. This is because (a) if we apply a single homomorphic contraction on a surface network then it only produces a “local simplified area" but not a “globally simplified surface”; (b) Homomorphic contraction overdoes generalisation. For instance if area A in a part of a surface needs to be simplified then homomorphic contractions wouldn't produce simplified A but a new form B of A by releasing some structural information. Generally, complicated contour maps are not a result of any redundant "critical points and critical lines" but due to the high frequency contour lines, for example rapidly changing elevations over short distances, such as in mountain gullies. High frequency contour lines can be better simplified by traditional line-simplification algorithms or by quadratic approximation kinds of smoothening types of filtering techniques, applied directly over DEM.

CONCLUSION AND SCOPE FOR FUTURE WORK

The authors believe that the actual potential of homomorphic contractions of surface networks is perhaps in the modelling and experimentation of evolution of landscapes, but certain more ideas need to be developed such as:

(i) Surface network will have to lose its straight line - graph like look and adopt a 3D visualisation as shown by *LandSerf* (meandering ridges and channels!)

(ii) Algorithms will need to be developed which would re-generate “topography” around a contracted area keeping in mind the new topological configuration/links. This is essential to maintain realistic visualisation of topographic surface as stressed in point (i) above.

After a surface network has been extracted by an automated routine, it would be necessary to test whether the surface network has all the topological properties proposed by Wolf (1988). Specific procedures would have to be developed to check the consistency of the surface network. Even if these procedures could be developed it appears that any form of shortcomings would require human intervention, perhaps in the form of addition/removal of critical points to maintain the consistency. It is during these sessions that an application like *STT* developed in this work would be most useful, after a raw surface network has been created. This kind of Surface Topology Toolkit could be modified to allow editing of the surface network by adding, deleting and redefining edges. A possible methodology for an automated extraction and generalisation of surface networks could be as follows:

Step 1. Extract critical points and lines from DEM
Step 2. Build surface network
Step 2. Perform homomorphic contraction of surface network.
Step 3. Re-generate the topography of the contracted area in the DEM around topological adjustments and new links. Some form of landform evolution model will have to be used to re-generate the contracted area.
Step 4. Repeat step 1 to 3 till surface network reaches desired stage.

The possibilities look immense and the authors hope that this work will break the long silence in the research on surface networks.

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[1] cdv by Jason Dykes. http://www.geog.le.ac.uk/jad7/cdv/
[2] RiverTools by Scott Peckham. ftp://cses.colorado.edu/pub/RiverTools/Appendix.ps

Appendix 1

Listing of Surface Network Data from Latschur Mountains, Austria (After Dr G.W.Wolf, University of Klagenfurt, Austria. Original Format Modified)

Pits Data Format

X Pit-ID X Y Z [0(surrounding) or 1(internal)]

Passes Data Format

Y Pass-ID X Y Z

Peaks Data Format

Z Peak-ID X Y Z [0(surrounding) or 1(internal)]

Edges Data Format

E Pass ID Pit1ID Pit2ID Peak1ID Peak2ID

X	1	37	113	1000	0
X	3	107	107	1000	0
X	6	147	98	1000	0
X	8	164	117	1000	0
X	9	174	121	1000	0
X	10	189	123	1000	0
X	11	186	135	1000	0
X	12	208	177	1000	0
X	13	258	170	1000	0
X	14	240	179	1000	0
X	15	209	199	1000	0
X	16	214	217	1000	0
X	17	209	252	1000	0
X	18	198	282	1000	0
X	19	228	275	1000	0
X	20	238	259	1000	0
X	21	252	254	1000	0
X	22	257	240	1000	0
X	23	289	230	1000	0
X	24	307	212	1000	0
X	25	326	206	1000	0
X	26	345	195	1000	0
X	58	36	75	1000	0
X	60	181	71	1250	1
X	61	203	55	1310	1
X	62	164	83	1151	1
X	63	283	121	1501	1
X	64	323	103	1505	1
X	65	329	102	1499	1
X	69	355	98	1190	1
X	70	364	96	1120	1
X	71	280	162	1201	1
X	72	212	110	1350	1
X	200	75	103	1000	0
X	204	148	105	1000	0
X	230	131	43	1000	0
X	237	215	62	1410	1
X	238	298	114	1699	1
Y	76	66	89	1450	
Y	77	98	78	1586	
Y	78	140	75	1253	
Y	79	170	60	1630	
Y	80	196	48	1550	
Y	81	219	46	1550	
Y	113	298	183	1620	
Y	114	330	169	1760	
Y	115	335	160	1820	
Y	116	336	148	2050	
Y	117	350	138	2030	
Y	118	373	135	1820	
Y	119	392	125	1620	
Y	120	427	112	1550	
Y	121	433	110	1480	
Y	122	444	109	1399	
Y	123	287	125	1503	
Y	124	334	85	1520	
Y	125	344	82	1520	
Y	126	355	79	1530	
Y	127	368	73	1250	
Y	180	161	85	1153	
Y	181	178	72	1252	
Y	182	201	56	1312	
Y	183	262	49	1201	
Y	184	263	54	1301	
Y	185	259	68	1501	
Y	186	208	111	1352	

X	27	363	180	1000	0	Y	82	234	55	1790	Y	187	275	164	1210	
X	28	376	173	1000	0	Y	83	246	76	1850	Y	188	327	103	1507	
X	29	374	163	1000	0	Y	84	247	85	2020	Y	189	332	102	1501	
X	30	400	165	1000	0	Y	85	258	90	2146	Y	190	356	98	1201	
X	31	417	154	1000	0	Y	86	277	86	2010	Y	191	366	95	1122	
X	32	421	141	1000	0	Y	87	300	93	2085	Y	192	36	97	1299	
X	33	448	135	1000	0	Y	88	283	73	1920	Y	195	460	104	1299	
X	34	452	123	1000	0	Y	89	296	57	1810	Y	239	211	60	1412	
X	35	455	122	1000	0	Y	90	303	47	1610	Y	240	294	116	1701	
X	36	455	88	1000	0	Y	91	313	45	1530	Z	128	51	96	1515	1
X	37	437	85	1000	0	Y	92	325	46	1510	Z	129	90	82	1650	1
X	38	410	72	1000	0	Y	94	312	108	1709	Z	130	118	85	1670	1
X	39	378	89	1000	0	Y	95	320	130	2030	Z	131	157	66	1636	1
X	40	388	75	1000	0	Y	96	331	140	2080	Z	132	182	54	1650	1
X	41	366	53	1000	0	Y	97	242	92	2140	Z	133	209	44	1620	1
X	42	344	61	1000	0	Y	98	221	94	1945	Z	134	232	54	1794	1
X	43	342	60	1000	0	Y	99	185	95	1797	Z	135	237	66	1890	1
X	45	328	27	1000	0	Y	100	169	102	1560	Z	136	247	83	2095	1
X	46	314	25	1000	0	Y	101	155	98	1170	Z	137	248	90	2180	1
X	47	301	23	1000	0	Y	102	241	114	2020	Z	138	241	100	2236	1
X	48	273	30	1000	0	Y	103	242	123	1950	Z	139	212	92	2002	1
X	49	261	51	1198	1	Y	104	248	130	1980	Z	140	183	97	1801	1
X	50	219	26	1000	0	Y	105	265	138	1750	Z	141	168	104	1569	1
X	51	204	29	1000	0	Y	107	213	271	1150	Z	142	152	99	1190	1
X	52	263	58	1299	1	Y	108	222	251	1150	Z	143	270	92	2221	1
X	53	172	38	1000	0	Y	109	241	225	1380	Z	144	282	78	2087	1
X	54	259	71	1498	1	Y	110	250	213	1570	Z	145	285	70	2008	1
X	56	101	59	1000	0	Y	111	257	206	1420	Z	146	300	55	1843	1
X	57	42	71	1000	0	Y	112	283	191	1680	Z	147	307	45	1652	1
Z	148	317	44	1540	1	E	96	71	65	153	E	88	52	43	144	145
Z	149	330	47	1517	1	E	97	237	63	137	E	89	49	43	145	146
Z	151	306	97	2218	1	E	98	237	72	138	E	90	47	43	146	147
Z	152	317	127	2054	1	E	99	60	9	139	E	91	46	43	147	148
Z	153	326	135	2130	1	E	100	62	8	140	E	92	45	43	148	149
Z	154	339	83	1524	1	E	101	62	204	141	E	94	63	64	151	152
Z	155	347	82	1535	1	E	102	72	63	138	E	95	71	64	152	153
Z	156	359	78	1549	1	E	103	11	63	158	E	195	35	35	36	179
Z	157	370	69	1280	1	E	104	12	63	159	E	239	61	61	237	139

```
Z  158  242  120  2024    1
Z  159  242  129  2060    1
Z  160  253  133  2046    1
Z  161  274  136  1805    1
Z  162  215  275  1180    1
Z  163  216  261  1247    1
Z  164  236  240  1391    1
Z  165  242  214  1643    1
Z  166  255  212  1580    1
Z  167  273  197  1770    1
Z  168  294  185  1720    1
Z  169  310  180  1780    1
Z  170  333  164  1840    1
Z  171  333  150  2059    1
Z  172  333  143  2142    1
Z  173  361  137  2039    1
Z  175  382  134  1830    1
Z  176  409  113  1720    1
Z  177  430  110  1564    1
Z  178  436  110  1483    1
Z  179  451  107  1401    1
Z  193   30   94  1416    1
Z  196  465  107  1301    1
E   76  200   57   128  129
E   77    3   56   129  130
E   78    6  230   130  131
E   79   62   53   131  132
E   80   61   51   132  133
E   81   61   50   133  134
E   82   61   49   134  135
E   83  237   52   135  136
E   84  237   54   136  137
E   85   54   63   137  143
E   86  238   54   143  144
E   87  238   43   144  151
E  105   13   63  160
E  107   18   19  162
E  108   17   20  163
E  109   16   21  164
E  110   15   22  165
E  111   14   23  166
E  112   13   24  167
E  113   71   25  168
E  114   71   26  169
E  115   71   27  170
E  116   71   28  171
E  117   69   29  172
E  118   70   30  173
E  119   39   31  175
E  120   38   32  176
E  121   37   33  177
E  122   36   34  178
E  123   63   71  152
E  124   42   64  151
E  125   42   65  154
E  126   41   69  155
E  127   41   40  156
E  180    6   62  131
E  181   60   62  132
E  182   60   61  132
E  183   48   49  134
E  184   49   52  135
E  185   52   54  136
E  186   10   72  139
E  187   13   71  161
E  188   64   65  153
E  189   65   69  155
E  190   69   70  156
E  191   70   39  156
E  192    1   58  128
E  240   63   63  238  143
```