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EXPERIMENTS ON THE GENERALISATION AND VISUALISATION OF SURFACE NETWORKS

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ABSTRACT

Parameterisation of a topographic surface into a framework of fundamental points and lines is a prominent research topic in geographic information science. Metric Surface Network also called Pfaltz's graph or surface network is an example of such a framework. Surface Network is a graph topological data structure and pits, passes, peaks are its three sets of vertices while ridges (lines from passes to peaks) and channels (lines from pits to passes) make up the edge set. Each point and line is assigned a weight, which indicates their importance in the surface. The most significant aspect of surface network is their ability to undergo generalisations by a graph theoretic technique called homomorphic contraction, which reduces the number of points and lines but preserves the topological structure of the corresponding topographic surface. This paper presents a review of the generalisation and visualisation methods by experimenting with two surface networks, one hypothetical and the other from Latschur region in Western Carinthia, Austria. Experiments revealed that the traditional weight measure i.e., difference in elevation between a pair of linked points, may not be able to provide desired generalisation and as an alternative three new weights measures namely edge lengths, edge slopes and degree (or valency) of points are proposed. This work also proposes the use of 3D visualisation for surface networks to understand the topological relations between points more clearly.

Introduction

Parameterisation of the continuously varying topographic surface into a discrete topological data structure or framework is an area of consistent investigation in spatial information science. A prominent example of such surface topology data structures is the Metric Surface Network also called Pfaltz's graph (Pfaltz, 1976; Wolf, 1988; Wolf, 1990) or simply surface network. Surface network is a graph theoretic based topological data structure, which has proved to be useful for both characterising and generalising the form and topology of topographic surfaces[1]. The basic aim of these topological data structures is to use less storage space and allow faster heuristics but yet be able to represent both the major form (geometry) and structure (topology) of the topography, which is particularly useful for applications such as generalisation, drainage network analysis and route planning. However in spite of their powerful capabilities of manipulating the structure of topography, not much work has been done to implement surface network and understand their properties. The aim of this paper is to revive this idea and report on the characteristics of the

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[1] Weighted Surface Network is the term used for a surface network, which only stores the topology relations between the fundamental points and lines, and thus not very useful for cartographic visualisation and morphometric calculations. Metric Surface Network is in fact WSN embedded in metric space.

generalisation methods and the use of 3D visualisation in surface networks. The next section presents a background on surface networks and also sets the aims of the experiments in this work.

Surface Networks - Background

In simple terms surface networks store topography as a graph made of the fundamental points (vertices) namely pits, passes and peaks, and the fundamental lines (edges) namely ridges (lines from passes to peaks) and channels (lines from pits to passes) (Figure 1) of a surface. However not all topographic surfaces and all such graphs can be used for surface networks but only the ones which satisfy the definition of topographic surface given below and the topological rules deduced by Pfaltz (1976) and Wolf (1988).

Definition of the topographic surface:

1. The surface should be twice continuously differentiable, which means it can not have sudden slope breaks or overhangs,
2. The surface should be defined over a domain, which is simply connected which means it can not have holes, and
3. The surface is bounded by a closed contour line, represented as either a surrounding pit or surrounding peak.

In nature, there are two other points on the surface namely junctions and bifurcations, which are not fundamental as an entity but have significant importance in structure of topography. Wolf (1990) proposed that channel junction/bifurcation and ridge junction/bifurcation could be represented as an infinitesimally close interpolated pair of pit-pass and pass-peak, respectively. Each vertex and edge is assigned a real positive number (a weight), which represents their importance in the surface. A typical weight measure is the difference in elevation between a pair of linked peak-pass and pass-pit. In case of junctions and bifurcations, an arbitrary low weight can be assigned to indicate their proximity, for example Wolf (1990) used a value of 2.

Apart from being compact and powerful data structures, due to the fundamental nature of surface network components and use of formal topological constraints, the most significant aspect of surface networks results from their property of being condensable by two homomorphic contractions called pit-pass contraction and pass-peak contraction. These contractions reduce the number of vertices and edges but preserve the topological structure of the corresponding topographic surface. In general, the pit-pass- and pass-peak- contractions cause the removal of an “internal” pit (the surrounding pit is not allowed to be selected for contraction) and an “internal” peak (the surrounding peak is not allowed to be selected for contraction) respectively with their lowest (in the case of pit) and highest (in the case of peak) adjacent pass together with all surface-specific lines incident with at least one of these critical points. The pass connected to the contracted pit (or peak), which becomes free, is then connected to the pit (or peak) originally linked to the contracted pass, thus restoring topological consistency (Figure 2). The complete mathematical proof of the contractions is provided in Pfaltz (1976) and Wolf (1989)**.** The selection of a pit or peak is based on an importance criterion, which depends upon the particular problem and the topography. Three possible importance criteria for the selection of pits and peaks, based on edge weights are:

- **The maximum of the elevation differences between a peak or pit and all its adjacent passes.** This measure can be used to remove peaks and pits ranked on the basis of the steepest ridge and channel linked to them.

- **The minimum of the elevation differences between a peak or pit and all its adjacent passes.** This measure can be used to remove peaks and pits ranked on the basis of the shallowest ridge and channel linked to them.

- **The sum of the elevation differences between a peak or pit and all its adjacent passes.** This measure can be used to selectively remove pits and peaks with low number of crossings.

The original ideas on generalisation and weight measures from Wolf (1988) however have limitations, which restrict the implementation of surface networks. The following list presents a compilation of the doubts raised by Rana (1998) and fresh proposals on weight measures.

Condensation:

> Sequential condensation of surface networks does not provide flexibility to the user to generate a desired topology and topography. Wolf (1989) experienced a typical limitation. He observed that the quality of condensed contour maps could be improved substantially if the step to eliminate a peak and its adjacent pass were shifted to a subsequent one. It is also apparent that vertex importance based selection criteria are insensitive to the ridge or channel structure at a peak or pit. This means that edges are solely selected for condensation, based on their weights and no consideration is given to the size or significance of the host structure (such as length of edges), which may not be suitable in some cases. Rana (1998) proposed the **User Defined Contraction (UDC),** in which a user can arbitrarily select an internal pit or peak for removal, which allows not only the flexibility desired above, but also the power to create experimental surface networks.

Weights:

  - A drop in elevation as a weight can be misleading regarding the importance of an edge. For example, long gradually sloping ridge or channels are significant structurally but they will be given a low weight, thus making them vulnerable for contractions.

- The sum of edge weights will not be able to differentiate between two equally weighted points but with a different number of edges. Thus it is not a good indicator of the ridge and channel crossings at a point.

This work proposes the use of edge lengths, edge slopes and degree (or valency) of points as alternative weights to avoid these limitations and examples of these will be shown in the next section. The use of edge length is also more effective for cartographic generalisation as it brings out high frequency elevation changes in surfaces for condensation.

- One more doubt in current ideas is the lack of suggestions to decide between equally weighted points.

Visualisation:

Due to the look of a straight-line graph, the visualisation of surface networks in 2-dimensions can give an impression of non-planarity to a user, when edges overlap during condensations. This also limits the user's scope of generalising the surface network. Rana (1998) proposed the use of 3D visualisation, which would show the relations between points clearly. This proposal has been implemented in this work and an example of its use will be shown in the next section.

Experimentation

Methodology

Experimentation involved carrying out condensation of two surfaces both generated from original data by Wolf (1991)(Figure 3a) and Wolf (1989)(Figure 4a). Figure 3a is a hypothetical surface while Figure 4a is a surface from an area in the Latschur Mountains of the Western Carinthia region in Austria. The surface networks (Figure 3b, Figure 4b) for the surfaces were created manually i.e., by identifying the fundamental points and their relations manually. It is worthwhile here to note that automated generation of surface networks has a number of difficulties still to be overcome for various reasons (Rana, 1998; Wood, 1998). The original format of the surface network was modified slightly into the following format.

Points						
Point	Column 1	Column 2	Column 3	Column 3	Column 4	Column 5
Pit	*X*	ID	x-co-ordinate	y-co-ordinate	z-co-ordinate	0 (if surrounding) or 1 (if internal)
Pass	*Y*	ID	-do-	-do-	-do-	
Peak	*Z*	ID	-do-	-do	-do	0 (if surrounding) or 1 (if internal)

Lines					
Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
E	Y_{ID}	$X1_{\mathrm{ID}}$	$X2_{\mathrm{ID}}$	$Z1_{\mathrm{ID}}$	$Z2_{\mathrm{ID}}$

For example part of the data for the Figure 4b surface network is as follows:

Y	y4	1.61	0.58	1150	
X	x5	0.77	0.45	1000	0
Z	z6	2.74	0.35	2200	1
E	y1	x1	x2	z1	z2

The generalisations were carried out using an application, *Surface Topology Toolkit* (STT), developed by the author in Tcl/Tk. Tcl/Tk is becoming a popular language amongst GIS programmers (Dykes, 1997). The highlight of Tcl/Tk functions is the provision of dynamically manipulating the properties of graphical objects with ease and speed, which is particularly useful for cartographic and other visualisation applications. Owing to the Graphic User Interface (Figure 5) and UDC present in *STT* a user is able to achieve considerable improvement over the earlier methods for the generalisation and visualisation of surface networks (Wolf, 1991). The other main advantages of *STT* are as follows:

- *STT* informs the user of every contraction (except for continuous contractions) so that a selection can be made more intuitively.
- Users can generalise the topography by a combination of importance measure rather than a single one and can also arbitrarily select an internal pit or peak for contraction.
- Users have the flexibility to undo a contraction to observe the changes in results for better generalisation.

An Arcview Avenue script has been implemented, which converts 2D surface networks into Arcview 3D-shape files so that they can be seen in 3D using Arcview 3D-Analyst extension.

This work has the following experimental aims:

1. To compare the effectiveness of drop in elevation, edge length and valency weight measures.
2. To compare the effect of maximum, minimum and sum of edge weights contraction criteria on the topography.
3. To use UDC to generate artificial landform changes.
4. To use 3-Dimensional visualisation to test topological consistency of surface network with overlapping edges.

Results

Effectiveness of drop in elevation, edge length and valency weight measure

As mentioned earlier the aim of using maximum and minimum edge weight criteria is the removal of peaks/pits based on respectively the steepest and shallowest ridges/channels linked to them. However, as would be expected a drop in elevation weight does not take into account the length of the edges, therefore it makes long edges vulnerable for condensation. For example, for the surface network shown in Figure 6a, the next maximum drop in elevation weight criterion based condensation will remove the ridge *[y1, z2]* (Figure 6b) although it is longer, thus more important, than some of the other ridges in the surface. On the other hand, maximum edge length weight criterion based condensation selects to remove the ridge *[y1, z1]* (Figure 6c) and therefore is a more sensible measure. However, it is important to note that even after a better decision the ridge *[y1, z2]* is still removed due to topology condensation rules, which proves the earlier stated proposal, that condensation solely based on weights, ignores the structure of ridge/channel networks.

Sum of edge weights and valency criteria are used to remove peaks/pits based on the ridge/channel crossings at them. The aim is to keep higher degree peaks or pits as they represent crossings of different ridge and channel lines and are therefore of great importance for the topography of the given area. A comparison of the condensation sequences based on the sum of drop in elevation weight criterion and valency weight criterion reveals that the later criterion identifies ridge/channel crossings more uniquely than the earlier criterion. Figure 7 shows the situation in which of sum of edge weight criterion selects to remove the ridge *[y4, z5]* (Figure 7b) although the peak *z5* has got the highest number of ridge crossings and is therefore a misleading condensation. On the other hand, valency weight criterion selects the ridge *[y5, z6]* (Figure 7c), which is closer to the expectation.

Effect of maximum-, minimum- and sum of edge weight contraction criteria on the topography

Surface networks from different contraction measures were collected after each 15 steps of contraction of the Latschur surface network (Figure 5a,b) to compare the generalisation of the corresponding topographic surface (Figure 8). The following striking observations were noted:

- Contractions based on minimum of edge weights measure produced intersections between edges very early in the generalisation process but it was the only one to maintain the continuity of the ridge structure through the entire area. Even after 60 contractions, the ridge structure could be observed in the surface network (Figure 8 viii). This indicates that minimum of edge weights measure would be the most suitable of the three measures for generalisation of topography based on a surrounding pit, if the major structure is to be retained.

- Contractions based on sum of edge weights (Figure 8 ix - xii) and valency result in the maximum and fastest contraction of the topographic structure (meaning loss of topography), moving from periphery (because of degree one edges) towards the centre.

Use of UDC to generate artificial landform changes

Study of landform evolution is a very useful topic of research in order to understand the geomorphic and tectonic phenomena in nature. Researchers use some form of landform models to simulate changes and predictions, but often require detailed mathematical analysis. As an alternative, this work proposes that UDC can be used to introduce similar changes more easily and quickly. An example of the generation of a NW-SE trending artificial valley in the Latschur surface network is shown in Figure 9. This valley was achieved simply by merging minor channels in this area and the removal of the intersecting ridges along these channels. However, as its apparent, the changes are purely topological and one of the main advantages of other landform evolution models is their ability to regenerate the topography.

Use of 3-Dimensional visualisation to view overlapping edges

A typical example of overlapping edges is shown in Figure 10a, taken at the stage of first intersection of edges (inside the dotted black circle) after the 34th step of contraction based on maximum of elevation differences measure. The same network in 3-Dimension (Figure 10b) clearly shows that the edges do not intersect and surface network remains topologically consistent.

Conclusion

Surface Networks have an immense potential for future research and some key areas which needs to be addressed are:

- Detailed understanding of the property and effects of each of the condensation criterion.
- Development of routines of the automated extraction of surface networks. An essential issue in this research will be to implement surface network for biased topographic forms such as glaciated-, karstic-, volcanic- surfaces as they would not have clearly defined surface network components.
- Development of models for the regeneration of topography incorporating topological settings. One aspect associated with this research will be the transformation of surface networks from a 2D straight-line graph like appearance to a more realistic (meandering ridge and channels) representation for realistic visualisation and morphometric analysis.

Acknowledgements

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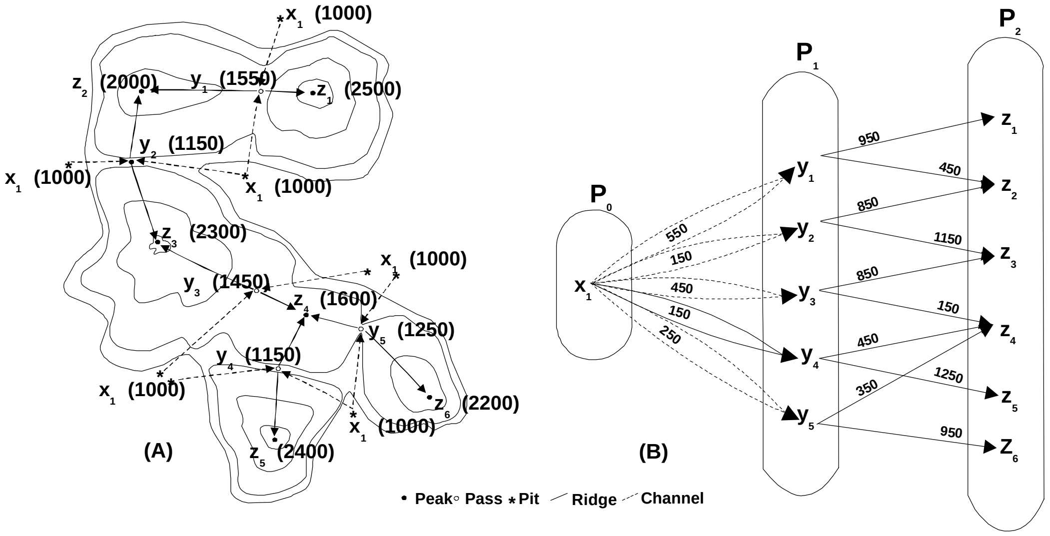


Figure 1. (A) Pits, passes, peaks, ridges and channel lines on a topographic surface (numbers in the parentheses indicate altitude of points) and its (B) Surface Network as a tripartite graph, where P_0, P_1, P_2 are the three vertex sets representing the sets of all pits, passes and peaks, respectively. Edge weights are indicated on the edges (From Wolf, 1991).

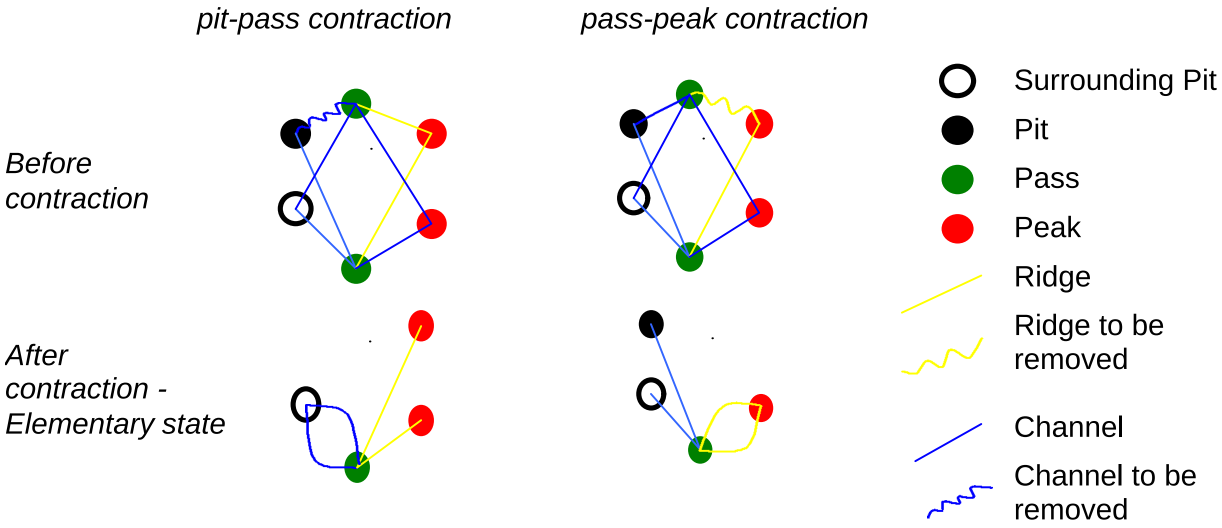


Figure 2. Mechanisms of Homomorphic Contractions.
Note: Legend on the right will be used hereafter for surface networks.

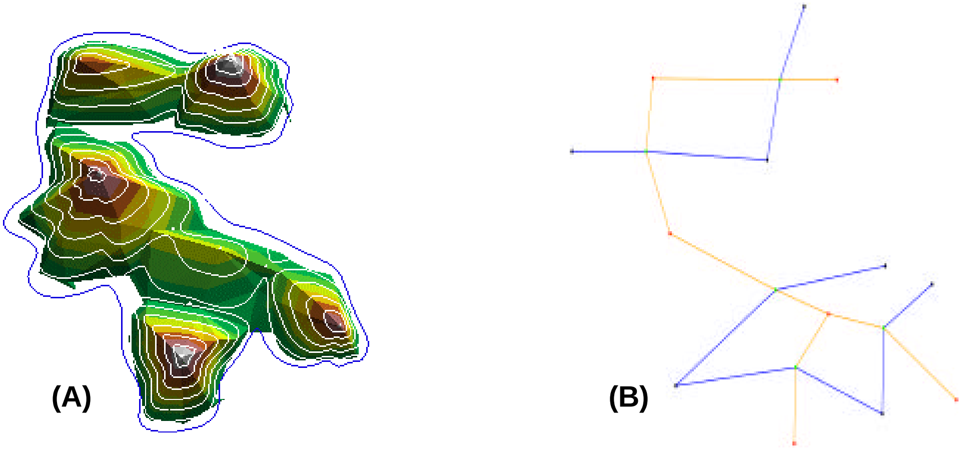


Figure 3. (A) Topographic surface (same as Figure 1) and its (B) surface network. Blue contour represents the surrounding pit.

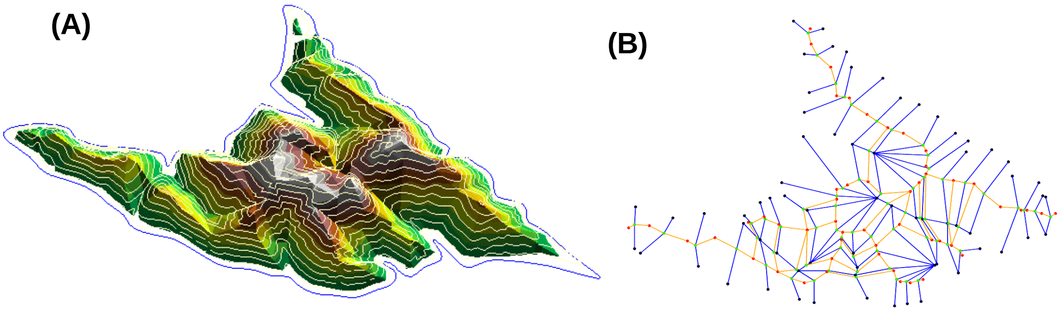
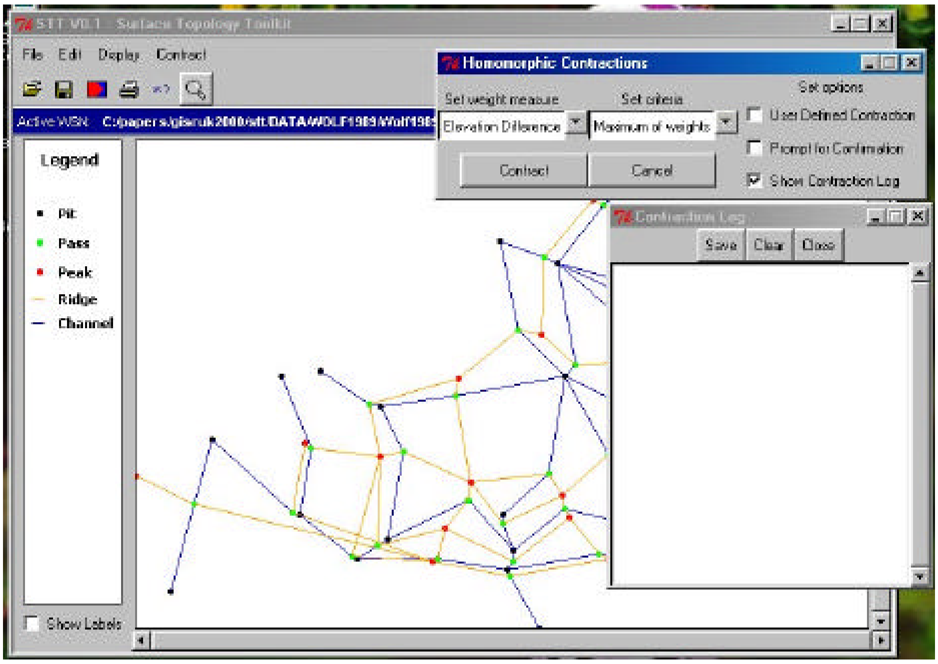


Figure 4. (A) Landscape of a portion of the Latschur Mountains in Western Carinthia, Austria and its (B) surface network. Blue contour represents the surrounding pit.



Active WSN: C:/papers/gisruk2000/stt/DATA/WOLF1989/Wolf198

Set weight measure: Elevation Difference

Set criteria: Maximum of weights

Set options
☐ User Defined Contraction
☐ Prompt for Confirmation
☑ Show Contraction Log

Figure 5. The Graphical User Interface of the Surface Topology Toolkit application with the functionalities available for contractions.

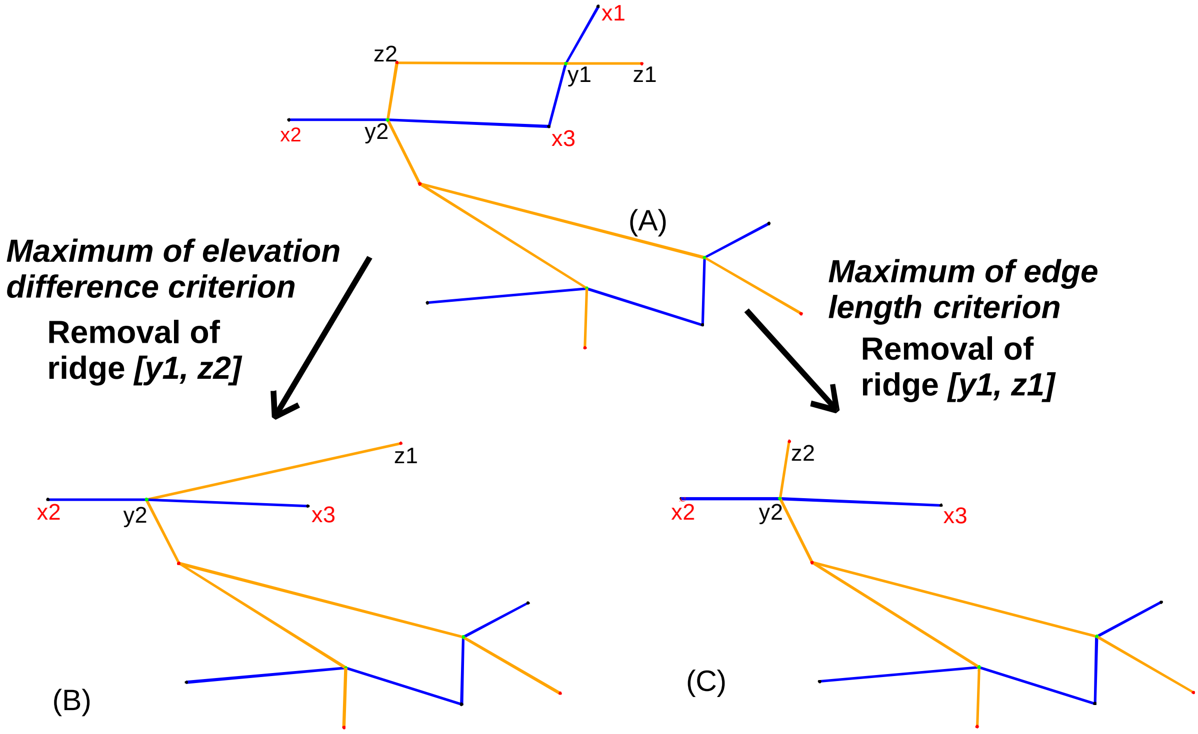


Figure 6. Comparison of the effectiveness for selection of points in the surface network (A), between maximum of elevation difference criterion (B) and maximum of edge length criterion (C). Note that criterion (B) selects a long ridge due to its low drop in elevation (350).

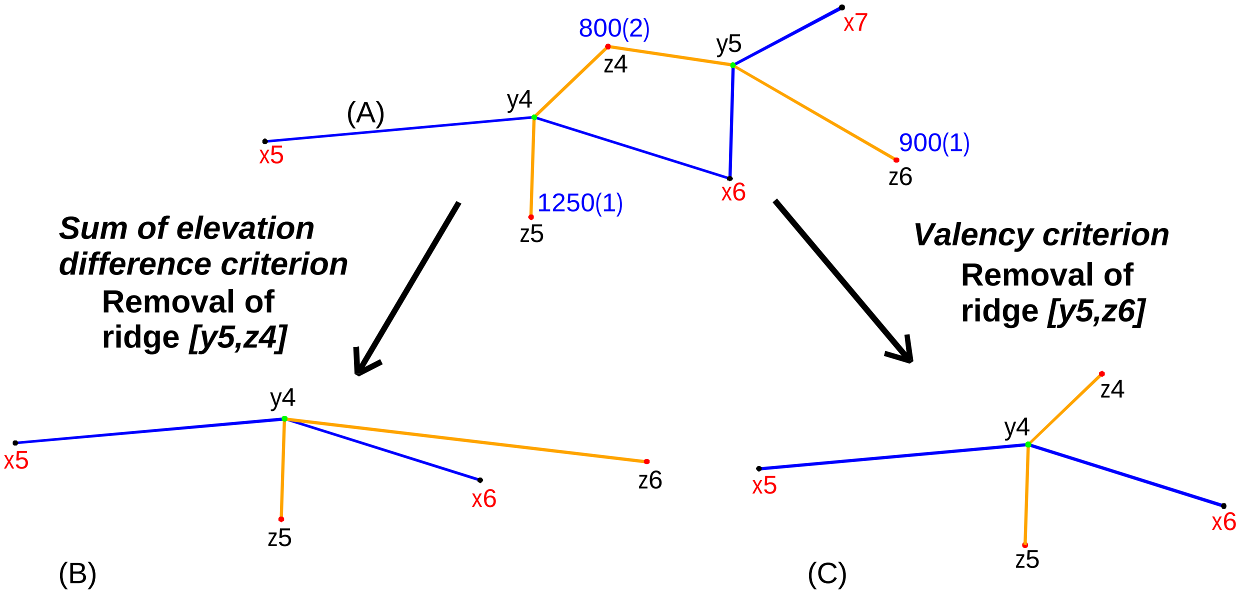


Figure 7. Comparison of the effectiveness for selection of points in a (A) surface network, between (B) sum of elevation difference criterion and (C) valency criterion, showing how criterion (B) can mislead about the ridge/channel crossings. Numbers at peaks in (A) are sum of elevation differences and their valencies (in parentheses).

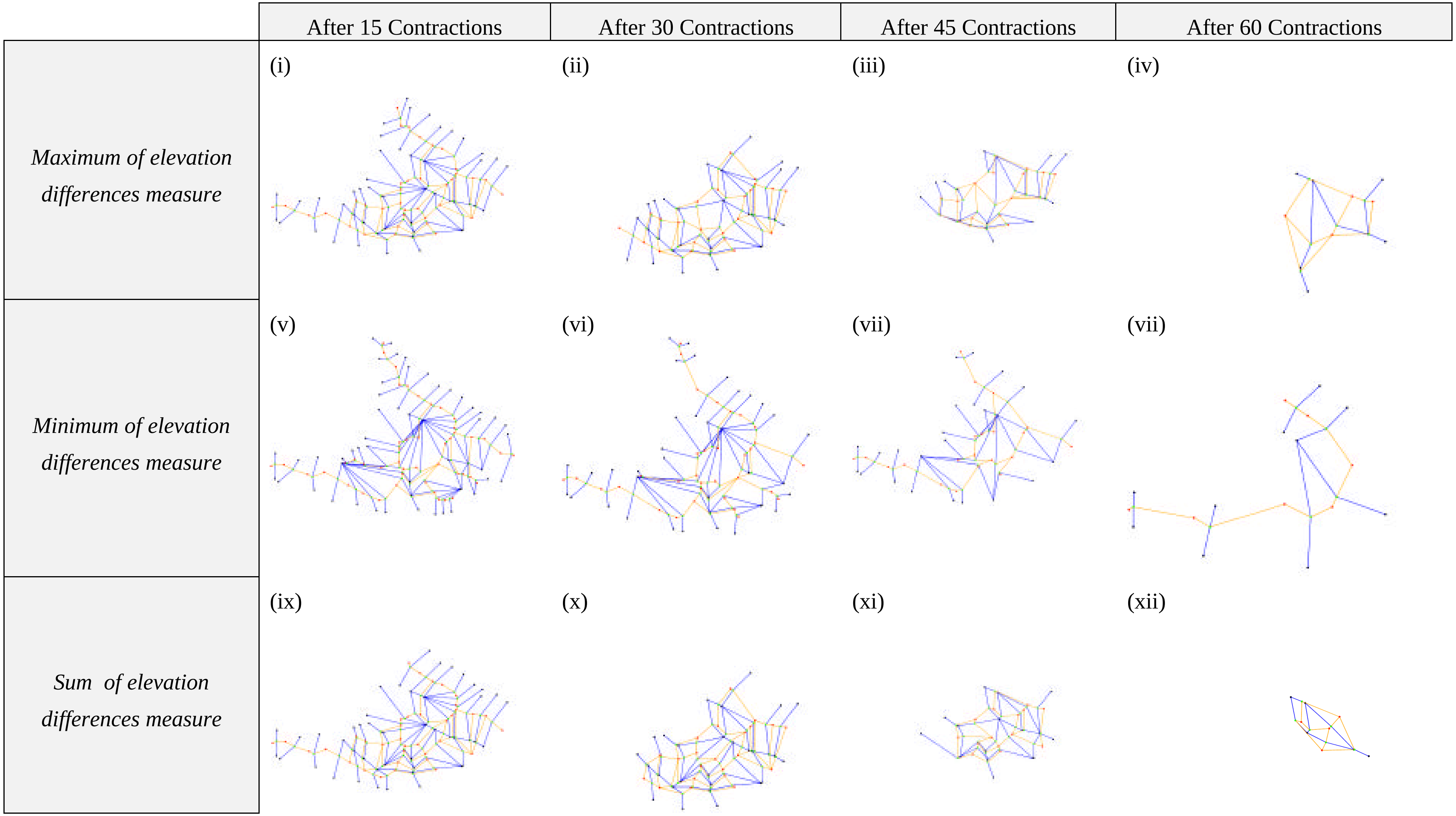


Figure 8. Surface Networks of Latschur area after each 15 contractions based on maximum of elevation differences criterion (i-iv), minimum of elevation differences criterion (v-viii) and sum of elevation differences criterion (ix-xii).

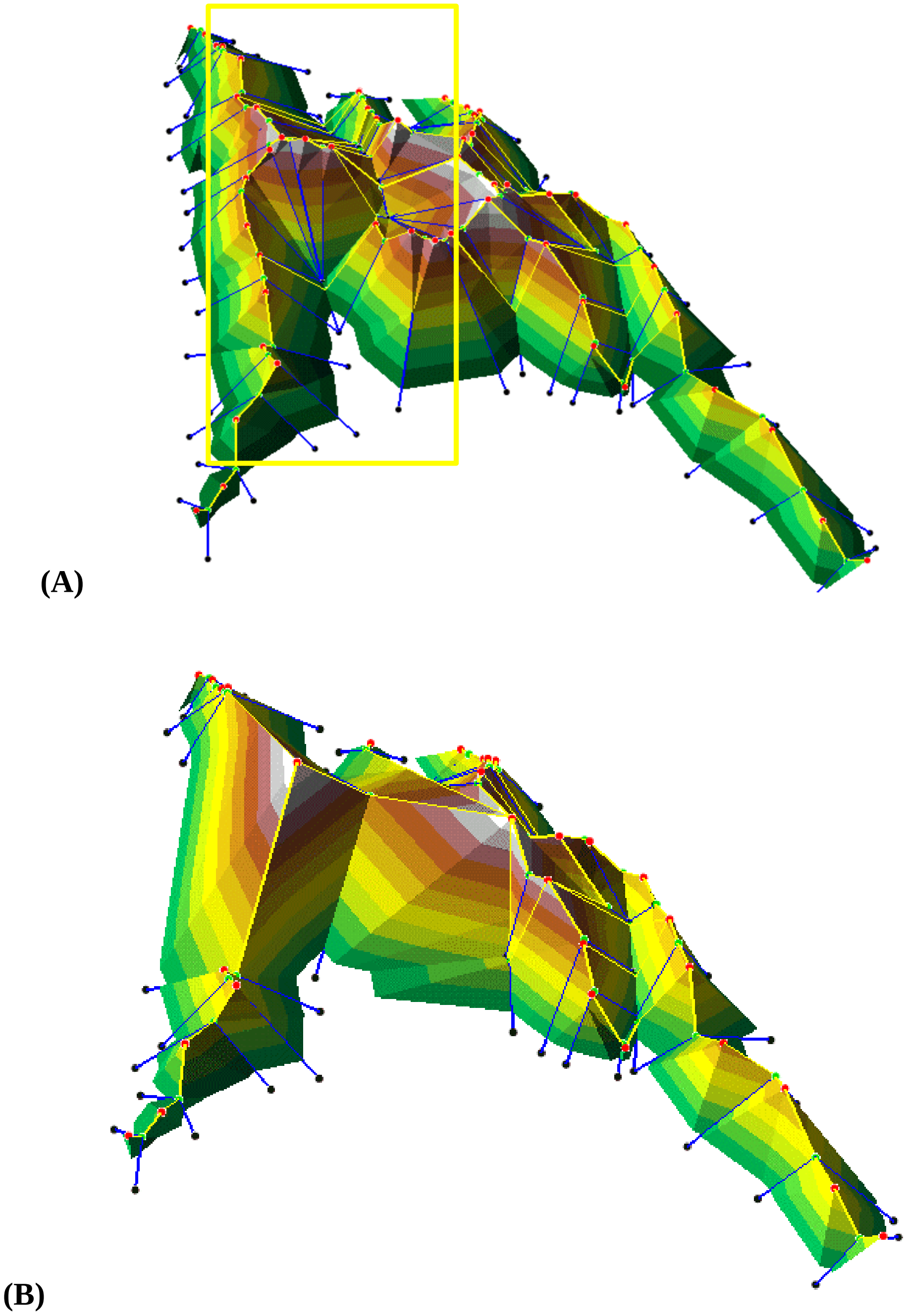


Figure 9. Generation of an artificial valley in the shaded region of a surface network.

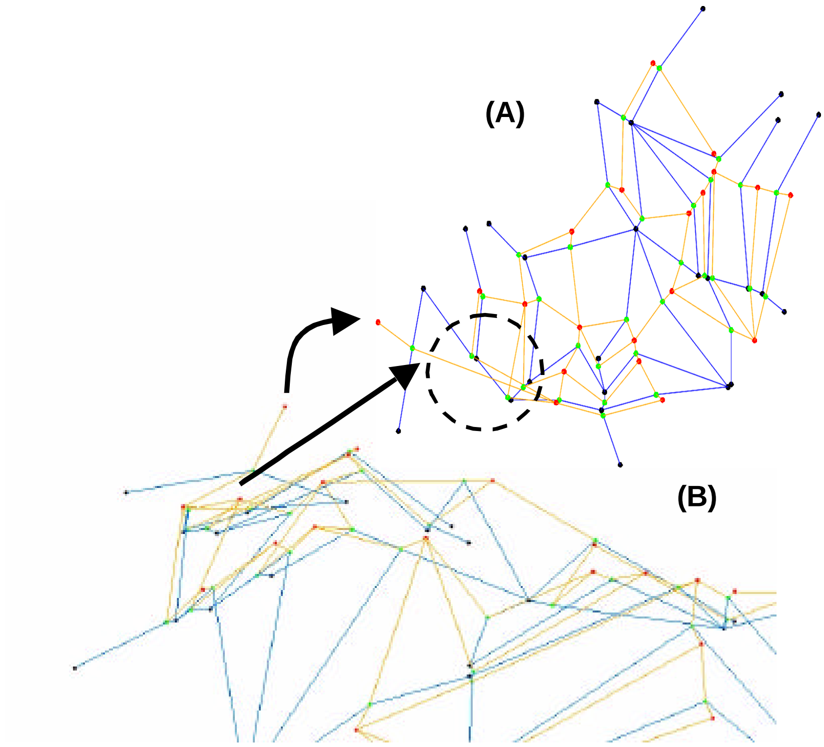


Figure 10. (A) Latschur surface network showing an apparent intersection of the edges in 2D inside the dotted circle. (B) The same surface network in 3D shows that the ridges are above the channels and the topological consistency is maintained. Arrows connect corresponding features in the surface networks.