UCL CENTRE FOR ADVANCED SPATIAL ANALYSIS

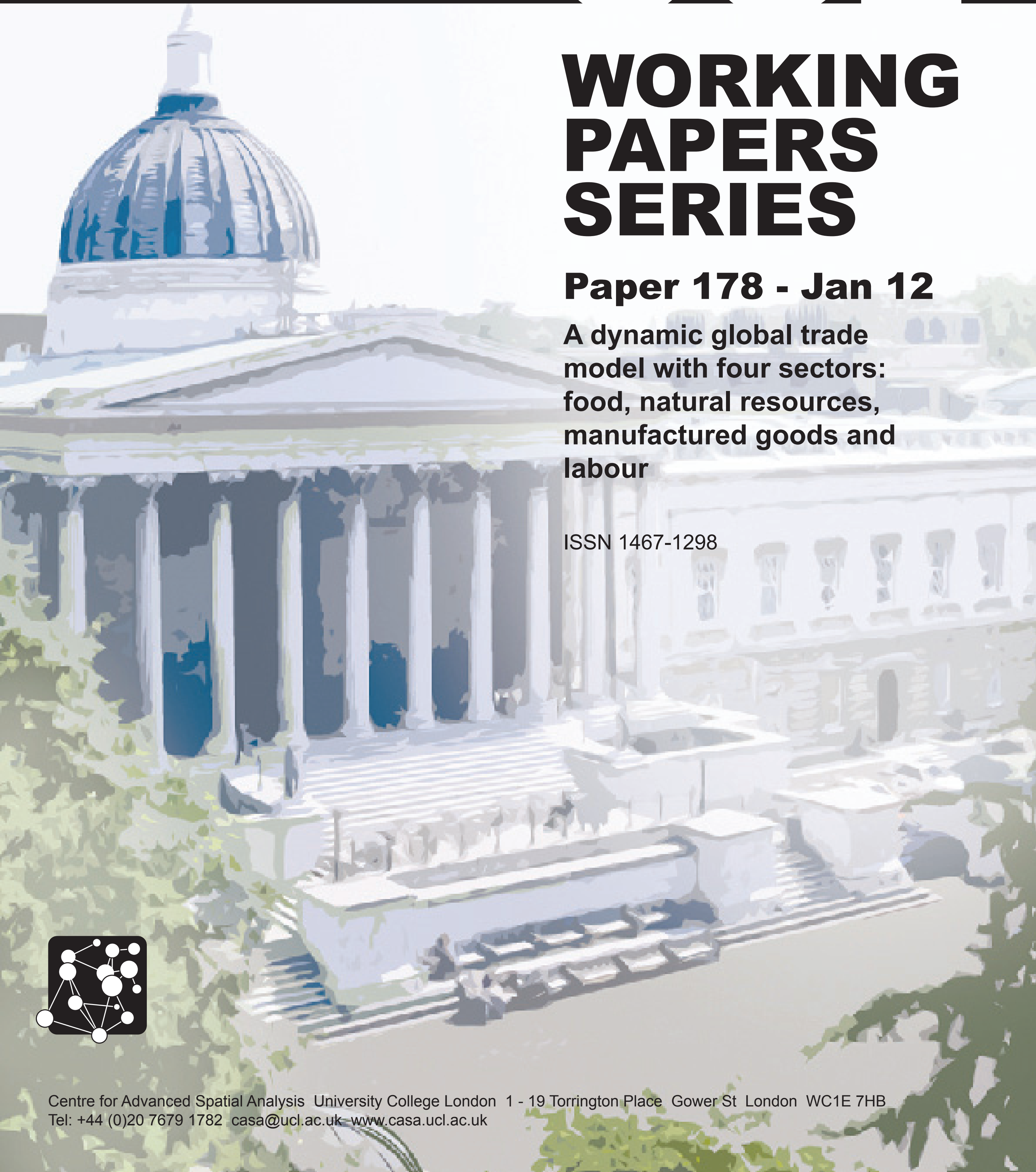
UCL

WORKING PAPERS SERIES

Paper 178 - Jan 12

A dynamic global trade model with four sectors: food, natural resources, manufactured goods and labour

ISSN 1467-1298



Centre for Advanced Spatial Analysis University College London 1 - 19 Torrington Place Gower St London WC1E 7HB
Tel: +44 (0)20 7679 1782 casa@ucl.ac.uk www.casa.ucl.ac.uk

A dynamic global trade model with four sectors: food, natural resources, manufactured goods and labour

Hannah Fry and Alan Wilson

Department of Mathematics and the Centre for Advanced Spatial Analysis,
University College London, WC1E 6BT

Abstract

An important and long-standing research task is the building of a model of international trade flows, anchored in a model of national economies. A demonstration model is presented which aims to exhibit the principal phenomena of the real system. Existing trade models, such as the Heckscher–Ohlin model, make certain kinds of simplifying assumptions, such as equal production functions and wages across all countries. We seek to make the model more realistic by making simplifying assumptions in a different way. Entropy-maximising spatial interaction submodels and Lotka-Volterra-type dynamics are deployed in a model that contains four representative sectors: food, natural resources, manufactured goods and labour. The model uses price as a mechanism to determine production levels. In this way, we incorporate country-specific dynamics of labour, consumption and individual income, while distinguishing between resource-rich/poor and GDP-rich/poor countries. The model is built with further disaggregation in mind, and possible extensions are discussed along with some experimental results [1]

1 Introduction

In economics and policy, it is important to understand the mechanisms of international trade, both in analysing existing links and in testing the impacts of, for example, changing technologies and network resilience. Ideally, this would be done on the basis of input-output models for each of the 200 or so countries, each of which would include import and export flows by sector and country of origin or destination respectively.

This can be done in theory, for example using the methods of [6] as applied in an inter-regional context by [3]. However, the data sources are to say the least imperfect and it is an enormous task to assemble what is known in these terms.

An objective of research in these areas therefore, is to create a model of national economies and trade flows that is feasible in scale, and capable of replicating the principle phenomena of the full system. In recent years, several such models have been presented with varying success. Gravity models for example which concentrate on the influence of spatial structure have been validated empirically, but may not take comparative advantage into account. The alternative Heckscher–Ohlin model which is based on comparative advantage, is capable of handling country specific capital and labour markets, but cannot include unemployment or wage discrepancies across countries and demonstrates poor predictive power on an international level.

[1]The authors acknowledge the financial support of the Engineering and Physical Sciences Research Council (EPSRC) under the grant ENFOLD-ing - Explaining, Modelling, and Forecasting Global Dynamics, reference EP/H02185X/1.

	Farming	Mining	Manufacturing
Product	Food	Resources	Goods
Consumed by	Population	Manufacturing	Population
Unit of production constraint	Volume	Volume	Money
Paid for by	Income	Goods prices	Income after food purchases
Cost prices depend on	Labour costs	Labour costs	Labour & natural resources costs

Figure 1: A summary of the input and outputs of the model.

In this paper we present a novel approach intended to tackle some of these issues, following the principles established in [7]. A key feature of the model is the distinction between countries that are rich and poor in terms of Gross Domestic Product (GDP) per capita, and those that are rich and poor in terms of natural resources such as oil or precious metals. This produces a 2x2 classification of countries and allows us to incorporate country specific dynamics of labour, unemployment, income, and production capacity.

We assume that the population needs food, but that food consumption is different for each country, reflecting income. Individuals also consume manufactured goods, and for the purposes of simplification, we assume that the balance of expenditure by the population on food is spent on manufactured goods.

The manufacturing industry on the other hand is the sole consumer of natural resources, while the three production sectors: food, natural resources and manufacturing, all require labour inputs. Thus, the four sectors are defined and the implicit input-output model is very simple. A summary of the system may be seen in Figure 1.

At the core of the trade model is a set of spatial interaction models for the three production sectors. These are built on entropy maximisation - see for example [2] as applied to retail models of consumer spending flows, [5], [1], [4] as models for inter-regional migration flows, and [3] [6] applied to trade flows. A description may also be found in [7], where the model presented here is both an extension and simplification of that work. This standard spatial interaction model, however, requires an adaptation to allow for varying production levels, and we introduce a variable selling price as a mechanism to do this.

At each time step, each sector of each country enters the global market place with a production capacity: a volume which they may not sell above, and a cost price which they may not sell below. The global competition then takes the form of an iterative algorithm: increasing the selling price whenever demand exceeds capacity and reducing the production whenever a sector cannot sell at capacity.

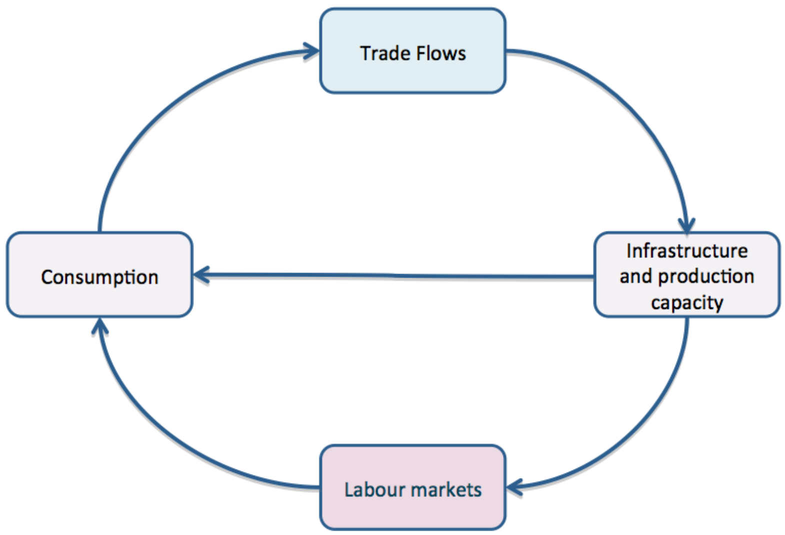


Figure 2: A schematic diagram of how various elements of the model interact

At the end of the trading process, the national government reinvests any profits into proportionally increasing the capacity of their best performing sectors, thus adding a dynamic element to the model. A schematic diagram of how the various elements of the model interact is given in Figure 2.

To put this work into context, the motivation for this model is as a first step in a global demonstration model, incorporating the coupled dynamics of migration, international aid and security. Thus the structure is built with future disaggregation in mind, and designed to cope with varying populations and national and international investment.

In Section 2 we define all the variables used to describe the system of interest. The pricing and trade flows algorithm which forms the core of the trade model is presented in Section 3. This algorithm requires inputs of production capacity, cost prices, a measure of the quality of goods and transport costs. Theoretically, these measures should be available from data and treated as exogenous. For the purposes of our demonstration model however, we have created estimates of these quantities, which are outlined in Section 4. Once these values are known, the pricing and trade flows algorithm, with minor modifications, may be applied to each of the producing sectors. The process follows in detail in Sections 5, 6, 7. Finally, the dynamics of the model are demonstrated in Section 8 and the results of an experimental run are described in Section 9.

2 Definition of variables for system description

The three producing sectors, as we have seen, are food, natural resources and manufacturing. These are labelled by a superscript $n = 1, 2, 3$ respectively.

The countries are each given a label denoted by subscript i when the country is acting as a seller, and subscript j when acting as a buyer. Thus at each time step (between t and $t + \delta t$) food, resources and manufactured goods flow from i to j. The variables of interest are as follows:

$\{P_i\}$ - Population of country i.

$\{u_i\}$ - Number of unemployed in i.

$\{\iota_i\}$ - Income per capita of country i.

$\{L_i^{(n)}\}$ -Work force of sector n in i (so that $\sum_n L_i^{(n)} + u_i = P_i$)

$\{\bar{X}_i^{(n)}\}$ - Capacity of production of n in i.

$\{X_i^{(n)}\}$ - Actual production of n in i over δt.

$\{\chi_i^{(n)}\}$ - Monetary value of $X_i^{(n)}$.

$\{Y_{ij}^{(n)}\}$ - Trade flows of n between i and j.

$\{\psi_{ji}^{(n)}\}$ - Money flows from sales of n between j and i/Monetary value of $Y_{ij}^{(n)}$

$\{Z_i^{(n)}\}$ - Consumption of n in i.

$\{\zeta_i^{(n)}\}$ - Monetary value of $Z_i^{(n)}$.

$\{q_i^{(n)}\}$ - Relative quality of product n in i

$\{v_i\}$ - Volume of food required per person in i

$\{m_i\}$ - Raw material cost per unit of goods production in i.

$\{\widehat{\phi}_i^{(n)}\}$ - Cost price per unit of n in i during δt.

$\{\phi_i^{(n)}\}$ - Sale price per unit of n in i during δt.

$\{D_{ij}\}$ - Distance between i and j.

$\{\theta^{(n)}\}$ - Transport costs of n per unit volume per unit distance.

$\{\mathcal{C}_i^{(n)}\}$ - Total cost of maintaining capacity of n in i.

$\{\Pi_i^{(n)}\}$ - Profit (or loss) made by sector n in i over δt.

Δ_i - National debt (if population cannot afford to feed itself)

$\mathcal{S}_i$ - Total surplus in δt.

Using this notation, the diagonal of the trade flow matrix $Y_{ii}^{(n)}$ will give the volume of n, consumed and produced in i. In the following we refer to this as local consumption.

3 The pricing and trade flows algorithm

Within each country and sector, the dynamics of income, global pricing and production levels are all governed by the trade flows which form the heart of the model. To simplify the explanation of the algorithm, the mechanism of the adapted spatial interaction model which derive these trade flows are described in a separate section. The principles outlined here are identical for all sectors so, for ease of notation, the superscripts (n) are temporarily dropped.

Assume, for the purpose of this section at least, that consumption, Z_j, production capacity, $\bar{X}_i$, product quality, q_i, and transport costs, θD_{ij}, of each country are known.

If the price, ϕ_i, were also known, the flow of goods, Y_{ij}, between seller i and buyer j, could be calculated directly and without iteration from a singly constrained spatial interaction model:

$$Y_{ij} = \frac{Z_j q_i^\alpha \exp\left[-\beta(\theta D_{ij} + \phi_i)\right]}{\sum_k q_k^\alpha \exp\left[-\beta(\theta D_{kj} + \phi_k)\right]}. \tag{3.1}$$

Using (3.1), the sum over all buyers, j, would then yield the total demand in i:

$$X_i = \sum_j Y_{ij}. \tag{3.2}$$

In some cases, this demand, X_i, will exceed the production capacity, $\bar{X}_i$, of i. Where this occurs, an alteration to the trade flows is needed. Using the usual theory of supply and demand, a variable price ϕ_i is introduced as a mechanism to deal with this adjustment. The aim is to increase the unit price ϕ_i of a product in any country where demand exceeds supply until the quantity demanded by consumers is balanced by the production capacity: $X_i = \bar{X}_i$. All other countries may not increase their prices and will sell at, or below, capacity. Assume also, for the time being, that a cost price, or price which a country cannot sell below, $\widehat{\phi}_i$, is known. For any country then, either the price is known (as $\widehat{\phi}_i$) and the production must be found (wherever $X_i < \bar{X}_i$), or the production is known, $X_i = \bar{X}_i$, and the price, $\phi_i > \widehat{\phi}_i$, must be found.

The algorithm to determine the trade flows will proceed as follows:

1. The spatial interaction model is run once using cost prices, $\widehat{\phi}_i$, to determine the demand on each country X_i.

$$X_i = \sum_j \frac{Z_j q_i^\alpha \exp\left[-\beta(\theta D_{ij} + \widehat{\phi}_i)\right]}{\sum_k q_k^\alpha \exp\left[-\beta(\theta D_{kj} + \widehat{\phi}_k)\right]}. \tag{3.3}$$

2. In any country where demand exceeds supply, the optimal prices - such that demand equals capacity - are calculated separately by rearranging (3.1) and taking $X_i = \bar{X}_i$. More formally:

$$\forall m \;\; \text{s.t.} X_m > \bar{X}_m, \tag{3.4}$$

$$\phi_m = \frac{1}{\beta}\ln\left[\frac{1}{\bar{X}_m}\right] + \frac{1}{\beta}\ln\left[\frac{Z_j q_i^\alpha \exp\left[-\beta\theta D_{mj}\right]}{\sum_k q_k^\alpha \exp\left[-\beta(\theta D_{kj} + \phi_k)\right]}\right] \tag{3.5}$$

   Which is solved iteratively since ϕ_m appears in the denominator of the last term.

3. The new prices contribute to the latest pricing vector ϕ_i, on which the spatial interaction model is re-run

$$X_i = \sum_j \frac{Z_j q_i^\alpha \exp\left[-\beta(\theta D_{ij} + \phi_i)\right]}{\sum_k q_k^\alpha \exp\left[-\beta(\theta D_{kj} + \phi_k)\right]}. \tag{3.6}$$

4. The new prices have the ability to push the demand in other countries over the capacity of production. If this is the case, the process is repeated until a global equilibrium is established and all production levels and prices are known.

This algorithm relies on knowing consumption, Z_j, production capacity, $\bar{X}_i$, product quality q_i, transport costs, θD_{ij}, and cost price, $\widehat{\phi}_i$. Once complete, it returns production, X_i, sale price, ϕ_i, and the trade flows, Y_{ij}.

To obtain a complete description of trade evolution, these inputs and a mechanism for dynamics based on the outputs must be determined for each sector. We present this in Sections 5, 6 and 7 for the farming, natural resources and manufacturing sectors respectively. However, first we briefly discuss some estimates of the exogenous variables required within the simulation.

4 Initial setup

To apply the trade flows algorithm we must determine the capacity, cost prices, income, quality and transport costs for each country. Theoretically, these could all be found from data sources, particularly if the the model was disaggregated to apply to individual products, such as oil, wheat, etc. In the present setting however, our main interest lies in demonstrating the workings of the system, rather than in obtaining realistic results, and so we proceed by outlining some simple functions to approximate the above variables.

Working through the variables in the order listed above, we take one unit of food, raw materials or goods as the amount one worker can produce or extract in a given time period. Using this formulation, the production capacity of a given sector, $\bar{X}_i^{(n)}$, will be equivalent to the labour force of that sector, $L_i^{(n)}$. Knowing the population, P_i, and the number of unemployed, u_i, from data, an initial guess of the work force of each sector in each country may be made, providing an initial $\bar{X}_i^{(n)}$. Cost prices are sector specific, but may be found from the production capacity using a simple expression. The exact formulations are outlined for each sector within the relevant sections.

The demonstration model does incorporate two readily accessible data sources: Gross Domestic Product (GDP) and distances between the countries of consideration calculated between centroids using the haversine formula. From these, we may approximate the remaining variables. Assuming that only the workers draw a salary, and that income is uniform across the population, an initial mean per-capita income may be found:

$$\iota_i = \frac{\mathrm{GDP}_i}{P_i - u_i}. \tag{4.1}$$

This value (4.1) will change during the simulation, as countries with a strong economy will invest in improving income. The method applied to facilitate this is within the dynamics section, equation (8.9). It is worth noting at this stage, that the simple expression (4.1) could easily be replaced with an income distribution which would not effect the workings of the algorithm. It is for simplicity in this case that this particular form has been chosen.

A measure of product quality within a country could also be derived from the relative income. We again propose a simple expression which normalises (4.1) by lowest income of the countries included:

$$q_i = \frac{\iota_i}{\max(\iota_k)}. \tag{4.2}$$

With this formulation, k is used as a dummy variable for i and $0 < q_i \leq 1$ for all i. Thus the richest country included within the model has $q = 1$.

We leave the transport costs, $\theta^{(n)}$, as a parameter to be calibrated within the modelling process. Generally speaking however, to ensure the argument of the exponential in (3.1) is dimensionless and $O(1)$, we would expect $\theta^{(n)}$ to roughly take the form:

$$\theta^{(n)} \sim \frac{1}{\beta \max(D_{ij})} \tag{4.3}$$

As mentioned above, the distances, D_{ij}, between each country may be found from data, while β is another parameter in (3.1) to be determined. Physically, β quantifies the relative importance of price, rather than quality, to the buyer.

These variable definitions provide enough information to proceed to the individual sector algorithms and flows, although one final adjustments is needed for the numerics. With the current definition, the pricing and trade flows algorithm (3.3) - (3.6) has no mechanism to stop global production capacity dipping below global demand. On occasions where this occurs, demand must be scaled back. To do so, we propose the additional assumption that in cases of global shortage, it is the poorest countries which see the largest reduction in consumption. Reduction in global demand is found by scaling according to income. To put this more formally, whenever:

$$\sum_j Z_j^{(n)} > \sum_i \bar{X}_i^{(n)}, \tag{4.4}$$

the reduction in consumption, $Z_j^{(n)}$, in country j will be relative to:

$$r_j = 1 - \frac{\iota_j}{\max(\iota_k)}. \tag{4.5}$$

This sets $r = 0$ for the richest country under consideration, and allows the new consumption vector to be found from the original consumption minus the relative reduction, times the total reduction:

$$Z_j^{(n)} = Z_j^{(n)} - \frac{r_j}{\sum_k r_k} \left(\sum_l Z_l^{(n)} - \sum_i \bar{X}_i \right), \tag{4.6}$$

It now remains to detail the full structure of the system for each individual sector. In the interests of clarity, we do so separately for each of the three sectors, in the following three sections.

5 The algorithm to determine farming trade flows

The method to determine the farming trade flows is largely based around the pricing and trade flows algorithm of Section 3. First we present some estimates for the sector-specific variables of consumption $Z_j^{(1)}$ and cost price $\widehat{\phi}_i^{(1)}$. These combined with $\bar{X}_i^{(1)}$ and $\theta^{(1)}$ discussed in the previous section, may be passed to the trade flows algorithm (3.3) - (3.6) to provide values for the actual production, $X_i^{(1)}$ (bounded above by $\bar{X}_i^{(1)}$), sale price, $\phi_i^{(1)}$ (bounded below by $\widehat{\phi}_i^{(1)}$), and the trade flows matrix Y_{ij}. To proceed with food consumption in country j, we define a parameter v_j to describe the volume consumed per capita, per unit income. This sets the total consumption in j as:

$$Z_j^{(1)} = v_j \iota_j P_j. \tag{5.1}$$

Although v_j is country specific, taking it as a constant across all models, as we do in our experimental run, implies that richer countries will consume more food than their poorer counterparts. The cost price per unit volume of food produced in i is calculated from the total labour costs of farming infrastructure $\iota_i L_i^{(1)}$, divided by the expected production output or sales:

$$\widehat{\phi}_i^{(1)} = \frac{\iota_i L_i^{(1)}}{X_i^{(1)}(t - \delta t)}. \tag{5.2}$$

We base the expected production output on the actual output of the previous time period. This removes the need for an extra iteration, and seems reasonable when the time periods under consideration are small. Indeed, $X_i^{(1)}(t) = X_i^{(1)}(t - \delta t)$ in the limit as $\delta t \to 0$.

As discussed, the current formulation defines one unit of food production as the amount which one worker can produce in one time period. Thus, it follows, that the production capacity of the farming sector is equal to the labour force of the sector.

$$\bar{X}_i^{(1)} = L_i^{(1)}. \tag{5.3}$$

It is this sector specific quantity, production capacity, which determines whether the country is resource rich or poor and allows for our 2x2 classification of countries. Equation (5.3) has the direct result that (5.2) may be rewritten

$$\widehat{\phi}_i^{(1)} = \frac{\iota_i \bar{X}_i^{(1)}}{X_i^{(1)}(t-\delta t)}. \tag{5.4}$$

Thus, cost price is income scaled by the ratio of the sales of the previous time period, to the capacity at the present time. Given this form for (5.4), it is necessary to define a minimum expected production value. Otherwise a country with little or no sales in the previous time period will have a very large or infinite cost price, making sector growth extremely difficult.

The now known values of $Z_j^{(1)}$, $\bar{X}_i^{(1)}$, $\widehat{\phi}_i^{(1)}$, $\theta^{(1)}D_{ij}$ are fed into the pricing algorithm of Section 3, providing results for $X_i^{(1)}$, $\phi_i^{(1)}$ and $Y_{ij}^{(1)}$:

$$Z_j^{(1)}, \bar{X}_i^{(1)}, \widehat{\phi}_i^{(1)}, \theta^{(1)}D_{ij} \longrightarrow X_i^{(1)}, \phi_i^{(1)} Y_{ij}^{(1)} \text{ via equations (3.3)-(3.6).} \tag{5.5}$$

With a set of farming flows $Y_{ij}^{(1)}$ found, the price which the buyer j pays, including transport costs for imports, is given by:

$$D_{ij}\theta^{(1)} + \phi_i^{(1)}, \tag{5.6}$$

and the total spent by the population on food may be determined from the per unit price of each import, times the volume imported and summed over all countries (including ones own):

$$\zeta_j^{(1)} = \sum_i \left(\theta^{(1)}D_{ij} + \phi_i^{(1)}\right) Y_{ij}^{(1)}. \tag{5.7}$$

As discussed in Section 1, any money left over will be spent on goods. All remaining expenditure is used, regardless of the volumes of goods the money buys. Thus we may define each country's goods consumption for the manufacturing sector in units of money:

$$\zeta_j^{(3)} = \mathrm{GDP}_j - \zeta_j^{(1)}. \tag{5.8}$$

There will be some $Z_j^{(3)}$ consumption term associated with (5.8) in volume units, although, as may be seen in Section 7, this does not feature explicitly in the problem.

If the population does not have enough money to feed itself, we want to avoid the computationally expensive additional iterative procedure of reducing consumption and repeating the pricing and trade flow calculations. To do so, we introduce national debt, which acts to keep track of any deficit:

$$\Delta_i = -\frac{\zeta_i^{(3)}(\mathrm{sgn}(\zeta_i^{(3)}) - 1)}{2}. \tag{5.9}$$

This structure is chosen as it is zero while $\zeta_i^{(3)}$ is positive, and equal to $-\zeta_i^{(3)}$ if $\zeta_i^{(3)}$ is negative. National debt is paid for by the profits of the three producing sectors, details of how this mechanism is applied are in Section 8.

5.1 The accounts for the farming industry

With all flows and sales prices determined, it is now possible to determine all costs, takings and profits for the farming industry as follows.

Costs of production (only labour costs within this sector) are given by,

$$\mathcal{C}_i^{(1)} = \iota_i \bar{X}_i^{(1)}. \tag{5.10}$$

Money taken/sales (sale price per unit times no. units):

$$\chi_i^{(1)} = \phi_i^{(1)} X_i^{(1)}. \tag{5.11}$$

Profit from farming sector (sales - cost) is,

$$\Pi_i^{(1)} = \phi_i^{(1)} X_i^{(1)} - \iota_i \bar{X}_i^{(1)} \tag{5.12}$$

5.2 A final point on the farming flows

It is worth noting that the total sales $\chi_i^{(1)}$ given in (5.11) will take one of two less general forms due to a subtlety in the pricing and trade flows algorithm of Section 3. The mechanism of that section relies on a two types of seller. If the first type, country i will sell below capacity, $X_i^{(1)} < \bar{X}_i^{(1)}$, at cost price $\phi_i^{(1)} = \widehat{\phi_i}^{(1)}$. In this case, given (5.4), the total sales of i (5.11) becomes:

$$\chi_i^{(1)} = \iota_i \bar{X}_i^{(1)} \frac{X_i^{(1)}}{X_i^{(1)}(t-\delta t)} \tag{5.13}$$

If the second type, i will sell at capacity $X_i^{(1)} = \bar{X}_i^{(1)}$, at a higher price than cost, $\phi_i^{(1)} > \widehat{\phi_i}^{(1)}$. In this case, (5.11) becomes:

$$\chi_i^{(1)} = {\phi_i}^{(1)} \widehat{X}_i^{(1)}. \tag{5.14}$$

This implies two corresponding forms for profit, originally defined in (5.12). The profits of the first type, like (5.13) will take the form

$$\Pi_i^{(1)} = \iota_i \bar{X}_i^{(1)} \left(\frac{X_i^{(1)}}{X_i^{(1)}(t-\delta t)} - 1 \right), \tag{5.15}$$

and is positive if sales exceed those of last year, negative otherwise. Meanwhile, the second type uses (5.14) to make (5.12):

$$\Pi_i^{(1)} = \widehat{X}_i^{(1)} \left({\phi_i}^{(1)} - \iota_i \right), \tag{5.16}$$

and profits are built from actual sales prices above the national income (theoretically equivalent in this case to the actual per-unit cost price as in (5.10). In both cases (5.15) and (5.16) are linearly related to the size of the current infrastructure. It is this feature which gives the dynamics a logistic, or Lotka-Volterra form, as discussed in Section 8.

6 The algorithm to determine the natural resources trade flows

The method to determine the natural resources flows largely mirrors the algorithm applied in the previous section to farming flows. For the sake of clarity, we include a brief description of the full process here. To begin, we must derive expressions for the consumption and cost prices. Since the sole customer of the natural resources sector is the manufacturing sector, the raw materials consumption in country j is linked to manufacturing production capacity. We introduce m_j so that the consumption is equal to the volume required per unit production times manufacturing production capacity.

$$Z_j^{(2)} = m_j \bar{X}_j^{(3)} \tag{6.1}$$

The per-unit cost price of raw materials in i, in units of money is the total labour cost of maintaining the mining infrastructure, divided by the expected production output (based on last years actual production).

$$\phi_i^{(2)} = \frac{\iota_i \bar{X}_i^{(2)}}{X_i^{(2)}(t - \delta t)} \tag{6.2}$$

Again, as in the farming flows, a minimum expected production value is found to be necessary to avoid a large cost price in small sectors. The trade flows are calculated in an identical procedure as before: $Z_j^{(2)}$, $\bar{X}_i^{(2)}$, $\widehat{\phi}_i^{(2)}$, $\theta^{(2)} D_{ij}$, are fed into the spatial interaction model outlined in Section 3, providing results for $X_i^{(2)}$, $\phi_i^{(2)}$ and $Y_{ij}^{(2)}$.

$$Z_j^{(2)}, \bar{X}_i^{(2)}, \widehat{\phi}_i^{(2)}, \theta^{(2)} D_{ij} \longrightarrow X_i^{(2)}, \phi_i^{(2)} Y_{ij}^{(2)} \quad \text{via equations (3.3)-(3.6).} \tag{6.3}$$

Once the flows and prices have been determined, the total per-unit price to the buyer, including transport costs is:

$$D_{ij}\theta^{(2)} + \phi_i^{(2)}, \tag{6.4}$$

which gives the money spent on raw materials by the manufacturing industry in j as the price per unit for each import times the volume imported, summed over all countries including one's own:

$$\zeta_j^{(2)} = \sum_i \left(D_{ij}\theta^{(2)} + \phi_i^{(2)} \right) Y_{ij}^{(2)}. \tag{6.5}$$

The total spend on raw materials, given by (6.5), will feature in the manufactured goods pricing of Section 7.

6.1 The accounts for the natural resources sector

Again, this follows the workings of the farming sector, so that costs of production are determined only by labour costs:

$$\mathcal{C}_i^{(2)} = \iota_i \bar{X}_i^{(2)}. \tag{6.6}$$

Money taken/sales, is sale price per unit times no. units:

$$\chi_i^{(2)} = \phi_i^{(2)} X_i^{(2)}. \tag{6.7}$$

Profit from mining sector (sales - cost) is

$$\Pi_i^{(2)} = \phi_i^{(2)} X_i^{(2)} - \iota_i \bar{X}_i^{(2)}. \tag{6.8}$$

As in (5.15) and (5.16), this profit will take one of two forms, both linearly related to $\bar{X}_i^{(2)}$.

7 The algorithm to determine manufacturing trade flows

Goods consumption in country j is calculated from the per capita income minus food purchases. We assume that people spend all their disposable income on goods, regardless of what volume of goods this buys them. The result, in units of money is given in equation (5.8) and repeated here for clarity:

$$\zeta_j^{(3)} = \text{GDP}_j - \zeta_j^{(1)}. \tag{7.1}$$

In the situation where a country cannot afford to feed themselves, there will be no remaining income to spend on manufactured goods. In such a case, the volume of food required is bought regardless, and a national debt term is introduced (see equation (5.9)). To allow for this scenario, we rewrite the amount of money each country has to spend on goods as:

$$\zeta_j^{(3)} = \left(\mathrm{GDP}_j - \zeta_j^{(1)}\right)(1 - \mathrm{sgn}(\Delta_i)), \tag{7.2}$$

where, by definition, debt, Δ_i, may only be zero or positive.

Within the manufacturing sector, the consumption (7.2) is expressed in terms of money, so that volumes do not feature explicitly in the problem. Thus, instead of applying the pricing and trade flows algorithm to solve the system ((3.3)-(3.6)) as before, an adapted version must be applied to take into account this special case of consumption in units of money. A per-unit cost price is still required however, where the form differs from that seen in farming (5.4) and mining (6.2) to allow for the natural resources costs to maintain the manufacturing infrastructure. This cost was found in the mining flows in (6.5). Thus the per-unit cost price of goods in i, in money units, is:

$$\widehat{\phi}_i^{(3)} = \frac{\iota_i \bar{X}_i^{(3)} + \zeta_i^{(2)}}{X_i^{(3)}(t - \delta t)}. \tag{7.3}$$

The adapted pricing and trade flows algorithm determines an initial demand on sales, by applying the standard spatial interaction model:

$$\chi_i^{(3)} = \sum_j \frac{\zeta_j^{(3)} q_i^\alpha \exp[-\beta(\theta^{(3)} D_{ij} + \widehat{\phi}_i^{(3)})]}{\sum_k q_i^\alpha \exp[-\beta(\theta^{(3)} D_{ik} + \widehat{\phi}_k^{(3)})]}. \tag{7.4}$$

Any countries where demand exceeds capacity are entitled to raise their sales prices. This price adjustment takes the form:

$$\forall m \text{ s.t } \chi_m^{(3)} > \widehat{\phi}_m^{(3)} \bar{X}_m^{(3)} \tag{7.5}$$

$$\phi_m^{(3)} = \frac{1}{\beta} \ln\left[\frac{1}{\phi_m^{(3)} \bar{X}_i^{(3)}}\right] + \frac{1}{\beta} \ln\left[\sum_j \frac{\zeta_j^{(3)} q_i^\alpha \exp[-\beta\theta^{(3)} D_{ij}]}{\sum_k q_i^\alpha \exp[-\beta(\theta^{(3)} D_{ik} + \widehat{\phi}_k^{(3)})]}\right], \tag{7.6}$$

which is solved iteratively as $\phi_m^{(3)}$ appears on the right-hand side of (7.6).

These adjusted prices form part of the latest pricing vector $\{\phi_i^{(3)}\}$, on which the spatial interaction model is re-run:

$$\chi_i^{(3)} = \sum_j \frac{\zeta_j^{(3)} q_i^\alpha \exp[-\beta(\theta^{(3)} D_{ij} + \phi_i^{(3)})]}{\sum_k q_i^\alpha \exp[-\beta(\theta^{(3)} D_{ik} + \phi_k^{(3)})]}. \tag{7.7}$$

The new prices $\{\phi_i^{(3)}\}$ have the ability to push other countries over production capacity, thus the process is repeated until a global equilibrium is established and all prices are known. Finally, the flow in money units from j to i is given from:

$$\psi_{ji}^{(3)} = \frac{\zeta_j^{(3)} q_i^\alpha \exp[-\beta(\theta^{(3)} D_{ij} + \phi_i^{(3)})]}{\sum_k q_i^\alpha \exp[-\beta(\theta^{(3)} D_{ik} + \phi_k^{(3)})]}, \tag{7.8}$$

and actual production may be found from

$$X_i^{(3)} = \frac{\chi_i^{(3)}}{\phi_i^{(3)}}. \tag{7.9}$$

Despite being a money flow, the price term and quality term allow the flows to differentiate between large volume, cheap, low quality exports and expensive, low volume high quality exports - the buyer is always driven towards the best deal. What is meant by "best" of course, is dictated by the calibration of the Lagrangian multipliers α and β.

7.1 The accounts for the manufacturing industry

Costs of production (within this sector costs include both labour and raw materials):

$$\mathcal{C}_i^{(3)} = \iota_i \bar{X}_i^{(3)} + \zeta_i^{(2)}. \tag{7.10}$$

Money taken/sales (here of course, $\chi_i^{(3)}$ is known from the flows, but the following should balance nonetheless):

$$\chi_i^{(3)} = \phi_i^{(3)} X_i^{(3)}. \tag{7.11}$$

Profit from manufacturing sector (sales - cost):

$$\Pi_i^{(3)} = \phi_i^{(3)} X_i^{(3)} - \iota_i \bar{X}_i^{(3)} - \zeta_i^{(2)}. \tag{7.12}$$

The last term here is not quite linearly related to $\bar{X}_i^{(3)}$ and the profits of the manufacturing sector do not take the same form as those seen previously.

8 The Dynamics

The profit of all sectors is now known and it remains to reinvest any profits to increase the capacity of each nation's best performing sectors. Before we do so however, any national debt accrued must be accounted for, as other countries have already been paid for the food which the population of i bought. The total surplus of all industry in i is the profit from each sector (which could be negative) plus any national debt:

$$\mathcal{S}_i = \Pi_i^{(1)} + \Pi_i^{(2)} + \Pi_i^{(3)} - \Delta_i. \tag{8.1}$$

Although we use the term 'surplus', $\mathcal{S}_i$ could be negative. As all surplus is reinvested, it would be possible at this stage to introduce national or international investment, and the main structure of the dynamics would remain unchanged. For the purposes of our demonstration model, we neglect this additional variable and leave surplus as defined in (8.1).

Assume that each country, and all sectors within it, is governed by one central body which reinvests in a manner best for everyone by splitting the surplus between sectors according to the performance in the latest time step. This reinvestment takes the form of both additional employment and wage increase. Such assumptions could well be adjusted at a later date, to include shareholders or allow for firms, for example. However this simplifying assumption works well for the purposes of our demonstration model.

With static populations, a country may not employ more people than the current unemployed and and cannot fire more people than the employed. So, the extra number of people who may be employed (or fired) is given by

$$E_i = \frac{1}{2}\left(1 + \mathrm{sgn}(\mathcal{S}_i)\right) \min\left[\frac{\epsilon_1 \mathcal{S}_i}{\iota_i}, u_i\right] - \frac{1}{2}\left(1 - \mathrm{sgn}(\mathcal{S}_i)\right) \min\left[\frac{-\epsilon_2 \mathcal{S}_i}{\iota_i}, P_i - u_i\right], \tag{8.2}$$

where, ϵ_1 and ϵ_2 are parameters to be calibrated.

The algorithm to assign this change in labour force to each sector is based on the sectors contribution to the overall profits

$$\delta L_i^{(n)} = \delta t \frac{\Pi_i^{(n)}}{\sum_k \Pi_i^{(k)}} E_i, \tag{8.3}$$

although, naturally this must be adjusted if it yields any negative results since a country cannot have a negative work force. This translates directly to a change in production capacity in the current model, since one unit of volume is equivalent the amount one worker can produce in a time period. Thus:

$$\delta \bar{X}_i^{(n)} = \delta L_i^{(n)}. \tag{8.4}$$

Note that the extra employment, E_i, in (8.2) takes the same sign as the country's overall profits $\sum_k \Pi_i^{(k)}$. If both are positive - that is if the country is in profit overall - any sector with losses $\Pi_i^{(n)}$ will still see a shrinking workforce, and vice-versa. Even if a country does badly, the mechanism (8.3) allows growth in a thriving sector.

In addition, given the two forms of profits discussed in Section 5, equations (5.15) and (5.16), the dynamics of production capacity 8.3 will take one of two forms within the farming sector. These will be,

$$\delta \bar{X}_i^{(1)} = \delta t \iota_i \bar{X}_i^{(1)} \left(\frac{X_i^{(1)}}{X_i^{(1)}(t - \delta t)} - 1 \right) \frac{E_i}{\sum_k \Pi_i^{(k)}} \tag{8.5}$$

$$\delta \bar{X}_i^{(1)} = \delta t \bar{X}_i^{(1)} \left({\phi_i}^{(1)} - \iota_i \right) \frac{E_i}{\sum_k \Pi_i^{(k)}} \tag{8.6}$$

to correspond with (5.15) and (5.16) respectively. Both of these, (8.5) and (8.6) , are therefore in Lotka-Volterra form. This is also true for the natural resources sector, as (5.15) and (5.16) apply when $n = 2$. For manufacturing however, the change in unit for the constraint on production capacity, and the interaction with the farming sector leads to a slightly different form. Specifically, combining (7.3) (7.12) and (8.3) gives the two possible forms as:

$$\delta \bar{X}_i^{(3)} = \delta t \frac{E_i}{\sum_k \Pi_i^{(k)}} \left\{ \iota_i \bar{X}_i^{(3)} \left(\frac{X_i^{(3)}}{X_i^{(3)}(t - \delta t)} - 1 \right) + \frac{\zeta_i^{(2)}}{X_i^{(3)}(t - \delta t)} \right\} \tag{8.7}$$

$$\delta \bar{X}_i^{(3)} = \delta t \frac{E_i}{\sum_k \Pi_i^{(k)}} \left\{ \bar{X}_i^{(3)} \left({\phi_i}^{(3)} - \iota_i \right) - \zeta_i^{(2)} \right\} \tag{8.8}$$

These expressions may be considered as 'forced' Lotka-Volterra.

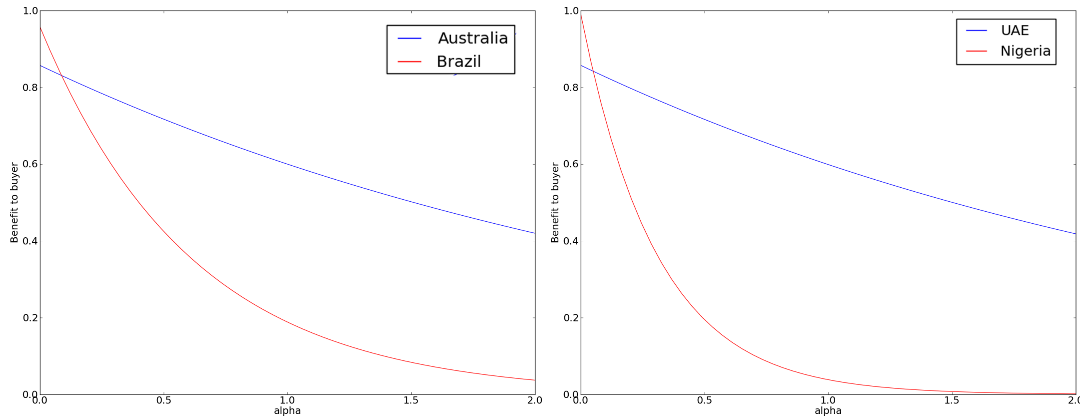
The national income also has a dynamic element which in turn affects both consumption and quality of products:

$$\delta \iota_i = \delta t \epsilon_3 \mathcal{S}_i \tag{8.9}$$

where ϵ_3 is a final parameter to be calibrated. Finally, latest GDP may be found as the sum of all incomes

$$GDP_i = \iota_i \sum_n L_i^{(n)} \tag{8.10}$$

and relative quality q_i may then be recalculated. The entire process may be repeated for subsequent time steps.



(a) (b)

Figure 3: A quantitative measure of benefit to a potential buyer against the parameter α. See equation (9.1).

9 Experimental results

The model generates an interesting balance of factors between countries. A poor country has the benefit of low cost of production due to lower wages, but (with the current assumptions on quality (4.2)) lower quality goods. The relative importance of these two factors to the buyer is determined by the weighting parameters α and β. In line with the derivation of the spatial interaction model of (3.1), this benefit to a potential buyer may be quantified as follows:

$$\text{Benefit} = \exp\left(\alpha \ln q_j - \beta\phi_j - \beta\theta D_{ij}\right). \tag{9.1}$$

Exploring the relationship between price and quality further, some plots are presented in Figure 3 of total benefit for a range of α values, at a fixed β. In the plots, we have taken $\theta = 0$, so that distance is deemed unimportant to the buyer. In reality, these curves would be distorted by θ, and different for each potential buyer. The examples chosen illustrate that there is a value of α below which cheaper items are valued, and above which, quality becomes more important. The countries selected form two pairs: Brazil and Australia, which are both big exporters of coal, timber and iron ore; and Nigeria and UAE which both export petroleum products. The mean income of UAE is 18 times that of Nigeria, while Australian citizens earn an average of 3.55 times more than Brazilians. Thus the four demonstrate examples of GDP rich (Australia, UAE) and poor countries (Brazil, Nigeria). In contrast, the production capacity (or equivalently, work force) of Brazil is five times that of Australia: resource rich and resource poor countries respectively. By our measure of relative quality given in (4.2) Australia has $q = 0.7$, Brazil $q = 0.20$, UAE $q = 0.7$ and Nigeria $q = 0.03$. Of course, a large variance in the quality of oil such as these is unrealistic, although in the context of our toy model the results serve the purpose of demonstrating the interesting potential bifurcations which the algorithm generates. In all plots, benefit tends to zero with large α, although this happens much faster for low quality products, such as those from Nigeria and Brazil.

Benefit is not the only factor in the trade model. Aside from distance between buyer and seller and cost to transport goods, consumption - both local and global - plays a key role in contributing to the demand on a nations produce. In Figure 4, we present a plot of the demand (as a ratio of capacity) on the natural resources products of the same four countries listed above, when offering goods at cost price. This is equivalent to the sum over j of the trade flows at the

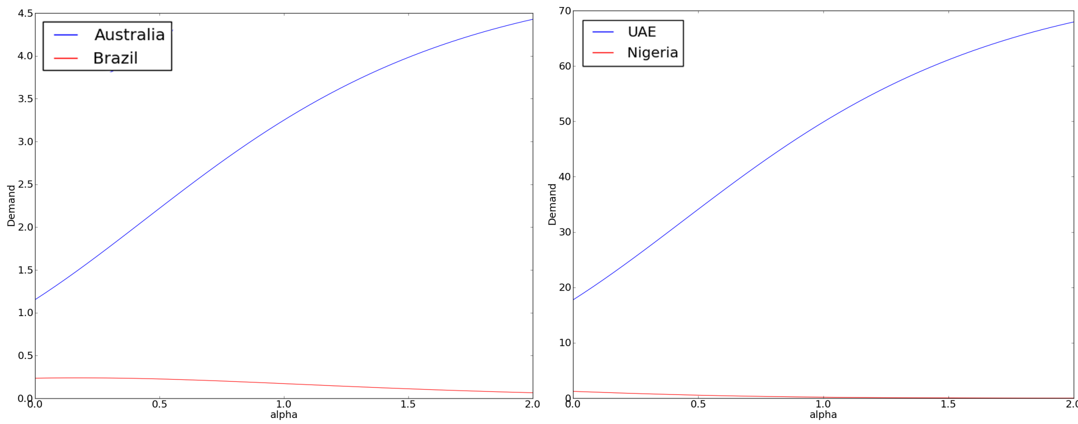


Figure 4: The demand on the natural resources produce for various values of alpha, normalised by production capacity. In the above plots, $\theta = 25$, $\beta = 5\text{x}10^{-6}$.

very first step in the pricing and trade flows algorithm - equation (3.1). In this example, $\theta = 25$ and so distance is taken into account in the buying process. The plots again demonstrate that high values of α correspond to quality being favoured, with the products of UAE and Australia increasing in demand. Given that quality and cost price of UAE and Australia are similar, the bigger demand on UAE is probably due to the large distances between Australia and its potential consumers. The relative demand of Nigeria and Brazil is at a maximum when $\alpha = 0$, and price dominates the decision process. The demand in both cases, however tends to zero with increasing α.

If demand exceeds supply, the countries may raise their sale price until production equals capacity. Our final plot of this type then is given in Figure 5 and shows the ratio of sale price to cost price for the same range of α and values of β, θ. As demand on Brazil never exceeds capacity in this simulation, they continue to sell at cost price, regardless of the value of α. This is not the case for Nigeria, where other countries are prepared to pay up to 140 times the cost price for low values of α. As the importance of quality, along with α increases, demand in Nigeria drops to below capacity, and they must sell at cost price. For UAE and Australia however, the relative sale price, like demand increases with α.

To empirically validate this model, its pricing algorithm, sector accounts and dynamics these parameters α, β and θ will have to be calibrated against data. In practice, the dynamics will be determined by a number of adjustments; the time path will be very much a function of the initial conditions, and variables at each stage. To explore the system, along with model refinements and extensions is our next task, but to demonstrate the goal of the demonstration model we include a map of simulated flows for the farming sector in Figure 6. The key is given within the caption of that figure. As one would expect, countries with higher GDP tend to produce a smaller percentage of their own food, and consume more per capita. In addition, trade links tend to form over shorter distances which matches with the results of other models. For example the gravity model. In general then, a great deal of work needs to be done to explore the feasible solutions, however, this first stage simulation looks to be very promising.

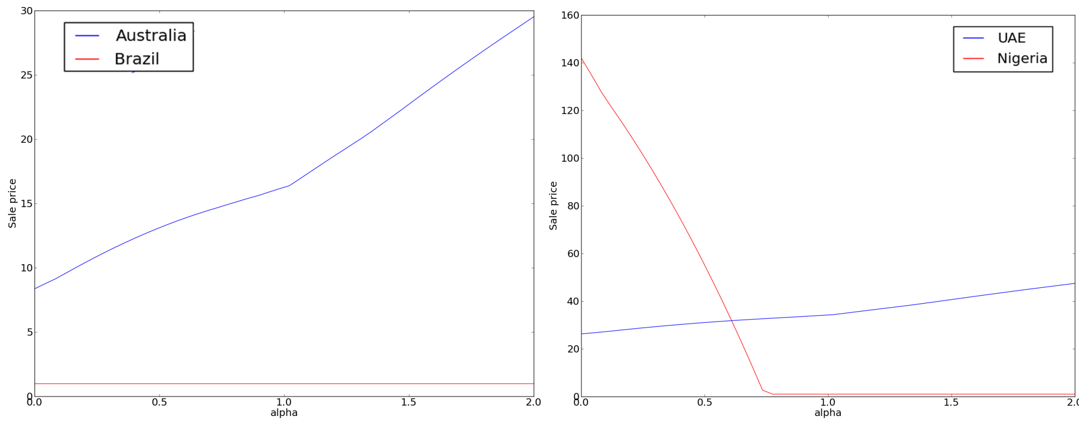


Figure 5: The actual sales price of natural resources for various values of α.

10 Future work

We believe that the model of trade flows presented here offers a good alternative to existing models of trade, not least for the various extensions and adaptations which the method can handle. Perhaps most importantly, the formulation allows us to pose the dynamic programming question: if we add, say, World Bank investment by country, and an objective if formulated to increase GDP per capita in a number of poor countries, what is the optimal path towards the objective? Indeed, is there a feasible solution?

The method also allows for variable migration, and connecting to an equally well developed model of migration should be relatively straightforward. Given the growing impact of piracy on trade flows, the model here could also be adjusted to test various scenarios, including the resilience of global flows to attacks on a given shipping route. Along these lines, the total money spent on shipping within the model is given by the relatively simple expression:

$$\sum_{ij} Y_{ij}^{(1)} D_{ij} \theta^{(1)} + Y_{ij}^{(2)} D_{ij} \theta^{(2)} + \frac{\psi_{ji}^{(3)}}{\phi_i^{(3)}} \theta^n D_{ji} \tag{10.1}$$

so that introducing a global shipping company into the model would be possible. Indeed, any number of firms or stakeholders could be introduced - the only adjustment being in the reinvestment dynamics of Section 8.

These extensions demonstrate the flexibility of the model, however the next step remains clear: to empirically validate this model, calibrate the parameters against real data and to explore the dependence of initial conditions on the dynamics.

References

[1] Rees P. Champion T. Kalogirou S. Fotheringham, A. S. and A. R. Tremayne. The development of a migration model for england and wales: overview and modelling out-migration. *Environment and Planning A*, 36:1633–72, 2004.

[2] B. Harris and A. G. Wilson. Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models. *Environment and planning. A*, 10:371–88, 1978.

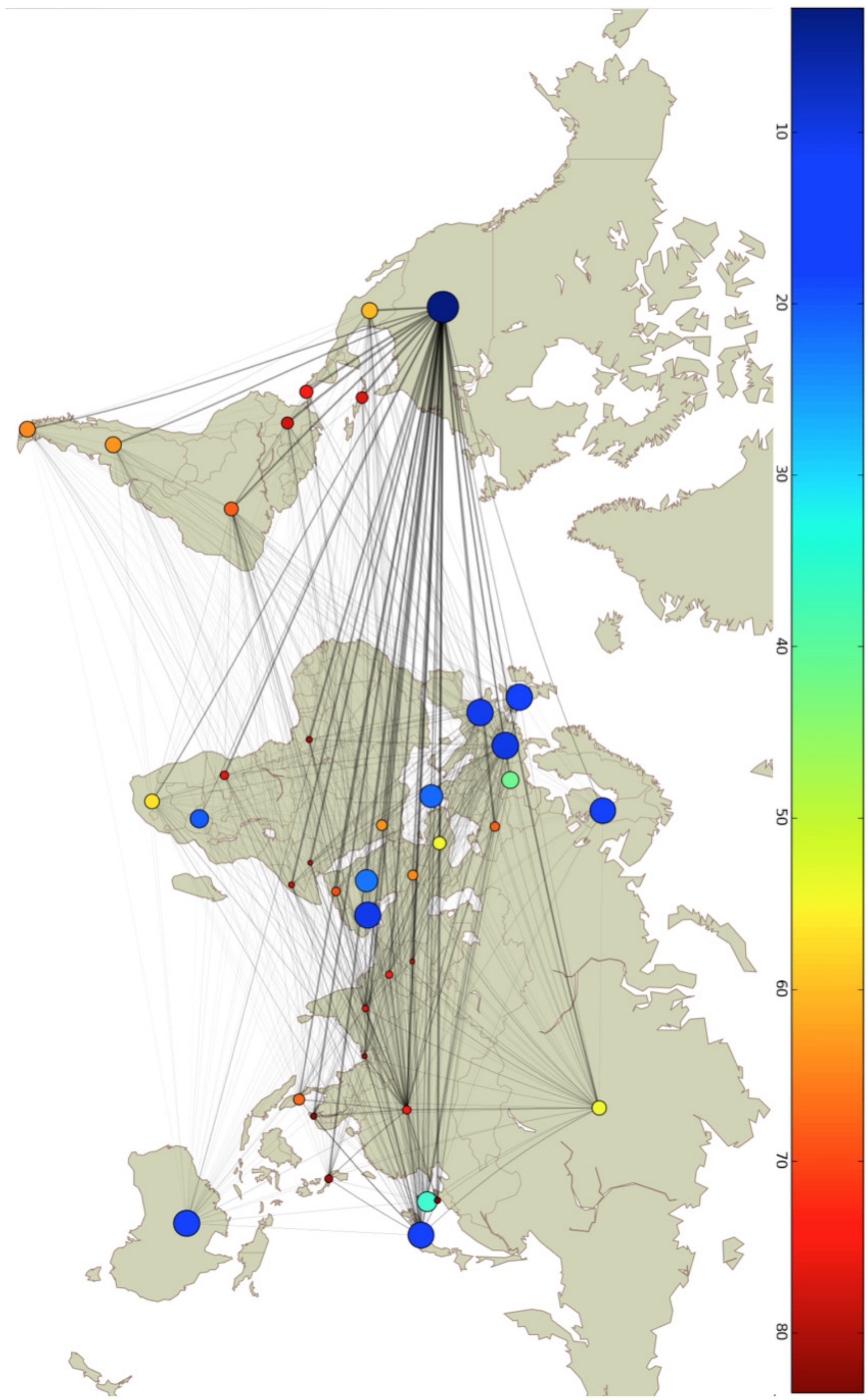


Figure 6: Some first results of simulation for the food flows. Here the node size is the consumption per person, edge width is the size of the inter-country flows, and node colour is the percentage of consumption met locally (see scale)

[3] J. H. Rho, D. E. Boyce, and T. J. Kim. Comparison of solution methods for wilson's interregional commodity flow model. *Geographical Analysis*, 21(3):259–267, 1989.

[4] J. Stillwell. Interzonal migration: some historical tests of spatial-interaction models. *Environment and Planning A*, 10:1187–200, 1978.

[5] F. Willekens. Modeling approaches to the indirect estimation of migration flows: from entropy to em. *Mathematical Population Studies*, 7:239–78, 1999.

[6] A. G Wilson. Interregional commodity flows: Entropy maximizing approaches. *Geographical Analysis*, 2(3):255–282, July 1970.

[7] A. G. Wilson. Urban and regional dynamics from the global to the local: 'hierarchies', 'dna', and 'genetic planning'. *Environment and planning B, Planning and Design*, 37(5):823–837, 2010.