UCL CENTRE FOR ADVANCED SPATIAL ANALYSIS

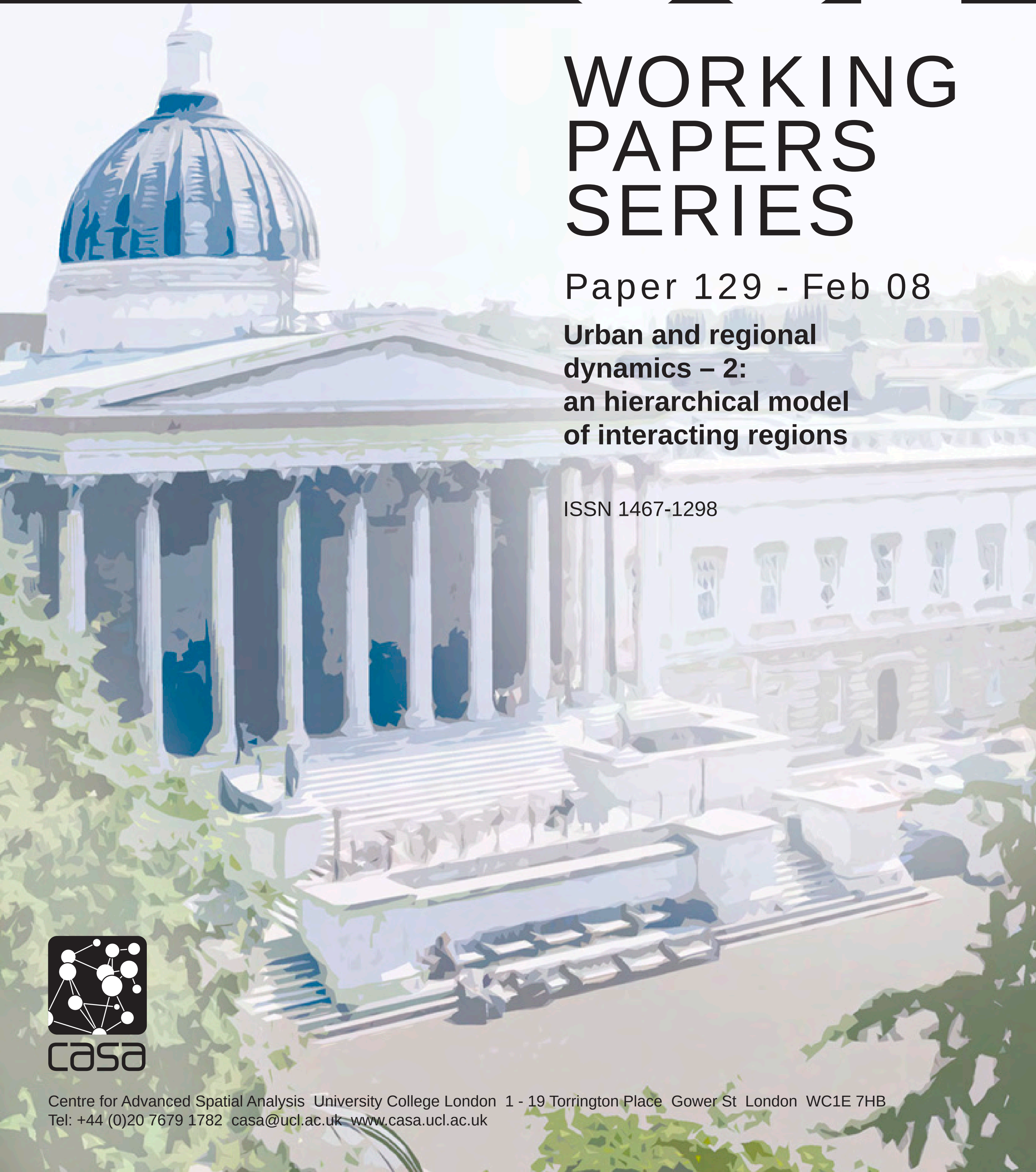
UCL

WORKING PAPERS SERIES

Paper 129 - Feb 08

Urban and regional dynamics – 2: an hierarchical model of interacting regions

ISSN 1467-1298



casa

Centre for Advanced Spatial Analysis University College London 1 - 19 Torrington Place Gower St London WC1E 7HB
Tel: +44 (0)20 7679 1782 casa@ucl.ac.uk www.casa.ucl.ac.uk

Urban and regional dynamics – 2: an hierarchical model of interacting regions

Alan Wilson
Centre for Advanced Spatial Analysis, University College London, WC1E 7HB

22 February 2008

Abstract

It is argued that there are at least four distinct levels at which it is appropriate to build urban and regional models, and that these are then linked. Two archetypal models are presented – at very coarse aggregated scales to illustrate the idea – one, regional, appropriate to the top three levels, in section 2, one appropriate for the lower urban level in section 3. A notation is developed in section 4 for the concise representation of the hierarchical system and some policy implications for the deployment of the system are outlined. Some next steps are briefly considered in the final section.

1. Introduction.

Urban and regional analysis has achieved the stage in its development at which it is possible, in principle, to model the evolution of spatial structure – see, for example, Harris and Wilson (1978), Wilson (2000, 2006, 2007-A, 2007-B). However, these models focus on one 'layer' of structure. It is important to recognise that urban and regional systems function through several levels, with some difference of governing phenomena at different levels. This paper seeks to address the following problem: to examine the links between the four spatial scales at which it is fruitful to examine the evolution of regions within a spatial system. These can be taken as:

(1) nations within the international system;
(2) regions within a country;
(3) cities within a region;
(4) intra-urban structure.

Since we are interested in the relative rates of growth and decline of different kinds of regions within a system, we will be seeking to build dynamic models. A main theme will be the interaction between these different hierarchical levels. The exploration of a system of models in a hierarchy is important for seeking an understanding of the core science, but also because different kinds of policy questions are formulated at different scales.

First, we seek to establish the simplest possible models that will provide the elements of the hierarchical structure and which will provide useful insights. It can be argued that the models for the top three levels have the same character but that the intra-urban model will be different. So we proceed by offering a model that will be applicable to the upper levels (section 2) followed by one that is applicable at the intra urban level (section 3). We then develop a notation (in section 4) for the hierarchical system and present an economical way of representing the model. Some 'next steps' are discussed briefly in section 5.

This is the second of three linked papers which are intended to chart the theoretical basis of a future research programme. In the first, Wilson (2008-A), a more detailed core model is developed – as the 'simplest' model which can capture the key interactions which had been presented in Wilson (2007-A). In the third, Wilson (2008-B), the focus is on the way in which the structural variables of an urban and regional system can be considered to be the system 'DNA' in such a way that a systematic typology can be developed. These two ideas can be deployed in the hierarchical model presented in this paper.

2. A core model for the top levels.

2.1. A framework

Consider a system divided into a set of regions, {i}. We will need a population and an economic model for each region and spatial interaction models to represent migration and trade. In each case, it would be possible to develop full-blown models in accord with the canons of the literature. However, what we seek to do here is to build extremely simple models to illustrate the possibilities of working within an hierarchical framework which can then be elaborated to whatever degree is needed. At this stage, therefore, we do not distinguish population or economic sectors but we do distinguish the degree of wealth of a region and the extent to which it is resource rich – by indicators $\gamma_i(t)$ and $\rho_i(t)$ respectively – at time t. We can then let the key elements of the model be functions of $\gamma_i(t)$ and $\rho_i(t)$ and in turn, we can model the dynamics of these parameters. Indeed, we will at least begin by taking $\gamma_i(t)$ as GDP in i per capita. This will be implicitly linked to education levels and at some stage we may wish to make this link explicit. $\rho_i(t)$ can initially be taken as constant and exogneous; or slowly varying to allow for resource depletion. The detailed mechanisms of the model can be specified at a later stage. This will allow a model to be developed which is in accord with intuition and which can be developed in more detail later.

The system of interest can then be described by the following variables, stock variables at time t and flow variables in the (t, t+1) period and in region i (where indicated):

$P(t)$: total population
$X(t)$: total economic activity in the system measured in money units
$P_i(t)$: population
$b_i(t)$: birth rate
$d_i(t)$: death rate
$X_i(t)$: economic activity
Qi(t): capacity for economic activity
$Z_i(t)$: consumption
$Y_{ij}(t)$: trade flows
$G_i(t)$: regional GDP
$g_i(t)$: consumption per capita
$p_i(t)$: an index of the unit cost of delivering economic activity at i
$C_i(t)$: cost of delivering $X_i(t)$
$I_i(t, t+1)$: investment in the period
$c_{ij}(t)$: a measure of impedance between i and j
$M_{ij}(t, t+1)$: migration flows

The formal relationships to be modelled can then be set out as follows:

$$\sum_i X_i(t) = X(t) \quad (1)$$
$$\sum_i X_i(t+1) = X(t+1) \quad (2)$$
$$\sum_i P_i(t) = P(t) \quad (3)$$
$$\sum_i P_i(t+1) = P(t+1) \quad (4)$$

These are the aggregate controlling relationships: the sum of the rates of change in regions cannot exceed the rates at a higher level.

$$P_i(t+1) = \mu P_i[P_i(t), b_i(\gamma_i), di(\gamma_i), M_{i*}(\gamma_j), M*i(\gamma_i)] \quad (5)$$

That is, the new population is a function of the initial population plus births minus deaths plus net migration; and the birth, death and migration rates are functions of the γ-parameters. μ is a normalising factor to ensure that (3) is satisfied.

In the case of economic activity, there are relationships to be satisfied for the period and then the dynamics are handled separately, providing the basis for the next period. Consumption per capita can be taken as

$$g_i(t) = g_i(t, \gamma_i, \rho_i) \quad (6)$$

and then

$$Z_i(t) = g_i(t)P_i(t) \quad (7)$$
$$X_i(t) = \nu[Z_i(t) + \sum_j Y_{ij}(t) - \sum_i Y_{ji}(t)] \quad (8)$$

which indicates that the total level of activity in i in a period is the consumption plus exports minus imports[1]. ν is a normalising factor to ensure that (1) is satisfied. GDP is then given by

$$G_i(t) = X_i(t) + I_i(t, t+1) \quad (9)$$

The interaction arrays are

$$Y_{ij}(t) = Y_{ij}(X_i, X_j, \gamma_i, \gamma_j, \rho_i, \rho_j) \quad (10)$$
$$M_{ij}(t, t+1) = M_{ij}[P_i, X_j, \gamma_i, \gamma_j, \rho_i, \rho_j) \quad (11)$$

Finally, we add the dynamics of the economic system. A reasonable assumption, recalling that $p_i(t, \gamma_i)$ is an index of the unit cost of maintaining the capacity for economic activity in i, is that

$$C_i(t) = p_i(t, \gamma_i)Q_i(t) \quad (12)$$

[1] The Y_{ii} terms cancel. We leave it like this for convenience, though at the point of further development it may be better to take the summations over j≠i.

Suppose $X_i(t) – C_i(t)$ constitutes savings in the period and that that, together with external investment, $\iota_i(t)$, provides the investment available for the next period, then

$$I_i(t+1) = X_i(t) – C_i(t) + \iota_i(t) \qquad (13)$$

Then we can take

$$\Delta Q_i(t) = \varepsilon^Q I_i(t+1) = \varepsilon^Q [X_i(t) – p_i(t,\gamma_i)Q_i(t)]Q_i(t) \qquad (14)$$

and

$$\Delta\gamma_i(t) = \varepsilon^\gamma I_i(t+1) = \varepsilon^\gamma [X_i(t) – p_i(t,\gamma_i)Q_i(t)]Q_i(t) \qquad (15)$$

so that

$$Q_i(t+1) = Q_i(t) + \Delta Q_i(t) \qquad (16)$$

and

$$\gamma(t+1) = \gamma(t) + \Delta\gamma_i(t, t+1) \qquad (17)$$

and these will feed into the economy for the next period.

2.2. The workings of the model.

Equation (5) is the core demographic model. The relationship of the new (t+1) population to the old via birth and death rates is straightforward in principle – but note that these rates have been made a function of the prosperity of the region. For the level 1 system, of course, these vary tremendously across nations. But the migration terms are particularly important. These will be difficult to model – as formally represented in equation (11) - in a contemporary situation because the migration flows are large and the data is poor.

The economic model is can be interpreted as follows. The core equations represent the workings of the economy in a given period, say (t, t+1). This representation is very important because, among other things, it will reveal divergences from equilibrium that determine much of the dynamics for example through the $X_i(t) – C_i(t)$ term in equation (13). We have already noted that equations (13) and (14) drive the economy forward. The amount of investment in (13) is determined by savings – based on a simple assumption – plus external (usually, but not necessarily, Government) investment. This in turn determines the increment in capacity through equation (14) and that in prosperity in equation (15) and hence the values of $Q_i(t+1)$ and $\gamma_i(t+1)$ in equations (16) and (17). These have the form of Lotka-Volterra equations and this reveals the possibilities of interesting phase transitions in the model.

These t+1 values for Q, γ and I then feed into equations (6)-(10), with t+1 replacing t. In a growing region, for example, $g_i(t+1)$ will grow through equation (6), then consumption through equation n(7), total economic activity through equation (8) [with the trade flows determined by equation (10)] and GDP will grow through equation (9) – and this, of course, incorporates the balance of payments through (8).

If we assume that appropriate submodels and functional relationships can be constructed, then there is currently only one exogenous function – the external investment vector $\{\iota_i(t)\}$. There is then an interesting control theory problem: if we identify a suitable objective function, what is the path of the $\{\iota_i(t)\}$ that optimises that function?

2.3. A demonstration model.

The next step in the argument is to show how to introduce explicit hypotheses for the relationships in equations (1)-(15) above. For convenience, all the equations are repeated and specific hypotheses are added where necessary.

The aggregate level accounting equations are the same:

$$\sum_i X_i(t) = X(t) \qquad (18)$$

$$\sum_i X_i(t+1) = X(t+1) \qquad (19)$$

$$\sum_i P_i(t) = P(t) \qquad (20)$$

$$\sum_i P_i(t+1) = P(t+1) \qquad (21)$$

A simple population model, using the earlier definitons is

$$P_i(t+1) = \mu\{P_i(t)[1 + b_i(\gamma_i) - di(\gamma_i)] - M_{i*}(\gamma_j) + M^*i(\gamma_i)\} \qquad (22)$$

In the case of economic activity, there are relationships to be satisfied for the period and then the dynamics are handled separately, providing the basis for the next period. Consumption per capita can be taken as

$$g_i(t) = \lambda\gamma_i^{\sigma}\rho_i^{1-\sigma} \qquad (23)$$

and then

$$Z_i(t) = g_i(t)P_i(t) \qquad (24)$$

with

$$X_i(t) = \nu[\, Z_i(t) + \sum_j Y_{ij}(t) - \sum_i Y_{ji}(t)] \qquad (25)$$

GDP is then given by

$$G_i(t) = X_i(t) + I_i(t, t+1) \qquad (26)$$

The interaction arrays are

$$Y_{ij}(t) = A_i^Y B_j^Y X_i(t)\, Xj(t)\exp(-\beta^Y c_{ij}) \qquad (27)$$

$$M_{ij}(t, t+1) = A_i^M B_j^M P_i(t)X_j(t)\exp(-\beta^M c_{ij}) \qquad (28)$$

Suppose we take

$$p_i(t, \gamma_i) = \kappa p_i(0)\gamma_i^{\omega} \qquad (29)$$

where κ and ω are appropriate constants

$$C_i(t) = p_i(t, \gamma_i)X_i(t) \qquad (30)$$

Then we can take

$$I_i(t+1) = X_i(t) - C_i(t) + \iota_i(t) \qquad (31)$$

and

$$\Delta Q_i(t) = \varepsilon^Q I_i(t+1) = \varepsilon^Q[X_i(t) - p_i(t,\gamma_i)Q_i(t)]Q_i(t) \quad (32)$$

and

$$\Delta\gamma_i(t) = \varepsilon^\gamma I_i(t+1) = \varepsilon^\gamma[X_i(t) - p_i(t,\gamma_i)Q_i(t)]Q_i(t) \quad (33)$$

so that

$$Qi(t+1) = Qi(t) + \Delta Qi(t) \quad (34)$$

and

$$\gamma(t+1) = \gamma(t) + \Delta\gamma_i(t, t+1) \quad (35)$$

and these feed back for the next period.

3. An intra-urban core model.

3.1. The core model.

When we shift to the lower level, we have to modify our approach. The elements of urban structure that interest us involve at least distinguishing residential from 'economic' zones; and it is also useful to distinguish retail (for which we will use the superscript R) from non-retail economic activity (N) – because their locational structures will be fundamentally different with high-value retail generating much higher land values than non-retail. We do this both in terms of activity levels (X_i^R and X_i^N) and of capacities (Q_i^R and Q_i^N) So far, we have used a measure related to GDP to represent economic activity. There is now the interesting question of how to measure GDP for small zones. Intuition suggests that we should switch, conceptually, to an income-related measure of GDP and that the measure in a residential zone should be 'incoming' income – mainly derived from employment. This would then generate an aggregate measure that was consistent with the next higher level. However, we can still then equate income with consumption as a first approximation. It is less clear how to handle imports and exports at this scale. As a first approximation, we could perhaps assume that all retail economic activity was imported and all non-retail exported. This implies that we should distinguish between 'R' and 'N' for trade flows. We should also note that the term 'migration' would normally be confined to ex-urban migration and that within the city, movement would be regarded as residential relocation. Thus the M-interaction array would, in this case, be a residential location model, though there would of course be external zones to handle migration in the normal sense. We can now amend the core model assumptions for the intra-urban case on the basis of this brief analysis. We again repeat equations, where appropriate, for convenience.

The aggregate level accounting equations are again the same:

$$\sum_i X_i(t) = X(t) \quad (36)$$

$$\sum_i X_i(t+1) = X(t+1) \quad (37)$$

$$\sum_i P_i(t) = P(t) \quad (38)$$

$$\sum_i P_i(t+1) = P(t+1) \quad (39)$$

A simple population model, using the earlier definitions still suffices, but with the migration terms reinterpreted:

$$P_i(t+1) = \mu\{P_i(t)[1 + b_i(\gamma_i) - di(\gamma_i)] - M_{i*}(\gamma_j) + M_{*i}(\gamma_i)\} \quad (40)$$

In the case of economic activity, there are relationships to be satisfied for the period and then the dynamics are handled separately, providing the basis for the next period. Consumption per capita (in this case expenditure of earned income by residents) can be taken as

$$g_i(t) = \lambda\gamma_i^{\sigma}\rho_i^{1-\sigma} \quad (41)$$

and then

$$Z_i(t) = g_i(t)P_i(t) \quad (42)$$

and it is convenient to define total retail activity at a zone j as the sum of the *inflows* as

$$X^R{}_j(t) = \nu[\textstyle\sum_i Y_{ij}{}^R(t)] \quad (43)$$

The $Z_i(t)$ will now function through the Y-arrays see equation (46) below. The non-retail sector is now

$$X^N{}_i(t) = \nu[\textstyle\sum_j Y_{ij}{}^N(t) - \sum_i Y_{ji}{}^N(t)] \quad (44)$$

GDP is then given by

$$G_i(t) = Z_i^R(t) + X_i^N(t) + I_i(t, t+1) \quad (45)$$

The interaction arrays are

$$Y_{ij}^R(t) = A_i^{YR}B_j^{YR}Z_i^R(t)\, X_j^R(t)\exp(-\beta^{YR}c_{ij}) \quad (46)$$

$$Y_{ij}^N(t) = A_i^{YN}B_j^{YN}X_i^N(t)\, X_j^N(t)\exp(-\beta^{YN}c_{ij}) \quad (47)$$

$$M_{ij}(t, t+1) = A_i^M B_j^M P_i(t)X_j^*(t)\exp(-\beta^M c_{ij}) \quad (48)$$

where

$$X_j^*(t) = X_i^R(t) + X_i^N(t) \quad (49)$$

and μ^M is the 'movement' rate in the period. That is, $\mu^M P_i(t)$ move in the period following t. We now need separate equations for p_i^R and p_i^N:

$$p_i^R(t, \gamma_i) = \kappa^R p_i^R(0)\gamma_i^{\omega R} \quad (50)$$

$$p_i^N(t, \gamma_i) = \kappa^R p_i^N(0)\gamma_i^{\omega R} \quad (51)$$

where κ and ω are appropriate constants. Then

$$C_i^R(t) = p_i^R(t, \gamma_i)X_i^R(t) \quad (52)$$

and

$$C_i^N(t) = p_i^N(t, \gamma_i)X_i^N(t) \quad (53)$$

Then we deal with new investment separately for retail and non-retail, but this time we nominally do this for economic activity as distinct from residential zones, and so we designate these as j-zones:

$$I_j^R(t+1) = X_j^R(t) - C_j^R(t) + \iota_j(t) \quad (54)$$
$$I_j^N(t+1) = X_j^N(t) - C_j^N(t) + \iota_j(t) \quad (55)$$

Then, in the spirit of the earlier analyses

$$\Delta Q_i^R(t+1) = [X_i^R(t) - C_i^R(t)]Q_i^R(t) \quad (56)$$

and

$$\Delta Q_i^N(t+1) = [X_i^N(t) - C_i^N(t)]Q_i^N(t) \quad (57)$$

Then we can take

$$\Delta \gamma_i(t) = \varepsilon^\gamma I_i(t+1) = \varepsilon^\gamma[X_i^R(t) + XiN(t) - p_i(t,\gamma_i)Q_i^R(t) - p_i(t,\gamma_i)Q_i^N(t)]\gamma_i(t) \quad (58)$$

so that

$$\gamma(t+1) = \gamma(t) + \Delta \gamma_i(t, t+1) \quad (60)$$

and these feed back for the next period.

3.2. The workings of the model.

Population change and movement is straightforward and follows from equation (40) together with (48): there are births, deaths and changes of location. There will also be one or more external zones – 'rest of the world' zones – which will handle migration proper, either exogenously, or through modified elements of equation (48) for these zones.

Equation (41) then indicates propensity to purchase as a function of zonal prosperity (through γ_i) and resource richness (ρ_i). However, we would expect σ to be nearly 1 in this case. Consumption can then be calculated from (42). We then note that $X_j^R(t)$ is the total retail activity in zone j by calculating the i-sums of the interaction matrix – the in-flows to j. This will provide the basis for the usual equilibrium model of retail structure later. The volume of activity in the non-retail sector (N) is given by equation (44) as the difference between exports and imports – given that we have assumed that at this scale, all production is exported. A zonal GDP can then be calculated as in equation (45).

The interaction arrays are straightforward in principle [equations (46)-(48)] but would obviously have to be refined in practice. Equations (50) and (51) give the price indices for retail and non-retail sectors in terms of the wealth indices – showing that it is the changes in the γ-indices that will drive the model. We can set up the costs of economic delivery by zone in equations (52) and (53) and investment in (54) and (55), this in terms of profit and external investment – that being a potentially controllable parameter as we saw earlier.

Equations (56)-(60) then determine the dynamics.

4. An hierarchical model.

4.1. Notation.

We now need subscripted location indices, where the subscripts indicate the different levels: i_1, i_2, i_3, i_4 or j_1, j_2, j_3, j_4 for 1 = international, so i_1 is a country label, 2 = a region (within country i_1), 3 = a city or rural area (within region i_2) and 4 = a zone (within city i_3); and similarly for the j-variables. If we then take, as in the illustrative core models, stock variables as X and Z and interaction arrays as Y, then we can define $X(i_1, i_2, i_3, i_4)$ as, say, the population of i_4 within i_3 within i_2 within i_1; and similarly $Z(j_1, j_2, j_3, j_4)$, say for a measure of economic activity. A general interaction variable, say a flow of workers at a given time, would be $Y(i_1, i_2, i_3, i_4, j_1, j_2, j_3, j_4)$.[2] Of course, not all the location labels would be needed in many instances. A national population could be represented as $X(i_1, *, *, *)$, where the asterisks denote summation over the other indices; and of course, these summations could be understood, and we could use $X(i_1)$. Similarly, we would never in practice need the full array of indices in the Y-definition: $Y(i_1, j_1)$ would be a matrix of international flows between two countries for example; $Y(i_4, j_4)$ could be commuting flows within a city, with the i_1, i_2, i_3 indices understood.

4.2. Representing the hierarchical model.

The next step is to devise a notation for representing the linked 4-level model without having to repeat the core model equations three times for the higher levels and once for the lowest level. The first step is to divide the variables and parameters into an endogenous set, calculated within the model, such as the X, Y, Z and P arrays and exogenous parameters such as the ε's. The endogenous set are the state variables, which we designate as a vector $\underline{S}(t)$ for time t and the parameters are the vector $\underline{r}(t)$. We then define an operator $\underline{M^{(k)}}(t, t+1)$ which represents the model equations at level k and which transforms $\underline{S^{(k)}}(t)$ to $\underline{S^{(k)}}(t+1)$. Formally:

$$\underline{S^{(k)}}(t+1) = \underline{M^{(k)}}[t, t+1, r^{(k)}(t+1)]\underline{S^{(k)}}(t) \qquad (61)$$

However, one of the features of the hierarchical model is that there is an interaction between levels in that the X and P totals at the higher level constrain these at the next level down. It is also important to make clear that the operator is a function of the state variables at time t – which represent the initial conditions for this transition. We should therefore show $\underline{M}$ as $M^{(k)}[t,t+1, r^{(k)}(t), S^{(k)}(t), S^{(k-1)}(t+1)]$ for k = 2, 3 and 4. Hence

[2] The location labels are listed on the main line for typographical convenience because they are themselves subscripted

$$S^{(k)}(t+1) = M^{(k)}[t,t+1, r^{(k)}(t), S^{(k)}(t), S^{(k-1)}(t+1)]S^{(k)}(t) \quad (62)$$

4.3. Policy implications of the hierarchical model.

We have discussed the workings of each of the core models presented in sections 2 and 3. We have seen that in each case, the presence of non-linear dynamics generates critical points. The additional feature of this type that appears when the different levels are connected is that the activity totals are constrained by the next higher level. Suppose that there is a set of exogenous parameters that represent the rates of economic growth – that is of X and Q variables in the models – at one or more higher levels.

Two issues follow from this: first, there may be bifurcation effects associated with these rates and the parameters could have critical values at these points; and secondly, there are major implications for the setting of policy and planning targets at lower levels. The two issues are linked: there may be critical points at which accelerated rates of development become possible.

Consider some major policy challenges at different levels, each associated with disadvantage - differential levels of wealth:

- between countries
- between regions within a country
- between areas within a region
- between small areas within a city.

In each case, there are, typically, huge disparities and there may be policy targets to reduce these. The hierarchical model provides the basis for assessing feasibility of targets of this kind.

5. Next steps.

There are four areas of future development:

- to test the model as presented;
- to add more detail to the model, in particular to disaggregate to the kinds of levels presented in Wilson (2008-A) which would include the incorporation of basic input-output relationships in the economic model;
- to explore the mathematical challenges offered by the operator $\underline{M}$: its dynamics and fixed points.
- to explore the implications of setting different kinds of policy targets at different scales to reduce the levels of disparity between areas.

References.

Harris, B. and Wilson, A. G. (1978) Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models, *Environment and Planning, A, 10*, pp 371-88.

Wilson, A. G. (2000) *Complex spatial systems: the modelling foundations of urban and regional analysis*, Prentice Hall, Harlow, 2000

Wilson, A. G. (2006) Ecological and urban systems' models: some explorations of similarities in the context of complexity theory, *Environment and Planning*, *A*, 38, pp. 633-646.

Wilson, A. G. (2007-A) A general representation for urban and regional models, *Computers, Environment and urban systems*, *31*, pp. 148-161.

Wilson, A. G. (2007-B). Boltzmann, Lotka And Volterra And Spatial Structural Evolution: An Integrated Methodology For Some Dynamical Systems, *Journal of the Royal Society, Interface. (doi: 10.1098/rsif.2007.1288)*.

Wilson, A. G. (2008-A) A core model for the analysis of urban dynamics, Working Paper 128, Centre for Advanced Spatial Analysis, University College London.

Wilson, A. G. (2008-B) Urban and regional 'DNA': the basis for constructing a typology of areas, Working Paper 130, Centre for Advanced Spatial Analysis, University College London.