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**Using Geo-spatial
Agent-Based Models
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Using Geo-spatial Agent-Based Models for Studying Cities

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The agent-based modelling (ABM) paradigm provides a mechanism for understanding the effects of interactions of individuals and through such interactions emergent structures develop, both in the social and physical environment of cities. This paper explores how through the use of ABM, and its linkage with complexity theory, allows one to create agent-based models for the studying cities from the bottom-up. Specifically the paper focuses on segregation and land-use change. Furthermore, it will highlight the growing interest between geographical information systems (GIS) and ABM. This linkage is allowing modellers to create spatially explicit agent-based models, thus relating agents to actual geographical places. This approach allows one to explore the link between socio-economic geography of the city and its built physical form, and can support decision-making regarding interventions within the social and physical environment.

1 Introduction

Cities play a critical role in our lives, providing habitats for more than half the world's population. The United Nations expects that over half (3.3 billion people) of the world's population will be located in urban areas by 2008 (United Nations, 2007) and this proportion is predicted to increase to over 75 percent by the year 2100. However, understanding such systems is not at all an easy task as they are composed of many parts which are dynamic, rapidly evolving, undergoing continual growth, change, decline and restructuring usually simultaneously (White and Engelen, 1993). Such change is a result of the interaction of large numbers of discrete actors interacting within space. This heterogeneous nature of cities makes it difficult to generalise localised problems from that of city-wide problems. Although our understanding of

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cities has increased throughout the twentieth century, incorporating ideas and theories from a diverse range of subjects including economics, geography, history, philosophy, mathematics and more recently computer science, it is now very clear that there are intrinsic difficulties in applying such understanding to policy analysis and decision making.

As Wilson (2000) writes, such understanding of cities represents “...one of the major scientific challenges of our time”. Human behaviour cannot be understood or predicted in the same way as in the sciences such as in the physical or chemical world. To understand urban problems such as sprawl, congestion, segregation, crime, migration and housing markets, researchers have recently focused on a bottom-up approach to urban systems, specifically researching the reasoning on which individual decisions are made. One such approach is agent-based modelling (ABM) which allows one to simulate the individual actions of diverse agents, measuring the resulting system behaviour and outcomes over time. This modelling approach provides an important medium for the study and management of urban systems affected by countless factors including economic, social, and environmental which are notoriously difficult to simulate (Torrens, 2000).

The remainder of the paper will provide a general overview of why there is a need for agent-based models for studying cities, how it links to how we believe cities operate through ideas of complexity theory, review and discuss a range of applications where agent-based models have been developed specifically focusing on urban phenomena at the individual level linking it to complexity theory where appropriate, and how such models lead to more aggregate structures developing in the social and physical environment. The paper will conclude with challenges modellers face when using agent-based models to study cities, and identify future avenues of research especially in relation to decision making.

2 Why the growth of agent-based models for cities?

The growth of ABM coincides with how our views and thinking about urban systems has changed. Rather than adopting a reductionist view of systems, whereby the modeller makes the assumption that cities operate from the top-down and results are filtered to the individual components of the system (see Torrens, 2004), people are

now adopting a reassembly approach to the system (O'Sullivan, 2004). This change follows the realisation that, planning and public policy do not always work in a top-down manner; aggregate conditions develop from the bottom-up, from the interaction of a large number of elements at a local scale (Pickles, 1995). Thus there is a move towards individualistic, bottom-up explanations of urban form and behaviour which links to what we know about complex systems. Such an approach is ABM, however, before discussing the advantages of ABM and how this relates to our understanding of cities, a brief examination of complexity science is first needed.

An exact definition of complexity is hard to pin down; as it has different meanings to different people. However, Manson's (2001; 2007) taxonomy helps to clarify the broad subject area by classifying complexity research into three broad categories: algorithmic (i.e. the complexity of a system lies in the difficulty faced in describing system characteristics), deterministic (i.e. unpredictable dynamic behaviour of relatively simple deterministic systems, where unpredictable refers to the sensitivity of outcomes based on initial conditions) and aggregate complexity (i.e. the study of phenomena characterised by interactions among many distinct components). These categories refer to aspects of phenomena that are not mutually exclusive and while these three major divisions allow a more coherent understanding of complexity theory, but these are not the only possible classifications (see for a debate: Reitsma, 2003; Manson, 2003).

Nonetheless, the main characteristics of complex systems – self-organisation, emergence, non-linearity, feedback and path dependence – provide a new way of thinking about cities and new tools for solving problems faced by cities. Emergent phenomena are characterised by stable macroscopic patterns arising from local interaction of individual entities (Epstein and Axtell, 1996). A small number of rules or laws, applied at a local level and among many entities, are capable of generating complex global phenomena: collective behaviours, extensive spatial patterns, hierarchies etc. are manifested in such a way that the actions of the parts do not simply sum to the activity of the whole. Thus, emergent phenomena can exhibit properties that are decoupled (i.e. logically independent) from the properties of the system's parts. For example, a traffic jam often forms in the opposing lane to a traffic accident, a consequence of 'rubber-necking'. Studying the behaviour of collections of

entities focuses attention on relationships between entities (O'Sullivan, 2004) because before change is noticed at the aggregate level, it has already taken place at the micro-level. Characteristics of emergent phenomena make them difficult to understand and predict, particularly as emergent outcomes can be counterintuitive (Epstein, 1999). Furthermore, the importance of history/path dependence make models based on such notions very sensitive to initial conditions and to small variations in interaction rules (Couclelis, 2002). Using such models for prediction can therefore be challenging. Despite this, complexity theory has brought awareness of the subtle, diverse, and interconnected facets common to many phenomena, and continues to contribute many powerful concepts, modelling approaches and techniques especially in relation to agent-based models (see below).

The use of complexity theory has numerous advantages with regard to our understanding and interpretation of cities. Cities happen to be problems of organised complexity they present situations in which half a dozen quantities are all varying simultaneously and in subtly interconnected ways (Jacobs, 1961). Change is only noticeable when different patterns become discernable, but before change at the macro-level can be seen, it is taking place at many micro-levels (subsystems) simultaneously, all of which interact separately, together forming a complex web of interactions (Holland, 1995). Understanding such systems from the 'bottom-up' is crucial with regard to urban planning (Batty, 1995). Urban geography provides many examples of self-organisation and emergence; for example, it is the local-scale interactive behaviour (commuting, moving) of many individual objects (vehicles, people) from which structured and ordered patterns emerge in the aggregate, such as peak-hour traffic congestion (Nagel *et al.*, 1997) and the large-scale spatial clustering of socioeconomic groups by residence (Schelling, 1971). In urban economies, large-scale economies of agglomeration and dispersion have long been understood to operate from local-scale interactive dynamics (Krugman, 1996). Additionally, cities exhibit several signatures, characteristic of complexity, including fractal dimensionality and self-similarity across scales, self organization, and emergence (see Batty and Longley, 1994; Allen, 1997; Portugali, 2000).

In summary, complexity science offers a new way of thinking about cities, especially when combined with ABM, and provides us with new tools to explore and analyse

urban systems from the 'bottom-up'. In a sense, agent-based models can be thought of as miniature laboratories where the key attributes and behaviour of agents, and the environment in which they are housed, can be altered and the repercussions observed over the course of multiple simulation runs, thus providing a tool to 'think with' and therefore supporting decision making.

But what is meant by ABM? While there is no universal agreement on a precise definition of the term 'agent', definitions tend to agree on more points than they disagree (Macal and North, 2005). Agent characteristics are difficult to extract from the literature in a consistent and concise manner, because they are applied differently within disciplines (Castle and Crooks, 2006). However, many agree that the main characteristic of ABM is that we represent the system with discrete individuals which interact with each other and their environment. Furthermore, the agent-based concept is a mindset more than a technology, where a system is described from the perspective of its constituent parts (Bonabeau, 2002). The concept of an agent is meant to be a tool for analysing a system, not an absolute classification where entities can be defined as agents or non-agents (Russell and Norvig, 2003). A detailed discussion about the definition and characteristics of agents is beyond the scope of this paper and readers are referred to writings of Wooldridge and Jennings (1995), Torrens (2004), Macal and North (2005), and Castle and Crooks (2006), for further discussions.

However, there are several key features of agents which make them attractive to studying cities and as a tool for complexity science in general. First is their ability to model multiple autonomous units (i.e. governed without the influence of centralised control), situated within a model or simulation environment. Animate (mobile) agents can be considered as agents who move about the systems, such as pedestrians. In contrast, inanimate (immobile) agents such as land parcels do not move but can change state. Secondly, ABM allows for the representation of a heterogeneous population therefore the notion of a mean individual is redundant, a common assumption of past urban models (Torrens, 2000). Agents permit the development of autonomous individuals. For example, an agent representing a human could have attributes such as age, sex, job etc. Groups of agents can exist, but they are spawned from the bottom-up, and are thus amalgamations of similar autonomous individuals. Such heterogeneity allows for the specification of agents with varying degrees of

rationality (see Axelrod, 2007). For example, throughout the social sciences, the dominant form of modelling is based upon the rational-choice paradigm (Axelrod, 2007). Rational-choice models generally assume that agents are perfectly rational optimisers with unfettered access to information, foresight, and infinite analytical ability (Parker *et al.*, 2003). These agents are therefore capable of deductively solving complex mathematical optimisation problems in order to maximise their well being; balancing long-run and short-run payoffs in the face of uncertainty. While rational-choice models can have substantial explanatory power, some of their axiomatic foundations are contradicted by experimental evidence, leading prominent social scientist to question their empirical validity (Axelrod, 2007). However, agents can be configured with 'bounded' rationality (through their heterogeneity), to circumnavigate the potential limitations of these assumptions (i.e. agents can be provided with fettered access to information at the local level). In affect, the 'perception' or in other words the knowledge of agents can be constrained. Thus, rather than implementing a model containing agents with optimal solutions that can fully anticipate all future states of which they are part of, agents make inductive, discrete, and adaptive choices that move them towards achieving goals. For instance, an agent may have knowledge of all building exit locations, but agents will be unaware if all exits are accessible (e.g. some may have become blocked through congestion) this is unlike those of rational-choice models (Castle and Crooks, 2006). This offers advantages over approaches that assume perfectly rational individuals, if they consider individuals at all. Thirdly, agents are active because they exert independent influence in a simulation. These autonomous units are capable of processing information and exchanging this information with other agents in order to make independent decisions. A relationship between agents is specified, linking agents to other agents and/or other entities within a system. Relationships may be specified in a variety of ways, from simply reactive (i.e. agents only perform actions when triggered to do so by some external stimulus) to goal-directed (i.e. seeking a particular goal). Furthermore, agents can also be designed to be adaptive, producing Complex Adaptive Systems (CAS; Holland, 1995). Agents can be designed to alter their state depending on their current state, permitting agents to adapt with a form of memory or learning.

The ability of agent-based models to describe the behaviour and interactions of a system additionally allows for system dynamics to be directly incorporated into the

model. This represents a movement away from the static nature of earlier styles of urban modelling which was one of their major failings (see Batty, 1976). However, while time in agent-based models is still discrete, i.e. it still moves in ‘snapshots’, the time steps may be small enough to approximate real time dynamics. Additionally, it is apparent that different processes occur in space and over different time scales (Liu and Andersson, 2004). For example, the location of residents and businesses is affected by long term processes, such as economic cycles and transport projects, and short term events in the form of daily commuting or hourly social interactions. Agent-based models can incorporate these different scale time processes into a single simulation by using a variety of automata clocks designed to mimic the temporal attributes of the specific urban process under study (Torrens, 2003), thus allowing the modeller to realistically simulate urban development (O’Sullivan, 2001). The choice of time in terms of both an event-scheduling approach and a temporal resolution can have important consequences for the behaviour of the model (see Brown *et al.*, 2005b for a more detailed discussion). In relation to urban dynamics, the ability to model different aspects of time is highly appealing. It is not just different temporal periods that can be incorporated within an agent-based model but different spatial scales can also be included. This flexibility is extremely important as it is the phenomena of interest which drives the scale to be used, not the modelling methodology. For example, from the micro movement of pedestrians within a building during an evacuation (e.g. Castle, 2007), to the movement of cars on a street network (e.g. Nagel *et al.*, 1999), to the study of urban growth (e.g. Brown *et al.*, 2005a). Additionally, as ABM allows for the representation of individual objects, it is therefore possible to combine these objects to represent phenomena at different scales within the same model. This means agent-based models can be useful tools for studying the effects of processes that operate at multiple scales and organisational levels (Brown, 2006). Furthermore, ABM incorporates many of the advances made in urban modelling such as dynamics, detail, usability, spatial flexibility and realism (see Torrens, 2000, 2001).

3 Example applications of agent-based models for cities

In many, cases ABM can be considered as a natural method within the social sciences for describing and simulating a system composed of real-world entities, especially when using object-orientated principles (see Castle and Crooks, 2006; Torrens, 2001)

where the focus is on individuals and how through individuals more aggregate properties emerge. The agent-based approach is more akin to reality than other modelling approaches, rendering ABM inherently suited to simulating people and objects in realistic ways. Agent-based simulations provide an opportunity to represent and test social theories which cannot easily be described using mathematical formula (Axelrod, 1997). Agent-based models often map more naturally to the structure of the problem than equation-based models (Parunak *et al.*, 1998) by specifying simple behavioural and transition rules attached to well defined entities, therefore providing a medium for the infusion of any geographic theory or methodology into the model. Furthermore, by modelling the behaviour of individual entities interacting, the agent-based approach enables users to study the aggregate properties of the system from the bottom-up. However, ABM is not without its challenges and problems, which we briefly discuss in section 8. Nevertheless, because of the advantages and reasons sketched out above ABM is increasingly being used as a tool to study a diverse range of phenomena. From archaeological reconstruction of ancient civilisations (Axtell *et al.*, 2002); size-frequency distributions for traffic jams (Nagel and Rasmussen, 1994); spatial patterns of unemployment (Topa, 2001), to name but a few. The remainder of this section explores a range of applications from the micro to the macro and demonstrates how ABM can be used to study a range of problems within cities with a particular emphasis on the social and physical environments. But before describing such models a caveat is needed, that it is impractical to comprehensively and thoroughly review the full range of ABM applications and provide adequate descriptions of each model within this paper. Within this section we therefore only explore a small number of models, chosen to demonstrate that the interaction of individual agents lead to the emergence of more aggregate patterns.

Despite the advantages of ABM as a tool for simulation, ABM has only recently started to be adopted in urban systems research. Thomas Schelling is credited with developing the first social agent-based model in which agents represent people, and agent interactions represent a socially relevant process. Schelling's (1971) model demonstrated that stark geographical segregated patterns can emerge from migratory movements among two culturally distinct, but relatively tolerant, types of household via mild discriminatory choices by individuals. (Schelling-type models and models inspired by it will be further explored below). Yet ABM did not begin to feature

prominently in the geographical literature until the mid-1990s, when Epstein and Axtell (1996) extended the notion of modelling people to growing entire artificial societies. The goal was to understand the emergence of patterns, trends, or other characteristics observable in a society and its geography. Epstein and Axtell's Sugarscape model demonstrated that agents could emerge with a variety of characteristics and behaviours suggestive of a rudimentary society (e.g. in terms of patterns of death, disease, trade, health, culture, conflict, war, etc).

4 Residential segregation

We start our exploration with segregation. Interest in such phenomena arises because people get separated along different lines and in different ways. There is segregation by sex, age, income, language, colour, taste, comparative advantage, and accidents of historical location. Some segregation is organised; some is economically determined; some results from specialised communication systems; and some results from the interplay of individual choices that discriminate and is seen in many cities. It is worth noting that it is not just residential groups that segregate, for segregation takes many other forms. Types of land-use, for example, residential, commercial, agricultural, are segregated in space. Types of businesses and industries are often segregated in clusters that indicate how they relate to one another. Interest in simple models such as Schelling's model for explaining how such complex phenomena might arise are due to the fact that while patterns of segregation are all too clear when one travels around any urban area, it is difficult to observe how such patterns might arise over time. For example, there are clear clusters of economic groups and residential groups based on ethnicity or social class. One might think that individuals must have strong preferences for these racially or economically homogeneous neighbourhoods to emerge. However, this is not the case. Empirical evidence suggests that individuals do not have strong racial preferences, but have rather mild preferences (see Clark, 1991; Antonovics *et al.*, 2003). Furthermore, to find clear examples of the segregation process taking place is difficult, because it only becomes noticeable when it is clearly underway, and by then a detailed chronology becomes impossible to reconstruct (Batty *et al.*, 2004). So while it is possible to quantify the degree of segregation within neighbourhoods (e.g. Reardon and O'Sullivan, 2004), it tells us little about the behaviour that leads to, or that leads away, from particular outcomes. To understand

this behaviour, we have to examine how individual choice leads to these outcomes, a process that can be explored through the use of ABM.

Schelling's model is excellent because it distils the key features enabling us to understand how segregation might arise. The model does not presume to tell us about the entire workings of the social and economic world, but focuses on the task at hand, namely to explain why weak individual preferences are consistent with strong and persistent patterns of segregation. The rules within the Schelling model are simple, simply stated all agents want to be located in areas where a certain percentage of their neighbours are like themselves. However, these simple rules give rise to complex and unanticipated behaviour in the system. This key feature of the model arises because the decisions of any one individual can impact in unexpected and unanticipated ways upon the decisions of others. A group of individuals can be perfectly happy in a neighbourhood. Unexpectedly, an agent arrives to fill an empty space. The newcomer may tip the balance – 'residential tipping' – so the agents who were previously content now decide to move. In turn, their moves may disrupt settled neighbourhoods elsewhere, and so the effects percolate through the system. No single individual intends this to happen or even necessarily desires this overall outcome, but local interactions between them produce global segregation.

What is important about this model and with many other agent-based models is that one cannot predict the precise outcome of a particular simulation, as the model is sensitive to initial conditions and interaction rules. When the model starts, we possess all the information that exists about it, for we know exactly how each individual behaves. At any stage of the simulation, we know exactly what has happened. Yet we cannot predict the exact outcome of any particular solution to the simulation. However, we know broadly that at each outcome, the agents will separate into distinct neighbourhoods surrounded by their own type and during the simulation neighbourhoods will change. This has important implications with respect to policy decision making. Since we cannot predict it, we cannot control it, even though we have full and complete information (Ormerod, 2005).

Unknowingly, Schelling was one of the pioneers in the field of ABM (Schelling, 2006). He emphasised the value of starting with rules of behaviour for individuals and

using simulations to discover the implications for large scale outcomes. His model highlights how peoples' actions may be influenced by others who act in a given way and how changes in individual behaviour alter the makeup of the population. Thus individuals' actions are both a response to some population statistic and contribute to that statistic. Schelling's model has generated important insights regarding how micro-level residential choice behaviour can produce complex aggregate-level patterns of ethnic residential segregation. Additionally, it has continued to inspire theory and research into the segregation phenomena. For example, Bruch and Mare (2005) compared Schelling's model with stated preference data on residential choice for various race-ethnic groups (e.g. Asians, Hispanics, whites and blacks) within American cities. The preference data showed that most people were unwilling to live in neighbourhoods in which their own race-ethnic group is the minority. However, Schelling's work has also received criticism; for example, Massey and Denton (1993) correctly point out that the 'residential-tipping' point model is not sufficient in itself as an explanation of segregation for many reasons. They comment that while it accurately captures the dynamic effects of prejudice, it accepts as a given the existence of racial discrimination. But what really matters is that individuals have preferences for both place and people. The remainder of this section will briefly explore some of the ABM applications which extend or are inspired by Schelling's original model.

Others have extended the Schelling model to incorporate other factors into their models, such as the inclusion of preferences for neighbourhood status and housing quality, and differing levels of socio-economic inequality within and between ethnic populations (see Fossett and Senft, 2004). Bruch (2006) explored the relationship between race and income, and how both interact to produce and maintain segregated neighbourhoods within Los Angeles. Within the model, agents were given a race and an income, and the model examined the probability of an agent moving into a neighbourhood of a given racial and economic composition. Crooks (2010) explores adding new agents and removing old agents from an existing population and how such change altered existing neighbourhood patterns. This phenomenon can be considered as the effect of immigration, or aging and the death of populations in urban areas.

Researchers from Tel Aviv University have been particularly active in the field of ABM, segregation and residential dynamics. They have investigated residential dynamics using agent-based models from abstract systems to real-world examples (see Benenson, 1998; Benenson *et al.*, 2002; Omer, 2005, Benenson and Or, 2009). Benenson (1998) explored how a theoretical city evolved when agents have both economic and cultural preferences. Omer (2005) extended the Schelling model to include a further hierarchical level; that is, the agents' ethnic identities are organised in a two-level hierarchy where each agent belongs to an ethnic group and a subgroup. For example, the British Asian community is multi-differentiated in terms of nationality, country of origin, religion, caste, class and language. Extending the Schelling model to include additional hierarchical level allows for further research dealing with the role of ethnic preferences on residential choice.

Of special interest is the study of fine scale residential segregation using individual census records and GIS data for representing streets and buildings (see Benenson and Omer, 2003). Benenson *et al.* (2002) have used this kind of detailed dataset to simulate ethnic residential dynamics between 1955-1995 in the Yaffo area of Tel Aviv. The model itself consists of two interacting layers, one layer representing mobile agents comprised of three cultural groups that of Jews, Arab Muslims, and Arab Christians, located on a physical environment layer representing streets and buildings. Each house is converted into a Voronoi polygon rather than using a regular cell space model (e.g. Fossett and Senft, 2004). The agents' residential behaviour within the model is affected by the ethnic composition of the neighbourhood defined using Voronoi polygons. A neighbour is a Voronoi polygon that has a common boundary and features such as roads act as barriers between these neighbourhoods.

Many of the models so far discussed, use cells to represent the agents environment. Within such cell space models neighbourhoods are often based on 'Moore' neighbourhood or 'von Neumann' neighbourhood or variations of these (Batty, 2005b). However, neighbourhoods mean different things to different people. Some may perceive a neighbourhood as houses that are directly attached to their home (e.g. Benenson *et al.*, 2002), while others may consider a street, or a collection of streets as their neighbourhood. A number of authors have demonstrated that neighbourhood sizes impact on the pattern of segregation (see for example, Laurie and Jaggi, 2003;

O'Sullivan *et al.*, 2003) but few take into account the impact of physical and spatial barriers (notable exceptions include Benenson *et al.*, 2002; Crooks, 2010). This is crucial for studying residential patterns within cities. For example, areas within cities are bounded by features such as highways, railway lines, rivers, lakes, and parks which can act as boundaries between residential groups (e.g. Rabin, 1987). Such divisions may promote numerous forms of separation such as residential segregation or influence urban form, yet are often overlooked in aggregate zonal analysis (Talen, 2003) and in ABM. Crooks (2010) explored the effect that such features have on the outcome of a Schelling type model and demonstrated how such features can be incorporated into this type of model.

The examples presented in this section can be viewed as a continuum between abstract demonstrations to real-world applications. Each one brings something new to the basic insights Schelling first presented. There are those that apply the Schelling model to empirical data (e.g. Bruch and Mare, 2005), those that explore the effect of differing neighbourhood sizes (e.g. O'Sullivan *et al.*, 2003) or shapes (e.g. Benenson *et al.*, 2002) or how through adding new agents and removing old agents from an existing population, altered existing neighbourhood patterns (e.g. Crooks, 2010), those that extend the Schelling model to incorporate subgroups (e.g. Omer, 2005) which has the potential to allow the model to be applied to different ethnic or socio-economic groups that makeup a city or region if so desired. Others introduce and explore other determinants of segregation such as income and housing quality (e.g. Fossett and Senft, 2004).

5 Residential location

Moving away from segregation, the paper explores more generally location choice within cities, and how agent-based models can be used to study such phenomena. Such interest arises as new and more established inhabitants and businesses within urban areas are faced with the fundamental decision of “where to locate?” This choice of location results from complex interrelationships between individual actions constrained by many social, political and economic factors. For example, for a resident, location is a trade-off between price of dwelling, type of residence and its location, both in terms of neighbourhood and in relation to place of work, all of which vary depending on age, sex, marital status and income.

There are various models and modelling techniques pertaining to the development of cities and regions (see Wilson, 2000), but one model that lies at the heart of urban economic theory is the trade-off between a consumer's demand to minimise distance travelled to various activities and a desire to capture as much living space as possible. This theory was first formally articulated by Alonso (1964) and can be seen as extending the work of von Thünen (1826). Alonso's model assumed that in the monocentric industrial cities, residents arranged their locations around the central business district (CBD) according to this trade-off between distance (travel cost) and space. As with Schelling's model, the model is simple, it abstracts key elements of the system to explain how land-use within a city is organised. The model illustrates that the structure of preferences and the market for various land-uses appears to lead to wealthy groups being able to capture more space at the edge of the city than the poorer groups who are confined to the inner areas around the CBD. However, the model does not explore dynamics per se, it simply assumes that the pattern of land-use is the result of an equilibrium based formula and leaves one to wonder how and why changes might occur.

By shifting our attention to ABM allows us to explore the evolution of land-use in urban areas from the interaction of many individuals rather than just providing a static snapshot. This approach is appealing as it has the potential to provide a detailed description and explanation of the evolution of urban spatial structure at differing scales. Additionally, this approach to modelling urban systems provides an improvement over past generations of models as it provides the flexibility which permits the consideration of many more factors. For example, in both the Alonso and von Thünen models, features of the landscape such as rivers and roads are often ignored, so that distance to the centre is the underlying determinant of land-use change. Additionally, Alonso's model fails to explain the complexity of the spatial and temporal patterns of urban growth (see Anas *et al.*, 1998 for a discussion). For example, it assumes all employment is centrally located, and it fails to take into account the distinctive nature of buildings and their use which are not easily changed, thus displaying a strong degree of inertia. Furthermore, the use of agent-based models allows us to model both imperfect competition and limited knowledge (see Tesfatsion and Judd, 2006) and how the decisions and actions of agents can be influenced by past

locational decisions (path dependence). The resulting land-use patterns reflect the actions of many individuals, all competing for the same area, and interacting over space and time.

There are numerous agent-based models examining land-use and land-use change and it is not the intention to reiterate these (see Parker, 2005). However, there are relatively few that examine the work of Alonso and von Thünen explicitly (e.g. Kii and Doi, 2005; Sasaki and Box, 2003 respectively). For example, Kii and Doi (2005) model two types of households and commercial firms with two different incomes. Within the model, agents are land-consuming entities, one agent can occupy one cell which is determined by which agent can pay the highest value. This competition between individual agents for the same space within the urban setting over time results in land-use patterns similar to ideas postulated by Alonso (1964). Sasaki and Box (2003) used von Thünen's model to demonstrate how a collection of autonomous individuals operating in a cellular space environment can contribute to the formation of an optimal land-use pattern as described by von Thünen by applying theories of positive feedback and lock-in. Hammam *et al.* (2004) have extended the Sasaki and Box (2003) model to include irregular cells representing farmers and these cells have the ability to change shape, growing or shrinking depending on competition for land. Such an approach has much potential as land parcels in urban areas change shape over time, for example, through changes in function or activity. Additionally, Parker and Meretsky (2004) used the von Thünen model as the basis of an agent-based model to explore conflicts arising between urban and agricultural land-uses which affect the value of particular land-uses. The common thread between the land-use models above is how urban form and function develops through the competition of agents. Furthermore, such models highlight how the ideas, concepts and techniques pertaining to 'classical' urban theory and modelling can be combined using ABM, thereby adding dynamics to such models and showing how urban structures emerge from the bottom-up therefore providing a blended modelling approach (North and Macal, 2007).

6 Abstract to 'real' world applications: linking GIS and ABM

Many of the models presented above represent space abstractly. However, there is a growing interest in the integration of GIS and ABM through coupling and embedding

(see Castle and Crooks, 2006; Brown *et al.*, 2005b; Parker, 2005; Benenson and Torrens, 2004; Gimblett, 2002; for reviews and applications). For agent-based modellers, this integration provides the ability to have agents that are related to actual geographic locations. This is of crucial importance with regard to urban modelling, as everything within a city or region is connected to a place. Furthermore, it allows modellers to think about how objects or agents and their aggregations, interact and change in space and time (Batty, 2005a). For GIS users, it provides the ability to model the emergence of phenomena through individual interactions of features on a GIS over time and space (Najlis and North, 2004). While the integration of ABM and GIS is clearly possible, allowing for a finer grain of urban models, there is no guarantee that by moving to a finer grain, the robustness of the aggregated results will be improved (Lee, 1994). For example, when going from total population to household types to individuals, there is no level at which behaviour (such as location choice) is better known. However, the creation of agent-based models allows one to build tools/models to explore such behaviour and how this manifests itself in aggregate form. A brief review of spatially explicit agent-based models will now follow.

ABM is increasingly being used as a tool for the spatial simulation of a wide variety of urban phenomena (some of which have been discussed above) including: urban housing dynamics (e.g. Benenson *et al.*, 2002); urban growth (e.g. Xie *et al.*, 2007), segregation (e.g. Crooks, 2010); residential and business location (e.g. Torrens, 2006; Barros, 2005) and gentrification (e.g. Torrens and Nara, 2007). Brown *et al.* (2005a) examine residential location with respect to land-use change at the urban-rural fringe. Focusing on how individual decision-making drives land-use decisions, such a modelling approach allows users formulate and test alternative policies and interventions that could reduce environmental costs and enhance environmental benefits. A similar model has also been developed by Yin and Muller (2007), who examine land-use-land-cover change at the urban-rural fringe incorporating households decision making in terms of preferences for accessibility, amenities, and scenic views. Additionally, Bossomaier *et al.* (2007) have developed an agent-based model to study house price evolution in Bathurst, Australia, which focuses on vendor/buyer behaviour. The agent's decisions of where to locate is affected by spatial attributes of actual land-parcels including distance from amenities such as

parks, area, elevation, orientation and environmental factors such as flood risk. These spatial factors combined with an agent's perceptions about the economy, new developments such as factories and roads, along with social trends in the desirability of house ownership and property investment then influence how buyers and sellers modify the price relative to the neighbourhood.

The ability to model and explore how agents move around their environment has allowed the study of micro-scale phenomena such as pedestrian models, which explore how agents move around their environment. Useful examples of spatially explicit models include: the simulation of pedestrians in the urban centres (e.g. Haklay *et al.*, 2001), the examination of crowd congestion (e.g. Batty *et al.*, 2003), emergency evacuation of buildings (e.g. Castle, 2007), or terrorist attacks within the built environment (Mysore *et al.*, 2006). In such models one can explore how the built environment impacts on movement of pedestrians, for example. Furthermore, these models demonstrate how micro interactions with many individuals lead to emergent patterns such as crowds. At a more macro level the ABM paradigm is increasingly being used to simulate the movement of traffic (e.g. Barrett *et al.*, 2002) and to explain features such as traffic jams during the morning rush hour or the effects that new roads might have on future traffic patterns. Furthermore, attempts have also been made to couple agent-based models of traffic to other models which explore different phenomena but still impact upon life within cities. For example, Thorp *et al.* (2006) have combined a spatial agent-based model of vehicle traffic with a cellular automata model of a spreading wildfire to evaluate how different evacuation options for residents Santa Fe, New Mexico. Such a model provides a lens on how residents might react in a wildfire event, thus providing a tool to evaluate different policy options such as the closing of specific roads or turning specific streets one way and how these can increase the flow of people evacuating the area.

7 Discussion

The models presented in this paper demonstrate the ability to move beyond a reductionist (or top-down) approach for studying systems. Instead of dissecting models into logically justified components, the focus lies on multiple interactions among simple basic units which correspond to physically existing entities such as people. This generative (or bottom-up) approach allows us to explore how a small

number of rules or laws, applied at a local level and among many entities, are capable of generating complex global phenomena at different temporal resolutions – collective behaviours, extensive spatial patterns, hierarchies – manifested in such a way that the actions of the parts do not simply sum to the activity of the whole. The richness of the system therefore lies in the way in which interactions between individual entities and their environment generate adaptations over time. Notwithstanding this, the examples also demonstrate how agent-based models provide a suitable means for exploring many aspects of urban phenomena, how human beings change their environment, and how they are affected by it. Such change occurs at the physical, social and economic level, a result of complex interactions between many different individual entities (Liu *et al.*, 2007).

Many of the ABM applications currently utilising geospatial data do so using a cellular space representation of reality. A regular cellular space is populated with agents that can migrate between cells (e.g. Portugali, 2000). Such models show the importance of considering mobility between cells when exploring the processes of segregation and immigration. Often, it is assumed that agents' movement behaviour depends on the properties of neighbouring cells and neighbours. This approach can be related to the supply of data in raster data formats, the computational power needed to compute complex geometries, and the lack of tools necessary to create agents operating in vector space. While agent-based models created using the cellular partition of space have provided valuable insights into urban phenomena, especially as they can capture geographic detail, they miss geometric detail. This area is critical to good applications but is barely touched upon in the literature (Batty, 2005a) with a few exceptions, (e.g. Benenson *et al.*, 2002; Crooks, 2010). The ability to represent the world as a series of points, lines and polygons allows the inclusion of geometry into the modelling process, therefore allowing for different sizes of features such as houses, roads and so on to be portrayed. Furthermore, this allows the use of land parcel datasets that are extensive and fine scale. However before exploring this, it needs to be stressed that vector representation is not necessarily more appropriate for modelling than raster representation. For example, Landis (2001) changed from vector-based polygons to raster-based grid cells in his Californian Urban Futures models to simplify computation. Additionally, Benenson *et al.* (2005) comment that while vector GIS can represent urban objects in spatially explicit models, for

theoretical models the points of a regular grid usually suffice. However, researchers have started using irregular spaces (e.g. Sembloni, 2000), and discovered that many models are sensitive to variations in the structure and size of neighbourhoods between locations in the grid (e.g. O'Sullivan, 2001). This is a topic that the author believes needs further investigation.

For example, many research topics in urban geography and planning explore interactions between spatial socio-economic processes and the built environment. Research into gentrification and social segregation for example, is closely linked to individuals buying and selling of buildings through the property market and urban form. Despite these links, direct measurement and analysis of the built environment is seldom employed in urban geography or ABM applications. The reasons for this omission are that the complexity of urban form data creates difficulties in compiling and analysing datasets; and that the aggregate methodologies used in geographical research do not integrate easily with the fine scale nature of urban form data.

Often the complexity of the built environment is minimised within many agent-based models. For example, buildings are represented as squares or agents movement being restricted to discrete cells. Never-the-less there is a growing interest in linking these geographical and geometrical approaches to provide an improved understanding of cities (Batty, 2007). Over the last decade there has been a continuing development of geographic information technologies and the emergence of rich fine scale digital data sources (Longley, 2003) such as Ordnance Surveys (OS) MasterMap[®] in the United Kingdom. These new detailed datasets have enhanced spatial and non-spatial information, which provides opportunities to model and analyse cities that were unimaginable in the past. These datasets are sufficiently intensive to analyse detailed patterns and morphologies but also sufficiently extensive to enable patterns to be generalised to entire metropolitan areas. It is now possible to link the aggregate socio-economic approach that forms the basis of geographical analysis to the geometric built environment approach that is employed in local urban planning. Batty (2007) has termed this process 'Geography and Geometry', the merging of iconic and symbolic urban models, and it opens up many possibilities for research. Such combined datasets will allow key indicators of urban form and structure –such as density, mix of uses and accessibility – to be measured and analysed. Fine scale relationships between

urban form, function and accessibility can be explored to provide an evidence base for research topics such as urban form and sustainability research, the housing property market, regeneration and gentrification, land-use change and neighbourhood definition (Galster, 2001) and act as a foundation for the creation and initialization of geospatial agent-based models for urban simulations which consider geometrical relationships directly in the simulation process.

For instance in the United Kingdom, there is a database on land parcels (e.g. building footprints) and associated land-uses (OS MasterMap Address Layer 2[®]), and road segment data (OS MasterMap Integrated Transport Network[™] Layer). Current GIS are capable of encoding these datasets into the foundations of a simulation along with providing methods for relating these objects based on their proximity, intersection, adjacency or visibility to each other. However, one major stumbling block in relation to ABM, and modelling more generally, is that there is potentially too much detail when studying an entire city instead of a small area, the problem can become too computationally intensive for the current generation of computers. This problem can be overcome by considering the level of abstraction needed to examine the phenomena of interest and the purpose of the model, for example, is 'all the detail needed?' (see Crooks *et al.*, 2008). Alternatively a series of smaller models could be created to examine specific aspects of the system. There is also a lack of personal data both for the present and the past. For example, in the UK, the smallest measure of individual data from the census is the output area which contains approximately 125 households, notwithstanding access to personal data (see Benenson *et al.*, 2002). One potential solution is to synthetically generate the population through microsimulation techniques (e.g. Birkin *et al.*, 2006).

8 Conclusion

Complexity now dominates our thinking about cities and this has changed our modelling approach. What becomes clear is that the processes at the core of urban modelling occur in space and change over time. We therefore need a different style of modelling coupled with new tools for studying urban systems (see Torrens, 2001). This has led our attention to shift from the aggregate to disaggregate, to that of modelling individuals with individual characteristics located in space whose behaviour has to be described over time. The applications reviewed in this paper

demonstrate how through the interaction of individual entities more complex aggregate structures develop. Examples include the economic distribution of land-uses or segregated neighbourhoods. The models range from explanatory models used to explore theory and generate hypotheses about urban change, to descriptive models concerned with making predictions, to how systems might evolve. Many consider the ABM paradigm as an electronic laboratory to test ideas and theory of urban change, to help understand and potentially predict future events, through analysis and experimentation in a controlled computer environment. This ability to test, refine and create numerous variations of models allows us create many models to explain the same phenomena based on the individual. However, one needs to balance the complexities of such models from insights gained from them in order to aid decision making. Perhaps one of the challenges arising from this is the need for ways of comparing such models. Attempts at devising ontologies and protocols for model comparison are being made, such as the ODD (overview, design concepts, details) protocol proposed by Grimm *et al.* (2006) might be one solution.

The growing interest in the integration of ABM and GIS was also discussed. Such integration allows agent-based models to be spatially explicit and capture model processes in both in time and space. However, this new generation of models is largely experimental in their development, and in many instances have not been applied in practice to the same extent as 'traditional' techniques, especially those of spatial interaction models. There is a need to move from explanatory models to more applied models and empirically based models (Parker *et al.*, 2003) if the ABM paradigm is to prove useful for policy makers. Additionally, when modelling urban systems it is argued that there needs to be consideration of the role of the built environment (geometry) in the simulation process. There are several additional challenges which ABM faces ranging across the spectrum of theory to practice, hypothesis to application (see Crooks *et al.*, 2008). Validation schemes are a classic example of this. One reason for this is simply a function of the degree to which micro-geography of urban systems is still largely unknown in many situations. Nevertheless, this style of modelling provides a tool for testing the impact of changes such as land use type or transportation in dense metropolitan areas. This approach is less focused on predicting the right future, but more on understanding and exploring the system. It focuses on its behaviour and prediction of possible outcomes based on informed

speculation incorporating individuals and dynamics. To this extent agent-based models may potentially assist policy makers in the same way as planning support systems do (see Brail and Klosterman, 2001). This is consistent with the notion that cities, and the societies they are part of, are intrinsically complex and inherently unpredictable (Batty, 2008). It is therefore virtually impossible to make meaningful predictions for such systems, or at least predictions that would form the basis of medium or long term policy-making (Batty, 2001). These models focus on the way local actions generate global outcomes, where system properties emerge from the bottom-up. This is in contrast to past generations of large scale urban models, which were economically driven, and focused on urban growth and transport infrastructure investment. This new style of modelling focuses on other issues which affect cities, specifically inequalities between the rich and poor, segregation along ethnic lines, redevelopment and so on. Such a move potentially offers a greater understanding of urban areas, to model future scenarios for cities, and prepare for challenges such as land-use, population, housing and employment change.

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