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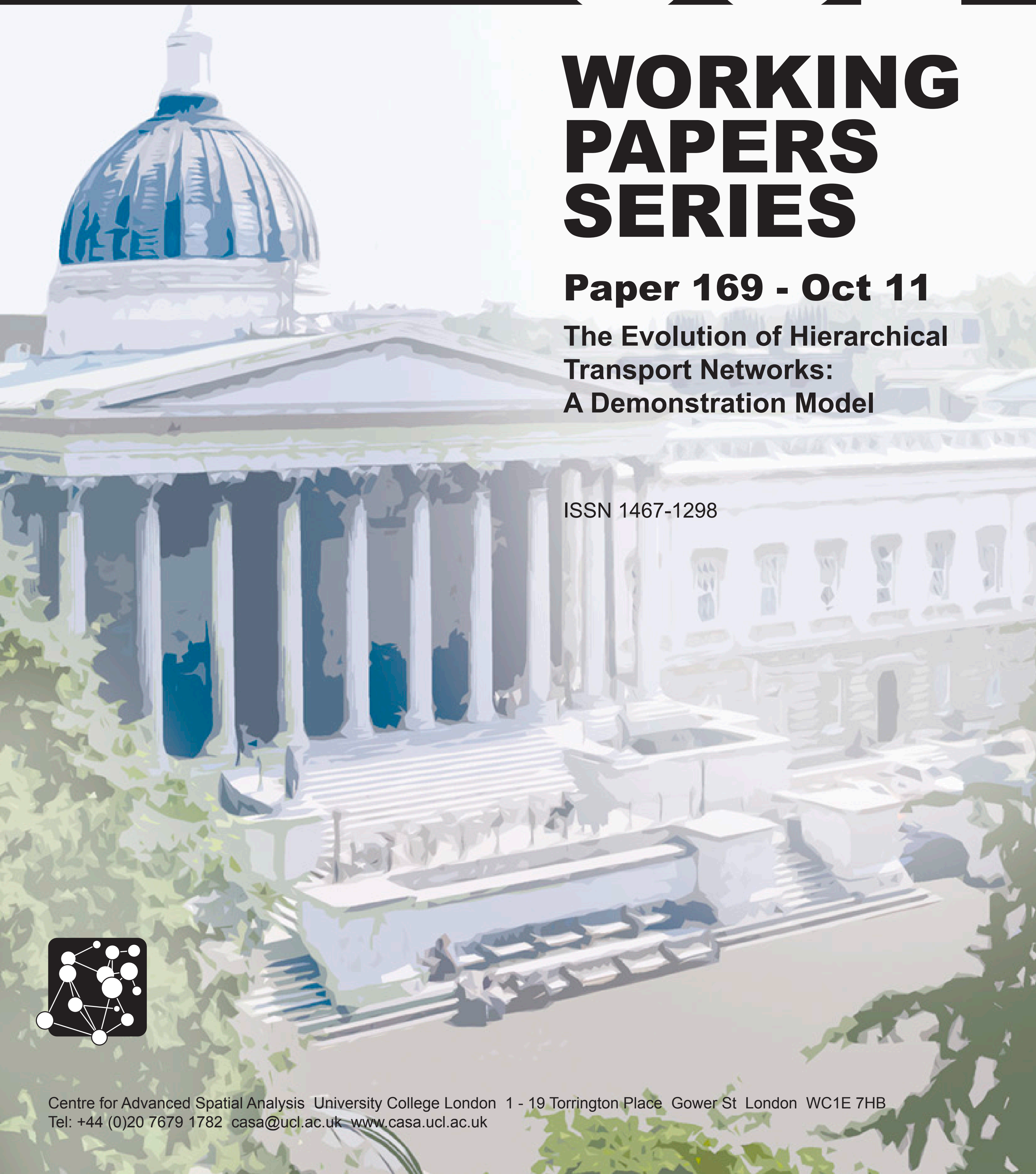

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The Evolution of Hierarchical Transport Networks: A Demonstration Model

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THE EVOLUTION OF HIERARCHICAL TRANSPORT NETWORKS:

A DEMONSTRATION MODEL

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Abstract

In order to take further the task of modelling the evolution of transport networks, the notion of an explicitly hierarchical dynamic model is introduced. This is developed first in the (simpler) retail context and then applied through a demonstration model to the evolution of transport networks.

1. Introduction.

A major challenge for urban modelling has been, and is, the development of dynamic models of the evolution of urban structure. Considerable progress has been made in relation to, for example, retail structure, but modelling the evolution of transport networks is more difficult. There is a higher degree of complexity in this case. In the retail example, the 'structure' is the vector of sizes of retail centres at points in space. In the transport case, the elements are not points but links of a network and the problem is further complicated by the fact that origin-destination flows are carried on routes which are sequences of links. It is also the case that 'size of link' does not capture the reality of the nature of network evolution. In practice, a multi-lane highway, for example, is grade-separated and of a different nature to lower 'levels' of link.

In order to make progress, we first consider the simpler retail problem by creating levels of a hierarchy (section 2) and then proceed to analyse the equivalent transport network problem (section 3). This enables us to proceed to build a simple demonstration model which we hope will open up opportunities for building realistic models of network evolution (section 4).

2. Hierarchical dynamics in a retail model

In the conventional dynamic retail model, there are extremes of configuration – large numbers of small centres or small numbers of large centres – and the 'hierarchy' is determined simply by size (Harris and Wilson, 1978). Consider food shopping. Suppose now that there are two kinds of centre: 'small' and supermarket with different cost structures, pulling powers etc. Can we model the evolution of this kind of system? The motivation for this is to provide insight to build a dynamic model of the evolution of a road transport network in which there are certainly at least three levels of hierarchy: (minor) arterial, 2-lane highway and multi-lane highway.

Suppose in the retail case we have two hierarchical levels, labelled by h = 1, 2, the second being the 'supermarket'. Let W_j^h be the size of the h-level in zone j. Let K^h be the cost per unit, say annually. Let e_i be the unit expenditure by the population of i and P_i the population. Let S_{ij}^h be the set of flows, measured in money units. Then a suitable model would be

$$S_{ij}^h = A_i e_i P_i (W_j^h)^{\alpha h} e^{-\beta^h c_{ij}} \tag{1}$$

where

$$A_i = 1 \Big/ \textstyle\sum_k (W_k^h)^{\alpha h} e^{-\beta^h c_{ij}} \tag{2}$$

to ensure that

$$\textstyle\sum_{j,h} S_{ij}^h = e_i P_i \tag{3}$$

The retail flows from zone i to zone j will be split between h = 1 and h = 2 according to the relative sizes of the Ws and the parameters α^h and β^h. The dynamics, in difference equation form, can be taken as

$$\Delta W_j^h = \varepsilon [D_j^h - K^h W_j^h] W_j^h \tag{4}$$

where

$$D_j^h = \textstyle\sum_i S_{ij}^h \tag{5}$$

We will assume that $K^{(2)}$ is greater than $K^{(1)}$ because of scale economies. However, these economies will only be achieved if W_j^h is greater than some minimum size. Say

$$W_j^{(2)} \geq W_j^{(2)min} \tag{6}$$

and we expect that $W_j^{(1)}$ is very much smaller than $W_j^{(2)min}$.

The model therefore consists of equations (1) – (6) run in sequence. In the first instance , the equilibrium solutions to

$$D_j^h = K^h W_j^h \tag{7}$$

should be explored. And then assumptions could be made about the trajectories of exogenous variables so that the dynamics could also be explored.

It would be quite likely that for certain sets of values of exogenous variables, there would be no $W_j^{(2)} > 0$ and as, say, income increased, we could model the emergence of supermarkets. This may be interesting in its own right, but as noted at the outset, the current motivation is to point the way to modelling the evolution of transport networks.

3. Extension to transport networks.

This argument follows that in Wilson (1983) but aims first to articulate the simplest transport model that would demonstrate the desired properties and into which we can insert explicit hierarchies. The 1983 paper contains a model that is rich is detail; it ends with a formulation of a simpler model, but one which does not have explicit hierarchies. In the simpler model of that paper, spider networks are deployed to seek to avoid the complexities of detailed real networks (which had been spelled out earlier in that paper). We use the same method here: spider networks are built by connecting nearby zone centroids with notional links.

Let {i} be a set of zones and these can also stand as names for zone centroids. Let h = 1, 2, 3 label three levels of hierarchy representing arterial, 2-lane highway and multi-lane highway links. We will also use j, u and v as centroid labels. For example, (u, v, h) may be a link on a route from i to j on the spider network at level h. $(u, v) \varepsilon R_{ij}^{min}$ is the set of links that make up the best route from i to j. This may involve a mix of links of different levels. Let Γ^h be a measure of the annual running cost per unit of a link of level h – including an annualised capital cost - and let ρ_{uv} be the 'length' of link (u, v). Let c_{ij} be the generalised cost of travel from i to j as perceived by consumers, and let $\{\gamma_{uv}^h\}$ be the set of link travel costs on the level h link (u, v). So:

$$c_{ij} = \sum_{(u,v,h)\, \varepsilon R_{ij_{min}}} \gamma_{u,v}^{h} \qquad (8)$$

If O_i and D_j are the total number of origins in zone i and destinations in zone j, and T_{ij} is the flow between i and j, then the standard doubly-constrained spatial interaction model is

$$T_{ij} = A_i B_j O_i D_j e^{-\beta c_{ij}} \qquad (9)$$

with

$$A_i = {}^{1}/_{\sum_k B_k D_k e^{-\beta c_{ik}}} \qquad (10)$$

$$B_j = {}^{1}/_{\sum_k A_k O_k e^{-\beta c_{ki}}} \qquad (11)$$

To model congestion, we need to know the flows on each link. Let q_{uv}^h be the flow on link (u, v, h) and let Q_{uv}^h be the set of origin-destination pairs at level h that use the (u, v, h) link. Then

$$q_{uvh} = \sum_{i,j \in Q_{uvh}} T_{ij} \quad (12)$$

γ_{uv}^h is a function of the flow:

$$\gamma_{uv}^h = \gamma_{uv}^h(q_{uv}^h) \quad (13)$$

For this simple demonstration model, we first assume that this function is the same for each link in a particular level of the hierarchy. This can be derived from standard speed-flow relationships. Then the equilibrium position of the network can be obtained by following the sequence of equations (8) – (13) and iterating. This will depend very much on the initial conditions and in particular, on the links that are in place at higher levels in the hierarchy. These initial conditions represent the 'DNA' of the system at this time – cf. Wilson (2010). The dynamics, in this case can be thought of as the addition of higher level links into the network, within a budget, to minimise an objective function – say, consumers' surplus, or for the sake of simplicity, total travel (generalised) costs measured in money units. This would be a mathematical programme of the form

$$\min \quad C = \sum_{i,j} T_{ij} c_{ij} \quad (14)$$

subject to (8) - (13) and

$$\sum_{uv,\gamma(t)_{uv}^h} \Gamma^h \rho_{uv} = \Gamma(t+1) \quad (15)$$

The summation in (15) is over possible links that do not exist at time t, so that new links can be added up to an incremental budget spend of $\Gamma(t+1)$.

The dynamic model would then take the form: run (14) – (15), insert the new γ_{uv}^h into (8) – (13) and iterate as usual (as an inner iteration) and then begin the outer iteration again with a repeat of (14) – (15). This is easy to formulate conceptually. The mathematical programme (14) – (15) is difficult to handle in practice and we use a simpler approach in the demonstration model that follows.

This leads to the possibility of a second dynamic model in which each link at each level of the hierarchy is given a capacity, x_{uv}^h and the link costs then become functions of this capacity as well as the flows:

$$\gamma_{uv}^h = \gamma_{uv}^h(q_{uv}^h, x_{uv}^h) \quad (16)$$

We can then think of a difference equation in Δx_{uv}^h. Suppose we take γ_{uv}^h as a measure of congestion – that is if the unit generalised cost on a link is high, we attribute this to congestion. (There will still be the problem of dealing with non-existent links where this is formally infinite.) We could control the costs in this case by increasing Γ^h in an iterative cycle to ensure that a budget constraint is met. This suggests

$$\Delta x_{uv}^h = \varepsilon^h [\gamma_{uv}^h - \gamma], [\gamma_{uv}^h - \gamma] > 0 \; ; \; \Delta x_{uv}^h = 0 \quad otherwise \quad (17)$$

where γ is taken as a threshold and ε is calculated to ensure that the budget constraint holds for this time period:

$$\sum_{uv,\gamma(t)_{uv}^{h}} \Gamma^{h} \rho_{uv} = \Gamma(t+1) \tag{18}$$

$x_{uv}^{h}(t+1)$ would then be fed back into (8) – (13) – with (16) replacing (13).

4. A demonstration model.

The demonstration network built for the testing stage is a simple spider grid with 8 centroids generating and receiving travel demand and 20 network nodes, connected by several bidirectional links (see Figure 1).

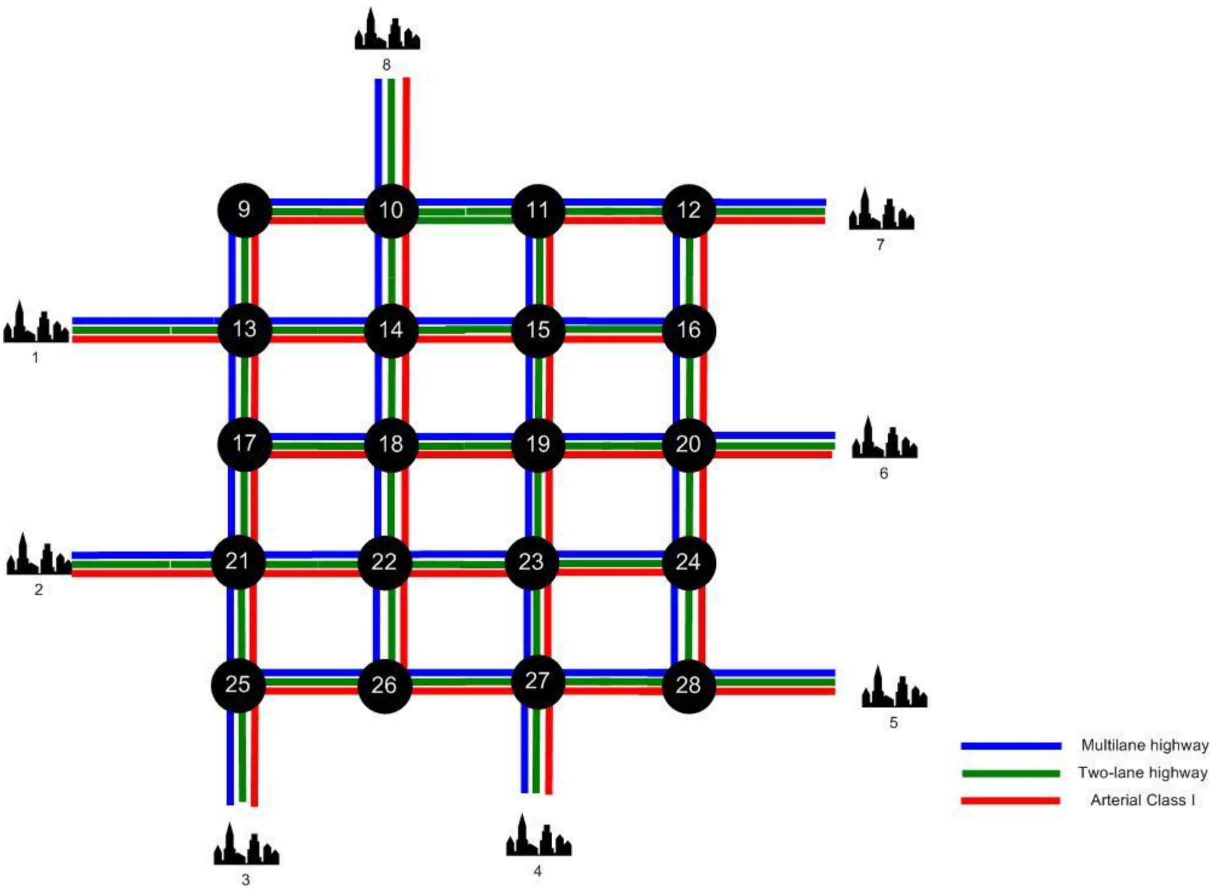

Figure 1. Centroids, nodes and interconnecting bidirectional links making up the spider network

The model has been implemented with MatLab® and MatLab Optimization Toolbox® (MatLab User Guide, 2007). The data set needs travel demand generated from centroids $\{O_i\}$, attracted flows to centroids $\{D_j\}$ and a travel cost parameter β, while the capacity of the generic link (u, v), q_{uv}, the links length l_{uv}, the free flow speed v_{0uv} , the speed at capacity v_{Cuv}, and the adjacency matrix A define the characteristics of the supply model. An incremental assignment process (Cascetta, 2009) has been used to investigate the evolution of the network. A fixed demand of 250 vehicles per OD pair is assigned initially, to the network iteratively. The assignment process at a particular iteration takes into consideration the assigned flows and the link costs generated at previous iterations. This constitutes a preload of the entire network by the assignment of this fixed demand before additional flows are added through an incremental assignment process. The preload demand is generated from each O_i while the attracted flows values D_i have been randomly chosen. To facilitate the interpretation of results, the additional-flow incremental assignment focuses on two OD pairs demand (OD 2-7 and OD 7-2). No demand modeling is then needed. In Table 1 preload and incremental demand values of generated and attracted flows are shown along with a summary of supply system principal characteristics.

Preload			***Incremental demand***		
centroids	O_i	D_i	***centroids***	O_i	D_i
1	250	200	1	0	0
2	250	100	2	250	250
3	250	400	3	0	0
4	250	300	4	0	0
5	250	400	5	0	0
6	250	100	6	0	0
7	250	200	7	250	250
8	250	300	8	0	0

Facility type	V_0 ***(km/h)***	V_c ***(km/h)***	q_{uv} ***(veic/h)***	α_1	α_2
ML Highway	96.5	88.5	Starting from 3000*	1	3
2L Highway	111 1	70.8	1800	1	3
Arterial Class I	80.5	53.1	1500	1	3

All q_{uv} values are intended per flows direction.

(*) depending on the number of lane (at least 2)

Table 1. Demand and supply system characteristics

Through a BPR function it is possible to obtain the cost of travel on the generic link γ_{uv} that at first step depends on preloaded flow (Cascetta, 2009).

$$\gamma_{u,v} = \frac{l_{u,v}}{v_{0u,v}} + \alpha_1 \left(\frac{l_{u,v}}{v_{Cu,v}} - \frac{l_{u,v}}{v_{0u,v}} \right) \left(\frac{F_{u,v}}{q_{u,v}} \right)^{\alpha_2} \tag{19}$$

where α_1 and α_2 are assigned values 1 and 3 respectively for the purposes of this demonstration (see Table 1).

Through a Dijkstra algorithm, all shortest paths R_{ij} are computed and path costs c_{ij} are deduced from

$$c_{ij} = \sum_{(u,v,h) \in R_{ij_{min}}} \gamma^{h}_{u,v} \quad (20)$$

The preload flows can be obtained through the assignment process of the generic travel demand T_{ij} from origin i to destination j that can be obtained through the equation (9) – (11) above, where β value is assumed to be 1 for simplicity. These are solved with the Quasi-Newton optimization algorithm (MatLab Optimization Toolbox, 2007). Link flows F_{uv} are defined by assigning the travel demand to the set of paths R_{ij} according to:

$$F_{uv} = \sum_{(u,v) \in R_{ij_{min}}} T_{ij} \quad (21)$$

The dynamic evolution of the network can be described through a differential equation – the link analogue of equation (4) in the retail case - with the following form:

$$\partial q_{uv}/\partial t = \varepsilon\,[\gamma_{uv}(q_{uv}, F_{uv}) - \gamma] \quad (22)$$

where γ is the average link speed:

$$\gamma = (\Sigma_{uv}\gamma_{uv})/\text{no. of links} \quad (23)$$

and ε is a function representing the response of the link capacity to variations from the average. It is convenient, as in the retail case, to transform this continuous relationship into discrete form and to consider the function as constant in the time interval.

$$\Delta q_{uv} = \varepsilon[\gamma_{uv}(q_{uv}, F_{uv}) - \gamma] \quad (24)$$

The evolution of the network by varying the considered type of road has been based on the results obtained by the simulation process of three different facility types (arterial, two-lane highway and multilane Highway) described in the Highway Capacity Manual (2010). In this case, we retain the principle represented by equation (24) but replace it by a simpler procedure: the facility type is upgraded each time the link Volume Over Capacity (VOC) ratio reaches the value of 0.85, which we assume to be the start of congested behaviour. We can then see the evolution of the network through incremental assignment of the additional travel demand. By the definition of the minimum path in terms of link travel cost, that in this case are the only link travel times, traffic flows are then assigned, and this procedure has been repeated for 40 time steps, adding 250 units at each step.

In Figure 2 the trend of the number of activated links during incremental assignment process is shown.

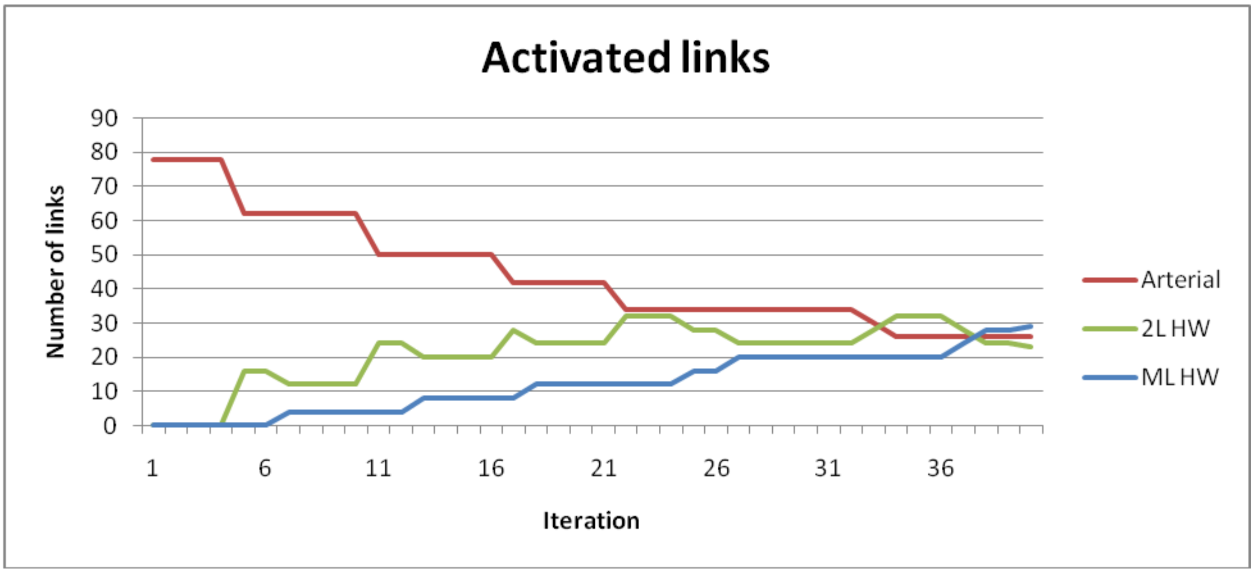

Figure 2. Number of activated links at each hierarchical level, at each step, during incremental assignment.

In the preload assignment the whole network has been activated, while during the incremental assignment phase 62 links has been used. This number is composed from links of different facility types, the ones previously mentioned, and during the incremental procedure no downgrade has been admitted. To simplify the presentation of the results, 7 milestones have been chosen (iterations 1, 10, 20, 25, 30, 35 and 40). It is at these milestones that 'jumps' occur in the evolution of the network: if any element of the figure is compared to the previous one, it will be seen that new higher level links appear. Table 2 reports the status of the network in terms of number of links (Total column) used during the incremental assignment and the facility type activated (arterial, 2 lane highway, multiLane highway), the activated capacity of the network (Q column) intended as the sum of the capacity of the activated links at a given iteration, and the saturation (Satur column) of the activated network intended as rate between the assigned flows, including preload flows, and the capacity of the activated network.

Iteration number	*number of used links*				*Q [veic/h]*	*Satur [Flows/Q]*
	Total	*Arterial*	*2L HW*	*ML HW*		
1	16	16	0	0	24000	0.172
10	28	12	12	4	51600	0.426
20	44	8	24	12	91200	0.383
25	48	4	28	16	104400	0.440
30	52	8	24	20	115200	0.493
35	52	0	32	20	117600	0.603
40	62	10	23	29	143400	0.495

Table 2. Network status at given milestones

Figure 3 and 4 represent the network status evolution. For clarity the bidirectional links have been split in two. We only show the links that are used for the flows between nodes 3 and 7. The results show very clearly the introduction of higher level links as the flows increase.

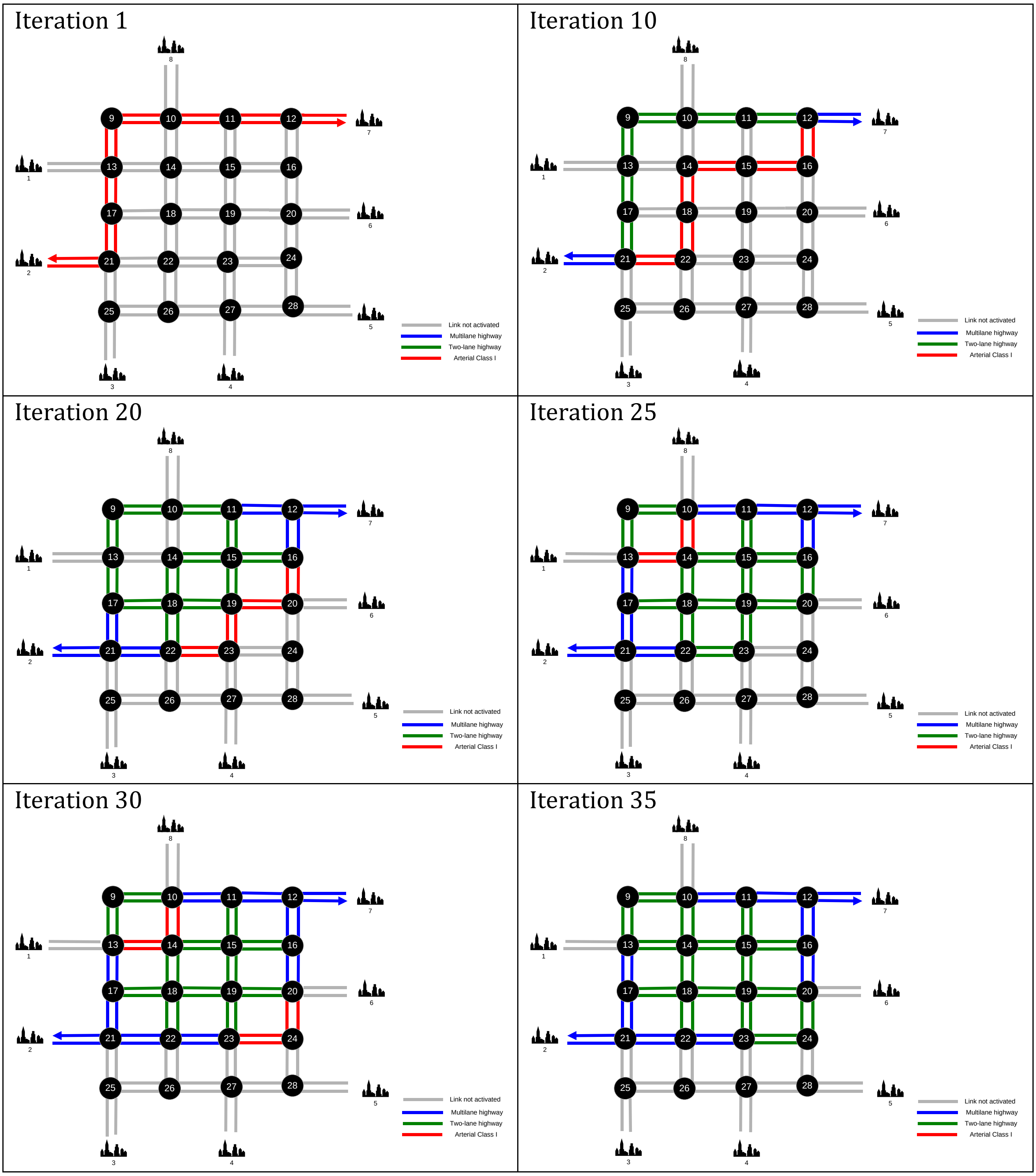

Figure 3. network status at iteration 1, 10, 20, 25, 30 and 35

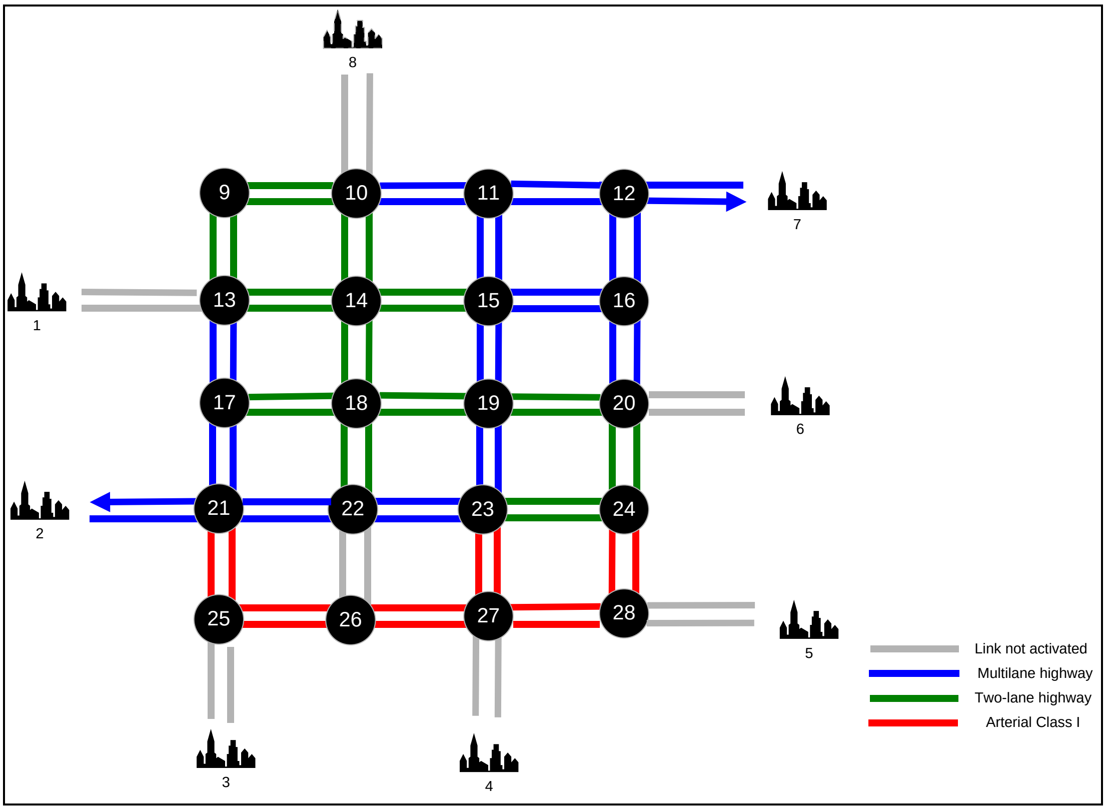

Figure 4. network status at final iteration 40

It is also useful to begin to consider investment costs. In Table 3, typical parametric costs are reported and in the last column the total link costs for each facility type.

	width [m]	*Cost [€/meter]*	*Cost each link [€]*
Arterials	9	217.11	434,220
2l Highway	10.5	771.87	1,543,740
ML Highway	47	3107.04	6,214,080

Source: Italian Public works Authority (2008)

Table 3. Network link costs

The costs are applied only to the used links and to their upgrading. In Table 4, the C_links column represent the total costs needed for network upgrading. The starting value considered is €33,864,480.00 that represent the whole network cost considering Arterial class as the facility type used. The values in the rows represent the difference in terms of € needed for upgrading the network to the selected iteration status starting from the selected one.

Iteration	*C_links*
1	0
10	14,727,040.00
20	66,500,960.00
25	29,294,560.00
30	20,417,440.00
35	8,877,120.00
40	41,703,200.00

Table 4. Network costs at milestones

If each iteration is considered to be a time period, then a challenge for further work is to select links for upgrading in such a way that budget constraints are satisfied – possibly by upgrading the most congested link first, and so on until the budget for that time period is exhausted.

5. Concluding comments.

We have shown through the explicit introduction of hierarchies into dynamic models of evolution that the ideas represented in the dynamic retail model can be transferred to the task of modelling the evolution of transport networks. Of course, there is much more work to be done: first, to introduce budget constraints as already noted; and secondly, to apply these ideas to real networks. These tasks will be undertaken in future papers.

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